EURASIP Journal on Applied Signal Processing 2002:1, 58–66

Estimating Position of Mobile Terminals with Survey Data

Michael McGuire

Edward S. Rogers Sr. Department of Electrical and Computer Engineering, University of Toronto, 10 King's College Road, Toronto, ON M5S 3G4, Canada
Email: mmcguire@dsp.toronto.edu

Konstantinos N. Plataniotis

Edward S. Rogers Sr. Department of Electrical and Computer Engineering, University of Toronto, 10 King's College Road, Toronto, ON M5S 3G4, Canada
Email: kostas@dsp.utoronto.ca

Anastasios N. Venetsanopoulos

Edward S. Rogers Sr. Department of Electrical and Computer Engineering, University of Toronto, 10 King's College Road, Toronto, ON M5S 3G4, Canada
Email: anv@dsp.toronto.edu

Received 30 July 2001 and in revised form 1 October 2001

Estimating the position of mobile terminals is an important problem for cellular networks. This paper describes low cost methods of locating mobile terminals in urban environments. These methods use data collected during propagation surveys of the network area. It will be shown how the maximum likelihood estimate and minimum mean-square error estimators can be approximated using survey data. Furthermore, the approximate solutions can provide low variance location estimates with low computational cost compared to other methods.

Keywords and phrases: radio location, non-parametric methods, cellular radio networks.

1. INTRODUCTION

The market for wireless networking services is undergoing fast growth. This growth is expected to continue with the proliferation of wireless data and digital multimedia devices. There are several reasons for a network provider to estimate the position of the mobile terminals in their network ranging from emergency communications to resource allocation.

Assisting emergency communications is the most immediate concern of cellular network operators with millions of emergency 911 calls being made with cellular telephones every year in the U.S. alone [1]. The FCC in the United States has mandated that cellular network providers must be able to provide an estimated location of terminals making E911 calls that is accurate to within 100 meters for 67% of calls for network-based solutions [2, 3].

For proposed third and fourth generation cellular networks, it is envisioned that wireless networks will be required to provide higher bandwidth multimedia data with strict quality of service requirements [4]. It has been argued that one method to provide these services is to use mobile terminal location and prediction to allocate resources to the terminals [5, 6]. Thus, mobile terminal location estimation will become an integral part of wireless network control systems.

Several methods have been proposed in the literature for the location of mobile terminals in wireless networks based on Angle of Arrival (AoA), Time of Arrival (ToA), or Received Signal Strength (RSS) measurements [7, 8]. Other options explored in the literature include adding GPS receiver hardware to the mobile terminals [9]. GPS can offer very high precision geo-location. This technology has the disadvantage that the mobile terminals have to be modified to be located. Also unmodified GPS does not work inside buildings or in outdoor areas where buildings or hills can block the signal from the GPS satellites [10]. Regions like this are quite common in the heavy urban centers of greatest interest to cellular network providers.

This paper will discuss the location of the mobile terminals based on RSS measurements. From the RSS measurements, the propagation path loss between the mobile terminal and the fixed location base stations are calculated. From a path loss measurement, it is possible to estimate the distance between the mobile terminal and a base station. If three or more such estimates are available, a unique estimate of the terminal's position is possible via triangulation techniques.

The RSS location method has the advantages that it does not require extra measurement hardware in the equipment, does not require strict synchronization between base stations, and can be used with all cellular network configurations with minimal modifications [7].

The most popular method in the literature for using the RSS based approach is to calculate the Maximum Likelihood Estimate (MLE) of the mobile terminal location. The estimated location, $\hat{\theta} = (\hat{x}, \hat{y})$, is the location value which maximizes $f_{\mathbf{Z}|\theta}(\mathbf{Z}|\hat{\theta})$, the conditional probability density function of the measured path loss values, $\mathbf{Z}$, given the location of the mobile terminal, θ.

The distributions for the path loss values at all locations in the network area are not known. Usually only median path loss values at specific points in the propagation environment are known. These values obtained from either computer models [11], or field surveys of the network area taken during the network planning stage [12]. The lack of closed form equations for the densities of the path losses can make direct use of the MLE difficult. At best, we can only estimate the densities and calculate an approximate MLE location.

The MLE is optimal when the relationship between the mean path loss values and location is linear, the deviation from the mean is a Gaussian random variable and there is no prior information about the parameters to be estimated, in this case, mobile terminal location [13].

In the wireless environment, the relationship between mean path loss value and location is rarely linear owing to the main propagation path for the base station to mobile signal commonly being a signal reflected off of a building or large geographic feature. The path loss is a function of the mobile terminal's position relative not only to the base station but also with respect to large obstacles in the propagation environment. There is switching between regions of different propagation effects when the direct propagation paths become obstructed for periods of time as the mobile terminal moves amongst buildings and geographic features.

The handoff algorithm of the cellular network determines which base station the mobile terminal communicates with. This decision is based on measurements of the received signal at the mobile terminal. The communicating base station is probably the closest base station to the mobile terminal [14]. Thus, the handoff algorithm provides some prior statistical information about the mobile terminal location.

The existence of prior information about mobile terminal location changes the criterion of optimality so that estimators other than the MLE can be considered. We propose the use of the Minimum Mean Square Error (MMSE) type estimator, which has the minimum mean-squared error of all location estimators.

In Section 2, the approximate MLE and approximate MMSE estimators will be described. A robust version of the MMSE that can deal with lack of knowledge about the propagation environment will also be described. Section 3 will describe the simulations used to evaluate the different location methods. Section 4 will give the results of the application of the estimation methods. Section 5 will give our conclusions.

2. ESTIMATION TECHNIQUE

The survey data can be described as a set of true locations, $\theta_1, \theta_2, \ldots, \theta_M$, and path loss vectors $\mathbf{Z}_1, \mathbf{Z}_2, \ldots, \mathbf{Z}_M$, where $\mathbf{Z}_j$ is a vector of measured path loss values sampled when the mobile terminal was at location θ_j. The length of the $\mathbf{Z}$ vectors is the number of base station path loss measurements used to locate the mobile terminal. A minimum of three base stations are required. The survey data sets are either obtained from computer propagation models or field survey measurements.

The measured path loss values (in decibels) are modeled as

$$\mathbf{Z} = \bar{\mathbf{Z}}(\theta) + \mathbf{V}, \tag{1}$$

where $\bar{\mathbf{Z}}(\theta)$ is the median path loss when the mobile terminal is at location θ and $\mathbf{V}$ is the measurement error which is assumed to be a zero mean Gaussian vector with a covariance matrix given by σ^2 multiplied by the properly sized unit matrix [15]. The function $\bar{\mathbf{Z}}(\theta)$ is an unknown nonlinear function.

It is assumed that the influence of fast fading, or multipath fading as it is sometimes called, is removed from the measurements so that the Gaussian assumption for the measurement noise is justified. The fast fading can be removed by use of a time averaging filter if the mobile terminal is in motion so that the fast fading magnitude is changing over time [16]. The requirement for motion can be relaxed if some form of diversity technique is being used to reduce the effect of fast fading on the base station to mobile terminal radio link such as slow frequency hopping as is proposed in the GSM standard [17], or the use of transmitter diversity at the base stations as is proposed for advanced IS-95/IS-2000 base stations [18].

The problem is then stated as estimating the location, θ, of the mobile terminal from the measured path loss vector, $\mathbf{Z}$, given a set of known locations, $\{\theta_1, \theta_2, \ldots, \theta_M\}$, that produced path loss measurements, $\{\mathbf{Z}_1, \mathbf{Z}_2, \ldots, \mathbf{Z}_M\}$. It is assumed that the known locations for the survey points and the unknown location of the mobile terminal are drawn from the same distribution. This assumption is valid because the use of handoff information allows survey points for the cell the mobile terminal is known to be residing in to be selected for use by the location estimation algorithm. The estimated location will be designated $\hat{\theta}$ with a subscript indicating the algorithm used to calculate the location, where required.

The MLE for location is given by

$$\hat{\theta}_{\text{MLE}} = \arg(\theta) \max f_{\mathbf{Z}|\Theta}(\mathbf{Z}|\theta; \mathbf{P}), \tag{2}$$

where $\mathbf{P}$ is the assumed propagation environment model. Previous work on location has assumed that the propagation environment $\mathbf{P}$ was Line of Sight (LoS), that is the shortest distance straight line propagation paths between the mobile terminals and base stations were unobstructed [7, 8]. This, unfortunately, is rarely the case in the urban micro-cell case where the LoS propagation path is often blocked by buildings.

In this work, we use the survey data to compute an approximation for **P** that is used to improve the location estimate.

An approximation to the MLE can be calculated using

$$\begin{aligned}\|\mathbf{Z}-\mathbf{Z}_j\|^2 &= \min\|\mathbf{Z}-\mathbf{Z}_i\|^2 \quad \forall i \in \{1,2,\dots,M\}\\ &\longrightarrow \hat{\theta}_{\text{MLE}} \approx \theta_j,\end{aligned} \tag{3}$$

where $\|\cdot\|^2$ specifies the Euclidean distance. The approximation improves as $M \to \infty$. This estimator has two disadvantages. First, it can only return estimated locations at positions of the survey points. Second, it makes limited use of information from survey points other than the one with the propagation path loss vector closest to that for the mobile terminal. It is these disadvantages that motivates the search for other location estimators.

The MMSE is given by $\hat{\theta}_{\text{MMSE}} = \mathrm{E}[\theta|\mathbf{Z}]$ where $\mathrm{E}[\cdot]$ designates the expectation operator [13]. If the mobile terminal location is assumed to be uniformly distributed over the region of interest and the selection of base stations providing path loss measurements is independent of the mobile terminal location then the MMSE location estimate can be approximated by

$$\begin{aligned}\hat{\theta}_{\text{MMSE}} &= \int_{\mathbf{S}} \theta f_{\theta|\mathbf{Z}}(\theta|\mathbf{Z})\,d\theta\\ &= \frac{\int_{\mathbf{S}} \theta f_{\mathbf{Z}|\Theta}(\mathbf{Z}|\theta) f_{\Theta}(\theta)\,d\theta}{\int_{\mathbf{S}} f_{\mathbf{Z}|\Theta}(\mathbf{Z}|\theta) f_{\Theta}(\theta)\,d\theta}\\ &\approx \frac{\sum_{j=1}^{M} \theta_j \hat{f}_{\mathbf{Z}|\Theta}(\mathbf{Z}|\theta_j)}{\sum_{j=1}^{M} \hat{f}_{\mathbf{Z}|\Theta}(\mathbf{Z}|\theta_j)},\end{aligned} \tag{4}$$

where the conditional density is approximated by

$$\hat{f}_{\mathbf{Z}|\Theta}(\mathbf{Z}|\theta_j) = \frac{1}{\sqrt{2\pi h^2}} \exp\left(-\frac{[\mathbf{Z}-\mathbf{Z}_j]^T[\mathbf{Z}-\mathbf{Z}_j]}{2h^2}\right). \tag{5}$$

S is the region where the mobile terminal is known to be residing, and h is a constant that determines the amount of smoothing in the estimator. The density in (5) matches the Gaussian density assumed for **V** in (1).

The region **S** would be determined by the handoff algorithm of the cellular network. It has been shown elsewhere that if Θ and **Z** are Gaussian and the number of survey points is *large*, then the optimal value of h is given by [19]

$$h = \left[\frac{8d(d+2)(d+4)(2\sqrt{\pi})^d}{(2d+1)C_d}\right]^{1/(d+4)}, \tag{6}$$

where d is the number of base stations used to locate the mobile terminal, and C_d is the volume of the hyper-sphere holding the survey data set. In the simulations below, the Euclidean distance is calculated from the measured path loss vector for the mobile terminal to each of the survey point's path loss vectors. The maximum of these distances is taken as the radius of the hyper-sphere containing all the survey points, a, and the value of C_d is then calculated using

$$C_d = \frac{a^d \pi^{d/2}}{\Gamma(d/2+1)}, \tag{7}$$

where $\Gamma(\cdot)$ is the Gamma function whose definition is found in any good calculus or probability textbook [20].

An equivalent way of thinking of this method is that the conditional density is being estimated as being the sum of M kernel functions, each of which has the form of the Gaussian d-variate density function [21]. For this reason, we will call the above estimator the Gaussian kernel estimator.

The Gaussian kernel estimator has the disadvantage that it requires calculation of the h value for good performance, while the MLE does not. More generalized weighting functions have been investigated for location estimates of the form

$$\hat{\theta} = \frac{\sum_{j=1}^{N} \theta_j w(\mathbf{Z},\mathbf{Z}_j)}{\sum_{j=1}^{N} w(\mathbf{Z},\mathbf{Z}_j)}, \tag{8}$$

where $w(\mathbf{Z},\mathbf{Z}_j)$ is a user selected weighting function [21]. One such estimator is the linear distance-based kernel estimator

$$w(\mathbf{Z},\mathbf{Z}_j) = Z_{\max} - \|\mathbf{Z}-\mathbf{Z}_j\|^2, \tag{9}$$

with $Z_{\max} = \max(\|\mathbf{Z}_j - \mathbf{Z}\|^2)$ for all $j \in \{1,2,\dots,N\}$. This estimator is computationally less expensive than the Gaussian kernel estimator since it does not involve the calculation of an exponential for each survey point. In some cases, obtaining the parameters for the hypersphere containing the survey points to calculate the value of h for the Gaussian kernel estimator can be costly.

A decision critical to the success of this technique is determining the number of survey points to be taken, M. Too small value for M will result in low accuracy while too large value of M will result in an expensive survey process with many of the survey points giving little benefit. The Cramer-Rao bound for an estimator gives a lower bound on the Mean-Square Error (MSE) of any unbiased estimator. The Cramer-Rao lower bound can be used to get a bound on the Root-Mean Squared Error (RMSE), the square root of the mean-squared distance between the true locations and estimated location of the mobile terminal. It will be shown below how this bound on RMSE can be used to estimate a good value for M.

The RMSE of the location estimates can be easily shown to be larger than the mean distance error

$$\begin{aligned}\operatorname{Var}\left(\sqrt{X^2+Y^2}\right) &\geq 0,\\ \mathrm{E}\left[\left(\sqrt{X^2+Y^2}\right)^2\right] - \mathrm{E}\left[\sqrt{X^2+Y^2}\right]^2 &\geq 0,\\ \mathrm{E}\left[\left(\sqrt{X^2+Y^2}\right)^2\right] &\geq \mathrm{E}\left[\sqrt{X^2+Y^2}\right]^2,\\ \mathrm{E}[X^2+Y^2] &\geq \mathrm{E}\left[\sqrt{X^2+Y^2}\right]^2,\\ \sqrt{\mathrm{E}[X^2+Y^2]} &\geq \mathrm{E}\left[\sqrt{X^2+Y^2}\right].\end{aligned} \tag{10}$$

In the inequalities above, X and Y are the x and y coordinate errors, respectively. Since the distance, D, between the true location and the estimated location of the mobile terminal is always positive, its distribution must satisfy the well-known

Markov inequality [20]

$$P[D \geq a] \leq \frac{\mathrm{E}[D]}{a}. \tag{11}$$

We can then use the inequality between the RMSE and mean distance error proven above to write

$$P[D \geq a] \leq \frac{\mathrm{E}[D]}{a} \leq \frac{\mathrm{RMSE}}{a}. \tag{12}$$

Using (12), bounds on the probability that the estimated location of the mobile terminal being some distance a from the true location can be calculated

$$P[D < a] \geq 1 - \frac{\mathrm{RMSE}}{a}. \tag{13}$$

Conversely, for a given probability, P, it is possible to calculate a radius, a, such that the probability that the estimated location is within distance a of the true location is of probability P or higher

$$a = \frac{\mathrm{RMSE}}{1 - P}. \tag{14}$$

Equation (14) shows how that survey points that are separated by a distance much less than the RMSE cannot be differentiated with high probability. Thus, the RMSE gives a rough estimate of a good separation distance between survey points. If we assume that the Cramer-Rao bound is sufficiently tight, so that

$$\mathrm{E}[D] \leq \sqrt{\text{Cramer-Rao bound}} \leq \mathrm{RMSE}, \tag{15}$$

then the square root of the Cramer-Rao bound can be used in place of the unknown RMSE in the relations above. The area of the probability regions defined by (14) will be proportional to the Cramer-Rao bound on the MSE. A first approximation to M is then to divide the area of $\mathbf{S}$ by the Cramer-Rao bound.

A reasonable propagation model is assumed and the variance of the location estimate is calculated. An urban LoS path loss model can be taken from [15] for micro-cell environments. (See the appendix for how the Cramer-Rao bound can be calculated from a path loss model.) The optimal value of M will be a factor of 2 to 10 times greater than the value calculated using the LoS path loss Cramer-Rao bound because of discontinuities in the propagation environment caused by buildings or geographic features which result in Non Line of Sight (NLoS) propagation. If NLoS propagation occurs between a mobile terminal and a given base station, it indicates that a large geographic feature or building lies between the mobile and base stations which gives some information about the mobile's location. Thus, NLoS propagation increases the information in the path loss measurements, increasing the optimal value of M.

It is possible to derive an estimator that is a compromise between the MLE and approximate MMSE estimators. The estimator given in (8) is used but only with the N survey points that have path loss vectors closest, as defined by Euclidean distance, to the measured path loss vector for the mobile terminal. Obviously, when $N = 1$, this estimators will give the approximate MLE result from (3).

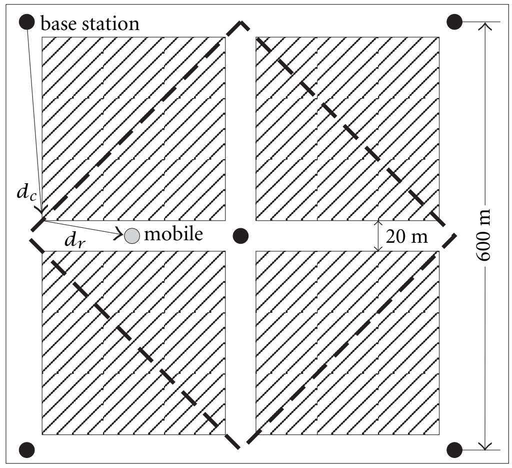

Figure 1: Manhattan propagation environment.

3. DESCRIPTION OF SIMULATION

The location estimation methods were evaluated using simulations. A regular Manhattan street micro-cell model was considered with dimensions and propagation characteristics taken from [7]. The environment is shown in Figure 1. The shaded areas are buildings. The origin of the coordinate systems is set at the central base station's position.

When the Line of Sight (LoS) or shortest distance path between the mobile terminal and base station is unobstructed the median path loss is

$$\bar{Z}(\theta) = 10\log_{10}\left[d(\theta)^a\left(1 + \frac{d(\theta)}{g}\right)^b\right]. \tag{16}$$

The value $d(\theta)$ is the distance between the mobile terminal and base station when the mobile terminal is at location θ. The values of a and b are constants determining the exponent of the path loss over distance. For the simulations described below, a and b are set to be 2. g is called the *breakpoint* distance and is a function of antennae height at the base station and the radio frequency used. The value of g is set to be 150 meters. These are typical values for urban environments [11, 15].

When the mobile terminal is in the street and the LoS path is blocked the median path loss is modeled by

$$\begin{aligned}\bar{Z}(\theta) = 10\log_{10}\Bigg\{&d_c(\theta)^a\left(1 + \frac{d_c(\theta)}{g}\right)^b \\ &\times \left(d_r(\theta)\right)^a\left[1 + \frac{d_r(\theta)}{g}\right]^b\Bigg\},\end{aligned} \tag{17}$$

where $d_c(\theta)$ is defined as the distance from the base station to the corner, and $d_r(\theta)$ is defined as the distance from the corner to the mobile terminal as shown in Figure 1.

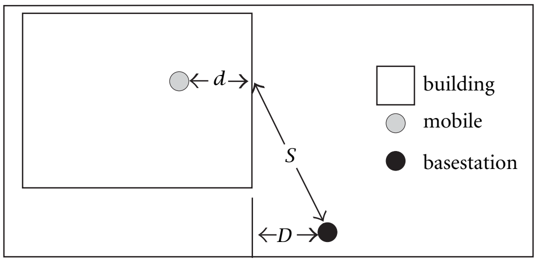

Figure 2: Building propagation distances.

The modeling of radio signal penetration inside of buildings is topic of much research in the propagation field [22, 23]. Many factors such as internal layout of the building, construction materials in the buildings, and the height of the mobile terminal above street level can all affect the path loss of the radio signal. We use a simple in-building penetration model. It is assumed that most interior walls inside of the building are perpendicular to the outside walls causing radio propagation inside of the building to be mostly perpendicular to the external walls. There is also a path loss factor added when the angle of the incoming signal to the external wall is small. The median path loss is modeled as [24]

$$\begin{aligned} \bar{Z}(\theta) = {} & 10\log_{10}\left[(S+d)^a\left(1+\frac{S+d}{g}\right)^b\right] \\ & + W_e + \left(1-\frac{D}{S}\right)^2\left(W_{Ge}+\alpha(d-2)\right). \end{aligned} \tag{18}$$

The distances are shown in Figure 2. The distance S is the distance from the base station to a point on the external wall where a propagation path to the mobile terminal perpendicular to the wall exists. The distance d is the perpendicular distance from the external wall of the building to the mobile terminal. The distance D is the perpendicular distance from building's external wall to the base station. W_e is the attenuation factor of the exterior wall when the signal path is normal to the wall surface. W_{Ge} is an attenuation factor associated with small grazing angles of radio signals. α gives the attenuation factor for radio signals traveling through the building. The values of these propagation constants are given in Table 1.

Table 1: Building propagation constants.

Constant	Value
W_e	10 dB
W_{Ge}	20 dB
α	0.6 dB/m

The in-building propagation equation above assumes that a LoS propagation path exists from the base station to an external wall of the building in which the mobile terminal resides. When this is not the case, it is assumed that the path loss is so high that the mobile terminal and base station cannot communicate. For cases where a LoS path exists to more than one external wall of the mobile terminal's building, when the base station is located near a corner of the building, the path loss is calculated for each of the walls with a LoS path and the lowest path loss value is used.

The path loss vector **Z** for a simulated location estimation trial is generated as follows. The median path loss is generated for each of the five base stations using formula (16), (17), or (18) depending on the relative location of the mobile terminal and base station. An independently generated noise value is then added to each path loss value. The noise value is sampled from a Gaussian density with zero mean and a standard deviation of 4 dB, a typical value for the long term fading in urban environments [15]. The central base station and two other base stations that have the lowest path loss values are used to locate the mobile terminal and their path loss values are placed in the measurement vector **Z**.

The diamond with the dashed boundary denotes the boundaries of the region where the mobile terminal is placed to evaluate the estimator performances. The mobile terminal locations are sampled from a uniform distribution over this region. This is the worst case distribution for cell location when no other information is known about mobile terminal location other than cell residency. The estimators are evaluated based on a survey set of 1000 points sampled from a uniform distribution over the diamond shaped region.

Two sets of simulations are performed. The first set of simulations evaluates the general performance of the location estimator algorithms. The second set of simulations evaluates the performance of each algorithm when the mobile terminal is located at specific positions.

For the first set of simulations, a total of 10000 Monte-Carlo runs are made to evaluate the performance of each of the estimators. We evaluated the linear distance-based kernel estimator with values of N ranging from 5 to 100 to see how many points the linear distance-based estimator requires to give good results.

In the second set of simulations, the mobile terminal is placed at fixed locations and 10000 Monte-Carlo runs are made. The average estimated location and variance of the estimates are calculated. The difference between the average estimated location and the true mobile terminal location is an estimate of the bias of the estimation algorithm at that point. The variances of the estimates of the x and y coordinates are added together to get a single value giving an indication of the magnitude of the variance of each of the estimators.

4. SIMULATION RESULTS

The figure of merit used to evaluate each estimation method is the Root Mean Squared Error (RMSE) which is the square root of the mean squared distance from the true mobile terminal location and estimated location. This value is estimated from the first set of simulations. The RMSE of the MLE estimator is 75.1 meters while the RMSE of the approximate MMSE estimator with the Gaussian kernel estimator is

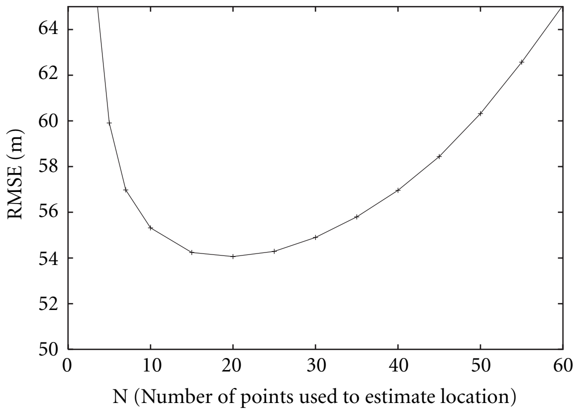

Figure 3: Linear distance-based estimator RMSE.

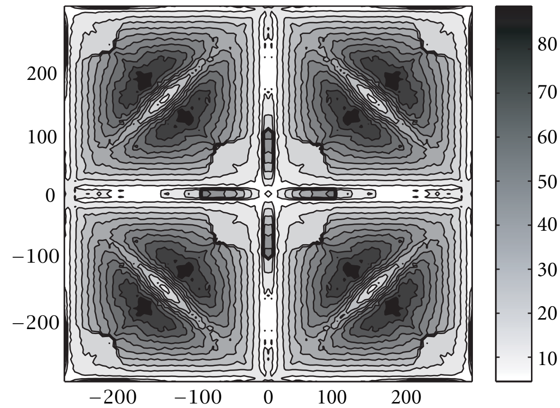

Figure 4: Gaussian kernel estimator bias magnitude contour plot.

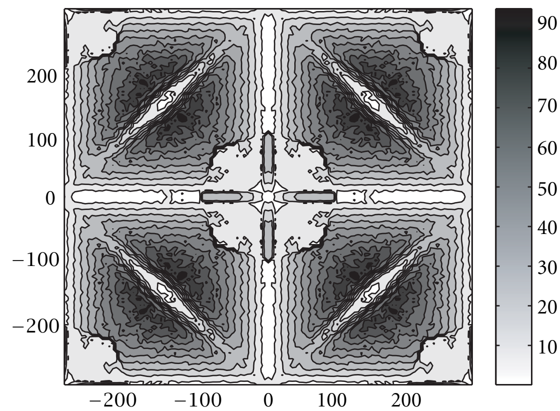

Figure 5: MLE bias magnitude contour plot.

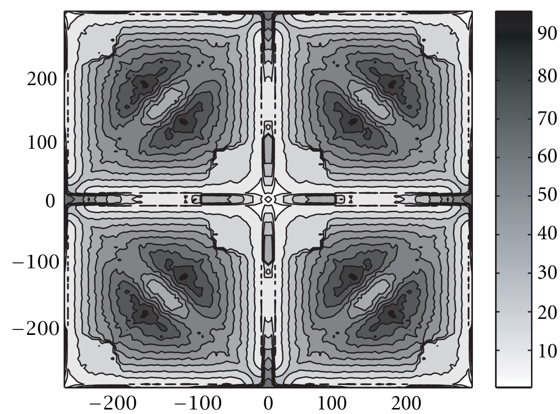

Figure 6: Distance-based kernel estimator bias magnitude contour plot.

52.0 meters, where h is calculated using (6). The RMSE for the linear distance-based kernel estimator is a function of N, the number of survey points used to estimate location. The results are shown in Figure 3.

The linear distance-based estimator with the optimal value of N gives a RMSE value almost as good as the Gaussian kernel estimator. The RMSE penalty for using non-optimal values of N is also fairly low. The worst case for a bad selection of N is RMSE performance equal to the MLE estimator. For the simulation using this estimator below, we will use the optimal value of $N = 20$.

When the mobile terminal is located at a position θ, the estimated position, $\hat{\theta}$ is a function of the random measurement noise vector, $\mathbf{V}$ and the estimator algorithm used. The estimated position is, therefore, also a random vector. The properties of the estimated position vector when the position of the mobile terminal is fixed are investigated to show the difference between the estimators.

It is well known that the MSE of an estimator has two components: the bias term and the variance term [13]. Therefore, the MSE when the mobile terminal is located at position θ is given by

$$\mathrm{MSE}(\theta) = \mathrm{bias}(\theta)^2 + \mathrm{variance}(\theta). \tag{19}$$

The bias term, $\mathrm{bias}(\theta)$, is the mean distance from the location of the mobile terminal, θ, to the estimated location $\hat{\theta}$. The variance term is the sum of the variances of the estimated x coordinate and the estimated y coordinate [25].

Figures 4, 5, and 6 show contour plots of the magnitude of the bias of the respective estimators. The areas for which the bias magnitude is graphed match the region depicted in Figure 1. The plots show that all the kernel estimators have similar bias magnitudes with the Gaussian kernel having the lowest and the linear distance-based kernel on average having the highest bias magnitude. The locations of highest bias are consistent between all the estimators. This is a result of street locations and in-building locations with median path loss vectors that are close to each other causing the estimators to have difficulty differentiating mobile terminal position between these locations.

Figures 7, 8, and 9 show plots of the variance of the estimators for the upper right hand corner section of Figure 1. It can be seen that for most locations the MLE has the highest variance and the Gaussian kernel estimator has the lowest

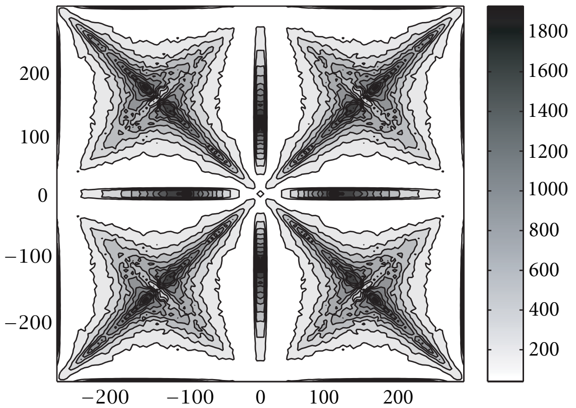

Figure 7: Gaussian kernel estimator variance contour plot.

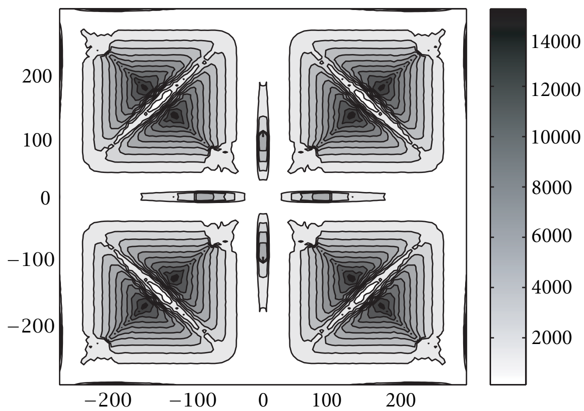

Figure 8: MLE variance contour plot.

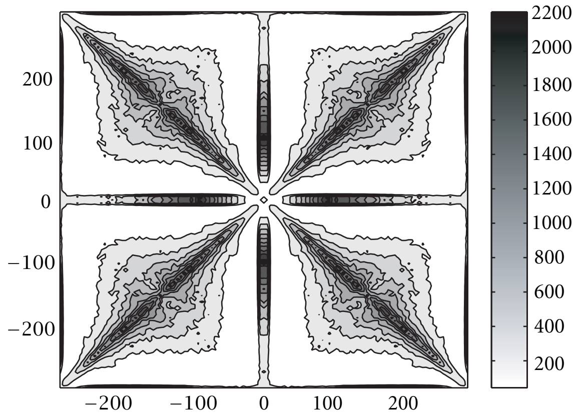

Figure 9: Linear distance-based kernel estimator variance contour plot.

variance, while the linear distance-based kernel's estimator is between the two.

This results agree with intuition that the linear distance-based kernel estimator is a compromise between the Gaussian kernel estimator and the MLE estimator. The MLE estimator has the highest bias because of its limited use of information from the survey set. Only the point with a measurement closest to the mobile terminal's measurement is used. The Gaussian kernel estimator performs better because it uses measurements from many survey points. This has an averaging effect which reduces the variance of the location estimate.

The kernel estimators deal with the LoS and NLoS propagation on the street location well provided the mobile terminal is located more than 50 meters from a base station. Close to a base station, there is a significant bias toward the base station location, as seen in the bias contour plots.

5. CONCLUSIONS

The results of this paper show that it is possible to get accurate estimates of the position of mobile terminal location using propagation path loss survey data of the mobile terminal environment. The MMSE location estimator is approximated by integrating over an estimated density function generated from a sum of kernel functions using survey data taken from the propagation environment.

The Gaussian kernel estimator was shown to provide lower RMSE than the MLE's location estimate but it requires the calculation of a smoothing parameter h. As well, an exponential function needs to be evaluated for every survey point.

These problems can be alleviated by using a linear distance-based kernel function. The kernel function requires less computational resources than the Gaussian kernel and can give RMSE values almost as good as the Gaussian kernel estimator provided only the N survey points with path loss closest to the measured path loss value of the mobile terminal are used. It was shown that this estimator is relatively insensitive to using values of N that are not equal to the optimal value. The disadvantage of this estimator is that it has a larger variance in estimated location position.

APPENDIX

CRAMER-RAO LOWER BOUND FOR LoS PROPAGATION LOCATION

The Cramer-Rao lower bound is a commonly used technique for obtaining a lower bound on the variance of an unbiased estimator. It does not, by itself, give any information on whether such an estimator exists or how to obtain it. The bound is based upon the inverse of the Fisher information matrix for the conditional density of the measurements given the true value of the parameters to be estimated [13]. For our case, the parameter, θ, is the location of mobile terminal. The log-likelihood of the measurements given the location with the LoS propagation described in (16) is

$$\begin{aligned} L &= \ln f_{\mathbf{Z}|\Theta}(\mathbf{Z}|\theta) \\ &= \frac{k}{2}\ln(2\pi\sigma^2) \\ &\quad - \frac{1}{2\sigma^2}\sum_{j=1}^{k}\left(z_j - 10\log_{10}\left({d_j}^a\left(1+\frac{d_j}{g}\right)^b\right)\right)^2, \end{aligned} \tag{A.1}$$

where

$$d_j = \sqrt{(x - x_j)^2 + (y - y_j)^2} \tag{A.2}$$

with (x_j, y_j) being the location of the jth base station. We can then calculate

$$\begin{aligned} L_{xx} &= \mathrm{E}\left[\frac{\partial^2 L}{\partial x^2}\right] \\ &= -\left(\frac{10}{\ln(10)\sigma}\right)^2 \sum_{j=1}^{k}\left[\frac{ag + (a+b)d_j}{{d_j}^2(g + d_j)}\right]^2 (x - x_j)^2, \\ L_{yy} &= \mathrm{E}\left[\frac{\partial^2 L}{\partial y^2}\right] \\ &= -\left(\frac{10}{\ln(10)\sigma}\right)^2 \sum_{j=1}^{k}\left[\frac{ag + (a+b)d_j}{{d_j}^2(g + d_j)}\right]^2 (y - y_j)^2, \\ L_{xy} &= \mathrm{E}\left[\frac{\partial^2 L}{\partial x \partial y}\right] \\ &= -\left(\frac{10}{\ln(10)\sigma}\right)^2 \sum_{j=1}^{k}\left[\frac{ag + (a+b)d_j}{{d_j}^2(g + d_j)}\right]^2 (x - x_j)(y - y_j). \end{aligned} \tag{A.3}$$

The Fisher information matrix is then

$$I(\theta) = \begin{bmatrix} -L_{xx} & -L_{xy} \\ -L_{xy} & -L_{yy} \end{bmatrix}. \tag{A.4}$$

From this the minimum variance of an unbiased estimated position can be calculated from $\mathrm{Var}(\hat{x}) \geq I^{-1}(\theta)_{11}$ and $\mathrm{Var}(\hat{y}) \geq I^{-1}(\theta)_{22}$.

ACKNOWLEDGEMENT

This work is partially supported by a grant from the Nortel Institute for Telecommunications.

REFERENCES

[1] FCC, "Enhanced 911 (E911) home page," Web page, http://www.fcc.gov/e911.

[2] FCC Docket 94-102, "Report and order and further notice of proposed rulemaking in the matter of revision of the commission's rules to ensure compatibility with enhanced 911 emergency calling systems," June 1996.

[3] FCC, "OET bulletin no. 71, guidelines for testing and verifying the accuracy of E911 location systems," April 2000.

[4] A. Acampora, "Wireless ATM: a perspective on issues and prospects," *IEEE Personal Communications*, vol. 3, no. 4, pp. 8–17, 1996.

[5] Z. J. Haas, J. H. Winter, and D. S. Johnson, "Simulation results of the capacity of cellular systems," *IEEE Transactions on Vehicular Technology*, vol. 46, no. 4, pp. 805–817, 1997.

[6] M. D. Kulavaratharasah and A. H. Aghvami, "Teletraffic performance evaluation of microcellular personal communication networks (PCN's) with prioritized handoff procedures," *IEEE Transactions on Vehicular Technology*, vol. 48, no. 1, pp. 137–152, 1999.

[7] J. J. Caffery, Jr. and G. L. Stüber, "Overview of radiolocation in CDMA cellular systems," *IEEE Communications*, vol. 36, no. 4, pp. 38–45, 1998.

[8] I. Jami, M. Ali, and R. F. Ormondroyd, "Comparison of methods of locating and tracking cellular mobiles," in *IEE Colloquium on Novel Methods of Location and Tracking of Cellular Mobiles and Their System Applications*, vol. 99/046, pp. 1/1–1/6, IEE Savoy Place, London, UK, 1999.

[9] M. J. Meyer, T. Jacobson, M. E. Palamara, E. A. Kidwell, R. E. Richton, and G. Vannucci, "Wireless enhanced 9-1-1 service—making it a reality," *Bell Labs Technical Journal*, vol. 1, no. 2, pp. 188–202, 1996.

[10] N. Bulusu, J. Heidemann, and D. Estrin, "GPS-less low cost outdoor localization for very small devices," *IEEE Personal Communications*, vol. 7, no. 5, pp. 28–34, 2000.

[11] D. Har, H. H. Xia, and H. L. Bertoni, "Path loss prediction model for microcells," *IEEE Transactions on Vehicular Technology*, vol. 48, no. 5, pp. 1453–1462, 1999.

[12] P. Bernardin, M. F. Yee, and T. Ellis, "Cell radius inaccuracy: a new measure of coverage reliability," *IEEE Transactions on Vehicular Technology*, vol. 47, no. 4, pp. 1215–1226, 1998.

[13] J. M. Mendel, *Lessons in Estimation Theory for Signal Processing, Communications and Control*, Prentice-Hall, New Jersey, 1995.

[14] G. P. Pollini, "Trends in handover design," *IEEE Communications*, vol. 34, no. 3, pp. 82–90, 1996.

[15] W. C. Y. Lee, *Mobile Communications Engineering*, McGraw-Hill, Toronto, 2nd edition, 1998.

[16] A. J. Goldsmith, L. J. Greenstein, and G. L. Foschini, "Error statistics of real-time power measurements in cellular channels with multipath and shadowing," *IEEE Transactions on Vehicular Technology*, vol. 43, no. 3, pp. 439–446, 1994.

[17] M. Mouly and M.-B. Pautet, *The GSM System for Mobile Communications*, Europe Media Duplication, France, 1993.

[18] A. Chheda, "On the forward link capacity of a cdma2000-1x system with transmit diversity," in *IEEE Vehicular Technology Conference*, vol. 2, pp. 618–629, September 2000.

[19] G. A. Babich and O. I. Camps, "Weighted parzen windows for pattern classification," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 18, no. 5, pp. 567–570, 1996.

[20] A. Leon-Garcia, *Probability and Random Processes for Electrical Engineering*, Addison-Wesley, Don Mills, Ontario, 2nd edition, 1994.

[21] D. W. Scott, *Multivariate Density Estimation: Theory, Practice, and Visualization*, John Wiley & Sons, New York, 1992.

[22] J. B. Andersen, T. S. Rappaport, and S. Yoshida, "Propagation measurements and models for wireless communications channels," *IEEE Communications*, vol. 33, no. 1, pp. 42–49, 1995.

[23] R. Hoppe, G. Wölfle, and F. M. Landstorfer, "Measurement of building penetration loss and propagation models for radio transmission into buildings," in *50th Vehicular Technology Conference (VTC)—Fall*, vol. 4, pp. 2298–2302, Amsterdam, The Netherlands, September 1999.

[24] J.-E. Berg, "Building penetration loss along urban street microcells," in *Proc. PIMRC'96*, vol. 3, pp. 795–797, October 1996.

[25] M. A. Spirito, "On the accuracy of cellular mobile station location estimation," *IEEE Transactions on Vehicular Technology*, vol. 50, no. 3, pp. 674–685, 2001.

Michael McGuire is presently a Ph.D. candidate in the Department of Electrical and Computer Engineering at the University of Toronto. He obtained the B. Engineering degree in Computer Engineering, and an M.A.Sc. from the University of Victoria in 1995 and 1997, respectively. His research interests are estimation and control algorithms for wireless cellular networks.

Konstantinos N. Plataniotis received the B. Engineering degree in Computer Engineering from the Department of Computer Engineering and Informatics, University of Patras, Patras, Greece in 1988 and the M.S. and Ph.D. degrees in Electrical Engineering from the Florida Institute of Technology (Florida Tech), Melbourne, Florida in 1992 and 1994, respectively. He was a Research Associate with the Computer Technology Institute (C.T.I), Patras, Greece from 1989 to 1991 and a Postdoctoral Fellow at the Digital Signal & Image Processing Laboratory, Department of Electrical and Computer Engineering University of Toronto, from 1995 to 1997. From August 1997 to June 1999 he was an Assistant Professor with the School of Computer Science at Ryerson Polytechnic University. While at Ryerson Prof. Plataniotis served as a lecturer in 12 courses to industry and Continuing Education programs. Since 1999 he has been with the University of Toronto as an Assistant Professor at the Department of Electrical & Computer Engineering, where he researches and teaches adaptive systems and multimedia signal processing. He co-authored, with A.N. Venetsanopoulos, a book on "Color Image Processing & Applications," Springer Verlag, May 2000, ISBN 3-540-66953-1, he is a contributor to three books, and he has published more than 100 papers in refereed journals and conference proceedings on the areas of adaptive systems, signal and image processing, and communication systems and stochastic estimation. Prof. Plataniotis is a member of the IEEE Technical Committee on Neural Networks for Signal Processing, and the Technical Co-Chair of the Canadian Conference on Electrical and Computer Engineering, CCECE 2001, May 13–16, 2001, Toronto, Ontario. His current research interests include: adaptive systems statistical pattern recognition, multimedia data processing, statistical communication systems, and stochastic estimation and control.

Anastasios N. Venetsanopoulos received the Diploma in Engineering degree from the National Technical University of Athens (NTU), Greece, in 1965, and the M.S., M.Phil., and Ph.D. degrees in Electrical Engineering from Yale University in 1966, 1968, and 1969, respectively. He joined the Department of Electrical and Computer Engineering of the University of Toronto in September 1968 as a lecturer and he was promoted to Assistant Professor in 1970, Associate Professor in 1973, and Professor in 1981. Since July 1997, he has been Associate Chair: Graduate Studies of the Edward S. Rogers Sr. Department of Electrical and Computer Engineering and was Acting Chair during the spring term of 1998-99. In 1999 a Chair in Multimedia was established in the ECE Department, made possible by a donation from Bell Canada, and Prof. A.N. Venetsanopoulos assumed the position as Inaugural Chairholder in July 1999. He has served as Chair of the Communications Group and Associate Chair of the Department of Electrical Engineering and Associate Chair: Graduate Studies for the Department of Electrical and Computer Engineering. Prof. A.N. Venetsanopoulos was on research leave at Imperial College of Science and Technology, the National Technical University of Athens, the Swiss Federal Institute of Technology, the University of Florence and the Federal University of Rio de Janeiro, and has also served as Adjunct Professor at Concordia University. He has served as lecturer in 138 short courses to industry and continuing education programs and as Consultant to numerous organizations; he is a contributor to twenty eight books, a co-author of *Color Image Processing and Applications*, ISBN 3-540-66953-1, Springer Verlag, August 2000, *Nonlinear Filters in Image Processing: Principles Applications*, ISBN-0-7923-9049-0, Kluwer, 1990, *Artificial Neural Networks: Learning Algorithms, Performance Evaluation and Applications*, ISBN-0-7923-9297-3, Kluwer, 1993, and *Fuzzy Reasoning in Information Decision and Control systems*, ISBN-0-7293-2643-1, Kluwer 1994. He has published 750 papers in refereed journals and conference proceedings on digital signal and image processing, and digital communications. Prof. Venetsanopoulos has served as Chair on numerous boards, councils and technical conference committees of the Institute of Electrical and Electronic Engineers (IEEE), such as the Toronto Section (1977–1979) and the IEEE Central Canada Council (1980–1982); he was President of the Canadian Society for Electrical Engineering and Vice President of the Engineering Institute of Canada (EIC) (1983–1986). He was a Guest Editor or Associate Editor for several IEEE journals and the Editor of the Canadian Electrical Engineering Journal (1981–1983). Prof. A.N. Venetsanopoulos is the Technical Program Co-Chair of the IEEE International Conference on Image Processing, ICIP-2001, to be held in Thessaloniki, Greece, October 7–10, 2001. He is a member of the IEEE Communications, Circuits and Systems, Computer, and Signal Processing Societies of IEEE, as well as a member of Sigma Xi, the Technical Chamber of Greece, the European Association of Signal Processing, the Association of Professional Engineers of Ontario (APEO) and Greece. He was elected as a Fellow of the IEEE "for contributions to digital signal and image processing," he is also a Fellow of the EIC, "for contributions to electrical engineering," and was awarded an Honorary Doctorate from the National Technical University of Athens, in October 1994. In October 1996 he was awarded the "Excellence in Innovation Award" of the Information Technology Research Centre of Ontario and Royal Bank of Canada, "for innovative work in color image processing."