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Research Article

An Energy-Based Similarity Measure for Time Series

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A new similarity measure, called SimilB, for time series analysis, based on the cross-Ψ_B-energy operator (2004), is introduced. Ψ_B is a nonlinear measure which quantifies the interaction between two time series. Compared to Euclidean distance (ED) or the Pearson correlation coefficient (CC), SimilB includes the temporal information and relative changes of the time series using the first and second derivatives of the time series. SimilB is well suited for both nonstationary and stationary time series and particularly those presenting discontinuities. Some new properties of Ψ_B are presented. Particularly, we show that Ψ_B as similarity measure is robust to both scale and time shift. SimilB is illustrated with synthetic time series and an artificial dataset and compared to the CC and the ED measures.

1. INTRODUCTION

A Time Series (TS) is a sequence of real numbers where each one represents the value of an attribute of interest (stock or commodity price, sale, exchange, weather data, biomedical measurement, etc.). TS datasets are common in various fields such as in medicine, finance, and multimedia. For example, in gesture recognition and video sequence matching using computer vision, several features are extracted from each image continuously, which renders them TSs [2]. Typical applications on TSs deal with tasks like classification, clustering, similarity search, prediction, and forecasting. These applications rely heavily on the ability to measure the similarity or dissimilarity between TSs [3]. Defining the similarity of TSs or objects is crucial in any data analysis and decision making process. The simplest approach typically used to define a similarity function is based on the Euclidean distance (ED) or some extensions to support various transformations such as scaling or shifting. The ED may fail to produce a correct similarity measure between TSs because it cannot deal with outliers and it is very sensitive to small distortions in the time axis [4]. The Pearson correlation coefficient (CC) is a popular measure to compare TSs. Yet, the CC is not necessarily coherent with the shape and it does not consider the order of time points and uneven sampling intervals. Furthermore, similarity measures using the ED or the CC do not include temporal information and the relative changes of the TSs. Thus, clustering algorithms based on these metrics, such as k-means, fuzzy c-means, or hierarchical clustering, cannot cluster TSs correctly [5]. In this paper, we introduce a new similarity measure, noted SimilB, which includes the temporal information and relative change of the TS. SimilB is based on the Ψ_B operator [1], a nonlinear similarity function which measures the interaction between two time-signals including their first and second derivatives [6]. Furthermore, the link established between Ψ_B operator and the cross Wigner-Ville distribution shows that Ψ_B and consequently SimilB are well suited to study nonstationary signals [1].

2. THE Ψ_B OPERATOR

To measure the interaction between two real time signals, the cross Teager-Kaiser operator (CTKEO) has been defined [7]. This operator has been extended to complex-valued signals noted $\Psi_{\mathbb{C}}$, in [1]. The CTKEO, applied to signals $x(t)$ and $y(t)$, is given by $[x, \dot{y}] \equiv \dot{x}y - x\dot{y}$, where $[x, \dot{y}]$ is the Lie bracket which measures the instantaneous differences in the relative rate of change between x and $\dot{y}$. In the general case, if x and y represent displacements in some generalized motions, $[x, \dot{y}]$ has dimensions of energy (per unit mass), it is viewed as a cross-energy between x and y [7]. Based on $\Psi_{\mathbb{C}}$ function, a symmetric and positive function, called cross-Ψ_{B}-energy operator, is defined [1]. We have shown that time-delay estimation problem between two signals is an example of interaction measure between these two signals by Ψ_{B} [6]. Let x and y be two complex signals, Ψ_{B} is defined as [1]

$$\Psi_{\mathrm{B}}(x,y) = \frac{1}{2}[\Psi_{\mathbb{C}}(x,y) + \Psi_{\mathbb{C}}(y,x)], \tag{1}$$

where $\Psi_{\mathbb{C}}(x,y) = (1/2)[\dot{x}^*\dot{y} + \dot{x}\dot{y}^*] - (1/2)[x\ddot{y}^* + x^*\ddot{y}]$. The $\Psi_{\mathrm{B}}(x,y)$ of complex signals x and y is equal to the sum of $\Psi_{\mathrm{B}}(x,y)$ of their real and imaginary parts [1]:

$$\Psi_{\mathrm{B}}(x,y) = \Psi_{\mathrm{B}}(x_r,y_r) + \Psi_{\mathrm{B}}(x_i,y_i), \tag{2}$$

where $x(t) = x_r(t) + jx_i(t)$ and $y(t) = y_r(t) + jy_i(t)$ and j denotes the imaginary unit. Subscripts r and i indicate the real and imaginary parts of the complex signal. According to (2), the $\Psi_{\mathrm{B}}(x,y)$ is a real quantity, as expected for an energy operator. To compute the analytic signals $x(t)$ or $y(t)$, the Hilbert transform is used. In the following we give the expression of Ψ_{B} for analytic signals.

3. EXPRESSION OF Ψ_{B} FOR ASSOCIATED ANALYTIC SIGNALS

Complex signals are used in various areas of signal processing. In the continuous time, they appear, for example, in the description for narrow-band signals. Indeed, the appropriate definition of instantaneous phase or amplitude of such signals requires the introduction of the analytic signal, which is necessarily complex. Let x and y be two real signals, and x_A and y_A, respectively, their corresponding analytic signals: $x_A = x + j\mathcal{H}(x)$ and $y_A = y + j\mathcal{H}(y)$, where $\mathcal{H}(\cdot)$ is the Hilbert transform.[1] By applying the relation

$$\dot{u}\dot{v} - \frac{1}{2}(u\ddot{v} + v\ddot{u}) = 2\dot{u}\dot{v} - \frac{1}{2}\frac{d^2uv}{dt^2} \tag{3}$$

in (2), for $(u,v) = (x,y)$ and $(u,v) = (\mathcal{H}(x),\mathcal{H}(y))$, respectively, it comes that $\Psi_{\mathrm{B}}(x_A,y_A)$ is expressed directly in terms of x, y, $\mathcal{H}(x)$ and $\mathcal{H}(y)$ as

$$\begin{aligned}\Psi_{\mathrm{B}}(x_A,y_A) &= 2[\dot{x}\dot{y} + \dot{\mathcal{H}}(x)\dot{\mathcal{H}}(y)] \\ &\quad - \frac{1}{2}\frac{d^2}{dt^2}[xy + \mathcal{H}(x)\mathcal{H}(y)].\end{aligned} \tag{4}$$

Equation (4) is used to calculate the interaction between continuous TSs.

4. DISCRETIZING THE CONTINUOUS-TIME Ψ_{B} OPERATOR

Discretized derivatives are combined to obtain from the continuous version of Ψ_{B} an expression closely related to discrete form of the operator noted Ψ_{B_d} and operating on discrete-time signals $x(n)$ and $y(n)$. Three sample differences are examined. For simplicity, we replace t by nT_s (T_s is the sampling period), $x(t)$ with $x(nT_s)$ or simply $x(n)$. Using the same reasoning as in [8] we obtain the following relations.

(i) Two-sample backward difference:

$$\begin{aligned}\dot{x}(t) &\longmapsto \frac{[x_k(n) - x_k(n-1)]}{T_s}, \\ \ddot{x}(t) &\longmapsto \frac{[x_k(n) - 2x_k(n-1) + x_k(n-2)]}{T_s^2}, \\ \Psi_{\mathrm{B}}(x_k(t),y_k(t)) &\longmapsto \frac{x_k(n-1)y_k(n-1)}{T_s^2} \\ &\qquad - \frac{0.5[x_k(n)y_k(n-2) + y_k(n)x_k(n-2)]}{T_s^2}, \\ \Psi_{\mathrm{B}}(x_k(t),y_k(t)) &\longmapsto \frac{\Psi_{\mathrm{B}_d}(x_k(n-1),y_k(n-1))}{T_s^2}, \quad k \in \{i,r\}.\end{aligned} \tag{5}$$

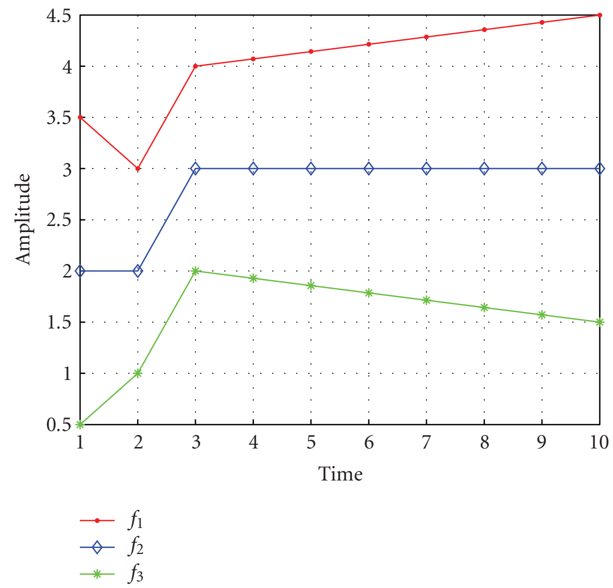


FIGURE 1: Three sampled TSs with different shapes.

TABLE 1: SimilB, the ED, the CC between f_2 and f_1, and f_2 and f_3 in Figure 1.

TSs	ED	CC	SimilB
(f_2, f_1)	3.9955	0.0917	0.930
(f_2, f_3)	3.9955	0.0917	0.750

TABLE 2: Classification errors of clustering task using the SimilB, the ED, and the CC for CBF.dat dataset.

SimilB	ED	CC
0.222	0.888	0.888

[1] $\mathcal{H}(x) = h \star x$, where the frequency response of h is $\hat{h}(f) = -j\,\mathrm{sign}(f)$.

Table 3: Estimated T_B value versus SNR signals $s_1(t)$ and $s_2(t)$ using SimilB.

SimilB	SNR = −6 dB	SNR = −2 dB	SNR = 1 dB	SNR = 3 dB	SNR = 5 dB	SNR = 9 dB
$(s_1(t), r_1(t))$	300 ± 1	300 ± 1	300	300	300	300
$(s_2(t), r_2(t))$	300 ± 2	300 ± 1	300 ± 1	300 ± 1	300 ± 1	300

Finally, the discrete form of $\Psi_B(x(t), y(t))$ is given by

$$\Psi_B(x(t), y(t)) \longmapsto \frac{[\Psi_{B_d}(x_r(n-1), y_r(n-1)) + \Psi_{B_d}(x_i(n-1), y_i(n-1))]}{T_s^2}, \quad (6)$$

where $\longmapsto$ denotes the mapping from continuous to discrete.

(ii) Two-sample forward difference:

$$\begin{aligned}
\dot{x}(t) &\longmapsto \frac{[x_k(n+1) - x_k(n)]}{T_s},\\
\ddot{x}(t) &\longmapsto \frac{[x_k(n+2) - 2x_k(n+1) + x_k(n)]}{T_s^2},\\
\Psi_B(x_k(t), y_k(t)) &\longmapsto \frac{x_k(n+1)y_k(n+1)}{T_s^2} - \frac{0.5[x_k(n+2)y_k(n) + y_k(n+2)x_k(n)]}{T_s^2},\\
\Psi_B(x_k(t), y_k(t)) &\longmapsto \frac{\Psi_{B_d}(x_k(n+1), y_k(n+1))}{T_s^2}, \quad k \in \{i, r\}.
\end{aligned} \quad (7)$$

Thus, from Ψ_B we obtain Ψ_{B_d} shifted by one sample to the right and scaled by T_s^{-2}. Finally, the discrete form of $\Psi_B(x(t), y(t))$ is given by

$$\Psi_B(x(t), y(t)) \longmapsto \frac{[\Psi_{B_d}(x_r(n+1), y_r(n+1)) + \Psi_{B_d}(x_i(n+1), y_i(n+1))]}{T_s^2}. \quad (8)$$

Note that for both asymmetric two-sample differences, Ψ_B is shifted by one sample and scaled by T_s^{-2}. If we ignore the one-sample shift and the scaling parameter, one can transform $\Psi_B(x(t), y(t))$ into $\Psi_{B_d}(x(n), y(n))$ as follows:

$$\Psi_B(x(t), y(t)) \longmapsto \Psi_{B_d}(x_r(n), y_r(n)) + \Psi_{B_d}(x_i(n), y_i(n)), \quad (9)$$

$$\begin{aligned}
\Psi_{B_d}(x_k(n), y_k(n)) &= x_k(n)y_k(n) - 0.5[x_k(n+1)y_k(n-1)\\
&\quad + y_k(n+1)x_k(n-1)], \quad k \in \{i, r\}.
\end{aligned} \quad (10)$$

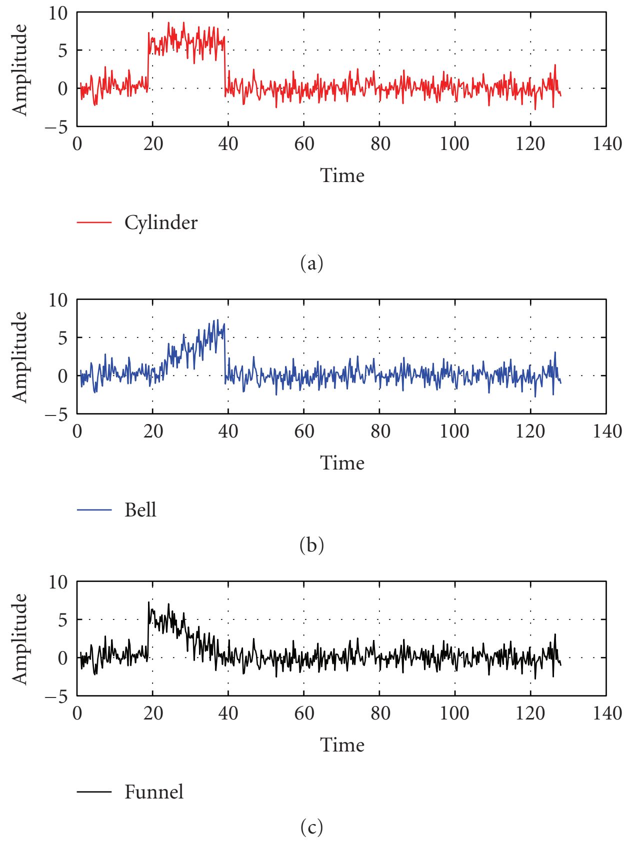
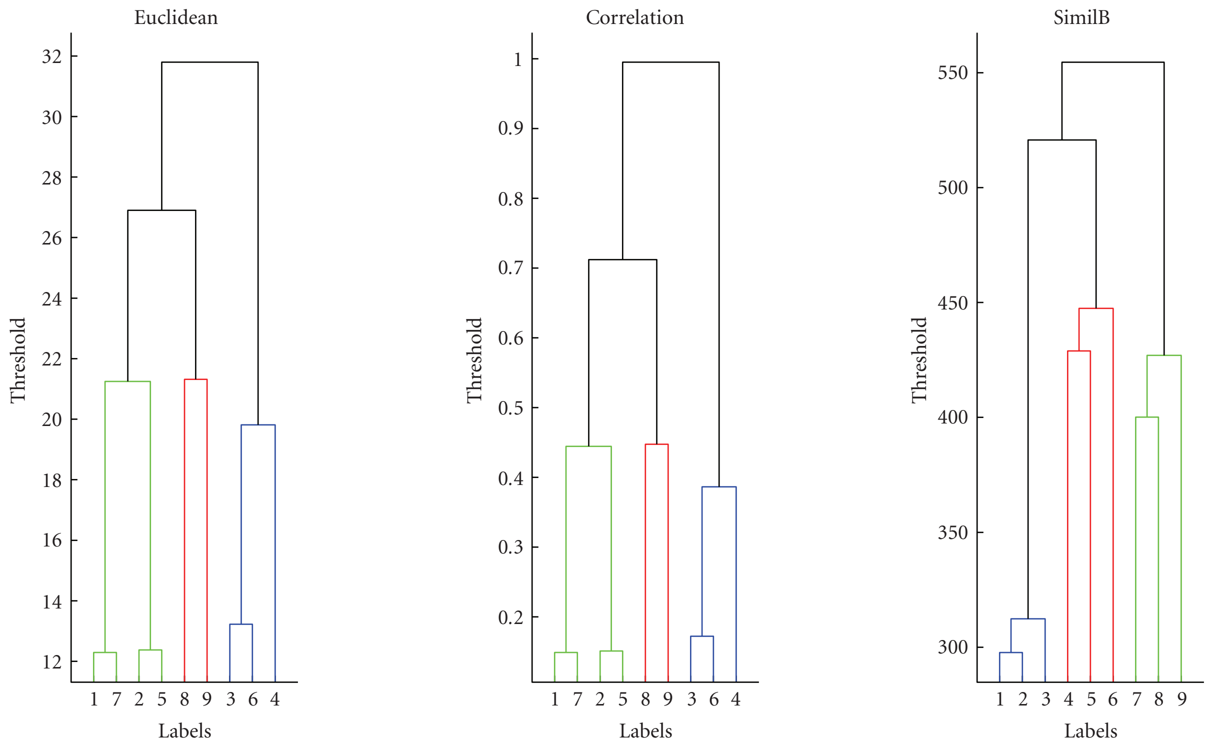


Figure 2: The Cylinder-Bell-Funnel dataset (CBF.dat) [10].

(iii) Three-sample symmetric difference:

$$\begin{aligned}
\dot{x}(t) &\longmapsto \frac{[x_k(n+1) - x_k(n-1)]}{2T_s},\\
\ddot{x}(t) &\longmapsto \frac{[x_k(n+2) - 2x_k(n) + x_k(n-2)]}{4T_s^2},\\
\Psi_B(x_k(t), y_k(t)) &\longmapsto \frac{2x_k(n)y_k(n)}{4T_s^2} - \frac{[x_k(n+1)y_k(n-1) + y_k(n+1)x_k(n-1)]}{4T_s^2},\\
&\quad \frac{x_k(n-1)y_k(n-1)}{4T_s^2} - \frac{0.5[x_k(n)y_k(n-2) + y_k(n)x_k(n-2)]}{4T_s^2}\\
&\quad + \frac{x_k(n+1)y_k(n+1)}{4T_s^2} - \frac{0.5[x_k(n+2)y_k(n) + y_k(n+2)x_k(n)]}{4T_s^2},\\
\Psi_B(x_k(t), y_k(t)) &\longmapsto [\Psi_{B_d}(x_k(n+1), y_k(n+1)) + 2\Psi_{B_d}(x_k(n), y_k(n))\\
&\quad + \Psi_{B_d}(x_k(n-1), y_k(n-1))]/4T_s^2, \quad k \in \{i, r\}.
\end{aligned} \quad (11)$$



(a) (b) (c)

Figure 3: Comparison of the SimilB, the ED, the CC on a clustering task. Labels (1,2,3), (4,5,6), and (7,8,9) correspond to Cylinder, Bell, and Funnel classes, respectively.

Compared to asymmetric two-sample differences, the three-sample symmetric difference leads to more complicated expression. Expression (11) corresponds to three-sample weighted moving average of $\Psi_{B_d}(x_k(n), y_k(n))$. Note if $x = y$, Ψ_{B_d} is reduced to the Teager-Kaiser operator (TKO): $\Psi_{B_d}(x(n), x(n)) = x^2(n) - x(n+1)x(n-1)$ (see [9]). Finally, the asymmetric approximation is less complicated for implementation and is faster than the symmetric one.

5. PROPERTIES OF Ψ_B

We provide here some new properties of Ψ_B [1]. We denote Ψ_B of $x(t)$ and $y(t)$ by $\Psi_B(x, y; t)$ and denote by "$\leftarrow$ " the affectation operation.

Similarity measure:

$$\Psi_B(x, y; t) = \Psi_B(y, x; t). \tag{12}$$

This is a basic requirement for most of similarity or distance measures.

Time shift:

$$\begin{aligned} x_1(t) &\longleftarrow x(t - t_0), \\ y_1(t) &\longleftarrow y(t - t_0). \end{aligned} \tag{13}$$

It is trivial that Ψ_B is time-shift invariant, that is, $\Psi_B(x_1, y_1; t) = \Psi_B(x, y; t - t_0)$. This property states that any time translations in the signals, $x(t)$ and $y(t)$, should be preserved in their measure of interaction, $\Psi_B(x, y; t)$. Thus, $\Psi_B(x, y; t)$ is robust to time shifts.

Amplitude scale:

$$\begin{aligned} x_1(t) &\longleftarrow \alpha \cdot x(t), \\ y_1(t) &\longleftarrow \beta \cdot y(t). \end{aligned} \tag{14}$$

It is easy to verify that $\Psi_B(x_1, y_1; t) = \alpha \cdot \beta \Psi_B(x, y; t)$. Thus, the time where Ψ_B peaks, corresponding to the maximum of interaction between $x(t)$ and $y(t)$, is robust to amplitude scale.

Time scale:

$$\begin{aligned} x_1(t) &\longleftarrow x(at), \\ y_1(t) &\longleftarrow y(at). \end{aligned} \tag{15}$$

It is easy to verify that $\Psi_B(x_1, y_1; t) = a^2 \Psi_B(x, y; t)$. This property states that if the time of the two signals is compressed by a scale a, then the energy of interaction is compressed by a^2.

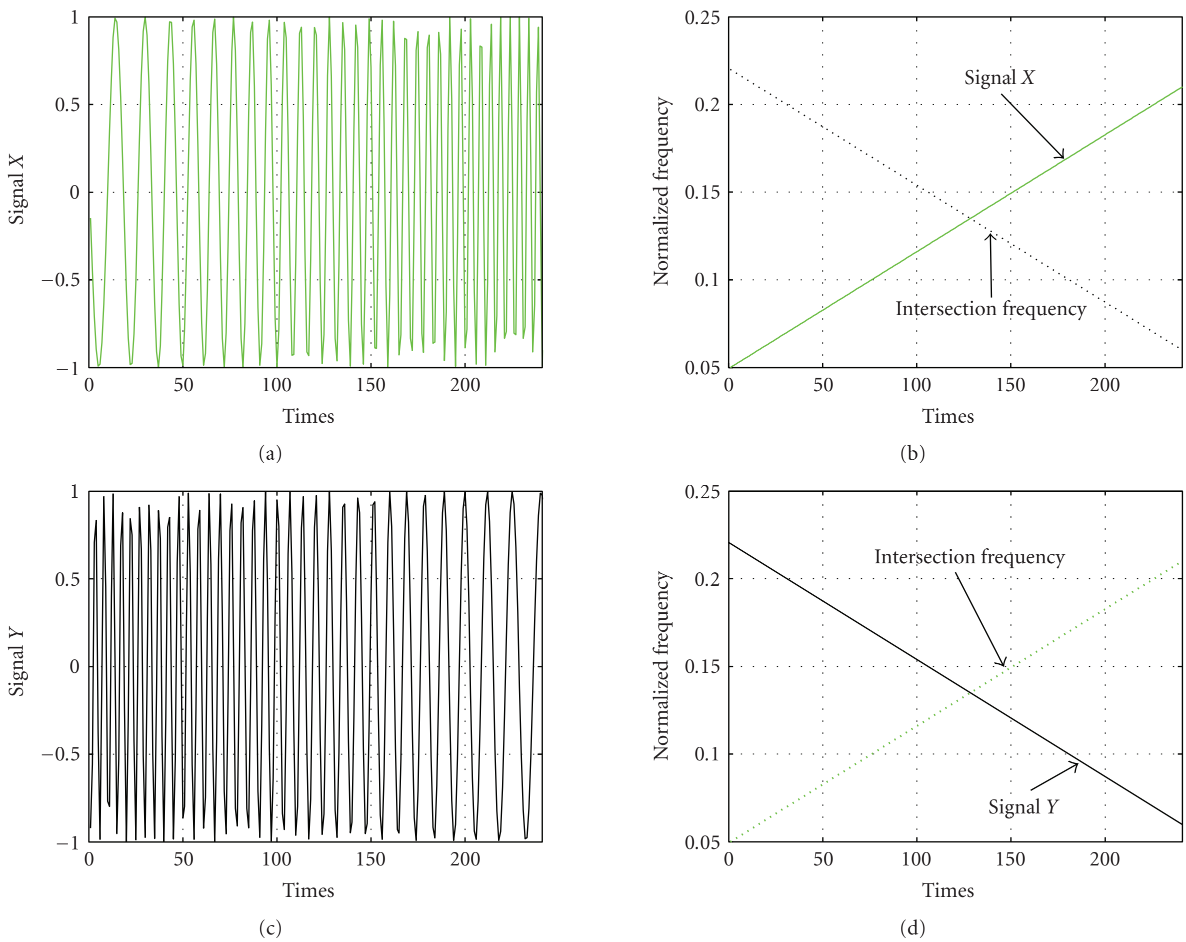


Figure 4: Linear chirp TSs (parabolic phase).

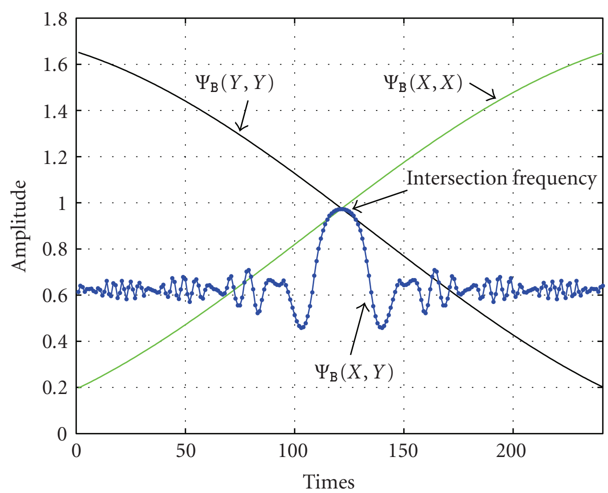


Figure 5: Similarity measure using SimilB with a sliding window analysis.

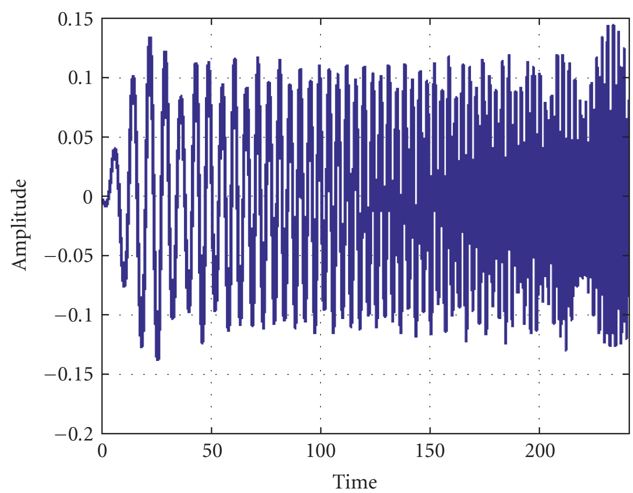
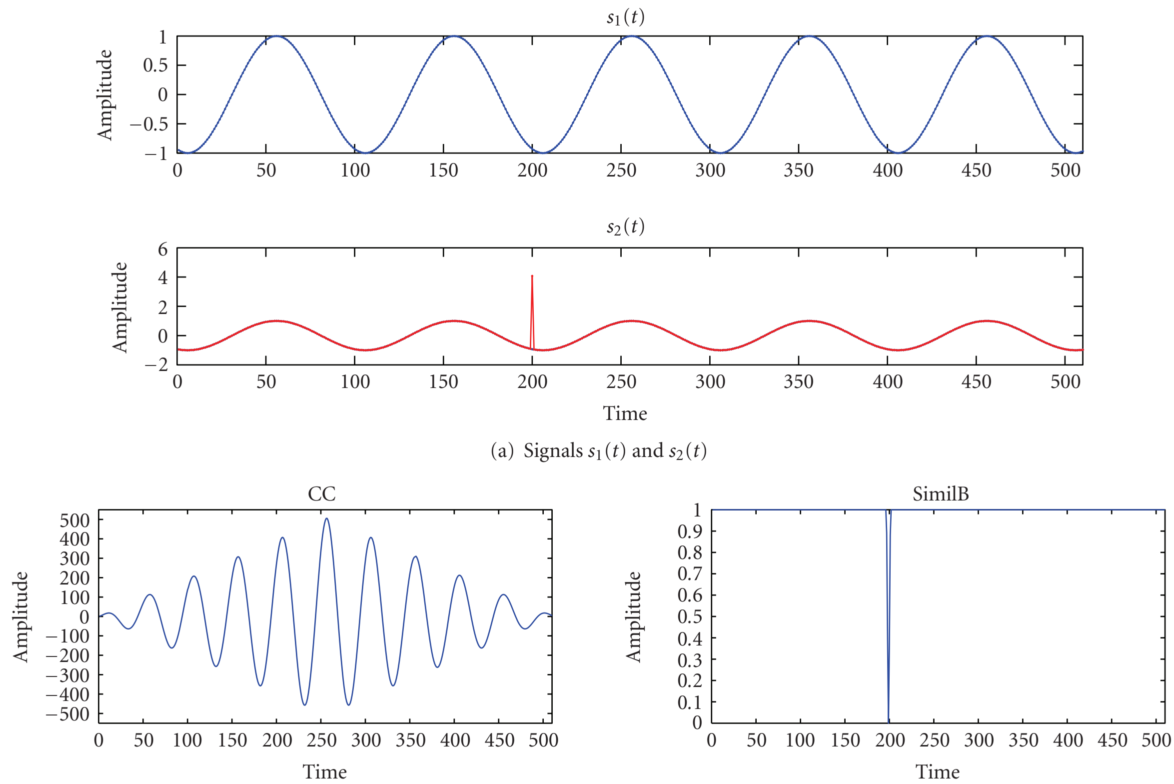


Figure 6: Similarity measure using CC with a sliding window analysis.

5.1. Ψ_B-based similarity measure

A similarity measure $S(x(t), y(t))$ is a function to compare the TSs $x(t)$ and $y(t)$. Conventionally, this measure is a symmetric function whose value is large when x and y are somehow similar. The proposed similarity measure based on $\Psi_B(x, y)$, between $x(t)$ and $y(t)$, uses their interaction. A



(a) Signals $s_1(t)$ and $s_2(t)$

(b) CC with a sliding window analysis

(c) SimilB with a sliding window analysis

FIGURE 7: Similarity measure using SimilB and CC of sinusoidal TSs.

larger value indicates more interaction in energy between TSs. If the input variables (or samples) of the TS $x(t)$ (or $y(t)$) have large range, then this can overpower the other input variables of $y(t)$ (or $x(t)$). Therefore, the proposed similarity measure, SimilB, is a normalized version of $\Psi_B(x, y)$ and is defined as follows:

$$\text{SimilB}(x, y) = \frac{\sqrt{2}\int_T \Psi_B(x, y)dt}{\int_T \sqrt{\Psi_B^2(x, x) + \Psi_B^2(y, y)}dt}. \tag{16}$$

T is the TS duration or the size of sliding window analysis. The similarity is symmetric when comparing two TSs:

$$\text{SimilB}(x, y) = \text{SimilB}(y, x) \quad \forall (x, y) \in \mathbb{C}^2. \tag{17}$$

It is a basic requirement for most of similarity or distance measures. Note that if $x = y$ then $\text{SimilB}(x, y) = 1$.

6. RESULTS

SimilB (equation (17)) is combined with relations (10) and (11), and relation (3) or (4) to process discrete (Figure 2) and continuous (Figures 1, 4, 7, and 8) data, respectively. The effects of temporal information and the inclusion of the signal derivatives are shown on nonstationary and stationary synthetic TSs. Figure 1 shows three TSs with different shapes to illustrate the limit of the ED and the CC. Since f_1, f_2, and f_3 have different shapes, then an appropriate similarity measure would show, for example, that the similarity values between f_1 and f_2 and that between f_3 and f_2 are different. Results of the SimilB, the ED, and the CC between f_2 and f_1 and that between f_2 and f_3 are reported in Table 1. These results show that SimilB is the unique measure which properly capture the temporal information in the comparison of the shapes. The most studied TS classification/clustering problem is the Cylinder-Bell-Funnel dataset (noted CBF.dat) [10]. It is a 3-class problem. Typical examples of each class are shown in Figure 2. The classes are generated by the equations [10]

$$\begin{aligned}
c(t) &= (6 + \eta)\cdot X_{[a,b]}(t) + \epsilon(t) \,//\, \text{Cylinder class},\\
b(t) &= (6 + \eta)\cdot X_{[a,b]}(t).\frac{(t - a)}{(b - a)} + \epsilon(t) \,//\, \text{Bell class},\\
f(t) &= (6 + \eta)\cdot X_{[a,b]}(t).\frac{(b - t)}{(b - a)} + \epsilon(t) \,//\, \text{Funnel class},\\
X_{[a,b]} &= 1 \quad \text{if } a \le t \le b,\ \text{else } X_{[a,b]} = 0,
\end{aligned} \tag{18}$$

where η and $\epsilon(t)$ are drawn from a standard normal distribution $\mathcal{N}(0, 1)$, a is an integer drawn uniformly from the range $[16, 32]$, and $(b - a)$ is an integer drawn uniformly from the range $[32, 96]$ (Figure 2). The task is to classify a TS as one of the three classes, Cylinder, Bell, or Funnel. We have performed an experiment classification on CBF.dat dataset consisting of 3 TSs of each class. TSs are clustered using group-average hierarchical clustering. The dendrograms are formed with nearest neighbor linkage for three of each type of TSs using SimilB measure, the ED, and the CC. We have averaged

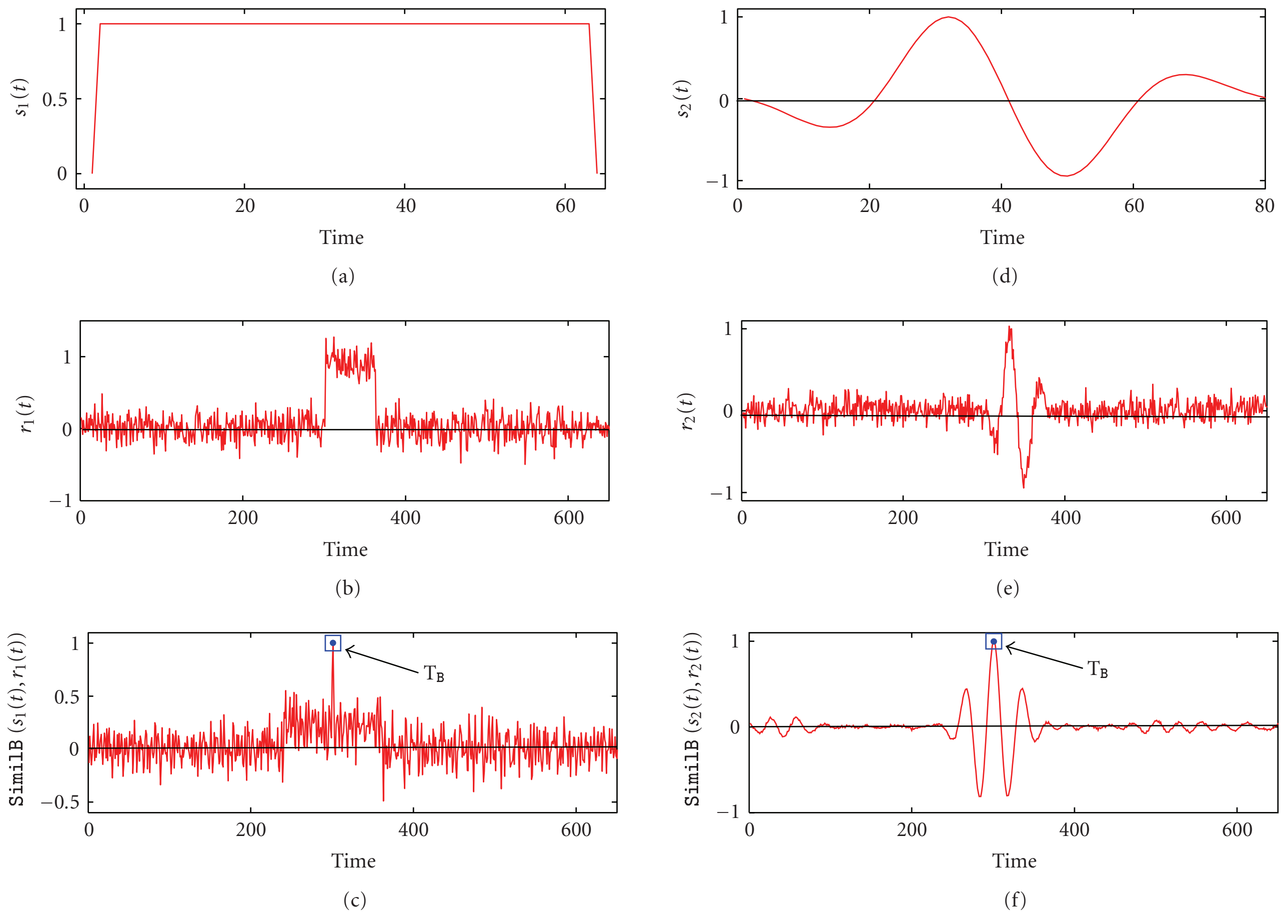


FIGURE 8: Similarity measure using SimilB of TSs of nonequal length.

the classification results over 45 runs. Figure 3 shows the result of these averaged runs where both the ED and the CC fail to differentiate between the three classes. SimilB distinguishes the three original classes as shown in Figure 3. Classification errors reported in Table 2 show that SimilB is more effective than the ED and the CC. These results are expected since the ED and the CC are not able to include the temporal information while SimilB using derivatives of the TS captures this kind of information. Moreover, these results may be due to the fact that Ψ_B is local operator [1, 6] while the ED and the CC are global ones. Figure 4 shows an example of nonstationary TSs (two linear FM signals), $x(t)$ and $y(t)$. The instantaneous frequency (IF) of $x(t)$ increases linearly with time while that of $y(t)$ decreases with time. The point where the IFs intercept (Figure 4), noted Q, is located at $t = 125$. Figure 5 shows the energy of each TS and the energy of their interaction obtained with a sliding window analysis of $T = 15$. The point Q corresponds to the maximum of similarity and also where the energy of $x(t)$ (SimilB(x, x)) and that of $y(t)$ (SimilB(y, y)) are equal. Away from Q, the amplitude of interaction decreases because there is less similarity between TSs (the TSs tend to be more and more different). As the IFs converge from the time origin to Q (the TSs tend to be equal), the interaction intensity of the TSs increases and the maximum of similarity is achieved at $t = 125$. Figure 6 shows that the maximum of similarity given by CC is located at $t = 240$. Thus, the CC fails to point out, as expected (Figure 4), the maximum of similarity at Q. The interaction measure using SimilB and CC is performed using a sliding window analysis of size T. Different T values ranging from 3 to 91 have been tested. Globally, we found comparable results. The CC is calculated with the same sliding window as for SimilB. Furthermore, as the IFs converge to Q or diverge from Q, the CC function has, globally, the same behavior and thus the similarity study of such TSs is difficult. This example shows that the SimilB is more effective to study nonstationary TSs than the CC. This may be due the fact that the Ψ_B is nonlinear operator while the CC is linear one. Figure 7(a) shows an example of two sinusoidal TSs, $s_1(t)$ and $s_2(t)$, of the same frequency and amplitude. TS $s_2(t)$ presents a discontinuity located at $t = 200$. Both CC and SimilB are calculated with T set to 17. CC measure fails to detect the discontinuity and shows a maximum of interaction at $t = 262$ (Figure 7(b)). The result of SimilB is expected (Figure 7(c)). Indeed, excepted for data point at $t = 200$, $s_1(t)$ and $s_2(t)$ are equal and Ψ_B behaves toward these two signals as the TKO applied to $s_1(t)$ ($s_2(t)$) and thus giving a constant output (square of the amplitude times the frequency) [9]. This example shows the interest of SimilB to track discontinuities (Figure 7(c)). Two synthetic signals, $s_1(t)$ and $s_2(t)$, of nonequal lengths with size window observation T of 65 and 81, respectively, are shown in Figures 8(a) and 8(d). These two signals are time shifted by 300 samples and corrupted by additive Gaussian noise. The obtained signals, $r_1(t)$ and $r_2(t)$, are shown in Figures 8(b) and 8(e), respectively. The attenuation coefficient is set to 0.7. For both signals $r_1(t)$ and $r_2(t)$, a similarity measure would show, in theory, a maximum of interaction located at $t = 300$. No warping process is used. We use the smallest TS length as a sliding window and calculate SimilB, inside this window, between two TSs of the same length. Outputs of SimilB are shown in Figures 8(c) and 8(f) indicating a net maximum at $t = \mathrm{T}_B$. As expected, both SimilB$(s_1(t), r_1(t))$ and SimilB$(s_2(t), r_2(t))$ peak to $\mathrm{T}_B = 300$. Table 3 lists the T_B values calculated for SimilB$(s_1(t), r_1(t))$ and SimilB$(s_2(t), r_2(t))$ for different SNRs ranging from -6 dB to 9 dB. Each value of Table 3 corresponds to the average of an ensemble of twenty five trials of T_B estimation. These results show that the performances of SimilB are very close to that of the theory and also that SimilB works correctly for moderately noisy TSs.

7. CONCLUSION

Relative change of amplitude and the corresponding temporal information are well suited to measure similarity between TSs. In this paper, a new nonlinear similarity measure for TS analysis, SimilB, which takes into account the temporal information is introduced. Using the first and second derivatives of the TS, SimilB is able to capture temporal changes and discontinuities of the TS. Some new properties of Ψ_{B} are presented showing, particularly, that the interaction measure is robust both to time shift and amplitude scale. It is also shown that if the time of the signals is scaled by a factor, the corresponding interaction energy is proportional to that of the original ones. Thus, the time corresponding to the maximum of interaction is unchanged by time scale. Note that SimilB is not a unique measure of similarity based on Ψ_{B} operator. Different similarity based on Ψ_{B} can be constructed. To process continuous analytic TSs an expression of Ψ_{B} is provided. The discrete version of Ψ_{B}, for its implementation, is presented and three derivative approximations are examined. Only the asymmetric approximation which is less complicated and less time consuming is implemented. Results of different synthetic TSs (stationary and nonstationary) show that SimilB performs better than the ED and the CC and show the interest to take into account the relative changes of the TSs. Compared to generative models (HMM, GMM, ...) or distance kernel-based methods, SimilB is nonparametric approach that does not require the specification of a kernel or the selection of a probability distribution. Furthermore, SimilB is fast and easy to implement. SimilB may be viewed as a data-driven approach because no a priori information about the signals or parameters setting is required. The processed TSs are either noiseless or moderately noisy. For very noisy TSs, the robustness of SimilB must be studied. In a future work, we plan to use smooth splines to give more robustness to SimilB [11]. We also plan to include the SimilB measure in a clustering process or algorithm such as fuzzy c-means or k-means for classification of TSs in different clusters. To confirm the presented results, a large class of real TSs datasets must be studied as well as the results compared to other methods particularly those including the temporal information.

REFERENCES

[1] J.-C. Cexus and A.-O. Boudraa, "Link between cross-Wigner distribution and cross-Teager energy operator," *Electronics Letters*, vol. 40, no. 12, pp. 778–780, 2004.

[2] J. Alon, S. Sclaroff, G. Kollios, and V. Pavlovic, "Discovering clusters in motion time-series data," in *Proceedings of IEEE Computer Society Conference on Computer Vision and Pattern Recognition (CVPR '03)*, vol. 1, pp. 375–381, Madison, Wis, USA, June 2003.

[3] R. Agrawal, C. Faloutsos, and A. Swami, "Efficient similarity search in sequence databases," in *Proceedings of the 4th International Conference on Foundations of Data Organization and Algorithms (FODO '93)*, vol. 730 of *Lecture Notes in Computer Science*, pp. 69–84, Chicago, Ill, USA, October 1993.

[4] S. Chu, E. Keogh, D. Hart, and M. Pezzani, "Iterative deepening dynamic time warping for time series," in *Proceedings of the 2nd SIAM International Conference on Data Mining*, Arlington, Va, USA, April 2002.

[5] C. S. Möller-Levet, F. Klawonn, K. H. Cho, and O. Wolkenhauer, "Fuzzy clustering of short time-series and unevenly distributed sampling points," in *Proceedings of the 5th International Symposium on Intelligent Data Analysis (IDA '03)*, vol. 2810 of *Lecture Notes in Computer Science*, pp. 330–340, Berlin, Germany, August 2003.

[6] Z. Saidi, A.-O. Boudraa, J.-C. Cexus, and S. Bourennane, "Time-delay estimation using cross-Ψ_{B}-energy operator," *International Journal of Signal Processing*, vol. 1, no. 1, pp. 28–32, 2004.

[7] P. Maragos and A. Potamianos, "Higher order differential energy operators," *IEEE Signal Processing Letters*, vol. 2, no. 8, pp. 152–154, 1995.

[8] P. Maragos, J. F. Kaiser, and T. F. Quatieri, "On amplitude and frequency demodulation using energy operators," *IEEE Transactions on Signal Processing*, vol. 41, no. 4, pp. 1532–1550, 1993.

[9] J. F. Kaiser, "Some useful properties of Teager's energy operators," in *Proceedings of IEEE International Conference on Acoustics, Speech, and Signal Processing (ICASSP '93)*, vol. 3, pp. 149–152, Minneapolis, Minn, USA, April 1993.

[10] N. Saito, *Local feature extraction and its application using a library of bases*, Ph.D. thesis, Yale University, New Haven, Conn, USA, 1994.

[11] D. Dimitriadis and P. Maragos, "An improved energy demodulation algorithm using splines," in *Proceedings of IEEE International Conference on Acoustics, Speech, and Signal Processing (ICASSP '01)*, vol. 6, pp. 3481–3484, Salt Lake, Utah, USA, May 2001.