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Research Article

Asymptotic Analysis of Large Cooperative Relay Networks Using Random Matrix Theory

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Cooperative transmission is an emerging communication technology that takes advantage of the broadcast nature of wireless channels. In cooperative transmission, the use of relays can create a virtual antenna array so that multiple-input/multiple-output (MIMO) techniques can be employed. Most existing work in this area has focused on the situation in which there are a small number of sources and relays and a destination. In this paper, cooperative relay networks with large numbers of nodes are analyzed, and in particular the asymptotic performance improvement of cooperative transmission over direction transmission and relay transmission is analyzed using random matrix theory. The key idea is to investigate the eigenvalue distributions related to channel capacity and to analyze the moments of this distribution in large wireless networks. A performance upper bound is derived, the performance in the low signal-to-noise-ratio regime is analyzed, and two approximations are obtained for high and low relay-to-destination link qualities, respectively. Finally, simulations are provided to validate the accuracy of the analytical results. The analysis in this paper provides important tools for the understanding and the design of large cooperative wireless networks.

1. INTRODUCTION

In recent years, cooperative transmission [1, 2] has gained considerable attention as a potential transmit strategy forwireless networks. Cooperative transmission efficiently takes advantage of the broadcast nature of wireless networks, and also exploits the inherent spatial and multiuser diversities of the wireless medium. The basic idea of cooperative transmission is to allow nodes in the network to help transmit/relay information for each other, so that cooperating nodes create a virtual multiple-input/multiple-output (MIMO) transmission system. Significant research has been devoted to the design of cooperative transmission schemes and the integration of this technique into cellular, WiFi, Bluetooth, ultrawideband, Worldwide Interoperability for Microwave Access (WiMAX), and ad hoc and sensor networks. Cooperative transmission is also making its way into wireless communication standards, such as IEEE 802.16j.

Most current research on cooperative transmission focuses on protocol design and analysis, power control, relay selection, and cross-layer optimization. Examples of representative work are as follows. In [3], transmission protocols for cooperative transmission are classified into different types and their performance is analyzed in terms of outage probabilities. The work in [4] analyzes more complex transmitter cooperative schemes involving dirty paper coding. In [5], centralized power allocation schemes are presented, while energy-efficient transmission is considered for broadcast networks in [6]. In [7], oversampling is combined with the intrinsic properties of orthogonal frequency division multiplexing (OFDM) symbols, in the context of maximal ratio combining (MRC) and amplify-and-forward relaying, so that the rate loss of cooperative transmission can be overcome. In [8], the authors evaluate cooperative-diversity performance when the best relay is chosen according to the average signal-to-noise ratio (SNR), and the outage probability of relay selection based on the instantaneous SNR. In [9], the authors propose a distributed relay selection scheme that requires limited network knowledge and is based on instantaneous SNRs. In [10], sensors are assigned for cooperation so as to reduce power consumption. In [11], cooperative transmission is used to create new paths so that energy depleting critical paths can be bypassed. In [12], it is shown that cooperative transmission can improve the operating point for multiuser detection so that multiuser efficiency can be improved. Moreover, network coding is also employed to improve the diversity order and bandwidth efficiency. In [13], a buyer/seller game is proposed to circumvent the need for exchanging channel information to optimize the cooperative communication performance. In [14], it is demonstrated that boundary nodes can help backbone nodes' transmissions using cooperative transmission as future rewards for packet forwarding. In [15], auction theory is explored for resource allocation in cooperative transmission.

Most existing work in this area analyzes the performance gain of cooperative transmission protocols assuming small numbers of source-relay-destination combinations. In [16], large relay networks are investigated without combining of source-destination and relay-destination signals. In [17], transmit beamforming is analyzed asymptotically as the number of nodes increases without bound. In this paper, we analyze the asymptotic (again, as the number of nodes increases) performance improvement of cooperative transmission over direct transmission and relay transmission. Relay nodes are considered in this paper while only beamforming in point-to-point communication is considered in [17]. Unlike [16], in which only the indirect source-relay-destination link is considered, we consider the direct link from source nodes to destination nodes. The primary tool we will use is random matrix theory [18, 19]. The key idea is to investigate the eigenvalue distributions related to capacity and to analyze their moments in the asymptote of large wireless networks. Using this approach, we derive a performance upper bound, we analyze the performance in the low signal-to-noise-ratio regime, and we obtain approximations for high and low relay-to-destination link qualities. Finally, we provide simulation results to validate the analytical results.

This paper is organized as follows. In Section 2, the system model is given, while the basics of random matrix theory are discussed in Section 3. In Section 4, we analyze the asymptotic performance and construct an upper bound for cooperative relay networks using random matrix theory. Some special cases are analyzed in Section 5, and simulation results are discussed in Section 6. Finally, conclusions are drawn in Section 7.

2. SYSTEM MODEL

We consider the system model shown in Figure 1. Suppose there are M source nodes, M destination nodes, and K relay nodes. Denote by $\mathbf{H}$, $\mathbf{F}$, and $\mathbf{G}$ the channel matrices of source-to-relay, relay-to-destination, and source-to-destination links, respectively, so that $\mathbf{H}$ is $M \times K$, $\mathbf{F}$ is $K \times M$, and $\mathbf{G}$ is $M \times M$. Transmissions take place in two stages. Further denote the thermal noise at the relays by the K-vector $\mathbf{z}$, the noise in the first stage at the destination by the M-vector $\mathbf{w}_1$ and the noise in the second stage at the destination by the M-vector $\mathbf{w}_2$. For simplicity of notation, we assume that all of the noise variables have the same power

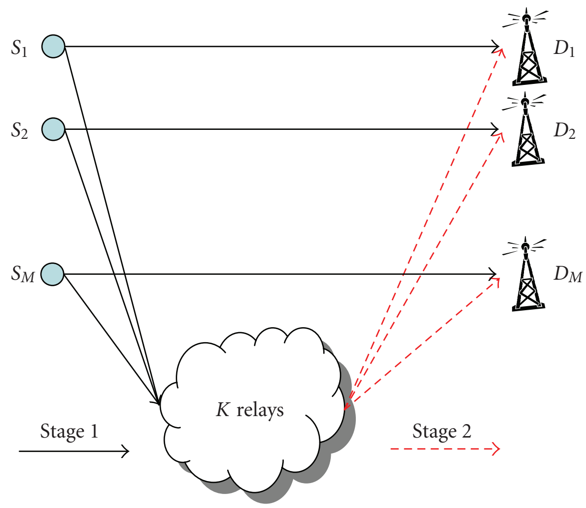

FIGURE 1: Cooperative transmission system model.

and denote this common value by σ_n^2, the more general case being straightforward. The signals at the source nodes are collected into the M-vector $\mathbf{s}$. We assume that the transmit power of each source node and each relay node is given by P_s and P_r, respectively. For simplicity, we further assume that matrices $\mathbf{H}$, $\mathbf{F}$, and $\mathbf{G}$ have independent and identically distributed (i.i.d.) elements whose variances are normalized to $1/K$, $1/M$, and $1/M$, respectively. Thus, the average norm of each column is normalized to 1; otherwise the receive SNR at both relay nodes and destination nodes will diverge in the large system limit. (Note that we do not specify the distribution of the matrix elements since the large system limit is identical for most distributions, as will be seen later.) The average channel power gains, determined by path loss, of source-to-relay, source-to-destination, and relay-to-destination links are denoted by g_{sr}, g_{sd}, and g_{rd}, respectively.

Using the above definitions, the received signal at the destination in the first stage can be written as

$$\mathbf{y}_{\mathrm{sd}} = \sqrt{g_{\mathrm{sd}} P_s}\mathbf{G}\mathbf{s} + \mathbf{w}_1, \tag{1}$$

and the received signal at the relays in the first stage can be written as

$$\mathbf{y}_{\mathrm{sr}} = \sqrt{g_{\mathrm{sr}} P_s}\mathbf{H}\mathbf{s} + \mathbf{z}. \tag{2}$$

If an amplify-and-forward protocol [16] is used, the received signal at the destination in the second stage is given by

$$\mathbf{y}_{\mathrm{rd}} = \sqrt{\frac{g_{\mathrm{rd}}\, g_{\mathrm{sr}} P_r P_s}{P_0}}\mathbf{F}\mathbf{H}\mathbf{s} + \sqrt{\frac{g_{\mathrm{rd}} P_r}{P_0}}\mathbf{F}\mathbf{z} + \mathbf{w}_2, \tag{3}$$

where

$$P_0 = \frac{g_{\mathrm{sr}} P_s}{K}\operatorname{trace}(\mathbf{H}\mathbf{H}^H) + \sigma_n^2, \tag{4}$$

namely, the average received power at the relay nodes, which is used to normalize the received signal at the relay nodes so that the average relays transmit power equals P_r. To see this, we can deduce the transmitted signal at the relays, which is given by

$$\mathbf{t}_{\mathrm{rd}} = \sqrt{\frac{g_{\mathrm{sr}}P_rP_s}{P_0}}\mathbf{Hs} + \sqrt{\frac{P_r}{P_0}}\mathbf{z}. \tag{5}$$

Then, the average transmit power is given by

$$\begin{aligned}\frac{1}{K}\operatorname{trace}\left[E\left[\mathbf{t}_{\mathrm{rd}}\mathbf{t}_{\mathrm{rd}}^H\right]\right] &= \frac{1}{K}\operatorname{trace}\left[\frac{g_{\mathrm{sr}}P_rP_s}{P_0}\mathbf{HH}^H + \frac{P_r\sigma_n^2}{P_0}\mathbf{I}\right]\\ &= \frac{P_r}{KP_0}\operatorname{trace}\left[g_{\mathrm{sr}}P_s\mathbf{HH}^H + \sigma_n^2\mathbf{I}\right]\\ &= P_r,\end{aligned} \tag{6}$$

where the last equation is due to (4).

Combining the received signal in the first and second stages, the total received signal at the destination is a $2M$-vector:

$$\mathbf{y} = \mathbf{Ts} + \mathbf{w}, \tag{7}$$

where

$$\begin{aligned}\mathbf{T} &= \begin{pmatrix}\sqrt{g_{\mathrm{sd}}P_s}\mathbf{G}\\ \sqrt{\dfrac{g_{\mathrm{sr}}g_{\mathrm{rd}}P_rP_s}{P_0}}\mathbf{FH}\end{pmatrix},\\ \mathbf{w} &= \begin{pmatrix}\mathbf{w}_1\\ \sqrt{\dfrac{g_{\mathrm{rd}}P_r}{P_0}}\mathbf{Fz} + \mathbf{w}_2\end{pmatrix}.\end{aligned} \tag{8}$$

The sum capacity of this system is given by

$$\begin{aligned}&C_{\mathrm{sum}}\\ &= \log\det\left(\mathbf{I} + \mathbf{T}^HE^{-1}\left[\mathbf{ww}^H\right]\mathbf{T}\right)\\ &= \log\det\Bigg[\mathbf{I} + \left(\sqrt{g_{\mathrm{sd}}P_s}\mathbf{G}^H, \sqrt{\frac{g_{\mathrm{sr}}g_{\mathrm{rd}}P_rP_s}{P_0}}\mathbf{H}^H\mathbf{F}^H\right)\\ &\qquad\times\begin{pmatrix}\sigma_n^2\mathbf{I} & 0\\ 0 & \sigma_n^2\left(\mathbf{I} + \dfrac{g_{\mathrm{rd}}P_r}{P_0}\mathbf{FF}^H\right)\end{pmatrix}^{-1}\begin{pmatrix}\sqrt{g_{\mathrm{sd}}P_s}\mathbf{G}\\ \sqrt{\dfrac{g_{\mathrm{sr}}g_{\mathrm{rd}}P_rP_s}{P_0}}\mathbf{FH}\end{pmatrix}\Bigg]\\ &= \log\det\Bigg[\mathbf{I} + \frac{g_{\mathrm{sd}}P_s}{\sigma_n^2}\mathbf{G}^H\mathbf{G}\\ &\qquad + \frac{g_{\mathrm{sr}}g_{\mathrm{rd}}P_rP_s}{P_0\sigma_n^2}\mathbf{H}^H\mathbf{F}^H\left(\mathbf{I} + \frac{g_{\mathrm{rd}}P_r}{P_0}\mathbf{FF}^H\right)^{-1}\mathbf{FH}\Bigg]\\ &= \log\det\left[\mathbf{I} + \gamma_1\mathbf{G}^H\mathbf{G} + \beta\gamma_2\mathbf{H}^H\mathbf{F}^H\left(\mathbf{I} + \beta\mathbf{FF}^H\right)^{-1}\mathbf{FH}\right].\end{aligned} \tag{9}$$

Here $\gamma_1 \triangleq g_{\mathrm{sd}}P_s/\sigma_n^2$ and $\gamma_2 \triangleq g_{\mathrm{sr}}P_s/\sigma_n^2$ represent the SNRs of the source-to-destination and source-to-relay links, respectively, and $\beta \triangleq g_{\mathrm{rd}}P_r/P_0$ is the amplification ratio of the relay.

We use a simpler notation for (9), which is given by

$$C_{\mathrm{sum}} = \log\det(\mathbf{I} + \boldsymbol{\Omega}) = \log\det\left(\mathbf{I} + \boldsymbol{\Omega}_s + \boldsymbol{\Omega}_r\right), \tag{10}$$

where $\boldsymbol{\Omega}_s \triangleq \gamma_1\mathbf{G}^H\mathbf{G}$ corresponds to the direct channel from the source to the destination; and

$$\boldsymbol{\Omega}_r \triangleq \beta\gamma_2\mathbf{H}^H\mathbf{F}^H\left(\mathbf{I} + \beta\mathbf{FF}^H\right)^{-1}\mathbf{FH} \tag{11}$$

corresponds to the signal relayed to the destination by the relay nodes. On denoting the eigenvalues of the matrix $\boldsymbol{\Omega}$ by $\{\lambda_m^\Omega\}_{m=1,2,\ldots}$, the sum capacity C_{sum} can be written as

$$C_{\mathrm{sum}} = \sum_{m=1}^{M}\log\left(1 + \lambda_m^\Omega\right). \tag{12}$$

In the following sections, we obtain expressions or approximations for C_{sum} by studying the distribution of λ_m^Ω.

We are interested in the average channel capacity of the large relay network, which is defined as

$$C_{\mathrm{avg}} \triangleq \frac{1}{M}C_{\mathrm{sum}}. \tag{13}$$

In this paper, we focus on analyzing C_{avg} in the large system scenario, namely, $K, M \to \infty$ while $\alpha \triangleq M/K$ is held constant, which is similar to the large system analysis arising in the study of code division multiple access (CDMA) systems [20]. Therefore, we place the following assumption on C_{avg}.

Assumption 1.

$$C_{\mathrm{avg}} \longrightarrow E\left[\log\left(1 + \lambda^\Omega\right)\right], \text{ almost surely}, \tag{14}$$

where λ^Ω is a generic eigenvalue of Ω, as $K, M \to \infty$.

This assumption will be validated by the numerical result in Section 6, which shows that the variance of C_{avg} decreases to zero as K and M increase. In the remaining part of this paper, we consider C_{avg} to be a constant in the sense of the large system limit, unless noted otherwise.

3. BASICS OF LARGE RANDOM MATRIX THEORY

In this section, we provide some basics of random matrix theory, including the notions of noncrossing partitions, isomorphic decomposition, combinatorial convolution, and free cumulants, which provide analytical machinery for characterizing the average channel capacity when the system dimensions increase asymptotically.

3.1. Freeness

Below is the abstract definition of freeness, which is originated by Voiculescu [21–23].

Definition 1. Let $\mathcal{A}$ be a unital algebra equipped with a linear functional $\psi : \mathcal{A} \to \mathbb{C}$, which satisfies $\psi(1) = 1$. Let $p_1, \ldots, p_k$ be one-variable polynomials. We call elements $a_1, \ldots, a_m \in \mathcal{A}$ *free* if for all $i_1 \neq i_2 \neq \cdots \neq i_k$, we have

$$\psi\left[p_1(a_{i_1})\cdots p_k(a_{i_k})\right] = 0, \tag{15}$$

whenever

$$\psi[p_j(a_{i_j})] = 0, \quad \forall j = 1, \ldots, k. \tag{16}$$

In the theory of large random matrices, we can consider random matrices as elements $a_1, \ldots, a_m$, and the linear functional ψ maps a random matrix $\mathbf{A}$ to the expectation of eigenvalues of $\mathbf{A}$.

3.2. Noncrossing partitions

A *partition* of a set $\{1, \ldots, p\}$ is defined as a division of the elements into a group of disjoint subsets, or *blocks* (a block is termed an i-block when the block size is i). A partition is called an r-partition when the number of blocks is r.

We say that a partition of a p-set is *noncrossing* if, for any two blocks $\{u_1, \ldots, u_s\}$ and $\{v_1, \ldots, v_t\}$, we have

$$u_k < v_1 < u_{k+1} \Longleftrightarrow u_k < v_t < u_{k+1}, \quad \forall k = 1, \ldots, s, \tag{17}$$

with the convention that $u_{s+1} = u_1$. For example, for the set $\{1, 2, 3, 4, 5, 6, 7, 8\}$, $\{\{1, 4, 5, 6\}, \{2, 3\}, \{7\}, \{8\}\}$ is noncrossing, while $\{\{1, 3, 4, 6\}, \{2, 5\}, \{7\}, \{8\}\}$ is not. We denote the set of noncrossing partitions on the set $\{1, 2, \ldots, p\}$ by $\mathbb{NC}_p$.

3.3. Isomorphic decomposition

The set of noncrossing partitions in $\mathbb{NC}_p$ has a partial ordering structure, in which $\pi \leq \sigma$ if each block of π is a subset of a corresponding block of σ. Then, for any $\pi \leq \sigma \in \mathbb{NC}_p$, we define the interval between π and σ as

$$[\pi, \sigma] \triangleq \{\psi \in \mathbb{NC}_p \mid \pi \leq \psi \leq \sigma\}. \tag{18}$$

It is shown in [21] that, for all $\pi \leq \sigma \in \mathbb{NC}_p$, there exists a canonical sequence of positive integers $\{k_i\}_{i\in\mathbb{N}}$ such that

$$[\pi, \sigma] \cong \prod_{j\in\mathbb{N}} \mathbb{NC}_j^{k_j}, \tag{19}$$

where $\cong$ is an isomorphism (the detailed mapping which can be found in the proof of Proposition 1 in [21]), the product is the Cartesian product, and $\{k_j\}_{j\in\mathbb{N}}$ is called the *class* of $[\pi, \sigma]$.

3.4. Incidence algebra, multiplicative function, and combinatorial convolution

The *incidence algebra* on the partial ordering structure of $\mathbb{NC}_p$ is defined as the set of all complex-valued functions $f(\psi, \sigma)$ with the property that $f(\psi, \sigma) = 0$ if $\psi \nleq \sigma$ [20].

The *combinatorial convolution* between two functions f and g in the incidence algebra is defined as

$$f \star g(\pi, \sigma) \triangleq \sum_{\pi \leq \psi \leq \sigma} f(\pi, \psi) g(\psi, \sigma), \quad \forall \pi \leq \sigma. \tag{20}$$

An important subset of the incidence algebra is the set of *multiplicative functions* f on $[\pi, \sigma]$, which are defined by the property

$$f(\pi, \sigma) \triangleq \prod_{j\in\mathbb{N}} a_j^{k_j}, \tag{21}$$

where $\{a_j\}_{j\in\mathbb{N}}$ is a series of constants associated with f, and the class of $[\pi, \sigma]$ is $\{k_j\}_{j\in\mathbb{N}}$. We denote by f_a the multiplicative function with respect to $\{a_j\}_{j\in\mathbb{N}}$. An important function in the incidence algebra is the zeta function ζ, which is defined as

$$\zeta(\pi, \sigma) \triangleq \begin{cases} 1, & \text{if } \psi \leq \sigma, \\ 0, & \text{else.} \end{cases} \tag{22}$$

Further, the unit function I on the incidence algebra is defined as

$$I(\pi, \sigma) \triangleq \begin{cases} 1, & \text{if } \psi = \sigma, \\ 0, & \text{else.} \end{cases} \tag{23}$$

The inverse of the ζ function, denoted by μ, with respect to combinatorial convolution, namely, $\mu \star \zeta = I$, is termed the *Möbius* function.

3.5. Moments and free cumulants

Denote the pth moment of the (random) eigenvalue λ by $m_p \triangleq E[\lambda^p]$. We introduce a family of quantities termed *free cumulants* [22] denoted by $\{k_p\}$ for Ω where $p$denotes the order. We will use a superscript to indicate the matrix for which the moments and free cumulants are defined. The relationship between moments and free cumulants is given by combinatorial convolution in the incidence algebra [21, 22], namely,

$$\begin{aligned} f_m &= f_k \star \zeta, \\ f_k &= f_m \star \mu, \end{aligned} \tag{24}$$

where the multiplicative functions f_m (characterizing the moments), f_k (characterizing the free cumulants), zeta function ζ, Möbius function μ, and combinatorial convolution $\star$ are defined above.

By applying the definition of a noncrossing partition, (24), can be translated into the following explicit forms for the first three moments and free cumulants:

$$\begin{aligned} m_1 &= k_1, \\ m_2 &= k_2 + k_1^2, \\ m_3 &= k_3 + 3k_1k_2 + k_1^3, \\ k_1 &= m_1, \\ k_2 &= m_2 - m_1^2, \\ k_3 &= m_3 - 3m_1m_2 + 2m_1^3. \end{aligned} \tag{25}$$

The following lemma provides the rules for the addition [22] (see (B.4)) and product [22] (see (D.9)) of two free matrices.

Lemma 1. *If matrices A and B are mutually free, one has*

$$f_{k^{A+B}} = f_{k^A} + f_{k^B}, \tag{26}$$

$$f_{k^{AB}} = f_{k^A} \star f_{k^B}. \tag{27}$$

4. ANALYSIS USING RANDOM MATRIX THEORY

It is difficult to obtain a closed-form expression for the asymptotic average capacity C_{avg} in (13). In this section, using the theory of random matrices introduced in the last section, we first analyze the random variable λ^{Ω} by characterizing its moments and providing an upper bound for C_{avg}. Then, we can rewrite C_{avg} in terms of a moment series, which facilitates the approximation.

4.1. Moment analysis of λ^{Ω}

In contrast to [16], we analyze the random variable C_{avg} via its moments, instead of its distribution function, because moment analysis is more mathematically tractable. For simplicity, we denote $\beta\mathbf{F}^H(\mathbf{I}+\beta\mathbf{F}\mathbf{F}^H)^{-1}\mathbf{F}$ by $\mathbf{\Gamma}$, which is obviously Hermitian. Then, the matrix $\mathbf{\Omega}$ is given by

$$\mathbf{\Omega} = \gamma_1\mathbf{G}^H\mathbf{G} + \gamma_2\mathbf{H}^H\mathbf{\Gamma}\mathbf{H}. \tag{28}$$

In order to apply free probability theory, we need as a prerequisite that $\mathbf{G}^H\mathbf{G}$, $\mathbf{H}^H\mathbf{H}$, and $\mathbf{F}^H(\mathbf{I}+\beta\mathbf{F}\mathbf{F}^H)^{-1}\mathbf{F}$ be mutually free (the definition of freeness can be found in [23]). It is difficult to prove the freeness directly. However, the following proposition shows that the result obtained from the freeness assumption coincides with [24, Theorem 1.1] (same as in (29)) in [24], which is obtained via an alternative approach.

Proposition 1. *Suppose $\gamma_1 = \gamma_2 = 1$ (note that the assumption $\gamma_1 = \gamma_2 = 1$ is for convenience of analysis; it is straightforward to extend the proposition to general cases). Based on the freeness assumption, the Stieltjes transform of the eigenvalues in the matrix Ω satisfies the following Marčenko-Pastur equation:*

$$m_{\Omega}(z) = m_{G^HG}\left[z - \frac{1}{\alpha}\int\frac{\tau d\mathcal{F}(\tau)}{1+\tau(z)m_{\Omega}(z)}\right], \tag{29}$$

where $\mathcal{F}$ is the probability distribution function of the eigenvalues of the matrix $\mathbf{\Gamma}$, and $m(z)$ denotes the Stieltjes transform [20].

Proof. See Appendix A. □

Therefore, we assume that these matrices are mutually free (the *freeness assumption*) since this assumption yields the same result as a rigorously proved conclusion. The validity of the assumption is also supported by numerical results included in Section 6. Note that the reason why we do not apply the conclusion in Proposition 1 directly is that it is easier to manipulate using the moments and free probability theory.

Using the notion of multiplicative functions and Lemma 1, the following proposition characterizes the free cumulants of the matrix Ω, based upon which we can compute the eigenvalue moments of Ω from (24) (or (25) explicitly for the first three moments).

Proposition 2. *The free cumulants of the matrix Ω in (28) are given by*

$$f_{k^{\Omega}} = f_{k^{\Omega_s}} + \left(\left(\left(f_{k^{\Gamma}}\star f_{k^{\tilde{H}}}\right)\star\zeta\right)\star\delta_{1/\alpha}\right)\star\mu, \tag{30}$$

where $k_p^{\Omega_s} = 1$, the free cumulant of $k_p^{\tilde{H}} = \gamma_2^p/\alpha$, $\forall p \in \mathcal{N}$, $\tilde{H} = \gamma_2\mathbf{H}\mathbf{H}^H$, and the multiplicative function $\delta_{1/\alpha}$ is defined as

$$\delta_{1/\alpha}(\tau,\pi) = \begin{cases}\dfrac{1}{\alpha}, & \text{if } \tau = \pi;\\ 0, & \text{if } \tau \neq \pi.\end{cases} \tag{31}$$

Proof. The proof is straightforward by applying the relationship between free cumulants and moments. The reasoning is given as follows:

(i) $f_{k^{\Gamma}}\star f_{k^{\tilde{H}}}$ represents the free cumulants of the matrix $\gamma_2\mathbf{\Gamma}\mathbf{H}\mathbf{H}^H$ (applying Lemma 1);
(ii) $(f_{k^{\Gamma}}\star f_{k^{\tilde{H}}})\star\zeta$ represents the moments of the matrix $\gamma_2\mathbf{\Gamma}\mathbf{H}\mathbf{H}^H$;
(iii) $((f_{k^{\Gamma}}\star f_{k^{\tilde{H}}})\star\zeta)\star\delta_{1/\alpha}$ represents the moments of the matrix $\gamma_2\mathbf{H}^H\mathbf{\Gamma}\mathbf{H}$;
(iv) $(((f_{k^{\Gamma}}\star f_{k^{\tilde{H}}})\star\zeta)\star\delta_{1/\alpha})\star\mu$ represents the free cumulants of the matrix $\gamma_2\mathbf{H}^H\mathbf{\Gamma}\mathbf{H}$;
(v) the final result is obtained by applying Lemma 1.

□

4.2. Upper bound of average capacity

Although in Section 4.1 we obtained all moments of λ^{Ω}, we did not obtain an explicit expression for the average channel capacity. However, we can provide an upper bound on this quantity by applying Jensen's inequality, which we summarize in the following proposition.

Proposition 3. *The average capacity satisfies*

$$C_{\text{avg}}^{(u)} \leq \log\left(1+\gamma_1+\frac{\alpha\beta\gamma_2}{\alpha+\beta}\right). \tag{32}$$

Proof. By applying Jensen's inequality, we have

$$\begin{aligned} E[\log(1+\lambda^{\Omega})] &\leq \log(1+E[\lambda^{\Omega}]) \\ &= \log(1+E[\lambda^{\Omega_s}]+E[\lambda^{\Omega_r}]). \end{aligned} \tag{33}$$

From [20], we obtain

$$E[\lambda^{\Omega_s}] = \gamma_1. \tag{34}$$

For $\mathbf{\Omega}$, we can show

$$E[\lambda^{\Omega_r}] = \frac{1}{\alpha}E[\lambda^{\Omega_r'}], \tag{35}$$

where

$$\Omega_r' = \beta\gamma_2\mathbf{F}^H(\mathbf{I}+\beta\mathbf{F}\mathbf{F}^H)^{-1}\mathbf{F}\mathbf{H}\mathbf{H}^H. \tag{36}$$

By applying the law of matrix product in Lemma 1, we can further simplify (35) to

$$E[\lambda^{\Omega_r}] = \frac{\gamma_2}{\alpha}E[\lambda^{HH^H}]E[\lambda^{\Gamma}] = \gamma_2 E[\lambda^{HH^H}]E\left[\frac{\beta\lambda^{FF^H}}{1+\beta\lambda^{FF^H}}\right]. \tag{37}$$

By applying Jensen's inequality again, we have

$$E[\lambda^{\Omega_r'}] \leq \gamma_2 E[\lambda^{HH^H}] \frac{\beta E[\lambda^{FF^H}]}{1+\beta E[\lambda^{FF^H}]} = \frac{\alpha\beta\gamma_2}{\alpha+\beta}, \tag{38}$$

where we have applied the facts $E[\lambda^{HH^H}] = \alpha$ and $E[\lambda^{FF^H}] = 1/\alpha$.

Combining the above equations yields the upper bound in (32). □

4.3. Expansion of average capacity

In addition to providing an upper bound on the average capacity, we can also expand C_{avg} into a power series so that the moment expressions obtained from Proposition 2 can be applied. Truncating this power series yields approximations for the average capacity.

In particular, by applying a Taylor series expansion around a properly chosen constant x_0, C_{avg} can be written as

$$C_{\text{avg}} = \log(1+x_0) + \sum_{k=1}^{\infty}(-1)^{k-1} E\left[\frac{(\lambda - x_0)^k}{k(1+x_0)^k}\right]. \tag{39}$$

Taking the first two terms of the series yields the approximation

$$C_{\text{avg}} \approx \log(1+x_0) + \frac{m_1 - x_0}{1+x_0} - \frac{m_2 - 2x_0 m_1 + x_0^2}{2(1+x_0)^2}. \tag{40}$$

We can set $x_0 = \gamma_1 + \alpha\beta\gamma_2/(\alpha+\beta)$, which is an upper bound for $E[\lambda^{\Omega}]$ as shown in Proposition 3. We can also set $x_0 = 0$ and obtain an approximation when λ^{Ω} is small.Equations (40) will be a useful approximation for C_{avg} in Sections 5.2 and 5.3 when β is large or small or when SNR is small.

5. APPROXIMATIONS OF C_{avg}

In this section, we provide explicit approximations to C_{avg} for several special cases of interest. The difficulty in computing C_{avg} lies in determining the moments of the matrix $\mathbf{\Gamma}$. Therefore, in the low SNR region (Section 5.1), we consider representing C_{avg} in terms of the average capacities of the source-destination link and the source-relay-destination link. Then, we consider the region of high (Section 5.2) or low β (Section 5.3), where $\mathbf{\Gamma}$ can be simplified; thus we will obtain approximations in terms of α, β, γ_1, and γ_2. Finally, higher-order approximation will be studied in Section 5.4.

5.1. Approximate analysis in the low SNR regime

Unlike Section 4 which deals with general cases, we assume here that both the source-to-destination and relay-to-destination links within the low SNR regime, that is, P_s/σ_n^2 and P_r/σ_n^2 are small. Such an assumption is reasonable when both source nodes and relay nodes are far away from the destination nodes.

Within the low-SNR assumption, the asymptotic average capacity can be expanded in the Taylor series expansion about $x_0 = 0$ in (40), which is given by

$$C_{\text{avg}} = E[\log(1+\lambda^{\Omega})] = \sum_{i=1}^{\infty}(-1)^{i+1}\frac{m_i^{\Omega}}{i}. \tag{41}$$

We denote the pth-order approximation of C_{avg} by

$$C_p = \sum_{i=1}^{p}(-1)^{i+1}\frac{m_i^{\Omega}}{i}, \tag{42}$$

which implies

$$m_i^{\Omega} = (-1)^{i+1} i (C_i - C_{i-1}). \tag{43}$$

We denote by $\{C_p^s\}$ and $\{C_p^r\}$ the average capacity approximations (the same as in (42)) for the source-destination link and the source-relay-destination link, respectively. Our target is to represent the average capacity approximations $\{C_p\}$ by using $\{C_p^s\}$ and $\{C_p^r\}$ under the low-SNR assumption, which reveals the mechanism of information combining of the two links.

By combining (25), (26), and (43), we can obtain

$$\begin{aligned} C_1 &= C_1^s + C_1^r, \\ C_2 &= C_2^s + C_2^r - C_1^s C_1^r, \\ C_3 &= C_3^s + C_3^r - C_1^s C_1^r + 4C_1^s C_1^r - 2C_1^s C_2^r - 2C_1^r C_2^s, \end{aligned} \tag{44}$$

where C_p^s and C_p^r denote the pth-order approximations of the average capacity of the source-destination link and the source-relay-destination link, respectively.

Equation (44) shows that, to a first-order approximation, the combined effect of the source-destination and source-relay-destination links is simply a linear addition of average channel capacities, when the low-SNR assumption holds. For the second-order approximation in (44), the average capacity is reduced by a nonlinear term $C_1^s C_1^r$. The third-order term in (44) is relatively complicated to interpret.

5.2. High β region

In the high β region, the relay-destination link has a better channel than that of the source-relay link. The following proposition provides the first two moments of the eigenvalues λ in Ω in this case.

Proposition 4. *As $\beta \to \infty$, the first two moments of the eigenvalues λ in Ω converge to*

$$\begin{aligned} m_1 &= \begin{cases} \gamma_1 + \alpha\gamma_2, & \text{if } \alpha \leq 1, \\ \gamma_1 + \gamma_2, & \text{if } \alpha > 1, \end{cases} \\ m_2 &= \begin{cases} 2(\gamma_1^2 + \alpha\gamma_2^2 + \alpha\gamma_1\gamma_2), & \text{if } \alpha \leq 1, \\ 2\gamma_1^2 + 2\gamma_1\gamma_2 + \gamma_2^2(1+\alpha), & \text{if } \alpha > 1. \end{cases} \end{aligned} \tag{45}$$

Proof. See Appendix B. □

5.3. Low β region

In the low β region, the source-relay link has a better channel than the relay-destination link does. Similar to the result of Section 5.2, the first two eigenvalue moments of Ω are provided in the following proposition, which can be used to approximate C_{avg} in (40).

Proposition 5. *Suppose $\beta\gamma_2 = D$. As $\beta \to 0$ and D remains a constant, the first two moments of the eigenvalues λ in Ω converge to*

$$\begin{aligned} m_1 &= \gamma_1 + D, \\ m_2 &= 2\gamma_1^2 + 2\gamma_1 D + D^2(\alpha + 2). \end{aligned} \tag{46}$$

Proof. See Appendix C. □

5.4. Higher-order approximations for high and low β regions

In the previous two subsections, taking a first order approximation of the matrix $\mathbf{\Gamma} = \beta\mathbf{F}^H(\mathbf{I} + \beta\mathbf{F}\mathbf{F}^H)^{-1}\mathbf{F}$ resulted in simple expressions for the moments. We can also consider higher-order approximations, which provide finer expressions for the moments. These results are summarized in the following proposition, a proof of which is given in Appendix D. Note that m_1 and m_2 denote the first-order approximations given in Propositions 4 and 5, and $\widetilde{m}_1$ and $\widetilde{m}_2$ denote the expressions after considering higher-order terms. Note that, when β is large, we do not consider the case $\alpha = 1$ since the matrix $\mathbf{F}\mathbf{F}^H$ is at a critical point in this case, that is, for any $\alpha < 1$, $\mathbf{F}\mathbf{F}^H$ is of full rank almost surely; for any $\alpha > 1$, $\mathbf{F}\mathbf{F}^H$ is singular.

Proposition 6. *For sufficiently small β, one has*

$$\begin{aligned} \widetilde{m}_1 &= m_1 - \gamma_2\beta^2\left(1 + \frac{1}{\alpha}\right) + o(\beta^2), \\ \widetilde{m}_2 &= m_2 - 2\gamma_2\beta^2(\gamma_1 + \beta\gamma_2)\left(1 + \frac{1}{\alpha}\right) + o(\beta^2). \end{aligned} \tag{47}$$

For sufficiently large β and $\alpha < 1$, one has

$$\begin{aligned} \widetilde{m}_1 &= m_1 - \frac{\gamma_2\alpha^2}{\beta(1-\alpha)} + o\left(\frac{1}{\beta}\right), \\ \widetilde{m}_2 &= m_2 - \frac{2\gamma_2\alpha^2(\gamma_1 + \alpha\gamma_2)}{\beta(1-\alpha)} + o\left(\frac{1}{\beta}\right). \end{aligned} \tag{48}$$

For sufficiently large β and $\alpha > 1$, one has

$$\begin{aligned} \widetilde{m}_1 &= m_1 - \frac{\alpha\gamma_2}{\beta(\alpha-1)} + o\left(\frac{1}{\beta}\right), \\ \widetilde{m}_2 &= m_2 - \frac{2\gamma_2\alpha(\gamma_1 + \gamma_2)}{\beta(\alpha-1)} + o\left(\frac{1}{\beta}\right). \end{aligned} \tag{49}$$

Proof. See Appendix D. □

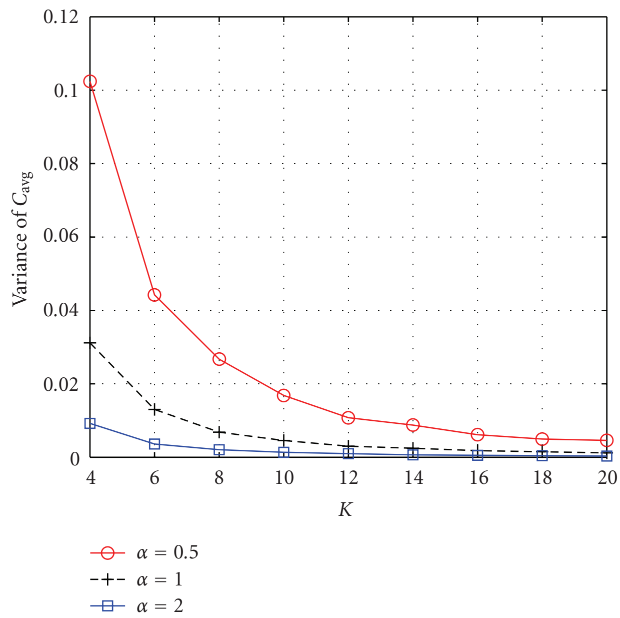

Figure 2: Variance of C_{avg} versus different K.

6. SIMULATION RESULTS

In this section, we provide simulation results to validate the analytical results derived in the previous sections. Figure 2 shows the variance of C_{avg} normalized by $E^2[C_{\text{avg}}]$ versus K. The configuration used here is $\gamma_1 = 1$, $\gamma_2 = 10$, $\beta = 1$, and $\alpha = 0.5/1/2$. For each value of K, we obtain the variance of C_{avg} by averaging over 1000 realizations of the random matrices, in which the elements are mutually independent complex Gaussian random variables. We can observe that the variance decreases rapidly as K increases. When K is larger than 10, the variance of C_{avg} is very small. This supports the validity of Assumption 1.

In the following simulations, we fix the value of K to be 40. All accurate values of average capacities C_{avg} are obtained from 1000 realizations of the random matrices. Again, the elements in these random matrices are mutually independent complex Gaussian random variables. All performance bounds and approximations are computed by the analytical results obtained in this paper.

Figure 3 compares the accurate average capacity obtained from (9) and the first three orders of approximation given in (44) with γ_1 ranging from 0.01 to 0.1. We set $\gamma_2 = \gamma_1$ and $\beta = 1$. From Figure 3, we observe that, in the low-SNR region, the approximations approach the correct values quite well. The reason is that the average capacity is approximately linear in the eigenvalues when SNR is small, which makes our expansions more precise. When the SNR becomes larger, the approximations can be used as bounds for the accurate values. (Notice that the odd orders of approximation provide upper bounds while the even ones provide lower bounds.)

In Figure 4, we plot the average capacity versus α, namely the ratio between the number of source nodes (or equivalently, destination nodes) and the number of relay nodes. The configuration is $\gamma_1 = 0.1$, $\gamma_2 = 10$, and $\beta = 10$.

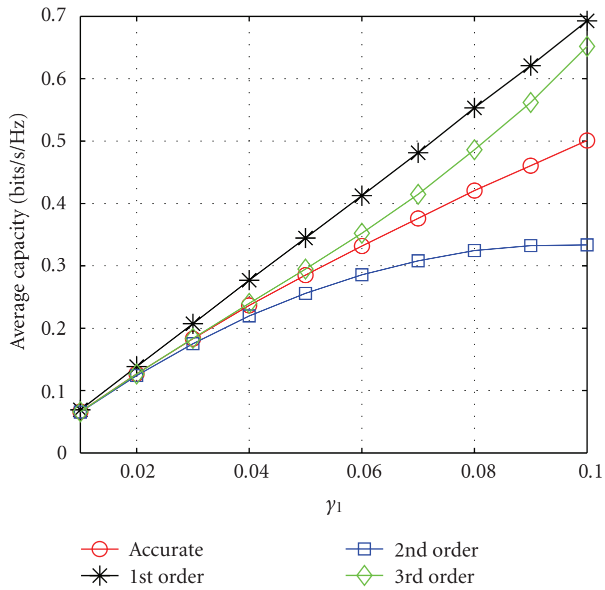

FIGURE 3: Comparison of different orders of approximation.

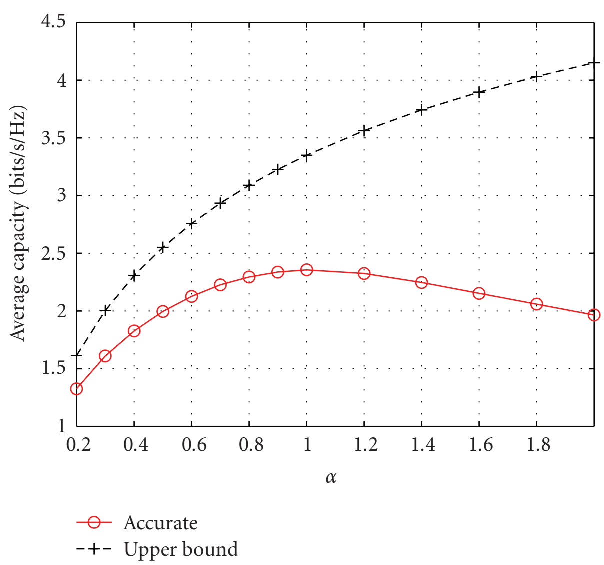

FIGURE 4: Performance versus various α.

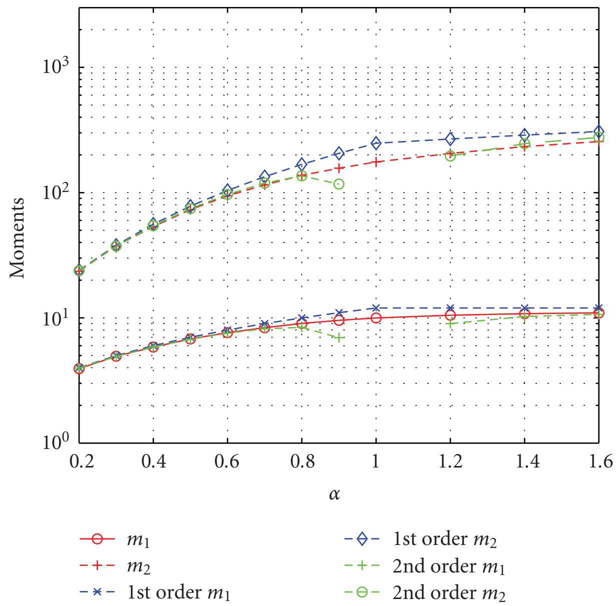

FIGURE 5: Eigenvalue moments versus various α in the high β region.

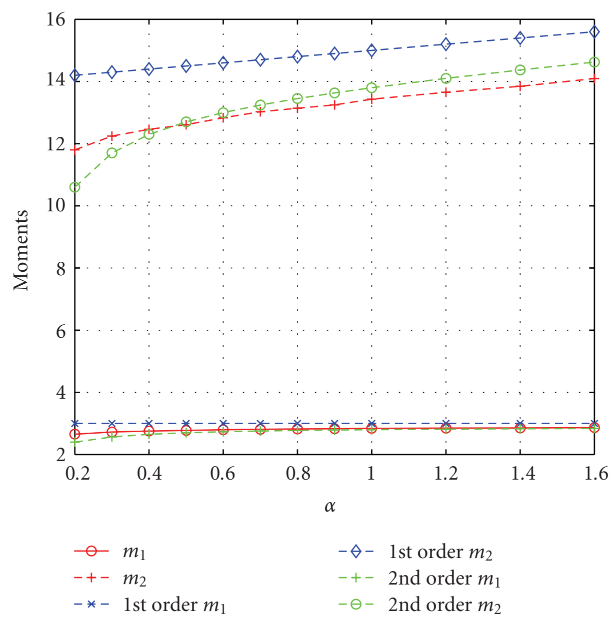

FIGURE 6: Eigenvalue moments versus various α in the low β region.

We observe that the average capacity achieves a maximum when $\alpha = 1$, namely, when using the same number of relay nodes as the source/destination nodes. A possible reason for this phenomenon is the normalization of elements in $\mathbf{H}$. (Recall that the variance of elements in $\mathbf{H}$ is $1/K$ such that the norms of column vectors in $\mathbf{H}$ are 1.) Now, suppose that M is fixed. When α is small, that is, K is large, the receive SNR at each relay node is small, which impairs the performance. When α is large, that is, K is small, we lose degrees of freedom. Therefore, $\alpha = 1$ achieves the optimal tradeoff. However, in practical systems, when the normalization is removed, it is always better to have more relay nodes if the corresponding cost is ignored. We also plot the upper bound in (32), which provides a loose upper bound here.

In Figures 5 and 6, we plot the precise values of m_1 and m_2 obtained from simulations and the corresponding first- and second-order approximations. The configuration is $\beta = 10$ (Figure 5) or $\beta = 0.1$ (Figure 6), $\gamma_1 = 2$ and $\gamma_2 = 10$. We can observe that the second-order approximation outperforms the first-order approximation except when α is close to 1 and β is large. (According to Proposition 6, the approximation diverges as $\alpha \to 1$ and $\beta \to \infty$.)

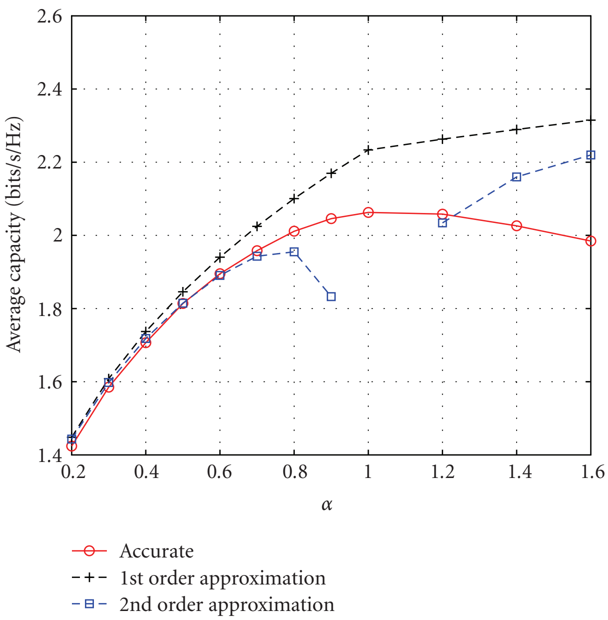

FIGURE 7: Performance versus various α in the high β region.

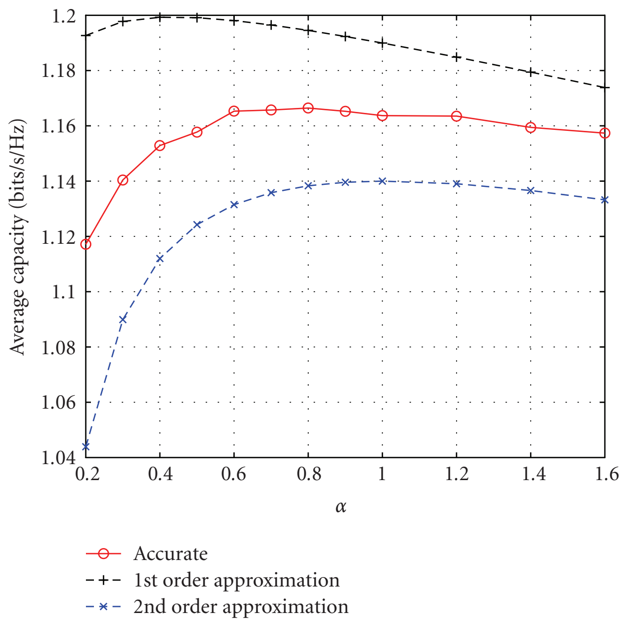

FIGURE 8: Performance versus various α in the low β region.

In Figure 7, we plot the average capacity versus α in the high β region, with configuration $\beta = 10$, $\gamma_1 = 2$, and $\gamma_2 = 10$. We can observe that the Taylor expansion provides a good approximation when α is small. Similar to Figure 7, the second-order approximation outperforms the first-order one except when α is close to 1. In Figure 8, we plot the average capacity versus α in the low β region. The configuration is the same as that in Figure 7 except that $\beta = 0.1$. We can observe that the Taylor expansion provides a good approximation for both small and large α. However, unlike the moment approximation, the error of the second-order approximation is not better than that of the first-order approximation. This is because (40) is also an approximation, and better approximation of the moments does not necessarily lead to a more precise approximation for the average capacity.

In Figure 9, we plot the ratio between the average capacity in (9) and the average capacity when the signal from the source to the destination in the first stage is ignored, as a function of the ratio γ_1/γ_2. We test four combinations of γ_2 and β. (Note that $\alpha = 0.5$.) We observe that the performance gain increases with the ratio γ_1/γ_2 (the channel gain ratio between source-destination link and source-relay link). The performance gain is substantially larger in the low-SNR regime ($\gamma_2 = 1$) than in the high-SNR regime ($\gamma_2 = 10$). When the amplification ratio β decreases, the performance gain is improved. Therefore, substantial performance gain is obtained by incorporating the source-destination link when the channel conditions of the source-destination link are comparable to those of the relay-destination link and the source-relay link, particularly in the low-SNR region. In other cases, we can simply ignore the source-destination link since it achieves marginal gain at the cost of having to process a high-dimensional signal.

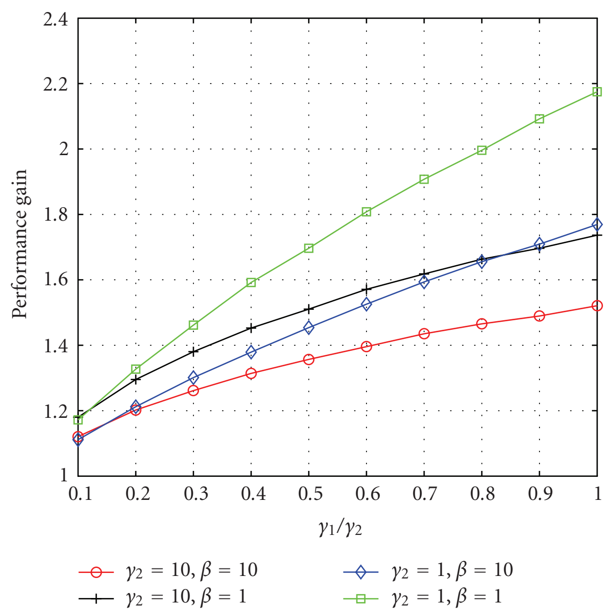

FIGURE 9: Performance gain by incorporating the source-destination link.

7. CONCLUSIONS

In this paper, we have used random matrix theory to analyze the asymptotic behavior of cooperative transmission with a large number of nodes. Compared to prior results of [23], we have considered the combination of relay and direct transmission, which is more complicated than considering relay transmission only. We have constructed a performance upper-bound for the low signal-to-noise-ratio regime, and have derived approximations for high and low relay-to-destination link qualities, respectively. The key idea has been to investigate the eigenvalue distributions related to capacity and to analyze eigenvalue moments for large wireless networks. We have also conducted simulations which validate the analytical results. Particularly, the numerical simulation results show that incorporating the direct link between the source nodes and destination nodes can substantially improve the performance when the direct link is of high quality. These results provide useful tools and insights for the design of large cooperative wireless networks.

APPENDICES

A. PROOF OF PROPOSITION 1

We first define some useful generating functions and transforms [22], and then use them in the proof by applying some conclusions of free probability theory [23].

A.1. Generating functions and transforms

For simplicity, we rewrite the matrix $\mathbf{\Omega}$ as

$$\Omega = \mathbf{G}^H\mathbf{G} + \mathbf{\Xi}\mathbf{\Gamma}\mathbf{\Xi}^H, \tag{A.1}$$

where $\mathbf{\Xi} \triangleq (1/\alpha)\mathbf{H}^H$ is an $M \times K$ matrix, in which the elements are independent random variables with variance $1/M$.

For a large random matrix with eigenvalue moments $\{m_i\}_{i=1,2,\ldots}$ and free cumulants $\{k_j\}_{j=1,2,\ldots}$, we define the following generating functions:

$$\Lambda(z) = 1 + \sum_{i=1}^{\infty} m_i z^i, \qquad C(z) = 1 + \sum_{j=1}^{\infty} k_j z^j. \tag{A.2}$$

We define the Stieltjes transform

$$m(z) = E\left[\frac{1}{\lambda - z}\right], \tag{A.3}$$

where λ is a generic (random) eigenvalue.

We also define a "Fourier transform" given by

$$D(z) = \frac{1}{z}(C(z) - 1)^{\langle -1 \rangle}, \tag{A.4}$$

which was originally defined in [25].

The following lemma provides some fundamental relations among the above functions and transforms.

Lemma 2. *For the generating functions and transforms in* (A.2)–(A.4)*, the following equations hold:*

$$\Lambda\left[\frac{zD(z)}{z+1}\right] = z + 1, \tag{A.5}$$

$$m\left[\frac{C(z)}{z}\right] = -z, \tag{A.6}$$

$$C(-m(z)) = -zm(z), \tag{A.7}$$

$$\Lambda(z) = -\frac{m(z^{-1})}{z}. \tag{A.8}$$

Note that we use subscripts to indicate the matrix for which the generating functions and transforms are defined. For example, for the matrix $\mathbf{M}$, the eigenvalue moment generating function is denoted by $\Lambda_{\mathbf{M}}(z)$.

A.2. Proof of Proposition 1

We first study the matrix $\mathbf{\Xi}\mathbf{\Gamma}\mathbf{\Xi}^H$ in (A.1). In order to apply the conclusions about matrix products, we can work on the matrix $\mathbf{J} = \mathbf{\Gamma}\mathbf{\Xi}^H\mathbf{\Xi}$ instead since we have the following lemma.

Lemma 3.

$$\Lambda_{\mathbf{\Xi}\mathbf{\Gamma}\mathbf{\Xi}^H}(z) - 1 = \frac{1}{\alpha}(\Lambda_{\Gamma\Xi^H\Xi}(z) - 1). \tag{A.9}$$

Proof. For any $n \in \mathcal{N}$, we have

$$\begin{aligned}\frac{1}{M}\text{trace}((\Xi\Gamma\Xi^H)^n) &= \frac{1}{M}\text{trace}((\Gamma\Xi^H\Xi)^n)\\ &= \frac{K}{M}\frac{1}{K}\text{trace}((\Gamma\Xi^H\Xi)^n).\end{aligned} \tag{A.10}$$

Letting $K, M \to \infty$, we obtain

$$m_n^{\Xi\Gamma\Xi^H} = \frac{1}{\alpha}m_n^{\Gamma\Xi^H\Xi}. \tag{A.11}$$

Then, we have

$$\begin{aligned}\Lambda_{\mathbf{\Xi}\mathbf{\Gamma}\mathbf{\Xi}^H}(z) - 1 &= \sum_{j=1}^{\infty} m_n^{\Xi\Gamma\Xi^H} z^n\\ &= \frac{1}{\alpha}\sum_{j=1}^{\infty} m_n^{\Gamma\Xi^H\Xi} z^n\\ &= \frac{1}{\alpha}(\Lambda_{\Gamma\Xi^H\Xi}(z) - 1).\end{aligned} \tag{A.12}$$

□

On denoting $\mathbf{\Xi}^H\mathbf{\Xi}$ by $\mathbf{B}$, the following lemma discloses the law of matrix product[22] and is equivalent to (27).

Lemma 4. *Based on the freeness assumption, for the matrix* $\mathbf{J} = \mathbf{\Gamma}\mathbf{B}$*, we have*

$$D_J(z) = D_\Gamma(z)D_B(z). \tag{A.13}$$

In order to use the "Fourier Transform," we need the following lemma.

Lemma 5. *For the matrix* $\mathbf{B}$*, we have*

$$D_B(z) = \frac{\alpha}{z + \alpha}. \tag{A.14}$$

Proof. Due to the definition of $\mathbf{\Xi}$, we have

$$\mathbf{\Xi}^H\mathbf{\Xi} = \frac{1}{\alpha}\mathbf{H}\mathbf{H}^H. \tag{A.15}$$

Then, it is easy to check that

$$\begin{aligned}m_n^{\Xi^H\Xi} &= \left(\frac{1}{\alpha}\right)^n m_n^{\mathbf{HH}^H},\\ k_n^{\Xi^H\Xi} &= \left(\frac{1}{\alpha}\right)^n k_n^{\mathbf{HH}^H},\end{aligned} \tag{A.16}$$

which is equivalent to

$$C_{\Xi^H\Xi}(z) = C_{\mathbf{HH}^H}\left(\frac{z}{\alpha}\right). \tag{A.17}$$

By applying the conclusion in [20], all free cumulants in $\mathbf{HH}^H$ are equal to α. Therefore,

$$C_{\Xi^H\Xi}(z) = C_{\mathbf{HH}^H}(z) = 1 + \frac{\alpha z}{1-z}. \tag{A.18}$$

The conclusion follows from computing the inverse function of $C_{\Xi^H\Xi}(z) - 1 = \alpha z/(\alpha - z)$. □

The following lemma relates $\Lambda_\Gamma(z)$ to $\mathcal{F}$. (Recall that $\mathcal{F}$ is the distribution of eigenvalues of the matrix $\mathbf{\Gamma}$.)

Lemma 6. *For the matrix $\mathbf{\Gamma}$, the following equation holds:*

$$\Lambda_\Gamma(z) - 1 = \int \frac{\tau z}{1-\tau z} d\mathcal{F}(\tau). \tag{A.19}$$

Proof. Based on the definition of $\Lambda_\Gamma(z)$, we have

$$\Lambda_\Gamma(z) - 1 = \sum_{j=1}^{\infty} m_j z^j = \sum_{j=1}^{\infty} E\left[\lambda^j z^j\right] = E\left[\sum_{j=1}^{\infty}(\lambda z)^j\right] = E\left[\frac{\lambda z}{1-\lambda z}\right], \tag{A.20}$$

from which the conclusion follows. □

Based on the above lemmas, we can show the following important lemma.

Lemma 7. *Based on the freeness assumption, for the matrix $\mathbf{\Xi\Gamma\Xi}^H$, we have*

$$C_{\Xi\Gamma\Xi^H}(z) = 1 + \frac{1}{\alpha}\int \frac{z\tau}{1-z\tau} d\mathcal{F}(\tau). \tag{A.21}$$

Proof. The lemma can be proved by showing the following series of equivalent equations:

$$C_{\Xi\Gamma\Xi^H}(z) = 1 + \frac{1}{\alpha}\int \frac{z\tau}{1-z\tau} d\mathcal{F}(\tau) \tag{A.22}$$

$$\Longleftrightarrow m_{\Xi\Gamma\Xi^H}(z) = \frac{1}{-z + (1/\alpha)\int\left(\tau/1 + \tau m_{\Xi\Gamma\Xi^H}(z)\right) d\mathcal{F}(\tau)} \tag{A.23}$$

$$\Longleftrightarrow \Lambda_{\Xi\Gamma\Xi^H}(z) = \frac{1}{1 - (1/\alpha)\int\left(z\tau/1 - \tau z\Lambda_{\Xi\Gamma\Xi^H}(z)\right) d\mathcal{F}(\tau)} \tag{A.24}$$

$$\Longleftrightarrow \Lambda_{\Xi\Gamma\Xi^H}(z) - \frac{1}{\alpha}\int \frac{z\tau\Lambda_{\Xi\Gamma\Xi^H}(z)}{1-\tau z\Lambda_{\Xi\Gamma\Xi^H}(z)} d\mathcal{F}(\tau) = 1 \tag{A.25}$$

$$\Longleftrightarrow \Lambda_{\Xi\Gamma\Xi^H}(z) - 1 = \frac{1}{\alpha}\left(\Lambda_\Gamma\left(z\Lambda_{\Xi\Gamma\Xi^H}(z)\right) - 1\right) \tag{A.26}$$

$$\Longleftrightarrow \Lambda_{\Gamma\Xi^H\Xi}(z) = \Lambda_\Gamma\left(z\left(\frac{1}{\alpha}\left(\Lambda_{\Gamma\Xi^H\Xi}(z) - 1\right)\right) + 1\right) \tag{A.27}$$

$$\Longleftrightarrow z + 1 = \Lambda_\Gamma\left(\frac{zD_{\Gamma\Xi^H\Xi}(z)}{z+1}\left(\frac{1}{\alpha}z + 1\right)\right) \tag{A.28}$$

$$\Longleftrightarrow z + 1 = \Lambda_\Gamma\left(\frac{zD_\Gamma(z)}{z+1}\right). \tag{A.29}$$

The equivalence of the above equations is explained as follows:

(i) substituting (A.6) into (A.22) yields (A.23);
(ii) substituting (A.8) into (A.23) yields (A.24);
(iii) equations (A.25) and (A.26) are equivalent due to Lemma 6;
(iv) equations (A.26) and (A.27) are equivalent due to Lemma 3;
(v) equations (A.27) and (A.28) are equivalent by substituting $z = zD_{\Gamma\Xi^H\Xi}(z)/(z+1)$ into (A.27) and applying (A.5);
(vi) equations (A.28) and (A.29) are equivalent due to Lemmas 4 and 5;
(vii) equation (A.29) holds due to (A.5). □

Based on Lemma 7, we can prove Proposition 1.

Proof. By applying (26) and the freeness assumption, we have

$$C_\Omega(z) = C_{G^HG}(z) + C_{\Xi\Gamma\Xi^H}(z)(z) - 1, \tag{A.30}$$

which implies

$$\frac{C_{G^HG}(z)}{z} = \frac{C_\Omega(z)}{z} - \frac{C_{\Xi\Gamma\Xi^H}(z)}{z} + \frac{1}{z}. \tag{A.31}$$

Taking both sides of (A.31) as arguments of $m_{G^HG}(z)$, we have

$$-z = m_{G^HG}\left(\frac{C_\Omega(z)}{z} - \frac{C_{\Xi\Gamma\Xi^H}(z)}{z} + \frac{1}{z}\right), \tag{A.32}$$

where the left-hand side is obtained from (A.6).

Letting $z = -m_\Omega(t)$ in (A.32), we have

$$\begin{aligned} m_\Omega(t) &= m_{G^HG}\left(\frac{C_\Omega(-m(t))}{-m(t)} - \frac{1 + (1/\alpha)\int\left(m_\Omega(t)\tau/(1 + m_\Omega(t)\tau)\right) d\mathcal{F}(\tau)}{-m_\Omega(t)} - \frac{1}{m_\Omega(t)}\right) \\ &= m_{G^HG}\left(t - \frac{1}{\alpha}\int \frac{\tau}{1 + m_\Omega(t)\tau} d\mathcal{F}(\tau)\right), \end{aligned} \tag{A.33}$$

where the first equation is based on (A.7). □

B. PROOF OF PROPOSITION 4

Proof. We first consider the matrix $\mathbf{\Gamma}' = \beta(\mathbf{I} + \beta\mathbf{FF}^H)^{-1}\mathbf{FF}^H$. When $K \geq M$, it is easy to check that $\mathbf{FF}^H$ is invertible almost surely since $\mathbf{F}$ is an $M \times K$ matrix. Then

$$\mathbf{\Gamma}' \longrightarrow \mathbf{I}, \tag{B.1}$$

as $\beta \to \infty$. Therefore, $m_p^{\Gamma'} = 1$, $\forall p \in \mathcal{N}$.

When $K \leq M$, let $\mathbf{F}\mathbf{F}^H = \mathbf{U}^H \mathbf{\Lambda} \mathbf{U}$, where $\mathbf{U}$ is unitary and $\mathbf{\Lambda}$ is diagonal. Then, we have

$$\begin{aligned} m_p^{\Gamma'} &= \frac{1}{M}\operatorname{trace}[(\mathbf{\Gamma}')^p] \\ &= \frac{1}{M}\operatorname{trace}[\beta(\mathbf{I}+\beta\mathbf{\Lambda})^{-p}\mathbf{\Lambda}^p] \\ &= \frac{K}{M}, \end{aligned} \tag{B.2}$$

where the last equation is due to the fact that only K elements in $\mathbf{\Lambda}$ are nonzero since $K \leq M$. Therefore, $m_p^{\Gamma'} = 1/\alpha$, $\forall p \in \mathcal{N}$.

Applying the same argument as in Lemma 3, we obtain

$$m_p^{\Gamma} = \begin{cases} 1, & \text{if } K \leq M, \\ \alpha, & \text{if } K \geq M, \end{cases} \quad \forall p \in \mathcal{N}, \tag{B.3}$$

which is equivalent to

$$\begin{aligned} k_1^{\Gamma} &= \begin{cases} 1, & \text{if } K \leq M, \\ \alpha, & \text{if } K \geq M, \end{cases} \\ k_2^{\Gamma} &= \begin{cases} 0, & \text{if } K \leq M, \\ \alpha - \alpha^2, & \text{if } K \geq M. \end{cases} \end{aligned} \tag{B.4}$$

Define $\mathbf{\Omega}_r' = \beta \mathbf{F}^H(\mathbf{I} + \beta\mathbf{F}\mathbf{F}^H)^{-1}\mathbf{F}\mathbf{H}\mathbf{H}^H$. Due to the law of the matrix product in Lemma 1, the free cumulants of $\mathbf{\Omega}_r'$ are given by

$$\begin{aligned} k_1^{\mathbf{\Omega}_r'} &= k_1^{\Gamma} k_1^{HH^H}, \\ k_2^{\mathbf{\Omega}_r'} &= k_2^{\Gamma}(k_1^{HH^H})^2 + k_2^{HH^H}(k_1^{\Gamma})^2. \end{aligned} \tag{B.5}$$

Then, combining (B.5), $k_1^{HH^H} = \alpha$ and $k_2^{HH^H} = \alpha$, we obtain

$$\begin{aligned} k_1^{\mathbf{\Omega}_r'} &= \begin{cases} \alpha^2, & \text{if } \alpha \leq 1, \\ \alpha, & \text{if } \alpha \geq 1, \end{cases} \\ k_2^{\mathbf{\Omega}_r'} &= \begin{cases} 2\alpha^3 - \alpha^4, & \text{if } \alpha \leq 1, \\ \alpha, & \text{if } \alpha \geq 1. \end{cases} \end{aligned} \tag{B.6}$$

which imply

$$\begin{aligned} m_1^{\mathbf{\Omega}_r'} &= \begin{cases} \alpha^2, & \text{if } \alpha \leq 1, \\ \alpha, & \text{if } \alpha \geq 1, \end{cases} \\ m_2^{\mathbf{\Omega}_r'} &= \begin{cases} 2\alpha^3, & \text{if } \alpha \leq 1, \\ \alpha + \alpha^2, & \text{if } \alpha \geq 1. \end{cases} \end{aligned} \tag{B.7}$$

Applying the same argument as in Lemma 3, we obtain

$$\begin{aligned} m_1^{\Omega_r} &= \begin{cases} \gamma_2\alpha, & \text{if } \alpha \leq 1, \\ \gamma_2, & \text{if } \alpha \geq 1, \end{cases} \\ m_2^{\Omega_r} &= \begin{cases} 2\gamma_2^2\alpha^2, & \text{if } \alpha \leq 1, \\ \gamma_2^2(1+\alpha), & \text{if } \alpha \geq 1. \end{cases} \end{aligned} \tag{B.8}$$

which is equivalent to

$$\begin{aligned} k_1^{\Omega_r} &= \begin{cases} \gamma_2\alpha, & \text{if } \alpha \leq 1, \\ \gamma_2, & \text{if } \alpha \geq 1, \end{cases} \\ k_2^{\Omega_r} &= \begin{cases} \gamma_2^2\alpha^2, & \text{if } \alpha \leq 1, \\ \gamma_2^2\alpha, & \text{if } \alpha \geq 1. \end{cases} \end{aligned} \tag{B.9}$$

The conclusion follows from the facts that $\forall p \in \mathcal{N}$, $k_p^{\Omega_s} = \gamma_1^p$ and $k_p^{\Omega} = k_p^{\Omega_s} + k_p^{\Omega_r}$. □

C. PROOF OF PROPOSITION 5

Proof. When $\beta \to 0$, we have (recall $D = \gamma_2\beta$)

$$\Omega = \gamma_1 \mathbf{G}^H\mathbf{G} + D\mathbf{H}^H\mathbf{F}^H\mathbf{F}\mathbf{H},$$

$$\begin{aligned} k_1^{F^HF} &= 1, \\ k_2^{F^HF} &= \frac{1}{\alpha}, \\ k_1^{HH^H} &= \alpha, \\ k_2^{HH^H} &= \alpha. \end{aligned} \tag{C.1}$$

Then, applying (B.5), we obtain

$$\begin{aligned} k_1^{F^HFHH^H} &= \alpha, \\ k_2^{F^HFHH^H} &= 2\alpha, \end{aligned} \tag{C.2}$$

which is equivalent to

$$\begin{aligned} m_1^{F^HFHH^H} &= \alpha, \\ m_2^{F^HFHH^H} &= \alpha^2 + 2\alpha. \end{aligned} \tag{C.3}$$

Then, for matrix $\mathbf{H}^H\mathbf{F}^H\mathbf{F}\mathbf{H}$, we have

$$\begin{aligned} m_1^{H^HF^HFH} &= 1, \\ m_2^{H^HF^HFH} &= \alpha + 2, \end{aligned} \tag{C.4}$$

which results in

$$\begin{aligned} k_1^{H^HF^HFH} &= 1, \\ k_2^{H^HF^HFH} &= \alpha + 1. \end{aligned} \tag{C.5}$$

The remaining part of the proof is the same as the proof of Proposition 4 in Appendix B. □

D. PROOF OF PROPOSITION 6

We first prove the following lemma which provides the impact of perturbation on m_1^{Γ} and m_2^{Γ}. We use $\widetilde{X}$ to represent the perturbed version of the quantity X.

Lemma 8. *Suppose the first and second moments of the matrix* $\mathbf{\Gamma}$ *are perturbed by small* δ_1 *and* δ_2*, respectively, where* δ_1 *and* δ_2 *are of the same order* $O(\delta)$*, namely,*

$$\tilde{m}_1^{\Gamma} = m_1^{\Gamma} + \delta_1, \qquad \tilde{m}_2^{\Gamma} = m_2^{\Gamma} + \delta_2. \tag{D.1}$$

Then, we have

$$\begin{aligned}\tilde{m}_1^{\Omega} &= m_1^{\Omega} + \gamma_2\delta_1,\\ \tilde{m}_2^{\Omega} &= m_2^{\Omega} + \alpha\gamma_2^2\delta_2 + 2\gamma_2\left(k_1^{\mathbf{\Omega}'_r}\gamma_2 - m_1^{\Omega_r} + k_1^{\Omega} + (1-\alpha)k_1^{\Gamma}\gamma_2\right)\delta_1 + o(\delta),\end{aligned} \tag{D.2}$$

where

$$\mathbf{\Omega}'_r = \beta\mathbf{F}^H\left(\mathbf{I} + \beta\mathbf{F}\mathbf{F}^H\right)^{-1}\mathbf{F}\mathbf{H}\mathbf{H}^H. \tag{D.3}$$

Proof. We begin from $\tilde{k}_1^{\Gamma}$ and $\tilde{k}_2^{\Gamma}$. Suppose small perturbations ϵ_1 and ϵ_2, which are both of order $O(\epsilon)$, are placed on k_1^{Γ} and k_2^{Γ}, namely,

$$\tilde{k}_1^{\Gamma} = k_1^{\Gamma} + \epsilon_1, \qquad \tilde{k}_2^{\Gamma} = k_2^{\Gamma} + \epsilon_2. \tag{D.4}$$

We have

$$\tilde{k}_1^{\mathbf{\Omega}'_r} = k_1^{\mathbf{\Omega}'_r} + \alpha\epsilon_1, \qquad \tilde{k}_2^{\mathbf{\Omega}'_r} = k_2^{\mathbf{\Omega}'_r} + \alpha^2\epsilon_2 + 2\alpha k_1^{\Gamma}\epsilon_1 + o(\epsilon), \tag{D.5}$$

which implies

$$\tilde{m}_1^{\mathbf{\Omega}'_r} = m_1^{\mathbf{\Omega}'_r} + \alpha\epsilon_1, \qquad \tilde{m}_2^{\mathbf{\Omega}'_r} = m_2^{\mathbf{\Omega}'_r} + \alpha^2\epsilon_2 + 2\alpha\left(k_1^{\Gamma} + k_1^{\mathbf{\Omega}'_r}\right)\epsilon_1 + o(\epsilon). \tag{D.6}$$

For $\mathbf{\Omega}_r = \gamma_2\beta\mathbf{H}^H\mathbf{F}^H\left(\mathbf{I} + \beta\mathbf{F}\mathbf{F}^H\right)^{-1}\mathbf{F}\mathbf{H}$, we have

$$\tilde{m}_1^{\Omega_r} = m_1^{\Omega_r} + \gamma_2\epsilon_1, \qquad \tilde{m}_2^{\Omega_r} = m_2^{\Omega_r} + \alpha\gamma_2^2\epsilon_2 + 2\gamma_2^2\left(k_1^{\Gamma} + k_1^{\Omega'_r}\right)\epsilon_1 + o(\epsilon), \tag{D.7}$$

which implies that we have

$$\tilde{k}_1^{\Omega_r} = k_1^{\Omega_r} + \gamma_2\epsilon_1, \qquad \tilde{k}_2^{\Omega_r} = k_2^{\Omega_r} + \alpha\gamma_2^2\epsilon_2 + 2\gamma_2\left(k_1^{\Gamma}\gamma_2 + k_1^{\Omega'_r}\gamma_2 - m_1^{\Omega_r}\right)\epsilon_1 + o(\epsilon). \tag{D.8}$$

Then, for $\mathbf{\Omega}$, we have

$$\tilde{k}_1^{\Omega} = k_1^{\Omega} + \gamma_2\epsilon_1, \qquad \tilde{k}_2^{\Omega} = k_2^{\Omega} + \alpha\gamma_2^2\epsilon_2 + 2\gamma_2\left(k_1^{\Gamma}\gamma_2 + k_1^{\Omega'_r}\gamma_2 - m_1^{\Omega_r}\right)\epsilon_1 + o(\epsilon), \tag{D.9}$$

which implies

$$\begin{aligned}\tilde{m}_1^{\Omega} &= m_1^{\Omega} + \gamma_2\epsilon_1,\\ \tilde{m}_2^{\Omega} &= m_2^{\Omega} + \alpha\gamma_2^2\epsilon_2 + 2\gamma_2\left(k_1^{\Gamma}\gamma_2 + k_1^{\Omega'_r}\gamma_2 - m_1^{\Omega_r} + k_1^{\Omega}\right)\epsilon_1 + o(\epsilon).\end{aligned} \tag{D.10}$$

Now, we compute ϵ_1 and ϵ_2. Equation (D.1) implies

$$\tilde{k}_1^{\Gamma} = k_1^{\Gamma} + \delta_1, \qquad \tilde{k}_2^{\Gamma} = k_2^{\Gamma} + \delta_2 - 2m_1^{\Gamma}\delta_1 + o(\delta), \tag{D.11}$$

which is equivalent to

$$\epsilon_1 = \delta_1, \qquad \epsilon_2 = \delta_2 - 2m_1^{\Gamma}\delta_1. \tag{D.12}$$

Combining (D.10) and (D.12), we obtain (D.2). □

Based on Lemma 8, we can obtain the following lemma, where δ_1 and δ_2 are defined the same as in Lemma 8. The proof is straightforward by applying the intermediate results in the proofs of Propositions 4 and 5.

Lemma 9. *For sufficiently high* β*,* (D.2) *is equivalent to*

$$\tilde{m}_1^{\Omega} = m_1^{\Omega} + \gamma_2\delta_1, \qquad \tilde{m}_2^{\Omega} = m_2^{\Omega} + \alpha\gamma_2^2\delta_2 + 2\gamma_2\left(\alpha\gamma_2 + \gamma_1\right)\delta_1 + o(\delta), \quad \text{when } \alpha \le 1, \tag{D.13}$$

or

$$\tilde{m}_1^{\Omega} = m_1^{\Omega} + \gamma_2\delta_1, \qquad \tilde{m}_2^{\Omega} = m_2^{\Omega} + \alpha\gamma_2^2\delta_2 + 2\gamma_2\left(\gamma_1 + \gamma_2\right)\delta_1 + o(\delta), \quad \text{when } \alpha \ge 1. \tag{D.14}$$

For sufficiently small β*, we have*

$$\tilde{m}_1^{\Omega} = m_1^{\Omega} + \gamma_2\delta_1, \qquad \tilde{m}_2^{\Omega} = m_2^{\Omega} + \alpha\gamma_2^2\delta_2 + 2\gamma_2\left(\gamma_1 + \beta\gamma_2\right)\delta_1 + o(\delta). \tag{D.15}$$

Now, we can prove the proposition by computing explicit expressions of δ_1 and δ_2.

Proof. We note that

$$E\left[\lambda^{\Gamma}\right] = \alpha E\left[\frac{\beta\lambda^{FF^H}}{1 + \beta\lambda^{FF^H}}\right], \tag{D.16}$$

which has been addressed in (37).

When β is sufficiently small, we have

$$E\left[\frac{\beta\lambda^{FF^H}}{1 + \beta\lambda^{FF^H}}\right] = \beta E\left[\lambda^{FF^H}\left(1 - \beta\lambda^{FF^H}\right) + o(\beta)\right] = \beta\left(\frac{1-\beta}{\alpha} - \frac{\beta}{\alpha^2}\right) + o(\beta), \tag{D.17}$$

where we have applied the facts that $E[\lambda^{FF^H}] = 1/\alpha$ and $E[(\lambda^{FF^H})^2] = 1/\alpha + 1/\alpha^2$. This implies

$$\delta_1 = -\beta^2\left(1+\frac{1}{\alpha}\right) + o(\beta). \tag{D.18}$$

Now, we consider the case of large β, for which we have

$$E\left[\frac{\beta\lambda^{FF^H}}{1+\beta\lambda^{FF^H}}\right] = E\left[\frac{1}{1/\beta\lambda^{FF^H}} \mid \lambda^{FF^H} > 0\right] = 1 - E\left[\frac{1}{\beta\lambda^{FF^H}} \mid \lambda^{FF^H} > 0\right] + o\left(\frac{1}{\beta}\right). \tag{D.19}$$

Therefore, we have

$$\delta_1 = -\alpha E\left[\frac{1}{\beta\lambda^{FF^H}} \mid \lambda^{FF^H} > 0\right] + o\left(\frac{1}{\beta}\right). \tag{D.20}$$

Then, we need to compute $E[1/\beta\lambda^{FF^H} \mid \lambda^{FF^H} > 0]$. An existing result for an $m \times n$ $(m > n)$ large random matrix $\mathbf{X}$ having independent elements and unit-norm columns is [26]

$$E\left[\frac{1}{\lambda^{X^H X}}\right] = \frac{1}{1-n/m}. \tag{D.21}$$

We apply (D.21) to (D.20). When $\alpha < 1$ $(M \leq K)$, all $\lambda^{FF^H} > 0$ almost surely. Therefore

$$E\left[\frac{1}{\beta\lambda^{FF^H}} \mid \lambda^{FF^H} > 0\right] = E\left[\frac{1}{\beta\lambda^{FF^H}}\right] = E\left[\frac{\alpha}{\beta\lambda^{\hat{F}^H\hat{F}}}\right] = \frac{\alpha}{\beta\alpha(1-\alpha)}, \tag{D.22}$$

where $\hat{\mathbf{F}} \triangleq \sqrt{\alpha}\mathbf{F}^H$ is a $K \times M$ matrix and $\mathbf{F}\mathbf{F}^H = (1/\alpha)\hat{\mathbf{F}}^H\hat{\mathbf{F}}$. This is equivalent to

$$\delta_1 = -\frac{\alpha^2}{\beta(1-\alpha)} + o\left(\frac{1}{\beta}\right). \tag{D.23}$$

When $\alpha > 1$ $(M > K)$, we have

$$P(\lambda^{FF^H} > 0) = \frac{1}{\alpha}. \tag{D.24}$$

Note that $\mathbf{F}^H\mathbf{F}$ is of full rank when $\alpha > 1$. Then we have

$$E\left[\frac{1}{\beta\lambda^{FF^H}} \mid \lambda^{FF^H} > 0\right] = \frac{1}{\alpha}E\left[\frac{1}{\beta\lambda^{F^H F}}\right] = \frac{1}{\alpha\beta}\frac{1}{1-1/\alpha} = \frac{1}{\beta(\alpha-1)}, \tag{D.25}$$

which implies

$$\delta_1 = -\frac{\alpha}{\beta(\alpha-1)} + o\left(\frac{1}{\beta}\right). \tag{D.26}$$

It is easy to verify that $\delta_2 = o(\beta^2)$ for small β and $\delta_2 = o(1/\beta)$ for large β. This concludes the proof. □

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