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Research Article

Time-of-Arrival Estimation in Probability-Controlled Generalized CDMA Systems

Itsik Bergel,[1] Efrat Isack,[2] and Hagit Messer[2]

[1] *School of Engineering, Bar-Ilan University, Ramat-Gan 52900, Israel*
[2] *School of Electrical Engineering, Tel-Aviv University, Tel-Aviv 69978, Israel*

Correspondence should be addressed to Itsik Bergel, bergeli@macs.biu.ac.il

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In recent years, more and more wireless communications systems are required to provide also a positioning measurement. In code division multiple access (CDMA) communication systems, the positioning accuracy is significantly degraded by the multiple access interference (MAI) caused by other users in the system. This MAI is commonly managed by a power control mechanism, and yet, MAI has a major effect on positioning accuracy. Probability control is a recently introduced interference management mechanism. In this mechanism, a user with excess power chooses not to transmit some of its symbols. The information in the nontransmitted symbols is recovered by an error-correcting code (ECC), while all other users receive a more reliable data during these quiet periods. Previous research had shown that the implementation of a probability control mechanism can significantly reduce the MAI. In this paper, we show that probability control also improves the positioning accuracy. We focus on time-of-arrival (TOA)-based positioning systems. We analyze the TOA estimation performance in a generalized CDMA system, in which the probability control mechanism is employed, where the transmitted signal is noncontinuous with a symbol transmission probability smaller than 1. The accuracy of the TOA estimation is determined using appropriate modifications of the Cramer-Rao bound on the delay estimation. Keeping the average transmission power constant, we show that the TOA accuracy of each user does not depend on its transmission probability, while being a nondecreasing function of the transmission probability of any other user. Therefore, a generalized, noncontinuous CDMA system with a probability control mechanism can always achieve better positioning performance, for all users in the network, than a conventional, continuous, CDMA system.

1. INTRODUCTION

In recent years, more and more wireless communications systems are required to provide also a positioning measurement of their mobile users. In this paper, we focus on time-of-arrival (TOA) positioning techniques for code division multiple access (CDMA) systems.

One of the main factors that limit the accuracy of TOA estimation in such communication systems is the multiple access interference (MAI). Research had shown that while MAI limits the system capacity (e.g., [1–3]) it also degrades the TOA estimation accuracy (e.g., [4]). The worst MAI scenario is known as the "near-far" problem. In this scenario, an interfering signal is received in much higher power than the desired signal.

The common way to mitigate the near far problem in CDMA systems is by using a power control mechanism [3, 5–7], which controls the users' transmitted powers in order to limit the amount of interference between users. Power control is currently implemented in almost any CDMA system, and can mitigate the interference very well in multiple access channels (in which all users receive the signal from the same antenna). In other scenarios, the power control is not always optimal, and typically systems performance is limited by the MAI.

Although our work is not limited to any frequency range, it is especially interesting in ultrawideband (UWB) communication and positioning systems. The large bandwidth of these systems can lead to a very good TOA estimation accuracy [8, 9]. However, most UWB communication systems are not planned for cellular deployment. Thus, power control is not efficient enough in such systems, and MAI severely reduces the positioning accuracy.

Recently, Bergel and Messer had suggested using a probability control mechanism to reduce the MAI [10–12]. Probability control mechanism can come in addition to or instead of a power control mechanism. If a user has an excess power, a probability control mechanism will choose not to transmit some of its symbols, while keeping its average power constant, such that a symbol is transmitted with probability $P < 1$, controlled by the system. The information in the nontransmitted symbols is recovered by an error correcting code (ECC). The advantage of this approach is that all other users in the system receive a more reliable data during these quiet periods and therefore improve their performance.

Probability control requires the transmission of noncontinuous CDMA signals. Bergel and Messer had termed these signals as generalized CDMA (GCDMA). The noncontinuity is achieved by setting some of the symbols to zero and transmitting the others. The percentage of transmitted symbols is termed the "transmission probability." Note that this symbol puncturing does not change the bandwidth of the signal, which remains identical to the bandwidth of a conventional CDMA signal (represented here by a transmission probability of 1).

As the importance of probability control mechanism for communication systems was proven and current research focuses on the implementation of probability control in practical CDMA systems, it is interesting to investigate the effect of the changes in transmission probability on the positioning performance. In this paper, we address this problem for TOA-based positioning.

Our derivation will follow the general lines of Botteron et al. [13], which derived bounds on the positioning accuracy in asynchronous CDMA systems with known transmitted data. As the relation between the bounds on unbiased estimation of the delay and the bounds on unbiased estimation of the position is already known [13], we limit the analysis herein to the effect of transmission probability on the delay estimation performance. We use the Cramer-Rao lower bound [14] to derive an achievable lower bound on the delay estimation error for any unbiased estimator. This bound depends on the transmitted data. Following [13], we also perform an asymptotic analysis (for large observation interval) to produce an asymptotic bound that does not depend on the transmitted data sequences, but only on the data statistics.

We use this novel bound to show that the TOA estimation mean square error (MSE) for each user does not depend on its transmission probability, while it is a nondecreasing function of the transmission probability of any other user. Therefore, any decrease in the transmission probability of any user in the network can only improve the positioning accuracy.

The system model and the definitions of the GCDMA transmitted and received signals are given in the following section. The bound derivation and its asymptotic form are given in Section 3. Section 4 contains the analysis of the effect of the transmission probability on the delay estimation bound. Section 5 includes simulation results, and Section 6 provides some concluding remarks.

2. SYSTEM MODEL

The GCDMA transmitted signal is a modification of the CDMA transmitted signal [15] where the symbols sequence is multiplied by a gating sequence. The gating sequence is modeled as an independent and identically distributed (i.i.d.) binary sequence, and the probability of the gating to be 1 is termed the transmission probability. The gating sequence determines whether a symbol is transmitted or not. The transmission probability determines the nature of the system, CDMA systems use transmission probability that equals 1, and the case of lower transmission probability reflects noncontinuous transmission.

The transmitted signal of the uth user is described by

$$s_u(t) = \sum_{k=-\infty}^{\infty} \frac{\sqrt{\varepsilon_u} d_{uk} g_{uk}}{\sqrt{\mathrm{SF}}} \sum_{\nu=0}^{\mathrm{SF}-1} c_{uk\nu} f\left(t - kT_s - \nu T_c\right), \qquad (1)$$

where $f(t)$ is the transmitted pulse shape with $\int f^2(t)dt = 1$, T_s is the symbol time, T_c is the chip time, and SF is the spreading factor. ε_u is the uth user peak power, d_{uk} is its kth data symbol, and $c_{uk\nu}$ its spreading sequence. g_{uk} is the uth user kth gating value, distributed as

$$g_{uk} = \begin{cases} 1 & w \cdot p \quad p_u, \\ 0 & w \cdot p \quad 1 - p_u, \end{cases} \qquad (2)$$

where p_u is the transmission probability of the uth user.

We assume that each receiver can only decode the information from its desired user (single user decoder). The desired user is indicated with index w, while the other users $(u = 1 \cdots U,\ u \neq w)$ are considered as interference. We will assume hereafter that the receiver knows the desired user transmitted symbols. This can correspond to positioning which is based on a pilot sequence (a known sequence which is transmitted periodically for synchronization purposes). Alternatively, this assumption also holds if the positioning is performed after the data has been detected with negligible probability of error.

Since we focus on single user decoder, we cannot assume any knowledge about the interfering users' data.[1] The common approach in previous works (e.g., [16]) was to treat the whole interference as a Gaussian-distributed additive noise. This approach simplifies the model but unfortunately, is not suitable for GCDMA systems. The reason is that probability control can cause the interference to be impulsive, and then the Gaussian approximation does not hold. In this paper, we consider each interferer individually and treat the data symbols as Gaussian distributed with zero mean and variance σ_d^2, $d_{uk} \sim N(0, \sigma_d^2)$. This assumption may also not be precise (e.g., if the data is binary data), however we use it as it simplifies the analysis. Although we model the CDMA chips and the gating sequence as random, in practical systems they are generated by pseudorandom predefined generators. We assume hereafter that there exists a central unit which informs all users what is the transmission powers and what pseudorandom gating sequence is used by each user.

[1] In pilot-based positioning, we assume that the transmitters are not synchronized, so that their pilot sequences do not overlap.

Assuming a frequency flat slow fading channel, the received signal is composed of the sum of the desireduser signal and the interferer signals

$$\mathbf{r}(t) = \alpha_w s_w(t-\tau_w) + \sum_{\substack{u=1\\u\neq w}}^{U} \alpha_u s_u(t-\tau_u) + \mathbf{n}(t), \tag{3}$$

where U is the number of users, α_u and τ_u are the uth user channel gain and channel delay, respectively, and $\mathbf{n}(t)$ is AWGN with zero mean and spectral density $N_0/2$.[2]

The delay of the desired user, τ_w, is the TOA parameter to be estimated, but since the receiver does not have prior knowledge of the other users delays and channel gains, we derive the bound on the error covariance matrix in joint estimation of the delays and gains of all users. Let $\vec{\tau}_u = [\tau_1, \ldots \tau_{w-1}, \tau_{w+1}, \ldots \tau_U]^T$ and $\vec{\alpha}_u = [\alpha_1, \ldots \alpha_{w-1}, \alpha_{w+1}, \ldots \alpha_U]^T$ be the vectors of interferers' delays and gains, respectively. The vector of parameters to be estimated is

$$\vec{\theta} = [\tau_w, \alpha_w, \vec{\tau}_u, \vec{\alpha}_u]^T, \tag{4}$$

where $\alpha_w, \vec{\tau}_u, \vec{\alpha}_u$ are nuisance parameters. We also collect the known parameters into the vector

$$\vec{\psi} = \left[\{d_{wk}\}_{k=-\infty,\ldots,\infty}, \{g_{uk}\}_{\substack{u=1,\ldots,U\\k=-\infty,\ldots,\infty}}, \{c_{ukv}\}_{\substack{u=1,\ldots,U\\k=-\infty,\ldots,\infty\\v=0,\ldots,\mathrm{SF}-1}} \right]^T. \tag{5}$$

Let $T = N \cdot T_s$ be the observation time, where N is the number of symbols in the observation interval. The receiver samples the received signal with Q samples per chip, so we get a total of $L = Q \cdot \mathrm{SF} \cdot N$ samples in the observation interval. The sampling interval is $T_i = T_c/Q$. The lth sample value is given by:

$$\begin{aligned}\mathbf{r}[l] &= \frac{1}{T_i}\int_{(l-1)T_i}^{lT_i} \mathbf{r}(t)dt = \alpha_w \frac{1}{T_i}\int_{(l-1)T_i}^{lT_i} s_w(t-\tau_w)dt \\ &\quad + \sum_{\substack{u=1\\u\neq w}}^{U} \alpha_u \frac{1}{T_i}\int_{(l-1)T_i}^{lT_i} s_u(t-\tau_u)dt + \mathbf{n}[l]\end{aligned} \tag{6}$$

for $l = 1, \ldots, L$, where the noise sample $\mathbf{n}[l] = (1/T_i)\int_{(l-1)T_i}^{lT_i} \mathbf{n}(t)dt$ has a Gaussian distribution with zero mean and variance $N_0/2T_i$.

Collecting the received samples, the received signal vector is the $L \times 1$ vector defined by

$$\vec{\mathbf{r}} = \sum_{u=1}^{U} \vec{s}_u + \vec{\mathbf{n}}, \tag{7}$$

[2] The analysis is based on baseband UWB systems and therefore assumes reception of real signals. The extension to bandpass complex systems is straight forward.

where the noise samples vector, $\vec{\mathbf{n}}$, is a Gaussian vector with zero mean and covariance matrix $\Lambda_n = (N_0/2T_i)I_L$, and $\vec{s}_u$ is the vector of the uth user transmitted signal after passing through the channel. Note that this vector contains only the part of the signal within the observation interval. We write $\vec{s}_u$ as

$$\vec{s}_u = \sum_{k=-\infty}^{\infty} \vec{s}_{uk}, \tag{8}$$

where $\vec{s}_{uk}$ is the vector describing the kth symbol of the uth user

$$\vec{s}_{uk} = \alpha_u \sqrt{\varepsilon_u} d_{uk} g_{uk} \vec{f}_{uk}, \tag{9}$$

and $\vec{f}_{uk}$ is the vector of the sampled pulse shape (with the appropriate delay for the kth symbols of the uth user), in which the lth element is

$$f_{uk}[l] = \frac{1}{\sqrt{\mathrm{SF}}} \sum_{v=0}^{\mathrm{SF}-1} c_{ukv} \frac{1}{T_i} \int_{(l-1)T_i}^{lT_i} f(t - kT_s - vT_c - \tau_u)dt. \tag{10}$$

In order to distinguish the desired user from the interference, we rewrite the received signal vector as

$$\vec{\mathbf{r}} = \vec{\mu}_w + \vec{\mathbf{q}}_w + \vec{\mathbf{n}}, \tag{11}$$

where $\vec{\mu}_w = \vec{s}_w$ is the desired user vector (in the following sections we will also use the notation: $\vec{\mu}_{wk} = \vec{s}_{wk}$) and $\vec{\mathbf{q}}_w = \sum_{u\neq w} \vec{s}_u$ is the interference vector. Note that, given $\tau_w, \alpha_w, \vec{\psi}$, only the interfering data symbols are random and therefore $\vec{\mu}_w$ is deterministic, while $\vec{\mathbf{q}}_w | \vec{\psi} \sim N(0, \Lambda_w)$ has a Gaussian distribution with

$$\begin{aligned}\Lambda_w &= E[\vec{\mathbf{q}}_w \vec{\mathbf{q}}_w^T \mid \vec{\psi}] \\ &= E\left[\sum_{\substack{u=1\\u\neq w}}^{U} \sum_{k=-\infty}^{\infty} \vec{\mathbf{s}}_{uk} \sum_{\substack{v=1\\v\neq w}}^{U} \sum_{j=-\infty}^{\infty} \vec{\mathbf{s}}_{vj}^T \mid \vec{\psi} \right] \\ &= \sum_{\substack{u=1\\u\neq w}}^{U} \sum_{k=-\infty}^{\infty} \Lambda_{uk},\end{aligned} \tag{12}$$

where

$$\Lambda_{uk} = E\left[\vec{\mathbf{s}}_{uk} \vec{\mathbf{s}}_{uk}^T \mid \vec{\psi}\right] = \alpha_u^2 \varepsilon_u \sigma_d^2 g_{uk}^2 \vec{f}_{uk} \vec{f}_{uk}^T, \tag{13}$$

is the covariance matrix of the interference caused by the kth symbol of the uth user, and the third equality in (12) results from the fact that $E[\vec{\mathbf{s}}_{uk}\vec{\mathbf{s}}_{vj}] = 0$ whenever $u \neq v$ or $k \neq j$.

As the received signal vector, (11), is the sum of a deterministic vector and independent Gaussian vectors, it also has a Gaussian distribution $\vec{\mathbf{r}} | \vec{\psi} \sim N(\vec{\mu}_w, \Lambda_{rw})$ with

$$\Lambda_{rw} = \Lambda_w + \Lambda_n. \tag{14}$$

Note that $\vec{\mu}_w$ depends only on the desired user parameters, while Λ_{rw} depends only on the interference and noise parameters.

3. THE ASYMPTOTIC BOUND

The Cramer-Rao bound [14] is a lower bound on the covariance of any unbiased estimator. As we assume that the receiver knows $\vec{\psi}$, we are only interested in bounds that are derived based on the conditional distribution of the received signal given $\vec{\psi}$. We therefore use a conditional version of the inequality and denote it by $R \geq \mathrm{CC}(\vec{\theta}|\vec{\psi})$, where $R = E_{\vec{\mathbf{r}};\vec{\theta}|\vec{\psi}}[(\hat{\vec{\theta}}(\vec{\mathbf{r}})\text{-}\vec{\theta})(\hat{\vec{\theta}}(\vec{\mathbf{r}})\text{-}\vec{\theta})^T \mid \vec{\psi}]$ is the estimator error covariance matrix, $\mathrm{CC}(\vec{\theta}|\vec{\psi}) = F^{-1}$ is the conditional bound, and F is the Fisher information matrix (FIM) given by

$$F = E_{\vec{\mathbf{r}};\vec{\theta}|\vec{\psi}}\left[\left(\frac{\partial \ln p(\vec{\mathbf{r}};\vec{\theta}|\vec{\psi})}{\partial\vec{\theta}}\right)\left(\frac{\partial \ln p(\vec{\mathbf{r}};\vec{\theta}|\vec{\psi})}{\partial\vec{\theta}}\right)^T \middle| \vec{\psi}\right]. \quad (15)$$

Note that since $\vec{\psi}$ is random, both the error covariance matrix, R, and the FIM, F, are random matrices that depend on $\vec{\psi}$, and the notation $R \geq \mathrm{CC}(\vec{\theta}|\vec{\psi})$ means $\Pr(R < \mathrm{CC}(\vec{\theta}|\vec{\psi})) = 0$.

The resulting bound is identical to the Cramer-Rao bound that is derived for the case that $\vec{\psi}$ is deterministic and known. However, the bound we use depends on the random vector $\vec{\psi}$ and therefore is itself a random variable. The bound holds for any unbiased estimator (satisfying $E_{\vec{\mathbf{r}};\vec{\theta}|\vec{\psi}}[\hat{\vec{\theta}}(\vec{\mathbf{r}})] = \vec{\theta},\ \forall\ \vec{\theta}, \vec{\psi})$. For more details about alternative derivations of the Cramer-Rao bound and their applicability see, for example, [17].

We divide F into the following blocks according to the components of $\vec{\theta}$:

$$F = \begin{bmatrix} F_{\tau_w\tau_w} & F_{\tau_w\alpha_w} & F_{\tau_w\tau_u} & F_{\tau_w\alpha_u} \\ F_{\alpha_w\tau_w} & F_{\alpha_w\alpha_w} & F_{\alpha_w\tau_u} & F_{\alpha_w\alpha_u} \\ F_{\tau_u\tau_w} & F_{\tau_u\alpha_w} & F_{\tau_u\tau_u} & F_{\tau_u\alpha_u} \\ F_{\alpha_u\tau_w} & F_{\alpha_u\alpha_w} & F_{\alpha_u\tau_u} & F_{\alpha_u\alpha_u} \end{bmatrix}. \quad (16)$$

As the received signal vector is Gaussian, each element in F can be calculated using the Bangs formula [18]

$$F_{ij} = \frac{\partial\vec{\mu}_w^T}{\partial\theta_i}\Lambda_{rw}^{-1}\frac{\partial\vec{\mu}_w}{\partial\theta_j} + \frac{1}{2}\mathrm{tr}\left(\frac{\partial\Lambda_{rw}}{\partial\theta_i}\Lambda_{rw}^{-1}\frac{\partial\Lambda_{rw}}{\partial\theta_j}\Lambda_{rw}^{-1}\right). \quad (17)$$

Since $\vec{\mu}_w$ only depends on the desired user parameters, while Λ_{rw} only depends on the interference and noise parameters, we get

$$\begin{aligned} F_{\tau_w\tau_w} &= \frac{\partial\vec{\mu}_w^T}{\partial\tau_w}\Lambda_{rw}^{-1}\frac{\partial\vec{\mu}_w}{\partial\tau_w}, & F_{\tau_w\alpha_w} &= \frac{\partial\vec{\mu}_w^T}{\partial\tau_w}\Lambda_{rw}^{-1}\frac{\partial\vec{\mu}_w}{\partial\alpha_w}, \\ F_{\alpha_w\tau_w} &= \frac{\partial\vec{\mu}_w^T}{\partial\alpha_w}\Lambda_{rw}^{-1}\frac{\partial\vec{\mu}_w}{\partial\tau_w}, & F_{\alpha_w\alpha_w} &= \frac{\partial\vec{\mu}_w^T}{\partial\alpha_w}\Lambda_{rw}^{-1}\frac{\partial\vec{\mu}_w}{\partial\alpha_w}. \end{aligned} \quad (18)$$

The blocks that correspond to the interferers parameters become

$$\begin{aligned} F_{\tau_u\tau_u} &= \frac{1}{2}\mathrm{tr}\left(\frac{\partial\Lambda_{rw}}{\partial\tau_u}\Lambda_{rw}^{-1}\frac{\partial\Lambda_{rw}}{\partial\tau_u}\Lambda_{rw}^{-1}\right), \\ F_{\tau_u\alpha_u} &= \frac{1}{2}\mathrm{tr}\left(\frac{\partial\Lambda_{rw}}{\partial\tau_u}\Lambda_{rw}^{-1}\frac{\partial\Lambda_{rw}}{\partial\alpha_u}\Lambda_{rw}^{-1}\right), \\ F_{\alpha_u\tau_u} &= \frac{1}{2}\mathrm{tr}\left(\frac{\partial\Lambda_{rw}}{\partial\alpha_u}\Lambda_{rw}^{-1}\frac{\partial\Lambda_{rw}}{\partial\tau_u}\Lambda_{rw}^{-1}\right), \\ F_{\alpha_u\alpha_u} &= \frac{1}{2}\mathrm{tr}\left(\frac{\partial\Lambda_{rw}}{\partial\alpha_u}\Lambda_{rw}^{-1}\frac{\partial\Lambda_{rw}}{\partial\alpha_u}\Lambda_{rw}^{-1}\right), \end{aligned} \quad (19)$$

and the blocks that include derivatives with respect to the parameters of both the interferers and the desired user become zero:

$$F_{\alpha_w\alpha_u} = 0\ , \qquad F_{\alpha_w\tau_u} = 0, \qquad F_{\alpha_u\tau_w} = 0, \qquad F_{\tau_u\tau_w} = 0\ . \quad (20)$$

Thus, the FIM becomes a block diagonal matrix, and the inverse of the matrix can be calculated by taking the inverse of each block. As we are only interested in the performance of the desired user, we can limit the analysis to the upper-left block defined as

$$F_w = \begin{bmatrix} F_{\tau_w\tau_w} & F_{\tau_w\alpha_w} \\ F_{\alpha_w\tau_w} & F_{\alpha_w\alpha_w} \end{bmatrix}, \quad (21)$$

and the bound is given by the top-left element of the inverse of this matrix $CC_{\tau_w}(\vec{\theta}|\vec{\psi}) = [F_w^{-1}]_{1,1}$.

As stated above, the resulting bound is a function of $\vec{\psi}$. Nevertheless, when the observation interval become long ($N\to\infty$), the elements in F_w/N converge to a limit that depend only on the statistics of the sequences in $\vec{\psi}$. We denote the asymptotic FIM by $\mathrm{As}F_w \triangleq \lim_{N\to\infty} F_w/N$ and the resulting asymptotic bound by $\mathrm{AsCC}_{\tau_w} = [\mathrm{As}F_w]_{1,1}$. In Appendix A, we prove that the asymptotic FIM is given by

$$\begin{aligned} \mathrm{As}F_w &= E[F_w] \\ &= \varepsilon_w\sigma_d^2 p_w E\left[\begin{bmatrix} \alpha w \dot{\vec{f}}^T{}_{wk} \\ \vec{f}_{wk}^T \end{bmatrix}\Lambda_{rw}^{-1}\begin{bmatrix}\alpha_w\dot{\vec{f}}_{wk} & \vec{f}_{wk}\end{bmatrix}\right], \end{aligned} \quad (22)$$

which can be evaluated numerically.

The asymptotic bound on the estimation error of the delay τ_w is given by

$$\mathrm{AsCC}_{\tau_w} = [\mathrm{As}F_w^{-1}]_{1,1} = \begin{bmatrix}1 & 0\end{bmatrix}\mathrm{As}F_w^{-1}\begin{bmatrix}1\\0\end{bmatrix}. \quad (23)$$

Note that as in [13], we can approximate the conditional bound for $N < \infty$ by

$$\mathrm{CC}_{\tau_w}(\vec{\theta}|\vec{\psi}) \approx \widetilde{\mathrm{CC}}_{\tau_w} = \frac{\mathrm{AsCC}_{\tau_w}}{N}. \quad (24)$$

This approximation becomes more accurate as the observation time increases and has the big advantage of not being dependant on the chips, gating, and data sequences.

It is also important to note that the asymptotic bound depends on the transmission probability directly while the conditional bound depends on the transmission probability only through a sample gating sequence. Therefore, the asymptotic bound also allows us to analyze the effect of the transmission probability.

4. THE EFFECT OF THE TRANSMISSION PROBABILITY

In this section, we prove that a decrease in any transmission probability can only decrease the delay estimation mean square error (MSE). Although a decrease in the transmission probability makes the transmitted signal more impulsive, it is important to note that it does not change the transmitted spectrum. Thus, the performance gain reported hereafter stems from the reduction in interference and not from a change in the signal bandwidth. In fact, it is easy to verify that the asymptotic bound, (23), depends on the desired user transmission probability only through the average transmission power $\varepsilon_w^{\text{av}} = \varepsilon_w p_w$. Therefore, changing a user transmission probability while keeping its average power constant will only affect the other users' performance.

We prove that the delay estimation MSE is a nondecreasing function of the transmission probability of any user by showing that the derivative of the desired user MSE w.r.t. any interferer transmission probability, when the average power is kept constant, is non negative. We use the following theorem.

Theorem 1. *If the asymptotic bound can by written as $AsCC_{\tau_w} = \vec{a}^T AsF_w^{-1}\vec{a}$ where $\vec{a}$ does not depend on the uth interferer's transmission probability and transmission power, then a sufficient condition for a GCDMA system to satisfy*

$$\left.\frac{dAsCC_{\tau_w}}{dp_u}\right|_{p_u\varepsilon_u=\varepsilon_u^{av}} \geq 0 \tag{25}$$

is

$$\frac{\partial^2 AsF_w}{\partial\varepsilon_{uk}^2} \geq 0, \tag{26}$$

where ε_{uk} is the power of the kth symbol of the uth user and the notations ≥ 0 mean that the matrix is nonnegative definite.

Proof of Theorem 1. See Appendix B.

Before we prove that the sufficient condition of Theorem 1, (26), is satisfied in our model, we verify that the theorem is applicable by inspecting (23) and setting $\vec{a} = [\ 1\ \ 0\ ]^T$. Next, we calculate the derivative of the asymptotic FIM, (22), with respect to the peak power of the kth symbol of the uth interferer. Noting that the only element that depends on the interferer power is Λ_{rw}^{-1}, we get

$$\frac{\partial^2 \text{As}F_w}{\partial\varepsilon_{uk}^2} = \varepsilon_w\sigma_d^2 p_w E\left[\begin{bmatrix}\alpha_w \dot{f}_{wk}^T \\ f_{wk}^T\end{bmatrix}\frac{\partial^2\Lambda_{rw}^{-1}}{\partial\varepsilon_{uk}^2}\begin{bmatrix}\alpha_w\dot{f}_{wk} & f_{wk}\end{bmatrix}\right]. \tag{27}$$

From the quadratic form in the expectation, we see a sufficient condition for the matrix $\partial^2\text{As}F_w/\partial\varepsilon_{uk}^2$ to be nonnegative definite. is that the matrix $\partial^2\Lambda_{rw}^{-1}/\partial\varepsilon_{uk}^2$ is always nonnegative definite.

Calculating the first derivative we have

$$\frac{\partial\Lambda_{rw}^{-1}}{\partial\varepsilon_{uk}} = -\Lambda_{rw}^{-1}\frac{\partial\Lambda_{rw}}{\partial\varepsilon_{uk}}\Lambda_{rw}^{-1}. \tag{28}$$

Before we calculate the second derivative, we note that Λ_{rw}, (14), is linear with ε_{uk}, and therefore $\partial\Lambda_{rw}/\partial\varepsilon_{uk}$ in (28) is independent of ε_{uk}. Using this fact, the second derivative is given by

$$\frac{\partial^2\Lambda_{rw}^{-1}}{\partial\varepsilon_{uk}^2} = 2\Lambda_{rw}^{-1}\frac{\partial\Lambda_{rw}}{\partial\varepsilon_{uk}}\Lambda_{rw}^{-1}\frac{\partial\Lambda_{rw}}{\partial\varepsilon_{uk}}\Lambda_{rw}^{-1}. \tag{29}$$

Again, the resulting expression has a quadratic form, and we only need to prove that the matrix Λ_{rw}^{-1} is nonnegative definite. This is guaranteed because this matrix is the inverse of the covariance matrix Λ_{rw} which is a positive definite matrix. Therefore, (25) is satisfied in our model.

Thus, Theorem 1 assures that the considered model satisfies $d\text{AsCC}_{\tau_w}/dp_u|_{p_u\varepsilon_u=\varepsilon_u^{\text{av}}} \geq 0$. Recalling that the bound on the TOA of the desired user depends only on its average transmitted power, we also have $d\text{AsCC}_{\tau_w}/dp_w|_{p_w\varepsilon_w=\varepsilon_w^{\text{av}}} = 0$, which shows that the asymptotic bound is a nondecreasing function of any transmission probability. Note that for a sufficiently large observation interval, the asymptotic bound is reachable, and therefore the bound indicates the achievable TOA estimation performance. As we always seek to reduce the estimation MSE, we conclude that, from the positioning performance point of view, the system would always prefer to reduce the transmission probabilities of all the users as much as possible.

Note that in practical systems that combine communication and positioning, the transmission probabilities will usually be chosen to maximize the communication performance. Yet, our results indicate that any decrease in the transmission probability can only increase the positioning performance. A system that employs probability control will typically use transmission probabilities which are less than 1, and therefore should be preferred, from the positioning point of view, over conventional CDMA systems.

5. SIMULATIONS

In order to demonstrate the results of the previous sections, we present in this section some simulation results over a simplified scenario. The simulated scenario includes two users. User 1 is the desired user while user 2 is the interferer. We assume known channel gains and a near-far scenario, characterized by the channel gains: $\alpha_1 = 1(0\,\text{dB}), \alpha_2 = 100(40\,\text{dB})$. Both users transmit the same average power ($E_1^{\text{av}} = E_2^{\text{av}}$), and the desired user signal-to-noise ratio is $E_1^{\text{av}}/N_0 = -9\,\text{dB}$ (so that the scenario is interference dominated).

The symbol time is set to $T_s = 1$ ns and the symbol shape was set as in [19] to be $f(t) = \sqrt{8/3t_n}[1 - 4\pi((t - T_s/2)/t_n)^2]\exp(-2\pi((t - T_s/2)/t_n)^2)$ with $t_n = 0.3$ ns. The number of samples per chip is $Q = 20$, and we start with no spreading (SF = 1). The users' delays are $\tau_1 = 0.35$ ns, $\tau_2 = 0.425$ ns.

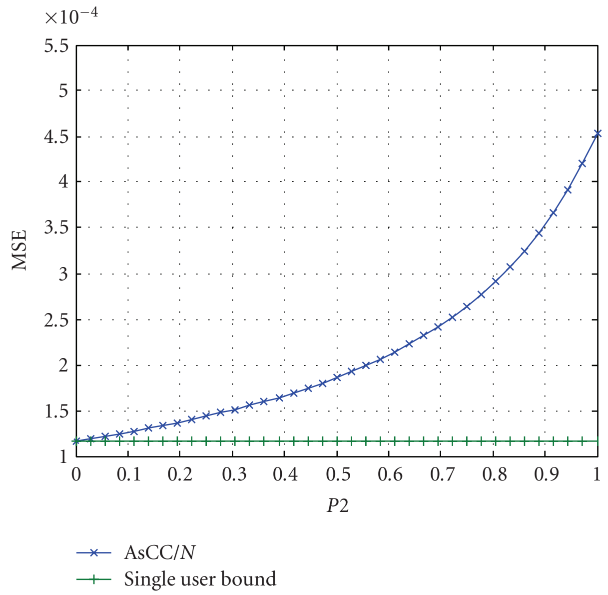

Figure 1: Asymptotic approximation of the bound versus interferer transmission probability. Observation interval contains $N = 100$ symbols.

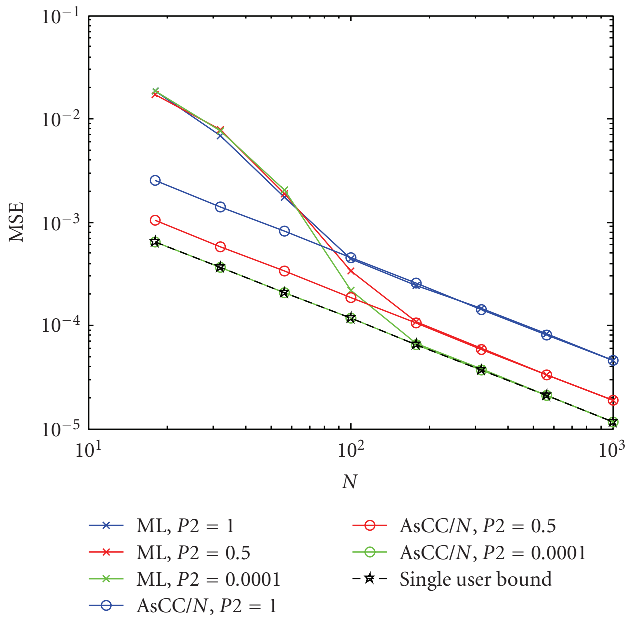

Figure 2: MSE of a ML estimator versus the number of symbols in the observation interval for different interferer's transmission probabilities. The figure also shows the asymptotic approximation of the bound and the single-user bound. (Results averaged over 20 000 simulations.)

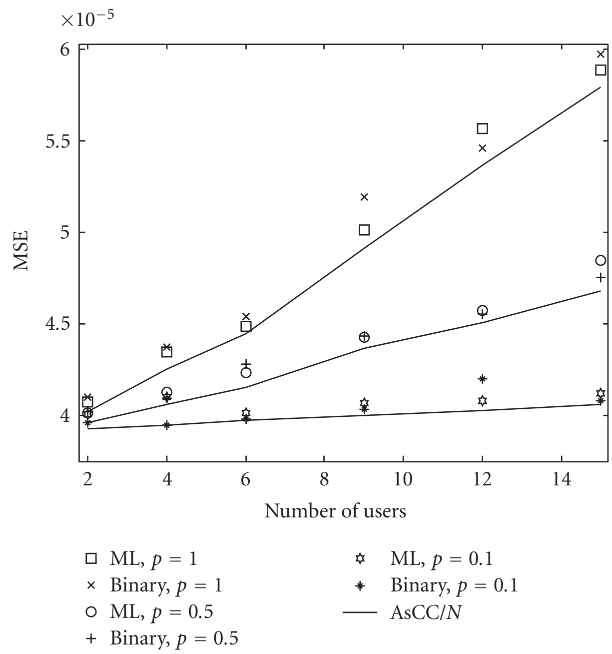

Figure 3: MSE of an ML estimator versus the number of users in the system, for different interferers' transmission probabilities, in a CDMA system with spreading factor of 6 and an observation interval of 300 symbols. The figure also shows the MSE of the estimation for binary-modulated signals and the asymptotic approximation of the bound. (Results averaged over 20 000 simulations.)

We use $P_1 = 1$ for the desired user transmission, and vary only the interferer transmission probability.

Figure 1 depicts the asymptotic approximation to the bound, (24), versus the interferer transmission probability. This figure demonstrates that the bound is monotonic increasing with the transmission probability P_2. For comparison, the figure also shows the single-user bound (the performance of user 1 in the absence of user 2). We can see that for small-enough transmission probability, the interference is practically suppressed and the desired user (user 1) can achieve the single user bound.

Figure 2 depicts the performance of a maximum likelihood (ML) estimator. The figure shows the MSE of the delay estimation versus number of symbols in the observation interval, N, for several values of the interferer transmission probability. The estimation MSE was calculated from 20 000 simulations. The figure also shows the approximated bound and the single-user bound. As expected, for all transmission probabilities, for large-enough number of symbols the ML performance converges to the bound. Again, we can see that the estimation error decreases as the transmission probability decreases. Comparing to the single-user bound, we also see that for small enough transmission probability, the interference can be significantly suppressed.

Turning to a more sophisticated system, Figure 3 depicts the performance of a CDMA system with spreading factor of 6 as a function of the number of users. As in the previous simulation scenario, all interfering users are 40 dB stronger than the desired user. The symbol time is $T_s = T_c \,\mathrm{SF} = 6$ ns, and the interfering users delays are uniformly distributed in the range $[0, T_s]$. Figure 3 depicts the asymptotic bound and the performance of an ML estimator with block size of 300 symbols, when all users transmit in probabilities of $P = 0.1$, 0.5, and 1. As the number of users grows, the amount of MAI increases and we can see an increase in the estimation errors.

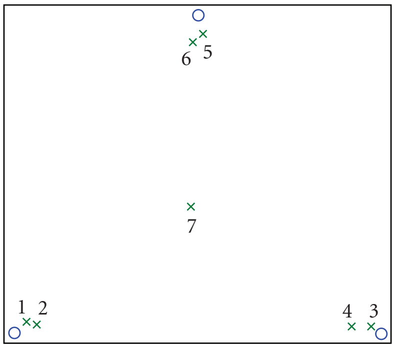

FIGURE 4: Simple positioning system. Circles indicate the location of bases and numbered x-marks indicate the location of mobiles. The distance between the bases is 1.7 meters.

TABLE 1: Received E_b/N_0 in dB by each base from each mobile in the positioning scenario of Figure 4.

Base mobile	1	2	3
1	−9.0	17.3	−9.1
2	−8.9	14.4	−8.8
3	−9.1	−9.1	20.1
4	−8.9	−8.6	12.2
5	16.3	−9.0	−8.9
6	13.4	−8.7	−8.9
7	−3.7	−4.8	−5.3

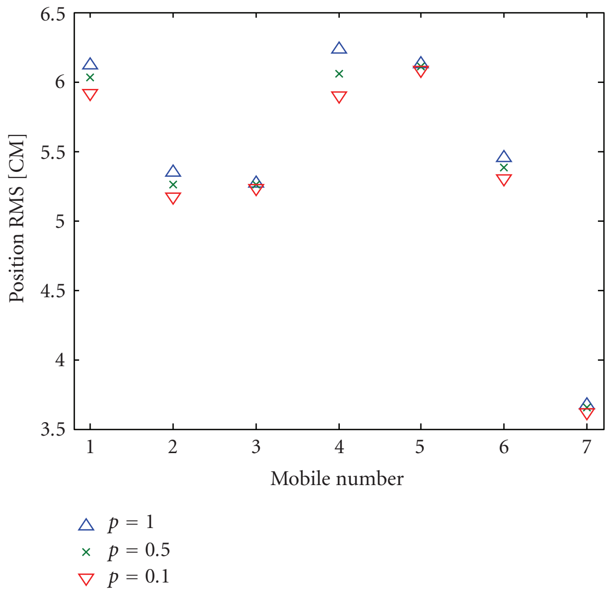

FIGURE 5: Root mean square (RMS) of positioning error in the system of Figure 4 for different transmission probabilities.

But, as expected, this increase strongly depends on the transmission probability. For lower probabilities, the estimation is much more accurate. For a transmission probability of 0.1 we see that the interference from other users has almost no effect on the desired user performance.

Figure 3 also depicts the performance of the same receiver when the transmitters use the common binary signaling (and not Gaussian, as assumed in the rest of the paper). As can be seen, the performance is almost identical to the performance with Gaussian signaling, and the asymptotic bound gives a good prediction of the actual performance with binary signaling. Receivers which are based on the assumption that the interference is Gaussian are common in practical systems as they give good tradeoff between complexity and performance. But we must note that this is not the optimal receiver for this case. In the case of binary signaling, the optimal receiver needs to consider all possible combinations of the transmitted bits from all users, which makes it impractical. On the other hand, the optimal receiver can perform much better, especially if the interference is very strong (in which case it can reliably detect the interference symbols, and therefore achieve the same performance as if the interference symbols were known).

Finally, although the relation between TOA estimation accuracy and positioning accuracy was already investigated [13], we show here a simple example of the effect of transmission probability on the positioning accuracy. We simulate the simple scenario of 3 base stations and 7 mobile users shown in Figure 4. The distance between the base stations is 1.7 meters. We assume an AWGN channel, and the channel gains are inversely proportional to the square of the distance. The E_b/N_0 received by each base from each mobile is summarized in Table 1. The positioning is based on TOA measurements that each base performs based on the reception of a block of 300 symbols. The root mean square of the positioning error in centimeters is shown in Figure 5. As can be seen, for some mobiles (e.g., 1 and 4) the reduction in transmission probability (keeping the average transmission power constant) causes a noticeable reduction in the positioning error. For other mobile, the effect of MAI is smaller, and therefore the effect of transmission probability is small. As proved above, for all users the reduction in transmission probability does not degrade the positioning accuracy. The actual improvement in positioning accuracy depends on the mobiles and bases locations, the propagation model, and the amount of MAI between users.

6. CONCLUSIONS

In this paper, we analyzed the asymptotic positioning performance of GCDMA systems with a probability control mechanism. We focused on positioning using TOA and used the asymptotic Cramer-Rao bound for time-delay estimation as the performance measure.

We proved that, keeping the average transmission powers constant, the asymptotic bound does not depend on the desired user transmission probability and is a nondecreasing function of the interferers' transmission probabilities. Since the bound is asymptotically achievable, this result indicates that the best TOA estimation accuracy in a GCDMA system is achieved by decreasing the transmission probabilities as much as possible (while keeping the average power constant). Conventional CDMA systems use transmission probability that equals 1, while probability-controlled systems would typically work in lower transmission probabilities. Therefore, a generalized CDMA system with a probability control mechanism can always achieve better positioning performance, for all users in the network, than a conventional CDMA system.

As this is the first work that analyzes the effect of the transmission probability on the delay estimation error, we chose the simplified frequency flat slow fading channel. For this channel, we were able to prove the basic results that estimation MSE is a nondecreasing function of the transmission probability. Further work will need to consider also frequency selective fading channels.

APPENDICES

A. EVALUATION OF THE ASYMPTOTIC FIM

In this appendix, we calculate the asymptotic FIM, $\mathrm{As}F_w = \lim_{N\to\infty} F_w/N$. Expanding (18), we get

$$\begin{aligned} F_{\tau_w\tau_w} &= \sum_{k=-\infty}^{\infty} \frac{\partial \vec{\mu}^T}{\partial \tau_w} \Lambda_{rw}^{-1} \frac{\partial \vec{\mu}_{wk}}{\partial \tau_w} + \sum_{k=-\infty}^{\infty} \sum_{\substack{j=-\infty \\ j\neq k}}^{\infty} \frac{\partial \vec{\mu}_{wk}^T}{\partial \tau_w} \Lambda_{rw}^{-1} \frac{\partial \vec{\mu}_{wj}}{\partial \tau_w} \\ &= \sum_{k=-\infty}^{\infty} \alpha_w^2 \varepsilon_w d_{wk}^2 g_{wk}^2 \dot{\vec{f}}_{wk}^T \Lambda_{rw}^{-1} \dot{\vec{f}}_{wk} \\ &\quad + \sum_{k=-\infty}^{\infty} \sum_{\substack{j=-\infty \\ j\neq k}}^{\infty} \alpha_w^2 \varepsilon_w d_{wk} d_{wj} g_{wk} g_{wj} \dot{\vec{f}}_{wk}^T \Lambda_{rw}^{-1} \dot{\vec{f}}_{wj}, \end{aligned} \tag{A.1}$$

$$\begin{aligned} F_{\alpha_w\alpha_w} &= \sum_{k=-\infty}^{\infty} \varepsilon_w d_{wk}^2 g_{wk}^2 \vec{f}_{wk}^T \Lambda_{rw}^{-1} \vec{f}_{wk} \\ &\quad + \sum_{k=-\infty}^{\infty} \sum_{\substack{j=-\infty \\ j\neq k}}^{\infty} \varepsilon_w d_{wk} d_{wj} g_{wk} g_{wj} \vec{f}_{wk}^T \Lambda_{rw}^{-1} \vec{f}_{wj}, \end{aligned} \tag{A.2}$$

$$\begin{aligned} F_{\alpha_w\tau_w} &= \sum_{k=-\infty}^{\infty} \alpha_w \varepsilon_w d_{wk}^2 g_{wk}^2 \vec{f}_{wk}^T \Lambda_{rw}^{-1} \dot{\vec{f}}_{wk} \\ &\quad + \sum_{k=-\infty}^{\infty} \sum_{\substack{j=-\infty \\ j\neq k}}^{\infty} \alpha_w \varepsilon_w d_{wk} d_{wj} g_{wk} g_{wj} \vec{f}_{wk}^T \Lambda_{rw}^{-1} \dot{\vec{f}}_{wj}, \end{aligned} \tag{A.3}$$

where $\dot{\vec{f}}_{wk} = (\partial/\partial\tau_w)\vec{f}_{wk}$ is the derivative of each element in the pulse-shape vector with respect to τ_w.

We begin by calculating the limit of the first element in $F_{\tau_w\tau_w}$, (A.1),

$$A = \lim_{N\to\infty} \frac{1}{N} \sum_{k=-\infty}^{\infty} \alpha_w^2 \varepsilon_w d_{wk}^2 g_{wk}^2 \dot{\vec{f}}_{wk}^T \Lambda_{rw}^{-1} \dot{\vec{f}}_{wk}^T. \tag{A.4}$$

Note that the summation is infinite because we assume the transmission of infinite number of symbols. On the other hand, the observation interval is limited to the duration of only N symbols. Thus, the observation interval contains the entire received signal of almost N of the transmitted symbols, while at the beginning and at the end of the observation interval there are some symbols for which only part of the received signal is included in the observation interval. However, when the observation interval is large enough, the effect of the clipped symbols at the edges is negligible for almost all of the symbols. Specifically, the term $\alpha_w^2 \varepsilon_w d_{wk_0}^2 g_{wk_0}^2 \dot{\vec{f}}_{wk_0}^T \Lambda_{rw}^{-1} \dot{\vec{f}}_{wk_0}$ has the same distribution for any symbol k_0 which is far enough from the observation interval edges (almost N symbols). Noting that the sequences d_w, g, c are independent and each of them is i.i.d, all terms in the sum in (A.4) are i.i.d, and we can apply the law of large numbers:

$$\begin{aligned} A &= E_{d_w,g,c}\left[\alpha_w^2 \varepsilon_w d_{wk_0}^2 g_{wk_0}^2 \dot{\vec{f}}_{wk_0}^T \Lambda_{rw}^{-1} \dot{\vec{f}}_{wk_0}\right] \\ &= \alpha_w^2 \varepsilon_w \sigma_d^2 p_w E_{g,c}\left[\dot{\vec{f}}_{wk_0}^T \Lambda_{rw}^{-1} \dot{\vec{f}}_{wk_0}^T\right]. \end{aligned} \tag{A.5}$$

The limit of the second part of $F_{\tau_w\tau_w}$, (A.1)is

$$B = \lim_{N\to\infty} \frac{1}{N} \sum_{k=-\infty}^{\infty} \sum_{\substack{j=-\infty \\ j\neq k}}^{\infty} \alpha_w^2 \varepsilon_w d_{wk} d_{wj} g_{wk} g_{wj} \dot{\vec{f}}_{wk}^T \Lambda_{rw}^{-1} \dot{\vec{f}}_{wj}. \tag{A.6}$$

Noting that $\sum_{j=-\infty}^{\infty} \dot{\vec{f}}_{wk}^T \Lambda_{rw}^{-1} \dot{\vec{f}}_{wj}$ is finite for any k, we can apply again the law of large numbers. But in this case, the expectation includes the expectation of two uncorrelated, zero-mean random variables, and therefore $B = 0$, and we have

$$\lim_{N\to\infty} \frac{F_{\tau_w\tau_w}}{N} = \alpha_w^2 \varepsilon_w \sigma_d^2 p_w E\left[\dot{\vec{f}}_{wk}^T \Lambda_{rw}^{-1} \dot{\vec{f}}_{wk}\right]. \tag{A.7}$$

In the same way, we calculate

$$\begin{aligned} \lim_{N\to\infty} \frac{F_{\alpha_w\tau_w}}{N} &= \alpha_w \varepsilon_w \sigma_d^2 p_w E\left[\vec{f}_{wk}^T \Lambda_{rw}^{-1} \dot{\vec{f}}_{wk}\right] = \lim_{N\to\infty} \frac{F_{\tau_w\alpha_w}}{N}, \\ \lim_{N\to\infty} \frac{F_{\alpha_w\alpha_w}}{N} &= \varepsilon_w \sigma_d^2 p_w E\left[\vec{f}_{wk}^T \Lambda_{rw}^{-1} \vec{f}_{wk}\right]. \end{aligned} \tag{A.8}$$

Summarizing the results above leads to the asymptotic FIM, (22).

B. PROOF OF THEOREM 1

In this appendix, we prove the sufficient condition of Theorem 1. Note that as we keep the average power constant, any change in the uth user transmission probability causes a change in its peak power according to $\varepsilon_u = \varepsilon_u^{\mathrm{av}}/p_u$.

Using the chain rule for derivatives,

$$\begin{aligned} \left.\frac{d\mathrm{AsCC}_{\tau_w}}{dp_u}\right|_{p_u\varepsilon_u=\varepsilon_u^{\mathrm{av}}} &= \frac{\partial \mathrm{AsCC}_{\tau_w}}{\partial p_u} + \frac{\partial \mathrm{AsCC}_{\tau_w}}{\partial \varepsilon_u}\frac{\partial \varepsilon_u}{\partial p_u} \\ &= \frac{1}{p_u}\left(\frac{p_u \partial \mathrm{AsCC}_{\tau_w}}{\partial p_u} - \frac{\varepsilon_u \partial \mathrm{AsCC}_{\tau_w}}{\partial \varepsilon_u}\right). \end{aligned} \tag{B.1}$$

Considering first the partial derivative with respect to the transmission probability, we use the chain rule again to write

$$\frac{\partial \mathrm{AsCC}_{\tau_w}}{\partial p_u} = \sum_{k=-\infty}^{\infty} \frac{\partial \mathrm{AsCC}_{\tau_w}}{\partial p_{uk}} \frac{\partial p_{uk}}{\partial p_u} = \sum_{k=-\infty}^{\infty} \frac{\partial \mathrm{AsCC}_{\tau_w}}{\partial p_{uk}}, \tag{B.2}$$

where p_{uk} is the transmission probability of the kth symbol of the uth user. Note that this is done only for the purpose of the derivation, and we still consider a single-transmission probability for each user. This means that we require $p_{uk} = p_u$ which results in the second equality in (B.2).

Calculating the partial derivative with respect to the peak power in the same manner, we get

$$\frac{\partial \mathrm{AsCC}_{\tau_w}}{\partial \varepsilon_u} = \sum_{k=-\infty}^{\infty} \frac{\partial \mathrm{AsCC}_{\tau_w}}{\partial \varepsilon_{uk}} \frac{\partial \varepsilon_{uk}}{\partial \varepsilon_u} = \sum_{k=-\infty}^{\infty} \frac{\partial \mathrm{AsCC}_{\tau_w}}{\partial \varepsilon_{uk}}, \quad \text{(B.3)}$$

where ε_{uk} is the power of the kth symbol of the uth user. Substituting (B.2) and (B.3) into (B.1), we can write

$$\frac{d\mathrm{AsCC}_{\tau_w}}{dp_u} = \sum_{k=-\infty}^{\infty} \frac{1}{p_{uk}} \Delta_{w,u,k}, \quad \text{(B.4)}$$

where

$$\Delta_{w,u,k} = p_{uk} \frac{\partial \mathrm{AsCC}_{\tau_w}}{\partial p_{uk}} - \varepsilon_{uk} \frac{\partial \mathrm{AsCC}_{\tau_w}}{\partial \varepsilon_{uk}}. \quad \text{(B.5)}$$

Now, a sufficient condition for the derivative, (25), to be nonnegative is that $\Delta_{w,u,k} \geq 0$ for any w, u, k. The derivatives in (B.5) satisfy

$$\frac{\partial \mathrm{AsCC}_{\tau_w}}{\partial p_{uk}} = -\vec{a}^T \cdot \mathrm{As}F_w^{-1} \frac{\partial \mathrm{As}F_w}{\partial p_{uk}} \mathrm{As}F_w^{-1} \cdot \vec{a}, \quad \text{(B.6)}$$

$$\frac{\partial \mathrm{AsCC}_{\tau_w}}{\partial \varepsilon_{uk}} = -\vec{a}^T \cdot \mathrm{As}F_w^{-1} \frac{\partial \mathrm{As}F_w}{\partial \varepsilon_{uk}} \mathrm{As}F_w^{-1} \cdot \vec{a}. \quad \text{(B.7)}$$

Writing the expectation in the definition of $\mathrm{As}F_w$, (22), as an explicit function of p_{uk}:

$$\mathrm{As}F_w = p_{uk} E[F_w \mid g_{uk} = 1] + (1 - p_{uk}) E[F_w \mid g_{uk} = 0], \quad \text{(B.8)}$$

we note that $E[F_w \mid g_{uk} = \gamma]$ does not depend on p_{uk} for $\gamma = 0, 1$. Thus, the derivative in (B.6) becomes

$$\begin{aligned} \frac{\partial \mathrm{As}F_w}{\partial p_{uk}} &= E[F_w \mid g_{uk} = 1] - E[F_w \mid g_{uk} = 0] \\ &= \frac{1}{p_{uk}} (\mathrm{As}F_w - E[F_w \mid g_{uk} = 0]). \end{aligned} \quad \text{(B.9)}$$

Since setting $g_{uk} = 0$ is equivalent to setting $\varepsilon_{uk} = 0$, we can write

$$\frac{p_{uk} \partial \mathrm{As}F_w}{\partial p_{uk}} = \mathrm{As}F_w - \mathrm{As}F_w|_{\varepsilon_{uk}=0} = \int_0^{\varepsilon_{uk}} f_{wuk}(\alpha) d\alpha, \quad \text{(B.10)}$$

where $f_{wuk}()$ denotes the derivative of the asymptotic FIM with respect to the u, k symbol power:

$$f_{wuk}(\alpha) = \left. \frac{\partial \mathrm{As}F_w}{\partial \varepsilon_{uk}} \right|_{\varepsilon_{uk}=\alpha}. \quad \text{(B.11)}$$

Substituting (B.10) into (B.6), we get

$$\frac{p_{uk} \partial \mathrm{AsCC}_{\tau_w}}{\partial p_{uk}} = -\int_0^{\varepsilon_{uk}} \vec{a}^T \cdot \mathrm{As}F_w^{-1} f_{wuk}(\alpha) \mathrm{As}F_w^{-1} \cdot \vec{a} \cdot d\alpha \quad \text{(B.12)}$$

and defining

$$\widetilde{f}(\alpha) = \vec{a}^T \cdot \mathrm{As}F_w^{-1} f_{wuk}(\alpha) \mathrm{As}F_w^{-1} \cdot \vec{a}, \quad \text{(B.13)}$$

where $f_{wuk}()$ is a matrix function and $\widetilde{f}()$ is a scalar function, we rewrite the derivative as

$$\frac{p_{uk} \partial \mathrm{AsCC}_{\tau_w}}{\partial p_{uk}} = -\int_0^{\varepsilon_{uk}} \widetilde{f}(\alpha) d\alpha. \quad \text{(B.14)}$$

The same functions ($f_{wuk}()$ and $\widetilde{f}()$ defined in (B.11) and (B.13), resp.) are used also to express the partial derivative with respect to the peak power in (B.7):

$$\begin{aligned} &\frac{\varepsilon_{uk} \partial \mathrm{AsCC}_{\tau_w}}{\partial \varepsilon_{uk}} \\ &\quad = -\varepsilon_{uk} \vec{a}^T \cdot \mathrm{As}F_w^{-1} f_{wuk}(\varepsilon_{uk}) \mathrm{As}F_w^{-1} \cdot \vec{a} = -\varepsilon_{uk} \widetilde{f}(\varepsilon_{uk}). \end{aligned} \quad \text{(B.15)}$$

Substituting (B.14) and (B.15) into (B.5), we have

$$\Delta_{w,u,k} = \varepsilon_{uk} \widetilde{f}(\varepsilon_{uk}) - \int_0^{\varepsilon_{uk}} \widetilde{f}(\alpha) d\alpha, \quad \text{(B.16)}$$

and a sufficient condition for that is

$$\frac{\partial \widetilde{f}(\alpha)}{\partial \alpha} \geq 0, \qquad \forall \alpha \in [0, \varepsilon_{uk}]. \quad \text{(B.17)}$$

Writing the derivation in (B.17) explicitly, we get

$$\begin{aligned} \frac{\partial \widetilde{f}(\alpha)}{\partial \alpha} &= \frac{\partial (\vec{a}^T \cdot \mathrm{As}F_w^{-1} f_{wuk}(\alpha) \mathrm{As}F_w^{-1} \cdot \vec{a})}{\partial \alpha} \\ &= \vec{a}^T \cdot \mathrm{As}F_w^{-1} \frac{\partial f_{wuk}(\alpha)}{\partial \alpha} \mathrm{As}F_w^{-1} \cdot \vec{a} \end{aligned} \quad \text{(B.18)}$$

and from the quadratic form of (B.18) we can see that a sufficient condition for $\partial \widetilde{f}(\alpha)/\partial \alpha \geq 0$ is that $\partial f_{wuk}(\alpha)/\partial \alpha \geq 0$. Recalling the definition of $f_{wuk}()$, (B.11), the sufficient condition becomes $\partial^2 \mathrm{As}F_w / \partial \varepsilon_{uk}^2 \geq 0$, which concludes the proof of Theorem 1.

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