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Research Article

Distributed Cooperation among Cognitive Radios with Complete and Incomplete Information

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This paper proposes that secondary unlicensed users are allowed to opportunistically use the radio spectrum allocated to the primary licensed users, as long as they agree on facilitating the primary user communications by cooperating with them. The proposal is characterized by feasibility since the half-duplex option is considered, and incomplete knowledge of channel state information can be assumed. In particular, we consider two situations, where the users in the scenario have complete or incomplete knowledge of the surrounding environment. In the first case, we make the hypothesis of the existence of a Common Control Channel (CCC) where users share this information. In the second case, the hypothesis of the CCC is avoided, which improves the robustness and feasibility of the cognitive radio network. To model these schemes we make use of theory of exact and Bayesian potential games. We analyze the convergence properties of the proposed games, and we evaluate the outputs in terms of quality of service perceived by both primary and secondary users, showing that cooperation for cognitive radios is a promising framework and that the lack of complete information in the decision process only slightly reduces system performance.

1. Introduction

Cognitive Radio is a new paradigm in wireless communications to enhance utilization of limited spectrum resources. It is defined as a radio able to utilize available side information, in a decentralized fashion, in order to efficiently use the radio spectrum left unused by licensed systems. The basic idea is that a secondary user (a cognitive unlicensed user) is able to properly sense the spectrum conditions, and, to increase efficiency in spectrum utilization, it seeks to underlay, overlay, or interweave its signals with those of the primary (licensed) users, without impacting their transmission [1]. The interweave paradigm was the original motivation for cognitive radio and is based on the idea of opportunistic communications. In fact, numerous measurement campaigns have demonstrated the existence of temporary space-time frequency voids, referred to as spectrum holes, which are not in constant use in both licensed and unlicensed bands and which can be exploited by secondary users (SUs) for their communications. The underlay paradigm encompasses techniques that allow secondary communications assuming that they have knowledge of the interference caused by its transmitter to the receivers of the primary users (PUs). Specifically, the underlay paradigm mandates that concurrent primary and secondary transmissions may occur as long as the aggregated interference generated by the SUs is below some acceptable threshold. The overlay paradigm allows the coexistence of simultaneous primary and secondary communications in the same frequency channel as long as the SUs somehow aid the PUs, for example, by means of advanced coding or cooperative techniques. In particular, in a cooperative scenario the SUs may decide to assign part of their power to their own secondary communications and the remaining power to relay the PUs transmission [2].

While the most important challenge of the interweave paradigm is that of spectrum sensing, in order to realize a reliable detection of the PUs, the significant challenge to face in the underlay paradigm is that of estimating the aggregated interference at the PUs receivers that is being caused by the opportunistic activity of multiple SUs. In literature, the analysis of the underlay paradigm for cognitive radio has often been realized by making use of game theoretic approaches where SUs are modeled as the players of a game. In this context, they make decisions in their own self interest by maximizing their utility function, while influenced by the other players decisions. Generally, the different controllable transmission parameters in the communication (e.g. transmission power, frequency channel, etc.) represent the strategies that can be taken by the players, and a function of, for example, the (Signal-to-Interference-and-Noise Ratio) SINR or the throughput is the utility of the game [3, 4]. The main drawback of this approach is that the maximization of the game utility function represents an incentive to reduce the interference at the PUs receiver, but not a guarantee that the aggregated interference generated by the SUs is maintained below a certain threshold, especially in scenarios where the spatial reuse is most challenging, for example, where PUs receivers are passive or where SUs transmitters are very close to PUs receivers. In this context, cooperation of SUs and PUs (overlay approach) can significantly reduce the interference at the PUs receivers. In particular, we propose a cognitive radio scenario where concurrent primary and secondary communications are allowed by exploiting spatial reuse as long as the SUs cooperate with the PUs by relaying their messages. We consider two different cooperation techniques: *decode and forward* (D&F) and *amplify and forward* (A&F). In the proposed system, decisions about channel selection and power allocation are taken distributively by the SUs according to the maximization of their individual utility. These decisions strongly depend on those made by the other SUs, since the PUs performances are limited by the aggregated interference generated by all the SUs simultaneously transmitting in their band. This is why the performance is analyzed using game-theoretic tools, already proven good at modeling interactions in decision processes. In particular, we define two games to model channel and power allocation for cognitive radios, underlay and overlay, which can be formulated as exact potential games converging to a pure strategy Nash equilibrium solution [5], and we compare the overlay to the underlay scheme to learn advantages and drawbacks of the proposed approach.

However, inherent in this approach, as in nearly all previous efforts, is the hypothesis of complete channel state information among SUs; that is, the wireless channel gains are assumed to be common knowledge across all SUs. This hypothesis implies the implementation of a common control channel (CCC) where the distributed SUs can share the information about their wireless channel gains. In literature, the hypothesis of such a fixed control channel in a cognitive radio context has often been rejected [6], since it requires a static assignment of licensed spectrum before deployment, which is basically against the very philosophy of cognitive radio. Additionally, this solution increases cost and complexity, limits scalability in terms of device and traffic density, and is not robust to, for example, jamming attacks. As a result, in an effort to model a more reliable, low-complexity and realistic self-organized cognitive radio system, in the second part of this paper we include uncertainty in the considered scenario, and we do not rely on the existence of a preassigned CCC. To this end, we propose a Bayesian Potential Game (BPG), converging to a Bayesian Nash Equilibrium, to model decentralized joint power and channel allocation for cooperative SUs with incomplete information. Simulation results will show that the more realistic hypothesis of incomplete information only slightly reduces performances of PUs and SUs, and that cooperation among SUs significantly improves performances of both PUs and SUs and that the improvement provided by the overlay scheme is higher as the SU is closer to the primary receiver. This results in a remarkable reduction of primary outage probability, since outages will typically occur in primary receivers with nearby SUs. The outline of the paper is organized as follows. Section 2 describes the system model. Section 3 presents the game theoretic model for the underlay and overlay games with complete (Section 3.1) and incomplete information (Section 3.2). Section 4 describes the simulation scenario. Section 5 discusses relevant simulation results. Finally, Section 6 summarizes the conclusion.

2. System Model

The cognitive radio network we consider consists of M transmitting-receiving PUs pairs, and N transmitting-receiving SUs pairs (Figure 1). In this paper we will indicate the transmission power levels of the PUs' transmitters as $p_i^P, i = 1, \dots, M$, and the transmission power levels of the SUs' transmitters as $p_j^S, j = 1, \dots, N$. PUs and SUs, both transmitters and receivers, are randomly and uniformly distributed in a circular coverage region of a primary network with radius $R_{\max}$. Primary communications can be characterized by a long distance between the transmitting and the receiving device, whereas secondary communications are in general characterized by short range. The nodes are either fixed or moving slowly (slower than the convergence of the proposed algorithm). The SUs are in charge of sensing the channel conditions and of choosing a transmission scheme which does not disrupt the communication of the PUs. In this paper we consider and compare two communication paradigms for cognitive radio: underlay and overlay. According to the underlay paradigm, an SU distributively selects the frequency channel and the transmission power level to maximize its throughput while at the same time not causing harmful interference to the PUs. On the other hand, based on the overlay paradigm, besides selecting the transmission power and the frequency channel, the SUs devote part of their transmission power for relaying the primary transmission. As a result, the SU's transmission power level is split in two parts: (1) a power level $p_j^{S'}, j = 1, \dots, N$ for its own transmission, and (2) a cooperation power level $p_j^{S''}, j = 1, \dots, N$ for relaying the PU's message on the selected band, where $p_j^S = p_j^{S'} + p_j^{S''}$. The cooperative scheme used by the SUs is shown in Figure 2. We assume that the PU transmission is divided into frames, and each frame further into slots. Relays are assumed to operate in half-duplex mode. Therefore, each relay listens to the primary transmission during one slot and transmits during the next. The relay will choose, as part of its strategy, whether to listen during even or odd slots. We define these two slot subsets as $\mathcal{S}_1$ and $\mathcal{S}_2$, respectively. The primary transmission

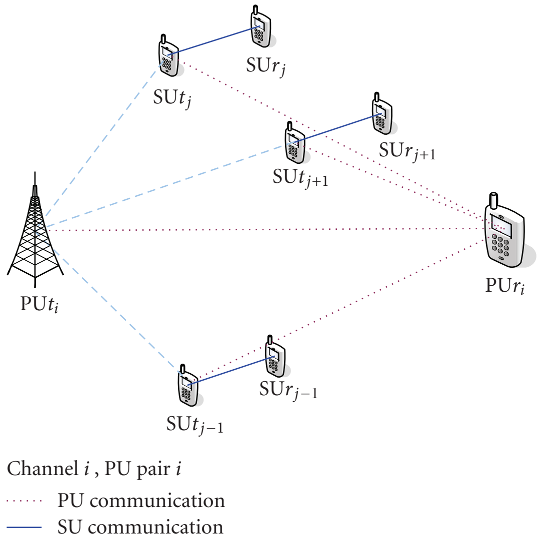

Figure 1: Cognitive system architecture.

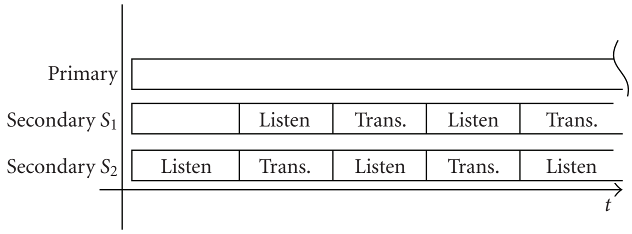

Figure 2: Half-duplex relaying scheme for secondary users. Each user chooses one slot to listen to the primary and retransmits in the following slot. Secondary users choose in which slot to transmit as a part of their strategy.

is continuous, and it does not interrupt to facilitate the relay operation of the SUs. In addition, we consider two different relaying techniques: D&F and A&F. In the D&F case the relay (secondary user) decodes the primary signal, regenerates it, and retransmits it during the next time slot. In the event that the relay is unable to decode, then it remains silent. In the A&F case, the relay simply stores the input during one slot, amplifies it, and retransmits it during the next. This technique has the advantage that the relay is not required to decode the signal. On the other hand, the relay amplifies input noise and interference as well as the useful signal. The performance of one technique or another will be better depending on the ability of the relays to decode the signal, and on the level of noise and interference at their input. The reader is referred to [7, 8] for a thorough performance comparison.

Notice that the overlay scheme proposed and evaluated in this paper is substantially different from the property-rights model presented in [2], where PUs own the spectral resource and may decide to lease part of it to SUs in exchange for cooperation. In fact, our overlay model does not require PUs to be aware of the presence and identity of SUs. It does, however, require the PUs to be able to decode the cooperative transmission scheme employed.

We shall analyze the network performance in terms of SINR and outage probability of both PUs and SUs. As for the notation, we indicate with h_{ij}^{PP} the link gain between a PU's transmitter i and a PU's receiver j, with h_{ij}^{PS} the link gain between a PU's transmitter i and an SU's receiver j, with h_{ij}^{SP} the link gain between a SU's transmitter i and a PU's receiver j, and with h_{ij}^{SS} the link gain between an SU's transmitter i and an SU's receiver j. Finally, σ^2 is the noise power (assumed to be equal in each channel).

2.1. Signal-to-Interference-and-Noise Ratio. In the following we calculate the expressions for the SINR for the underlay and overlay cases. Notice that, for the PUs' transmission, we will consider an (Frequency Division Multiplexing) FDM scheme, so that only one PU is active per frequency channel.

In the underlay paradigm, the SINR $\gamma_i^{PU,u}$ for a pair i of PUs in a frequency channel c_i is given by

$$\gamma_i^{PU,u} = \frac{p_i^P h_{ii}^{PP}}{\sum_{j=1}^{N} p_j^S h_{ji}^{SP} f\left(c_j, c_i\right) + \sigma^2}, \quad i = 1, \ldots, M, \tag{1}$$

where f is defined as

$$f\left(c_i, c_j\right) \doteq \begin{cases} 1, & \text{if } c_i = c_j, \\ 0, & \text{if } c_i \neq c_j. \end{cases} \tag{2}$$

Additionally, the SINR for the SUs is given by

$$\gamma_i^{SU,u} = \frac{p_i^S h_{ii}^{SS}}{\sum_{j=1, j\neq i}^{N} p_j^S h_{ji}^{SS} f\left(c_j, c_i\right) + \sigma^2}, \quad i = 1, \ldots, N. \tag{3}$$

In (3) it is assumed that the primary signal is known either at the secondary receiver or at the secondary transmitter. In the first case, the interference of the primary signal can be eliminated at the secondary receiver through a successive decoding strategy. In the second case, it can be eliminated through dirty paper coding.

For the overlay paradigm the expression of the SINR of the PUs depends on the relaying technique used.

2.1.1. D&F. In the following we will use the notation sl_i to refer to the slot subset chosen by SU i, and we define the function f'

$$f'\left(sl_i, sl_j\right) \doteq \begin{cases} 1, & \text{if } sl_i = sl_j, \\ 0, & \text{if } sl_i \neq sl_j. \end{cases} \tag{4}$$

In the D&F approach, the SU must be able to correctly decode the primary signal to relay it. In order to do that, the SINR of the primary signal, from PU j at SU transmitter i, which is given by

$$\gamma_i^{PS} = \frac{p_j^P \hat{h}_{ji}^{PS}}{\sigma^2 + \sum_{k=1, k\neq i}^{N} p_k^S \hat{h}_{ki}^{SS} f(c_k, c_i) f'(sl_k, sl_i)}, \quad i = 1, \ldots, N \tag{5}$$

must be above the sensitivity threshold, ρ. In the equation, we use $\hat{h}_{ji}$ and $\hat{h}_{ki}$ to denote the channel gains to the SU transmitter, rather than the SU receiver, of SU pair i. We define the function

$$f''\left(\gamma_i^{PS} > \rho\right) \doteq \begin{cases} 1, & \text{if } \gamma_i^{PS} > \rho, \\ 0, & \text{otherwise.} \end{cases} \tag{6}$$

If $\gamma_i^{PS} > \rho$, then the SU may relay the primary signal. We assume that the SU uses an encoding strategy that is able to contribute to the received SINR. Since the PU will continue to transmit information, the scheme must implement a distributed space-time coding scheme. In order to be realistic in terms of implementation, we do not assume that PU and SUs may transmit phase-synchronously (i.e., to perform distributed beamforming); therefore, their received power adds up in incoherently. The description of a specific distributed space-time coding scheme is beyond the scope of this paper, and the reader is referred to [2, 9, 10] and references therein for specific designs. The SINR of the PU i will be time-varying on the two slot subsets $\mathcal{S}_1$, $\mathcal{S}_2$ and is given by

$$\gamma_i^{PU,o}(\mathcal{S}_i) = \frac{p_i^P h_{ii}^{PP} + \sum_{j=1}^N p_j^{S''} h_{ji}^{SP} f\left(c_j, c_i\right) f'\left(sl_j, \mathcal{S}_i\right) f''\left(\gamma_j^{PS} > \rho\right)}{\sum_{j=1}^N p_j^{S'} h_{ji}^{SP} f\left(c_j, c_i\right) f'\left(sl_j, \mathcal{S}_i\right) + \sigma^2}, \quad i = 1, \dots, M. \tag{7}$$

As conservative choice, in our performance evaluation we consider the minimum SINR in any of the two slot subsets, as it normally dominates the error rate. Notice that, unlike the underlay approach, part of the SU power contributes to increasing the SINR by increasing the useful signal power at the receiver (cooperation power). The SINR of the SU is given by

$$\gamma_i^{SU,o} = \frac{p_i^{S'} h_{ii}^{SS}}{\sum_{j=1, j\neq i}^N p_j^{S'} h_{ji}^{SS} f\left(c_j, c_i\right) f'\left(sl_j, sl_i\right) + \sigma^2}, \quad i = 1, \dots, N. \tag{8}$$

2.1.2. A&F. In the A&F mode, the SU retransmits the analog signal received during the previous time slot. The received signal at SU j is given by

$$r_j = p_i^P \hat{h}_{ij}^{PS} + \sum_{k=1, k\neq j}^{N} p_k^S \hat{h}_{kj}^{SS} f\left(c_k, c_j\right) f'\left(sl_k, sl_j\right) + \sigma^2, \quad i = 1, \dots, M, \quad j = 1, \dots, N. \tag{9}$$

Define the useful signal fraction of the transmitted primary signal as

$$R_j = \frac{p_i^P \hat{h}_{ij}^{PS}}{p_i^P \hat{h}_{ij}^{PS} + \sum_{k=1, k\neq j}^N p_k^S \hat{h}_{kj}^{SS} f\left(c_k, c_j\right) f'\left(sl_k, sl_j\right) + \sigma^2}, \quad i = 1, \dots, M, \quad j = 1, \dots, N \tag{10}$$

and the noise amplification fraction as

$$I_j = \frac{\sum_{k=1, k\neq j}^N p_k^S \hat{h}_{kj}^{SS} f\left(c_k, c_j\right) f'\left(sl_k, sl_j\right) + \sigma^2}{p_i^P \hat{h}_{ij}^{PS} + \sum_{k=1, k\neq j}^N p_k^S \hat{h}_{kj}^{SS} f\left(c_k, c_j\right) f'\left(sl_k, sl_j\right) + \sigma^2}, \quad i = 1, \dots, M, \quad j = 1, \dots, N. \tag{11}$$

In A&F mode, it is not possible to implement a space-time coding scheme since the relay may not do any processing of its received signal. Therefore, the relay retransmits the signal in the same format as it was received, which creates an *artificial multipath* channel for the receiver. We assume that the PU is able to take advantage of this multipath effect using similar techniques as those employed in conventional multipath resulting from propagation effects of the wireless medium. As for the D&F case, the SINR of the PU will be time-varying on the two slot subsets, and again we consider the minimum SINR in any of the two. For slot set $\mathcal{S}_i$,

$$\gamma_i^{PU,o}(\mathcal{S}_i) = \frac{p_i^P h_{ii}^{PP} + \sum_{j=1}^N p_j^{S''} R_j h_{ji}^{SP} f\left(c_j, c_i\right) f'\left(sl_j, \mathcal{S}_i\right)}{\sum_{j=1}^N \left(p_j^{S'} + p_j^{S''} I_j\right) h_{ji}^{SP} f\left(c_j, c_i\right) f'\left(sl_j, \mathcal{S}_i\right) + \sigma^2}, \quad i = 1, \dots, M. \tag{12}$$

Finally, the SINR of the SUs is given by

$$\gamma_i^{SU,o} = \frac{p_i^{S'} h_{ii}^{SS}}{\sum_{j=1, j\neq i}^N \left(p_j^{S'} + p_j^{S''} I_j\right) h_{ji}^{SS} f\left(c_j, c_i\right) f'\left(sl_j, sl_i\right) + \sigma^2}, \quad i = 1, \dots, N. \tag{13}$$

It is worth noting that in all the SINR expressions, the power relay and interference terms are not supposed to add up coherently. This assumption relaxes the synchronization requirements of primary and secondary users.

2.2. Outage Probability. Outage probability is defined as the probability that a user i perceives an SINR $\gamma_i < \gamma\ dB$, where the threshold is set according to the primary receiver sensitivity.

3. Game Theoretic Model

Game theory constitutes a set of mathematical tools to analyze interactions in decision making processes. In this paper we model joint channel and transmission power selection in a cognitive radio scenario as the output of a game where the players are the N SUs, the strategies are the choice of the transmission power and of the frequency channel, and the utility is a function of, (1) the interference each SU causes to the surrounding PUs and SUs simultaneously operating in the same frequency channel, (2) the interference each SU receives from the surrounding SUs simultaneously operating in the same frequency channel, and (3), the satisfaction of each SU. The SUs are aware of the interference they receive, but to evaluate the interference they cause to the surrounding PUs and SUs, they need information about the wireless channel gains of their neighbors. To retrieve this information, we consider two cases. In the first case, we foresee the existence of a CCC where all the users in the scenario share their transmission information, so that the decisions of the SUs are made with complete information. Much attention has recently been paid to this kind of channels; some examples are the Cognitive Pilot Channel (CPC) [11] proposed by the E2R2/E3 consortium [12] or the radio enabler proposed by the P1900.4 Working Group [13]. In the second case, taking into account that the hypothesis of the existence of a CCC has often been rejected in the cognitive radio literature, we provide a more realistic and feasible proposal by avoiding the need of the CCC and assuming that the decisions of the SUs are made with incomplete information. In this section we introduce two games modeling the underlay and the overlay games, for both the cases of complete (see Section 3.1) and incomplete (see Section 3.2) information.

3.1. An Exact Potential Game Formulation: Underlay and Overlay Games with Complete Information. We model this problem as a normal form game, which can be mathematically defined as $\Gamma = \{N, \{S_i\}_{i\in N}, \{u_i\}_{i\in N}\}$, where N is the finite set of players (i.e., the N SUs), and S_i is the set of strategies s_i associated with player i. We define $S = \times S_i$, $i \in N$ as the strategy space and $u_i : S \to \Re$ as the set of utility functions that the players associate with their strategies. For each player i in game Γ, the utility function u_i is a function of s_i, the strategy selected by player i and of the current strategy profile of the other players, which is usually indicated with s_{-i}. The players make decisions in a decentralized fashion, and independently, but they are influenced by the other players decisions. In this context, we are interested in searching an equilibrium point for the joint power and channel selection problem of the SUs from which no player has anything to gain by unilaterally deviating. This equilibrium point is known as Nash equilibrium. In the following we introduce two games, representative of the underlay and overlay paradigms, and we formulate them as Exact Potential games.

3.1.1. Underlay Game. The underlay game is defined as follows.

(i) N is the finite set of players, that is, the SUs.

(ii) The strategies for player $i \in N$ are

(a) a power level p_i^S in the set of power levels $P^S = (p_1^S, \dots, p_m^S)$;

(b) a channel c_i in the set of channels $C = (c_1, \dots, c_l)$.

These strategies can be combined into a composite strategy $s_i = (p_i^S, c_i) \in S_i$.

(iii) The utility of each player i is defined as follows:

$$u(s_i, s_{-i}) = -\sum_{j=1}^{M} p_i^S h_{ij}^{SP} f\left(c_i, c_j\right) - \sum_{j=1, j\neq i}^{N} p_j^S h_{ji}^{SS} f\left(c_j, c_i\right) - \sum_{j=1, j\neq i}^{N} p_i^S h_{ij}^{SS} f\left(c_i, c_j\right) + b\log\left(1 + p_i^S h_{ii}^{SS}\right). \tag{14}$$

The expression presented in (14) consists of five terms. The first and the third terms account for the interference the user i is causing to the PUs and SUs simultaneously operating in the same frequency channel. The second term accounts for the interference received by player i from the SUs simultaneously transmitting in the same frequency channel. Finally, the fourth term only depends on the strategy selected by player i and provides an incentive for individual players to increase their power levels. It is in fact considered that the players' satisfaction increases logarithmically with their transmission power. We weight this term by a coefficient b to give it more or less importance than the other terms of the utility function.

3.1.2. Overlay Game. The overlay game is defined as follows

(i) N is the finite set of players, that is, the SUs.

(ii) The strategies for player $i \in N$ are

(a) a power level p_i^S in the set of power levels $P^S = (p_1^S, \dots, p_m^S)$;

(b) the power level $p_i^{S'}$ that the player devotes to its own transmissions, in the set of power levels $P^{S'} = (p_1^{S'}, \dots, p_q^{S'})$, where q is the order of set $P^{S'}$;

(c) the cooperative power level $p_i^{S''}$ that the player devotes to relaying a PU transmission and which is computed as $p_i^{S''} = p_i^S - p_i^{S'}$. The set of these power levels, $P^{S''}$, is the same as $P^{S'}$;

(d) a channel c_i in the set of channels $C = (c_1, \dots, c_l)$.

(e) a slot subset sl_i from the two possible subsets $\mathcal{S}_1$ (even) and $\mathcal{S}_2$ (odd).

These strategies can be combined into a composite strategy $s_i = (p_i^S, p_i^{S'}, p_i^{S''}, c_i, sl_i) \in S_i$. We define $S = \times S_i, i \in$ N as the strategy space.

(iii) The utility of each player i is defined as follows:

$$
\begin{aligned}
u(s_i, s_{-i}) = &-\sum_{j=1}^{M} p_i^{S'} h_{ij}^{SP} f\left(c_i, c_j\right) \\
&- \sum_{j=1, j\neq i}^{N} p_j^{S'} h_{ji}^{SS} f\left(c_j, c_i\right) f'\left(sl_j, sl_i\right) \\
&- \sum_{j=1, j\neq i}^{N} p_i^{S'} h_{ij}^{SS} f\left(c_i, c_j\right) f'\left(sl_i, sl_j\right) \\
&+ b\log\left(1 + p_i^{S'} h_{ii}^{SS}\right) \\
&+ \sum_{j=1}^{M} p_i^{S''} h_{ij}^{SP} f\left(c_i, c_j\right) f''\left(\gamma_i^{PS} > \rho\right).
\end{aligned} \tag{15}
$$

The expression presented in (15) consists of five terms. The first and the third terms account for the interference perceived by the PUs and by the other SUs in c_i from player i, which only consists of the power the user i devotes to the secondary transmission (i.e., $p_i^{S'}$). In case of SUs, $p_i^{S'}$ only affects users active in c_i and in the same slot subset. The second term accounts for the interference generated on player i by the SUs active in channel c_i and in the same slot subset as player i, sl_i. The fourth term represents an incentive for the individual players to increase the power level devoted to their own communications. We weight this term by a coefficient b to give it more or less importance than the other terms of the utility function. Finally, the last term is a positive contribution to the utility function and accounts for the benefit provided to the PUs by the relaying realized by the SUs. This term is positively defined to encourage SUs to cooperate with PUs in exchange for using their frequency channel. Note that the term $f''(\gamma_i^{PS} > \rho)$ in the last term, which takes value 1 if the condition is satisfied and 0 otherwise, only applies to the D&F scheme. It determines if the relay is not able to decode, and then it does not increase its utility by cooperating, as it is not able to do so. For the A&F scheme, the relay always cooperates, and therefore the term $f''(\gamma_i^{PS} > \rho)$ is always 1.

3.1.3. Existence of a Nash Equilibrium. In order to have good convergence characteristics for the above described games, some mathematical properties have to be imposed on the utility functions. In particular, certain classes of games have shown to always converge to a Nash Equilibrium when a best response adaptive strategy is applied. An example of them is the class of Exact Potential Games. A game $\Gamma = \{N, \{S_i\}_{i\in N}, \{u_i\}_{i\in N}\}$ is an Exact Potential game if there exists a function $\mathrm{Pot} : S \to \mathfrak{R}$ such that, for all $i \in N$, $s_i, s_i' \in S_i$,

$$
\mathrm{Pot}(s_i, s_{-i}) - \mathrm{Pot}(s_i', s_{-i}) = u(s_i, s_{-i}) - u(s_i', s_{-i}) \tag{16}
$$

The function Pot is called *Exact Potential Function* of the game Γ. The potential function reflects the change in utility for any unilaterally deviating player. As a result, if *Pot*Pot is an exact potential function of the game Γ, and $s^* \in \{\mathrm{argmax}_{s\in S}\mathrm{Pot}(s)\}$ is a maximizer of the potential function, then s^* is a Nash equilibrium of the game. In particular, the best reply dynamic converges to a Nash Equilibrium in a finite number of steps, regardless of the order of play and the initial condition of the game, as long as only one player acts at each time step, and the acting player maximizes its utility function, given the most recent actions of the other players. For the previously formulated underlay and overlay games, we can define two exact potential functions, $\mathrm{Pot}_u(S)$ and $\mathrm{Pot}_o(S)$.

(i) *Underlay game Potential function*:

$$
\begin{aligned}
\mathrm{Pot}_u(s_i, s_{-i}) = &\sum_{i=1}^{N}\left(-\sum_{j=1}^{M} p_i^{S} h_{ij}^{SP} f\left(c_i, c_j\right)\right) \\
&+ \sum_{i=1}^{N}\left(-a \sum_{j=1, j\neq i}^{M} p_j^{S} h_{ji}^{SS} f\left(c_j, c_i\right) \right.\\
&\qquad \left. -(1-a) \sum_{j=1, j\neq i}^{N} p_i^{S} h_{ij}^{SS} f\left(c_i, c_j\right)\right) \\
&+ \sum_{i=1}^{N} b\log\left(1 + p_i^{S} h_{ii}^{SS}\right).
\end{aligned} \tag{17}
$$

(ii) *Overlay game Potential function*:

$$
\begin{aligned}
&\mathrm{Pot}_o(s_i, s_{-i}) \\
&= \sum_{i=1}^{N}\left(-\sum_{j=1}^{M} p_i^{S'} h_{ij}^{SP} f\left(c_i, c_j\right)\right) \\
&\quad + \sum_{i=1}^{N}\left(-a \sum_{j=1, j\neq i}^{N} p_j^{S'} h_{ji}^{SS} f\left(c_j, c_i\right) f'\left(sl_j, sl_i\right) \right.\\
&\qquad \left. -(1-a) \sum_{j=1, j\neq i}^{N} p_i^{S'} h_{ij}^{SS} f\left(c_i, c_j\right) f'\left(sl_i, sl_j\right)\right) \\
&\quad + \sum_{i=1}^{N} b\log\left(1 + p_i^{S'} h_{ii}^{SS}\right) \\
&\quad + \sum_{i=1}^{N}\sum_{j=1}^{M} p_i^{S''} h_{ij}^{SP} f\left(c_i, c_j\right) f''\left(\gamma_i^{PS} > \rho\right),
\end{aligned} \tag{18}
$$

where $a < 1$. The proof that the underlay and overlay games, with utility functions defined in (14) and (15) and with the potential functions defined in (17) and (18), are exact potential games is given in the appendix.

3.2. A Bayesian Potential Game Formulation: Underlay and Overlay Games with Incomplete Information. In a more realistic and feasible scenario, we should not rely on the existence of a CCC where SUs share their transmission information. As a result, we consider a situation where incomplete knowledge is available at the decision making agents. In this section we model joint channel and transmission power selection for cognitive radios with incomplete information as the output of a Bayesian Potential game. In particular, we consider two games of incomplete information, the underlay and overlay. Each one of these games is defined as $\Gamma = \{N, \{S_i\}_{i\in N}, \{\eta_i\}_{i\in N^+}, \{f_{H_i}(\eta_i)\}_{i\in N}, \{u_i\}_{i\in N}\}$ where

(i) N is the finite set of players, that is, the SUs, and N^+ is a finite set with $N^+ \supseteq N$, and $N^+ \setminus N$ is the set of outside players (i.e., the PUs);

(ii) for every $i \in N$, S_i is the set of strategies of player i, which have already been introduced in case of complete knowledge for the underlay game in Section 3.1.1 and for the overlay game in Section 3.1.2;

(iii) a game of incomplete information, with respect to a game of complete information, is characterized by the player's type, which embodies any information that is not common knowledge to all players and is relevant to the players' decision making. This may include the player's utility function, his belief about other player's utility functions, and so forth. For every $i \in N^+$, H_i is the finite set of possible types of player i, $\eta_i = (h_{1i}^{SS}, \dots, h_{i-1i}^{SS}, h_{i+1i}^{SS}, \dots, h_{Ni}^{SS}) \in \mathrm{H}_i$, which includes the wireless channel gains of player i. Each player is assumed to observe perfectly its type but is unable to observe the types of its neighbors;

(iv) $f_{H_i}(\eta_i)$ is a probability distribution on $H = \times H_i, i = 1, \dots, \mathrm{N}$, with the a priori probability density function (PDF) on H defining the wireless channel gain PDF;

(v) for every $i \in N$, $u_i : S\times H \to \mathfrak{R}$ is the utility function of player i.

The utility functions for player i, for the underlay and overlay games with incomplete information, are very similar to those defined in (14) and (15), but besides being functions of player i's chosen strategy $s_i \in S_i$ and other players' strategies (s_{-i}), they are functions of player i's realized channel gains $\eta_i \in H_i$ and other SUs and PUs' channel gains (i.e., η_{-i}). In particular, for the underlay game with incomplete information,

$$\begin{aligned} u(s_i, s_{-i}; \eta_i, \eta_{-i}) = &-\sum_{j=1}^{M} p_i^S h_{ij}^{SP} f(c_i, c_j) \\ &- \sum_{j=1, j\neq i}^{N} p_j^S h_{ji}^{SS} f(c_j, c_i) \\ &- \sum_{j=1, j\neq i}^{N} p_i^S h_{ij}^{SS} f(c_i, c_j) \\ &+ b\log\left(1 + p_i^S h_{ii}^{SS}\right), \end{aligned} \tag{19}$$

and for the overlay game with incomplete information,

$$\begin{aligned} u(s_i, s_{-i}; \eta_i, \eta_{-i}) = &-\sum_{j=1}^{M} p_i^{S'} h_{ij}^{SP} f(c_i, c_j) \\ &- \sum_{j=1, j\neq i}^{N} p_j^{S'} h_{ji}^{SS} f(c_j, c_i) f'(sl_j, sl_i) \\ &- \sum_{j=1, j\neq i}^{N} p_i^{S'} h_{ij}^{SS} f(c_i, c_j) f'(sl_i, sl_j) \\ &+ b\log\left(1 + p_i^{S'} h_{ii}^{SS}\right) \\ &+ \sum_{j=1}^{M} p_i^{S''} h_{ij}^{SP} f(c_i, c_j) f''\left(\gamma_i^{PS} > \rho\right). \end{aligned} \tag{20}$$

It can be easily demonstrated (see the appendix) that the games with utility functions defined in (19) and (20) are Bayesian Potential games, if the following Potential functions are considered, for the underlay (21) and overlay (22) games with incomplete information, respectively:

$$\begin{aligned} &\mathrm{Pot}_{uB}(s_i, s_{-i}; \eta_i, \eta_{-i}) \\ &= \sum_{i=1}^{N}\left(-\sum_{j=1}^{M} p_i^S h_{ij}^{SP} f(c_i, c_j)\right) \\ &\quad + \sum_{i=1}^{N}\Bigg(-a\sum_{j=1, j\neq i}^{N} p_j^S h_{ji}^{SS} f(c_j, c_i) \\ &\qquad -(1-a)\sum_{j=1, j\neq i}^{N} p_i^S h_{ij}^{SS} f(c_i, c_j)\Bigg) \\ &\quad + \sum_{i=1}^{N} b\log\left(1 + p_i^S h_{ii}^{SS}\right), \end{aligned} \tag{21}$$

$$\begin{aligned} &\mathrm{Pot}_{oB}(s_i, s_{-i}; \eta_i, \eta_{-i}) \\ &= \sum_{i=1}^{N}\left(-\sum_{j=1}^{M} p_i^{S'} h_{ij}^{SP} f(c_i, c_j)\right) \\ &\quad + \sum_{i=1}^{N}\Bigg(-a\sum_{j=1, j\neq i}^{N} p_j^{S'} h_{ji}^{SS} f(c_j, c_i) f'(sl_j, sl_i) \\ &\qquad -(1-a)\sum_{j=1, j\neq i}^{N} p_i^{S'} h_{ij}^{SS} f(c_i, c_j) f'(sl_i, sl_j)\Bigg) \\ &\quad + \sum_{i=1}^{N} b\log\left(1 + p_i^{S'} h_{ii}^{SS}\right) \\ &\quad + \sum_{i=1}^{N}\sum_{j=1}^{M} p_i^{S''} h_{ij}^{SP} f(c_i, c_j) f''\left(\gamma_i^{PS} > \rho\right). \end{aligned} \tag{22}$$

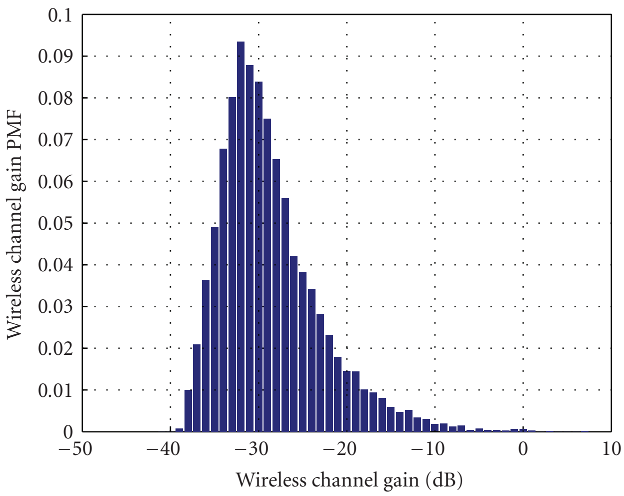

Figure 3: Wireless channel gain PMF derived by discretizing the wireless channel gain PDF.

As for the game with complete information, we need to find an equilibrium point from which no player has anything to gain by unilaterally deviating. In a Bayesian game, this point is a Bayesian Nash equilibrium; that is, a Bayesian Nash equilibrium is a Nash equilibrium of a Bayesian game. In particular, a strategy profile $s^* = (s_1^*, \dots, s_N^*)$ is a Bayesian Nash equilibrium if $s_i^*(\eta_i)$ solves (23), assuming that types of different players are independent:

$$s_i^*(\eta_i) \in \arg\max_{s_i \in S} \sum_{\eta_{-i}} f_H(\eta_{-i}) u_i(s_i, s_{-i}; \eta_i, \eta_{-i}). \tag{23}$$

As it is proven in [14], the existence of a Bayesian Nash equilibrium is an immediate consequence of the Nash existence theorem. As a result, considering that the potential games have shown to always converge to a Nash Equilibrium when a best response adaptive strategy is applied, it can be derived that for the Bayesian Potential game Γ there exists a Bayesian Nash equilibrium, which maximizes the expected utility function defined in (23).

4. Simulation Scenario

The scenario considered to evaluate the proposed framework consists of a circular area with radius $R_{\max}$=150 m. With respect to the strategy space, the set of power levels $P^S = (p_1^S, \dots, p_m^S)$ is defined as $P^S = (0, 5, 10, 15, 20)$ dBm, that is, $m = 5$. On the other hand, the SUs can be scheduled over $l = 4$ available frequency channels, so that the set of channels $C = (c_1, \dots, c_l)$ is defined as $C = (1, 2, 3, 4)$. Each channel is assumed to have a bandwidth $B_c = 200$ KHz. We consider $M = 4$ PUs pairs, one pair for each frequency channel, and N SUs pairs, which at simulation start are randomly distributed over the l frequency channels. The PUs pairs are randomly located in the scenario. Specifically, the maximum distance between a PU transmitter and a PU receiver is randomly selected depending on their random position in the coverage area. On the other hand, the maximum distance between a SU transmitter and receiver is 20 m. We consider a wireless channel gain of $h_{ii} = (10/d_{ii}^2)$, where d_{ii} is the distance from transmitter i to receiver i. The transmission power of a PU is 43 dBm. The minimum SINR for a user not to be in outage is $\gamma = 3$dB. In order to define the PDF of the wireless channel gains, we proceed by simulations. We discretize the random variable R representing the distance between two nodes, and accordingly the possible values of wireless channel gains, into K equally spaced values. In this way we generate a path loss probability mass function (PMF) of the wireless channel gains, which is represented in Figure 3.

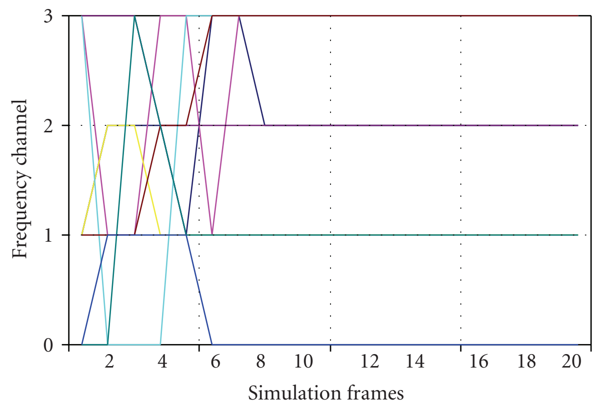

Figure 4: Convergence of SUs pairs–frequency channel.

5. Discussion

In this section we present simulation results to evaluate the performances of the proposed joint power and channel allocation algorithm for underlay and overlay approaches in both cases of complete and incomplete information. First of all, we illustrate the convergence properties of the proposed algorithms. The convergence of action updates in the overlay game for the case of $N = 8$ SUs in the scenario, and D&F relay mode, is shown in Figures 4, 5, and 6. In particular, Figure 4 represents the choice of frequency channels, and Figures 5 and 6 depict the selection of the transmission power, for the overlay game, which is split in two parts, the first one devoted to the secondary communication, and the second one to relaying the primary communication. Notice how the players choose a variety of power levels and disperse, so as to transmit on a variety of frequency channels. The convergence of action updates of the underlay game is not shown since they are very similar to those of the overlay game. Second, Figure 7 compares the behavior of the Bayesian Potential Game with incomplete information (BPG) to the Exact Potential Game with complete information (EPG). It can be noticed how the lack of complete information only slightly reduces performances in terms of SINR for both PUs and SUs.

In the following, we compare performance results of the underlay and overlay approaches, taking as a reference the D&F mode and the incomplete information case, since this is the most feasible option. Figure 8 compares performance

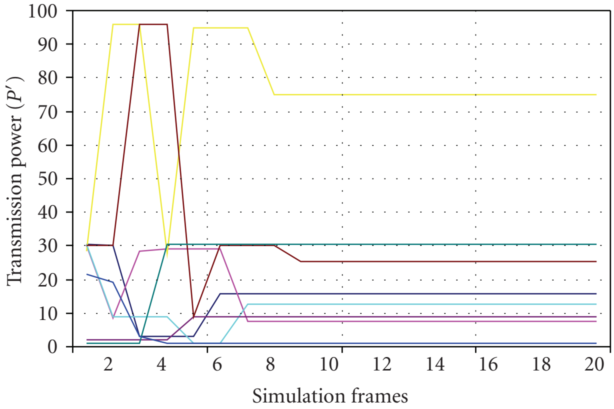

Figure 5: Convergence of SUs pairs–transmission power devoted to secondary communication.

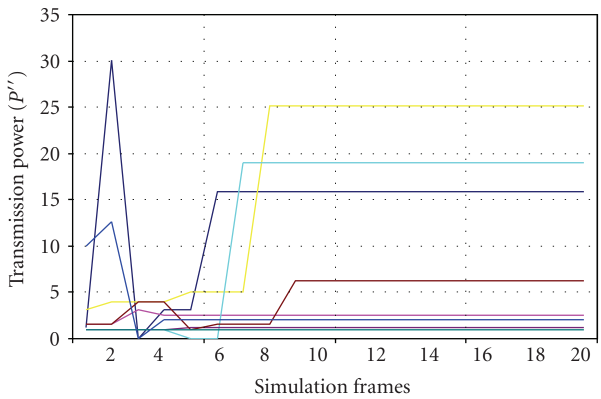

Figure 6: Convergence of SUs pairs –transmission power devoted to primary communication.

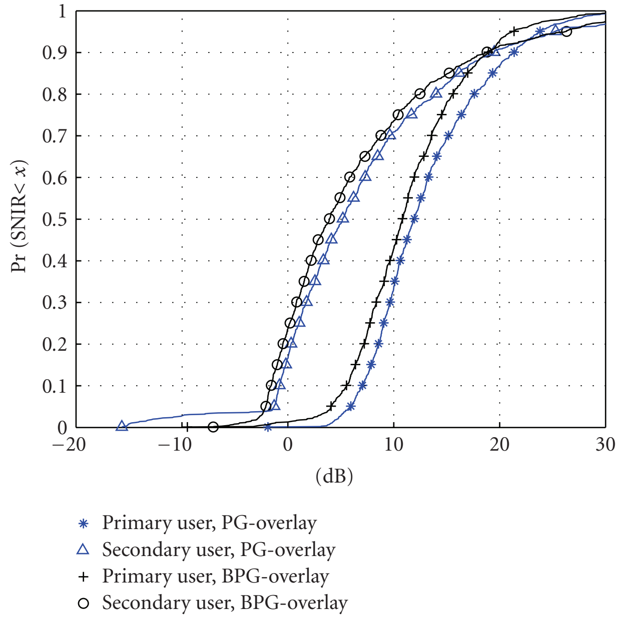

Figure 7: SINR results: Bayesian Potential game with incomplete information versus Exact Potential game with complete information, for PUs and SUs.

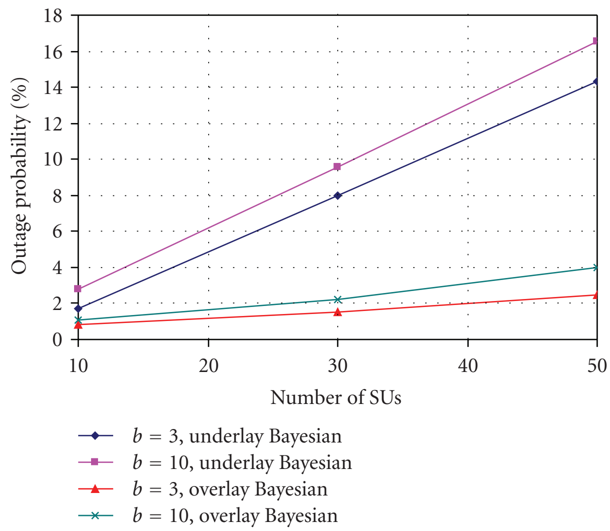

Figure 8: PUs Outage probability for overlay and underlay games.

results in terms of outage probability obtained by the underlay and the overlay paradigms, as a function of the number of SUs in the scenario. It can be observed that the overlay paradigm outperforms the underlay scheme in terms of outage. One of the reasons is that, in situations characterized by the proximity of an SU transmitter to a PU receiver, which are very critical for the underlay scheme, the benefit gained by the cooperative approach increases. In fact, the message relayed by the SU is received with a higher quality by the PU receiver. Additionally, it is worth noting that different results are obtained for different values of b. In particular, the lower b, the more the SUs are discouraged from increasing their transmission power at the expense of the interference caused on the other users. On the other hand, **Figure 9** compares SINR results for both PUs and SUs. It can be observed again that the overlay approach benefits PUs but reduces the SUs performances, which is the price to pay for being allowed to access primary channels. Let us now consider two different values of b for which both the overlay and underlay games provide the PUs with less than 3% of outage probability, (i.e., $b = 10$, for the overlay game with incomplete information and $b = 0.001$ for the underlay game with incomplete information). It can be observed from **Figure 10** that even if the PUs results in terms of outage are comparable, the SUs performances are reduced, when considering a lower b, due to their lower transmission power levels. This demonstrates that, under the condition of limited interference on the PUs, also the SUs are benefited by cooperation. In fact, they are allowed to transmit with higher power levels, as long as they devote a part of it for relaying primary communications; the results are more favorable to them than not cooperating and reducing the b parameter of the game.

Finally, **Figure 11** compares outage performances for the D&F and A&F relay modes, for the overlay game with

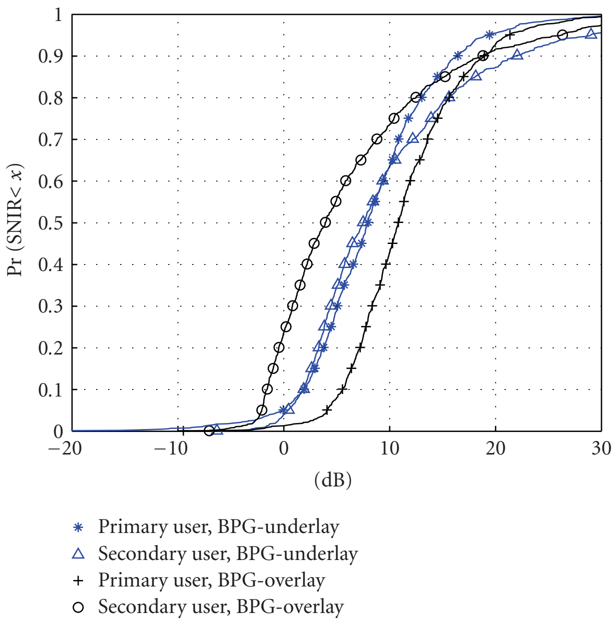

Figure 9: SINR results: overlay versus underlay, for PUs and SUs.

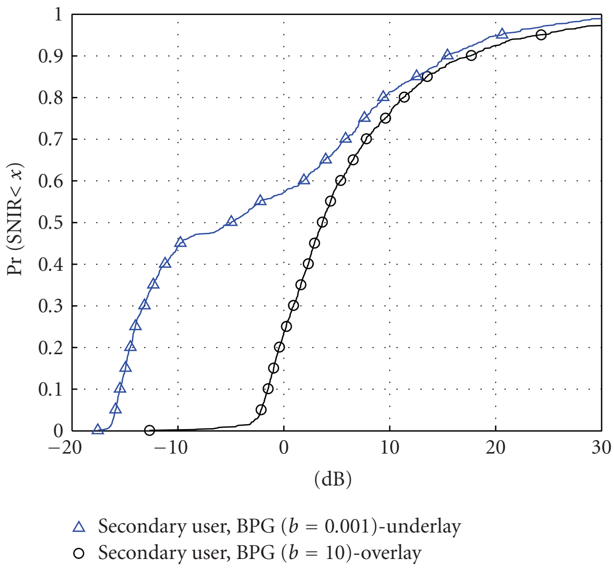

Figure 10: SINR results for SUs considering different values of b: underlay versus overlay, when the outage probability of PUs is 3%.

incomplete information, when different values of detection probability of the primary message at the SUs' receivers are considered. It can be observed that when the SUs are able to decode the PUs' signals with a probability equal to 1, the D&F relaying approach provides better performances than the A&F scheme. On the other hand, when the probability of decoding the PU's messages is reduced, it is also reduced the probability that the SUs are able to cooperate with the PUs. Consequently, the A&F approach provides better performances than the D&F.

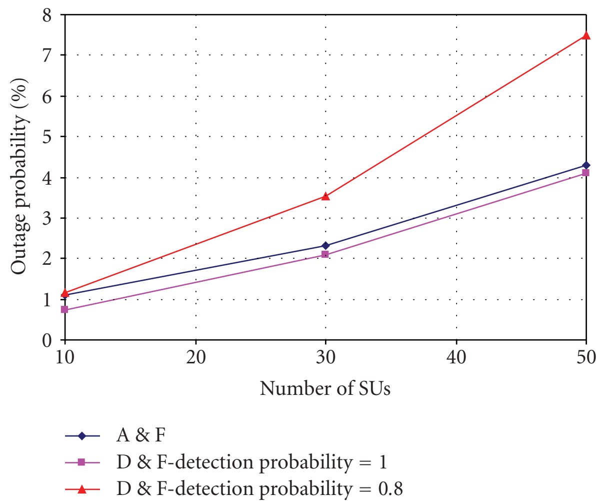

Figure 11: Comparison of D&F and A&F outage performance results.

6. Conclusion

In this paper we have introduced potential games to model joint channel and power allocation for cooperative and noncooperative cognitive radios. Particular emphasis has been given to the feasibility of the proposed approach. In fact, both the hypothesis of complete and incomplete information about the wireless channel gains is taken into account and compared, and the half-duplex option is considered for both D&F and A&F relay options of cooperative cognitive radios. More in particular, we have proposed a cooperative scheme where SUs are allowed to use licensed channels as long as they provide compensation to PUs by means of cooperation (overlay approach), and we have compared it to a scheme where cooperation between SUs and PUs is not considered (underlay approach). We have modeled these schemes by means of two Potential games, which are always characterized by a pure Nash equilibrium. In addition to this, in order to avoid the implementation of a CCC, which would increase cost and complexity, we have considered the hypothesis of incomplete information, where SUs are unaware of the wireless channel gains of the other PUs and SUs. Taking into account this additional hypothesis, both the underlay and overlay schemes have been modeled by means of Bayesian potential games converging to a pure Bayesian Nash equilibrium. Simulation results have shown that cooperation benefits both PUs and SUs and that the hypothesis of incomplete information only slightly reduces performance results with respect to the case of complete information.

Appendix

We prove that the game with the utility function defined in (20) and the potential function $\mathrm{Pot}_{oB}(S, H)$ defined in (22) is a Bayesian potential game. The same demonstration is also valid for the case of complete information with utility function (15) and potential function (18).

The proposed potential function consists of four contributions:

$$\mathrm{Pot}_{oB}(S,H) = W(S,H) + Z(S,H) + X(S,H) + Y(S,H), \tag{A.1}$$

where, for $i = 1,\ldots,N$,

$$\begin{aligned}
W(S,H) &= W(s_i,s_{-i};\eta_i,\eta_{-i}) \\
&= \sum_{i=1}^{N}\left(-\sum_{j=1}^{M} p_i^{S'} h_{ij}^{SP} f(c_i,c_j)\right), \\
Z(S,H) &= Z(s_i,s_{-i};\eta_i,\eta_{-i}) \\
&= \sum_{i=1}^{N}\left(\sum_{j=1}^{M} p_i^{S''} h_{ij}^{SP} f(c_i,c_j) f''\left(\gamma_i^{PS} > \rho\right)\right), \\
X(S,H) &= X(s_i,s_{-i};\eta_i,\eta_{-i}) \\
&= \sum_{i=1}^{N}\Bigg(-a\sum_{j=1,j\neq i}^{N} p_j^{S'} h_{ji}^{SS} f(c_j,c_i) f'(sl_j,sl_i) \\
&\qquad -(1-a)\sum_{j=1,j\neq i}^{N} p_i^{S'} h_{ij}^{SS} f(c_i,c_j)\Bigg), \\
Y(S,H) &= Y(s_i,s_{-i};\eta_i,\eta_{-i}) \\
&= \sum_{i=1}^{N} b\log\left(1+p_i^{S'} h_{ii}^{SS}\right).
\end{aligned} \tag{A.2}$$

The first term $W(s_i,s_{-i};\eta_i,\eta_{-i})$ can be rewritten in the following way:

$$\begin{aligned}
W(s_i,s_{-i};\eta_i,\eta_{-i}) &= -\sum_{j=1}^{M} p_i^{S'} h_{ij}^{SP} f(c_i,c_j) \\
&\quad + \sum_{k=1,k\neq i}^{N}\left(-\sum_{j=1}^{M} p_k^{S'} h_{kj}^{SP} f(c_k,c_j)\right) \\
&= -\sum_{j=1}^{M} p_i^{S'} h_{ij}^{SP} f(c_i,c_j) + W(s_{-i},\eta_i,\eta_{-i}),
\end{aligned} \tag{A.3}$$

where

$$W(s_{-i};\eta_i,\eta_{-i}) = \sum_{k=1,k\neq i}^{N}\left(-\sum_{j=1}^{M} p_k^{S'} h_{kj}^{SP} f(c_k,c_j)\right), \tag{A.4}$$

and it does not depend on the strategy of player i.

The second term $Z(s_i,s_{-i};\eta_i,\eta_{-i})$ can be rewritten as

$$\begin{aligned}
&Z(s_i,s_{-i};\eta_i,\eta_{-i}) \\
&= \sum_{j=1}^{M} p_i^{S''} h_{ij}^{SP} f(c_i,c_j) f''\left(\gamma_i^{PS} > \rho\right) \\
&\quad + \sum_{k=1,k\neq i}^{N}\left(\sum_{j=1}^{M} p_k^{S''} h_{kj}^{SP} f(c_k,c_j) f''\left(\gamma_k^{PS} > \rho\right)\right) \\
&= \sum_{j=1}^{M} p_i^{S''} h_{ij}^{SP} f(c_i,c_j) f''\left(\gamma_i^{PS} > \rho\right) \\
&\quad + Z(s_{-i},\eta_i,\eta_{-i}),
\end{aligned} \tag{A.5}$$

where

$$Z(s_{-i};\eta_i,\eta_{-i}) = \sum_{k=1,k\neq i}^{N}\left(\sum_{j=1}^{M} p_k^{S''} h_{kj}^{SP} f(c_k,c_j) f''\left(\gamma_i^{PS} > \rho\right)\right), \tag{A.6}$$

and it does not depend on the strategy of player i.

As for the third term, it can be rewritten as follows:

$$\begin{aligned}
&X(s_i,s_{-i};\eta_i,\eta_{-i}) \\
&= -a\sum_{j=1,j\neq i}^{N} p_j^{S'} h_{ji}^{SS} f(c_j,c_i) f'(sl_j,sl_i) \\
&\quad -(1-a)\sum_{j=1,j\neq i}^{N} p_i^{S'} h_{ij}^{SS} f(c_i,c_j) f'(sl_i,sl_j) \\
&\quad + \sum_{k=1,k\neq i}^{N}\Bigg(-a\sum_{j=1,j\neq k}^{N} p_j^{S'} h_{jk}^{SS} f(c_j,c_k) f'(sl_j,sl_k) \\
&\qquad -(1-a)\sum_{j=1,j\neq k}^{N} p_k^{S'} h_{kj}^{SS} f(c_k,c_j) f'(sl_k,sl_j)\Bigg) \\
&= -a\sum_{j=1,j\neq i}^{N} p_j^{S'} h_{ji}^{SS} f(c_j,c_i) f'(sl_j,sl_i) \\
&\quad -(1-a)\sum_{j=1,j\neq i}^{N} p_i^{S'} h_{ij}^{SS} f(c_i,c_j) f'(sl_i,sl_j) \\
&\quad + \sum_{k=1,k\neq i}^{N}\Big(-a p_i^{S'} h_{ik}^{SS} f(c_i,c_k) f'(sl_i,sl_k) \\
&\qquad -(1-a) p_k^{S'} h_{ki}^{SS} f(c_k,c_i)\Big) f'(sl_k,sl_i) \\
&\quad + \sum_{k=1,k\neq i}^{N}\Bigg(-a\sum_{j=1,j\neq k,j\neq i}^{N} p_j^{S'} h_{jk}^{SS} f(c_j,c_k) f'(sl_j,sl_k) \\
&\qquad -(1-a)\sum_{j=1,j\neq k,j\neq i}^{N} p_k^{S'} h_{kj}^{SS} f(c_k,c_j) f'(sl_k,sl_j)\Bigg)
\end{aligned}$$

$$
\begin{aligned}
&= (-a-1+a)\sum_{j=1, j\neq i}^{N} p_j^{S'} h_{ji}^{SS} f\left(c_j, c_i\right) f'\left(sl_j, sl_i\right) \\
&\quad - (1-a+a)\sum_{j=1, j\neq i}^{N} p_i^{S'} h_{ij}^{SS} f\left(c_i, c_j\right) f'\left(sl_i, sl_j\right) \\
&\quad + \sum_{k=1, k\neq i}^{N}\Bigg(-a\sum_{j=1, j\neq k, j\neq i}^{N} p_j^{S'} h_{jk}^{SS} f\left(c_j, c_k\right) f'\left(sl_j, sl_k\right) \\
&\qquad -(1-a)\sum_{j=1, j\neq k, j\neq i}^{N} p_k^{S'} h_{kj}^{SS} f\left(c_k, c_j\right) f'\left(sl_k, sl_j\right)\Bigg).
\end{aligned} \tag{A.7}
$$

The last term does not depend on s_i, so that

$$
\begin{aligned}
X(s_i, s_{-i}; \eta_i, \eta_{-i}) = &-\sum_{j=1, j\neq i}^{N} p_j^{S'} h_{ji}^{SS} f\left(c_j, c_i\right) f'\left(sl_j, sl_i\right) \\
&-\sum_{j=1, j\neq i}^{N} p_i^{S'} h_{ij}^{SS} f\left(c_i, c_j\right) f'\left(sl_i, sl_j\right) \\
&+ X(s_{-i}; \eta_i, \eta_{-i}).
\end{aligned} \tag{A.8}
$$

Finally, $Y(s_i, s_{-i}; \eta_i, \eta_{-i})$ can be rewritten as

$$
Y(s_i, s_{-i}; \eta_i, \eta_{-i}) = b\log\left(1 + p_i^{S'} h_{ii}^{SS}\right) + Y(s_{-i}; \eta_i, \eta_{-i}), \tag{A.9}
$$

where $Y(s_{-i}; \eta_i, \eta_{-i}) = \sum_{k=1, k\neq i}^{N} b\log(1 + p_k^{S'} h_{kk}^{SS})$, and it does not depend on s_i. As a result,

$$
\begin{aligned}
\mathrm{Pot}_{oB}(s_i, s_{-i}; \eta_i, \eta_{-i}) = &-\sum_{j=1}^{M} p_i^{S'} h_{ij} f\left(c_i, c_j\right) \\
&+ \sum_{j=1}^{M} p_i^{S''} h_{ij} f\left(c_i, c_j\right) f''\left(\gamma_i^{PS} > \rho\right) \\
&- \sum_{j=1, j\neq i}^{N} p_j^{S'} h_{ji}^{SS} f\left(c_j, c_i\right) f'\left(sl_j, sl_i\right) \\
&- \sum_{j=i, j\neq i}^{N} p_i^{S'} h_{ij}^{SS} f\left(c_i, c_j\right) f'\left(sl_j, sl_i\right) \\
&+ b\log\left(1 + p_i^{S'} h_{ii}^{SS}\right) + W(s_{-i}; \eta_i, \eta_{-i}) \\
&+ Z(s_{-i}; \eta_i, \eta_{-i}) + X(s_{-i}; \eta_i, \eta_{-i}) \\
&+ Y(s_{-i}; \eta_i, \eta_{-i}),
\end{aligned} \tag{A.10}
$$

that is,

$$
\mathrm{Pot}_{oB}(s_i, s_{-i}; \eta_i, \eta_{-i}) = u(s_i, s_{-i}; \eta_i, \eta_{-i}) + F(s_{-i}; \eta_i, \eta_{-i}), \tag{A.11}
$$

where

$$
\begin{aligned}
F(s_{-i}; \eta_i, \eta_{-i}) = &\; W(s_{-i}; \eta_i, \eta_{-i}) + Z(s_{-i}; \eta_i, \eta_{-i}) \\
&+ X(s_{-i}; \eta_i, \eta_{-i}) + Y(s_{-i}; \eta_i, \eta_{-i}),
\end{aligned} \tag{A.12}
$$

and it is a function that does not depend on the strategy of player i. As a result, if player i changes its strategy from s_i to s_i', then we obtain that

$$
\mathrm{Pot}_{oB}(s_i', s_{-i}; \eta_i, \eta_{-i}) = u(s_i', s_{-i}; \eta_i, \eta_{-i}) + F(s_{-i}; \eta_i, \eta_{-i}), \tag{A.13}
$$

and consequently

$$
\begin{aligned}
&\mathrm{Pot}_{oB}(s_i, s_{-i}; \eta_i, \eta_{-i}) - \mathrm{Pot}_{oB}(s_i', s_{-i}; \eta_i, \eta_{-i}) \\
&\quad = u(s_i, s_{-i}; \eta_i, \eta_{-i}) - u(s_i', s_{-i}; \eta_i, \eta_{-i}).
\end{aligned} \tag{A.14}
$$

In order to prove that the underlay game is also an exact potential game, we define $p_i^S \equiv p_i^{S'}$ and restrict the cooperative power to take only the zero value: $p_i^{S''} \in \{0\}$. Then it can be easily seen that the potential function of the overlay game matches that of the underlay game in (21).

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