

Research Article

Cooperative Communications over Flat Fading Channels with Carrier Offsets: A Double-Differential Modulation Approach

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We propose double-differential (DD) modulation for the amplify-and-forward protocol over Nakagami- m fading channels with carrier offsets. We propose an emulated maximum ratio combining (EMRC) decoder, which could be used by the double-differential receiver in the absence of exact channel knowledge. Approximate bit error rate (BER) analysis is performed for the proposed double-differential modulation-based cooperative communication system. The proposed double-differential system is immune to random carrier offsets, whereas the conventional single-differential modulation-based cooperative system breaks down. In addition, the proposed scheme is able to perform better than the same rate training-based cooperative system which utilizes training data for finding estimates of carrier offsets and channel gains.

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1. INTRODUCTION

Cooperative communications has several promising features to become a main technology in future wireless communications systems. It has been shown in the literature [1, 2] that the cooperative communication can avoid the difficulties of implementing actual antenna arrays and convert the single-input single-output (SISO) system into a virtual multiple-input multiple-output (MIMO) system. In this way, cooperation between the users allows them to exploit the diversity gain and other advantages of MIMO system in an SISO wireless network. Most of the existing works within cooperative communications assume that there is no carrier offset present over any link [1–3]. However, this assumption is not justified as cooperative communications are proposed for wireless mobile system, where the mobile users are *moving*. Moreover, the transmit and receive oscillators can never achieve perfect matching. Another practically infeasible assumption is that all nodes in the cooperative network have perfect knowledge about the channel coefficients of all the links in the network. Several single-differential strategies for cooperative communications have been proposed so far to avoid the estimation of the channel coefficients at the receiver side [4–7]. All of these conventional differential

schemes assume that the channel is constant over at least *two* consecutive time intervals. However, in the presence of carrier offsets, the flat fading wireless channel does *not* remain constant over two consecutive time intervals and these differential schemes experience substantial performance loss.

Double-differential (DD) modulation [8–11] is a key differential technique to remove the effect of carrier offset from the communication system. It differs from single-differential modulation in a sense that the decoder uses *three* consecutively received data samples for decoding the current symbol. Two levels of single-differential modulation are employed at the transmitter as shown in Figure 2(a) and a simple heuristic decoder [9, Equation (15)] is used at the receiver to find the estimate of the transmitted data as shown in Figure 2(b). It has been shown in [9, Section III] that the heuristic decoder coincides with the maximum likelihood decoder (MLD) under the assumption that the product of two zero-mean white circularly symmetric Gaussian noise terms in the decision variable is also zero-mean white circularly symmetric Gaussian. Symbol error rate (SER) expressions for the double-differentially modulated data over Rayleigh and Ricean fading SISO channels with carrier offsets are provided in [11]. In [12], a double-differential orthogonal space-time block code for time-selective MIMO channels

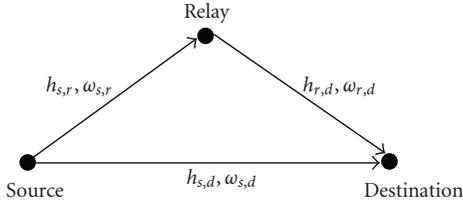


FIGURE 1: Cooperative communication system with carrier offsets.

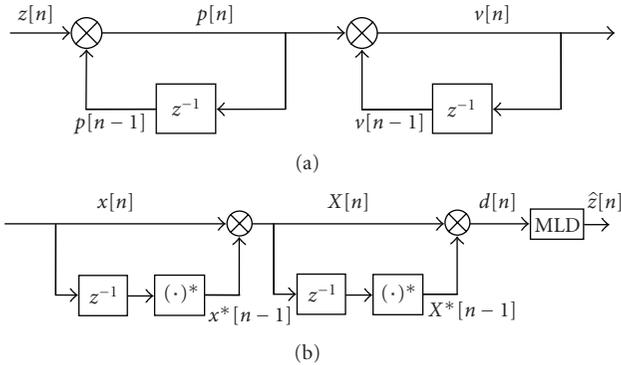


FIGURE 2: Double-differential (a) encoder and (b) decoder.

is proposed. A distributed double-differential modulation based on [12] with *regenerative* relays over cooperative Rayleigh channels is proposed and an upper bound of the pairwise error performance (PEP) is obtained in [13]. It is assumed in [13] that the flat-fading part of the time-varying channel remains constant over 10^4 time intervals [13, Sections IV and VI].

In this paper, we consider DD modulation for cooperative communications with *nonregenerative* relays over flat-fading Nakagami- m channels with random carrier offsets. The Nakagami- m family of distributions [14], also known as “ m -distribution,” contains Rayleigh fading ($m = 1$) as a special case; along with cases of fading that are more severe than Rayleigh ($1/2 \leq m < 1$) as well as cases less severe than Rayleigh ($m > 1$). In contrast to [13], the proposed scheme works with the basic assumption of DD modulation for SISO system [8–11] that the flat-fading part of the time-varying channel is constant over at least *three* time intervals.

The main contributions of this paper are the following. (1) A double-differential modulation-based amplify-and-forward (AAF) cooperative wireless communication is proposed to improve the performance over flat-fading Nakagami- m channels with random carrier offsets. (2) Approximate analytical bit error rate (BER) of the DD modulation with AAF protocol (DDAAF) is obtained. (3) Based on the approximate BER analysis, we determine the numerical power allocation for the DD cooperative systems.

The rest of this paper is organized as follows. In Section 2, the system model, channel model, and DD modulation for an SISO link are discussed. Section 3 implements DD modulation in the AAF cooperative communications. The approximate BER performance expressions for DD modulation with AAF protocol are found in Section 4.

Training-based cooperative communication is discussed in Section 5. In Section 6, the analytical and simulation results are discussed and details of numerical power allocation for DDAAF cooperative system is provided. Section 7 contains some conclusions. The article contains two appendices, which provide details of the derivations.

2. SYSTEM MODEL

We consider a basic cooperative communication system, which consists of one source, one relay, and one destination terminal as shown in Figure 1. Each of them can either transmit or receive a signal at a time. The transmission of the data from the source to the destination terminal is furnished in two phases. In the first phase, the source broadcasts data to the destination and the relay. The relay amplifies the received data and retransmits it to the destination, in the second phase. To avoid the interference, source and relay use orthogonal channels for transmission [3]. For ease of presentation, we assume that in both phases, the source and relay transmit stream of data through time-division multiplexing (TDMA). In the TDMA scheme, the source has to remain silent in the second phase in order to maintain the orthogonality between the transmissions. However, in the frequency-division multiplexing (FDMA) or the code-division multiplexing (CDMA) schemes, the source and the relay can transmit simultaneously.

2.1. Channel model

All links are assumed to be Nakagami- m distributed with the following probability density function (pdf) [15, Equation (2.21)]:

$$f_{\gamma_{p,q}}(\gamma_{p,q}) = \frac{m_{p,q}^{m_{p,q}} \gamma_{p,q}^{m_{p,q}-1}}{\bar{\gamma}_{p,q}^{m_{p,q}} \Gamma(m_{p,q})} \exp\left(-\frac{m_{p,q} \gamma_{p,q}}{\bar{\gamma}_{p,q}}\right), \quad \gamma_{p,q} \geq 0, \quad (1)$$

where $\Gamma(\cdot)$ is the gamma function [16, Equation (8.310.1)], $m_{p,q} \geq 1/2$ is the Nakagami- m fading parameter, $\gamma_{p,q} = P_p |h_{p,q}|^2 / \sigma^2$ is the instantaneous signal to noise ratio (SNR), $P_p \in \{P_s, P_r\}$ is the power transmitted by source or relay with $P_s = P_1$ and $P_r = P_2$, $h_{p,q}$ is a *zero-mean* Nakagami- m channel coefficient, σ^2 is the variance of the additive white Gaussian noise (AWGN), and $\bar{\gamma}_{p,q} = P_p \sigma_{p,q}^2 / \sigma^2$ is the average SNR over the link between p and q terminals in the cooperative system, where $\sigma_{p,q}^2 = E\{|h_{p,q}|^2\}$ is channel variance and $E\{\cdot\}$ represents expectation. If we represent the source by s , the relay by r , and the destination by d , then $(p, q) \in \{(s, d), (s, r), (r, d)\}$. The channel of each link is assumed to be a block fading channel, which remains constant for *at least three* consecutive time intervals and all the channel coefficients are assumed to be independent of each other. It is assumed that all three links are perturbed by different carrier offsets $\omega_{p,q} = 2\pi f_{p,q} T_s$ [17], where $f_{p,q}$ is the physical carrier frequency offset (CFO) in Hertz, $f_{p,q} \in [-1/(2T_s), 1/(2T_s))$, and T_s is the sampling period in seconds. Apparently, $\omega_{p,q} \in [-\pi, \pi)$ and the maximum value of $\omega_{p,q}$ corresponds to the offset of 50% of the carrier.

We assume that the carrier offsets $\omega_{p,q}$ are random and uniformly distributed over $[-\pi, \pi)$, however, in general, there is no restriction over the probability distribution of the carrier offsets and they could have any probability distribution. We have assumed that these carrier offsets remain fixed for *at least three* consecutive time intervals. The presence of carrier offsets makes all three block-fading channels behave as *time-varying channels*, which do *not* remain stationary over two consecutive time intervals. Since the phase term $e^{j\omega_{p,q}n}$ is multiplied with the channel coefficient $h_{p,q}$, the effective channel is time-varying even though $\omega_{p,q}$ and $h_{p,q}$ stay constant for the same three consecutive time instants.

2.2. Double-differential modulation [8–11]

Let $z[n]$ denote the symbols belonging to the unit-norm M -PSK constellation \mathcal{A} to be transmitted at the time n . In a DD modulation-based system [8–11], the transmitted signal $v[n]$ is obtained from $z[n]$ as shown in Figure 2(a):

$$\begin{aligned} p[n] &= p[n-1]z[n], \\ v[n] &= v[n-1]p[n], \quad n = 2, 3, \dots, \end{aligned} \quad (2)$$

with $v[0] = v[1] = p[1] = 1$. As $|z[n]| = 1$ for the unit-norm M -PSK symbols, it follows from (2) that $|v[n]| = |p[n]| = 1$. We consider a flat fading SISO channel with carrier offset described by

$$x[n] = \sqrt{\rho} h e^{j\omega n} v[n] + e[n], \quad n = 0, 1, \dots, \quad (3)$$

where $x[n]$ is the received signal, ρ is the transmitted signal power, h is the channel gain, $e[n]$ is complex-valued AWGN noise, and $\omega \in [-\pi, \pi)$ is the unknown frequency offset. The receiver makes a decision variable, $d[n] = X[n]X^*[n-1]$, where $X[n] = x[n]x^*[n-1]$ as shown in Figure 2(b). The decoding of $z[n]$ is performed in the following way [9, Equation (15)]:

$$\hat{z}[n] = \arg \max_{z \in \mathcal{A}} \text{Re}\{X[n]X^*[n-1]z^*\}. \quad (4)$$

The decoding of (4) corresponds to maximum-likelihood decoding (MLD) in [9] under the assumption that the product of two zero-mean white circularly symmetric Gaussian noise terms in the decision variable is also zero-mean white circularly symmetric Gaussian.

3. DOUBLE-DIFFERENTIAL MODULATION FOR AAF COOPERATIVE COMMUNICATION SYSTEM

If we use DD modulation in the cooperative communication system, the data received during the *first phase* at the destination is

$$x_{s,d}[n] = \sqrt{P_1} h_{s,d} e^{j\omega_{s,d}n} v[n] + e_{s,d}[n], \quad n = 0, 1, \dots, \quad (5)$$

and at the relay is

$$x_{s,r}[n] = \sqrt{P_1} h_{s,r} e^{j\omega_{s,r}n} v[n] + e_{s,r}[n], \quad n = 0, 1, \dots, \quad (6)$$

where P_1 is the power transmitted by the source, $h_{s,d}$ and $h_{s,r}$ are the channel gains, and $\omega_{s,d}$ and $\omega_{s,r}$ are the carrier

offsets between source and destination, and source and relay, respectively, and $e_{s,d}[n]$ and $e_{s,r}[n]$ are complex-valued circular symmetric AWGN noise on the two links. During the *second phase*, the relay amplifies the received data of (6) and retransmits such that the received signal by the destination in the second phase is

$$x_{r,d}[l] = \sqrt{\hat{P}_2} h_{r,d} e^{j\omega_{r,d}l} x_{s,r}[l] + e_{r,d}[l], \quad l = 0, 1, \dots, \quad (7)$$

where l is the time index which is used in the place of n to show the difference in time of first and second phases, $h_{r,d}$ is the channel gain, $\omega_{r,d}$ is the carrier offset between relay and destination, $e_{r,d}[l]$ is the AWGN noise, and \hat{P}_2 is the amplification factor which ensures constant average transmission power during the second phase. It can be seen from (6) that the average power of $x_{s,r}[l]$ is $P_1\sigma_{s,r}^2 + \sigma^2$, where $\sigma_{s,r}^2$ is variance of $h_{s,r}$, hence, \hat{P}_2 is given by

$$\hat{P}_2 = \frac{P_2}{P_1\sigma_{s,r}^2 + \sigma^2}, \quad (8)$$

where P_2 is the average power transmitted by the relay. It is also assumed that $P_1 + P_2 = P$, where P is the total transmitted power.

Next, we propose the following maximal ratio combining (MRC) [18] based scheme for a DDAAF receiver

$$\begin{aligned} d[k] &= \alpha_1 (x_{s,d}[n]x_{s,d}^*[n-1]) (x_{s,d}[n-1]x_{s,d}^*[n-2])^* \\ &\quad + \alpha_2 (x_{r,d}[l]x_{r,d}^*[l-1]) (x_{r,d}[l-1]x_{r,d}^*[l-2])^*, \end{aligned} \quad (9)$$

where $k = n = l$, that is, the data received by the destination during the same time interval with respect to the beginning of each phase is combined, and α_1 and α_2 are given by

$$\begin{aligned} \alpha_1 &= \frac{1}{(2P_1 |h_{s,d}|^2 + \sigma^2)\sigma^2}, \\ \alpha_2 &= \frac{(P_1\sigma_{s,r}^2 + \sigma^2)^2}{\kappa}, \end{aligned} \quad (10)$$

where

$$\begin{aligned} \kappa &= 2P_1P_2^2 |h_{r,d}|^4 |h_{s,r}|^2 \sigma^2 + 2P_1P_2 \\ &\quad \times (P_1\sigma_{s,r}^2 + \sigma^2) |h_{r,d}|^2 |h_{s,r}|^2 \sigma^2 + P_2^2 |h_{r,d}|^4 \sigma^4 + 2P_2 \\ &\quad \times (P_1\sigma_{s,r}^2 + \sigma^2) |h_{r,d}|^2 \sigma^4 + (P_1\sigma_{s,r}^2 + \sigma^2)^2 \sigma^4. \end{aligned} \quad (11)$$

The normalization factors can be found as $\alpha_1 = 1/E_1$ and $\alpha_2 = 1/E_2$, where E_1 and E_2 are the average noise powers of $X_{s,d}[n] = x_{s,d}[n]x_{s,d}^*[n-1]$ and $X_{r,d}[n] = x_{r,d}[n]x_{r,d}^*[n-1]$, respectively, as we intend to use DD modulation, the destination and relay are not expected to have knowledge of the exact channel coefficients, therefore, we can use emulated maximum ratio combining (EMRC) by replacing the channel coefficients $|h_{s,r}|^2$, $|h_{r,d}|^2$, and $|h_{s,d}|^2$ by their variances $\sigma_{s,r}^2$, $\sigma_{r,d}^2$, and $\sigma_{s,d}^2$, respectively, in (10). Then, the data is decoded as

$$\hat{z}[n] = \arg \max_{z \in \mathcal{A}} \text{Re}\{d[k]z^*\}, \quad (12)$$

where $n = k$. It is shown by simulation in Section 6.2 that the EMRC scheme performs very close to the MRC scheme from moderate to high SNR. Hence, the approximation of using the variances in the place of instantaneous channel values is reasonable from moderate to high SNR.

4. BER PERFORMANCE ANALYSIS

The EMRC obtained by replacing channel gains by their variances in (9) will perform worse than the ideal MRC scheme given by (9) and (10) [18]. For simplicity, we assume that the instantaneous signal-to-noise ratio (SNR) of the EMRC scheme is

$$\gamma = \gamma_{s,d} + \gamma_{s,r,d}, \quad (13)$$

where $\gamma_{s,d}$ and $\gamma_{s,r,d}$ are the instantaneous SNRs of the direct link between source and destination (s, d), and cooperative link between source and destination through relay (s, r, d), respectively. This assumption is justified by the simulation results in Section 6.2 as the EMRC scheme performs close to the ideal MRC scheme.

4.1. Analogy between double-differential and single-differential modulations

In single-differential modulation, $p[n]$ is obtained from $z[n]$ as shown in the first line of (2) with $p[0] = 1$. The received data when $p[n]$ was sent over a channel h without carrier offset is

$$x[n] = \sqrt{\rho}hp[n] + e[n], \quad n = 0, 1, \dots \quad (14)$$

The ML decoding of $z[n]$ is performed as follows [10]:

$$\hat{z}[n] = \arg \max_{z \in \mathcal{A}} \text{Re}\{x[n]x^*[n-1]z^*\}. \quad (15)$$

It can be seen by comparing (4) and (15) that the decoding of *double-differentially* modulated signal depends upon $X[n]$ in the similar manner as the decoding of *single-differentially* modulated signal depends upon $x[n]$. Therefore, we can approximate the performance of DDMPK by the BER expressions of DMPSK with the SNR of $X[n]$ under the assumption that the product of two zero-mean white circularly symmetric Gaussian noise terms in $X[n]$ is also zero-mean white circularly symmetric Gaussian. This connection is shown in more detail in [8, 10, 11]. It can be shown using (3) that

$$X[n] = \rho e^{\omega} |h|^2 p[n] + \sqrt{\rho} h e^{\omega n} v[n] e^* [n-1] + \sqrt{\rho} h^* e^{-\omega n} v^* [n-1] e[n] + e[n] e^* [n-1]. \quad (16)$$

From (16), we can find the SNR of $X[n]$ as

$$\frac{E_s}{E_N} = \frac{\gamma'}{2 + (\gamma')^{-1}}, \quad (17)$$

where E_s is the signal power, E_N is the total noise power, and γ' is SNR of $x[n]$ in (12). We may further take the following high SNR approximation to maintain the mathematical feasibility of the analysis:

$$\frac{\gamma'}{2 + (\gamma')^{-1}} \approx \frac{\gamma'}{2} - \frac{1}{4}. \quad (18)$$

As a cross-check, we have compared the exact and approximate SNRs in Figure 3, and it is satisfying to see that the approximate SNR follows closely the exact one for ($\gamma' \geq 5$) dB, which is the region of γ' values of most practical interest.

4.2. Average BER of DDAAF system

From (6), (7), and (8), it can be shown that the SNR of the cooperative link between source and destination through relay (s, r, d) under double-differential modulation is given by

$$\gamma_{s,r,d} = \frac{P_1^2 P_2^2 |h_{s,r}|^4 |h_{r,d}|^4}{\kappa}, \quad (19)$$

where κ is defined in (11). After some manipulations it can be shown that

$$\gamma'_{s,r,d} = \frac{\gamma'_{s,r,d}}{2 + (\gamma'_{s,r,d})^{-1}} \approx \frac{\gamma'_{s,r,d}}{2} - \frac{1}{4}, \quad (20)$$

where

$$\gamma'_{s,r,d} = \frac{P_1 P_2 |h_{s,r}|^2 |h_{r,d}|^2}{(P_1 \sigma_{s,r}^2 + P_2 |h_{r,d}|^2 + \sigma^2) \sigma^2}. \quad (21)$$

It can be seen from [19, Equation (6)] that $\gamma'_{s,r,d}$ is the instantaneous SNR of a dual-hop fixed gain relay transmission scheme. Let

$$\begin{aligned} \gamma_{s,d} &= \frac{P_1 |h_{s,d}|^2}{\sigma^2}, & \gamma_{s,r} &= \frac{P_1 |h_{s,r}|^2}{\sigma^2}, \\ \gamma_{r,d} &= \frac{P_2 |h_{r,d}|^2}{\sigma^2}, & \bar{\gamma}_{s,d} &= \frac{P_1 \sigma_{s,d}^2}{\sigma^2}, \\ \bar{\gamma}_{s,r} &= \frac{P_1 \sigma_{s,r}^2}{\sigma^2}, & \bar{\gamma}_{r,d} &= \frac{P_2 \sigma_{r,d}^2}{\sigma^2}. \end{aligned} \quad (22)$$

It can be seen from (1) that for Nakagami- m independent fading channels, $|h_{s,d}|^2$, $|h_{s,r}|^2$, and $|h_{r,d}|^2$ are independent gamma random variables [20] with parameters $(m_{s,d}, 1/\sigma_{s,d}^2)$, $(m_{s,r}, 1/\sigma_{s,r}^2)$, and $(m_{r,d}, 1/\sigma_{r,d}^2)$, respectively.

Theorem 1. The pdf of $\gamma'_{s,r,d}$ can be written as

$$\begin{aligned}
f_{\gamma'_{s,r,d}}(\gamma) &= \frac{2\gamma^{m_{s,r}-1}}{\Gamma(m_{s,r})\Gamma(m_{r,d})} \left(\frac{m_{s,r}\sigma^2}{P_1\sigma_{s,r}^2} \right)^{m_{s,r}} \\
&\times \left(\frac{m_{r,d}\sigma^2}{P_2\sigma_{r,d}^2} \right)^{m_{r,d}} \exp\left(-\frac{m_{s,r}\sigma^2\gamma}{P_1\sigma_{s,r}^2}\right) \\
&\times \sum_{k=0}^{\infty} \left\{ (-1)^k \frac{[-m_{s,r}]_k}{k!} \left(\frac{P_1\sigma_{s,r}^2 + \sigma^2}{\sigma^2} \right)^k \right. \\
&\times \left(\frac{m_{s,r}(P_1\sigma_{s,r}^2 + \sigma^2)P_2\sigma_{r,d}^2\gamma}{m_{r,d}P_1\sigma_{s,r}^2\sigma^2} \right)^{(m_{r,d}-k)/2} \\
&\times \left. K_{m_{r,d}-k} \left(2\sqrt{\frac{m_{s,r}m_{r,d}(P_1\sigma_{s,r}^2 + \sigma^2)\sigma^2\gamma}{P_1P_2\sigma_{s,r}^2\sigma_{r,d}^2}} \right) \right\}, \quad (23)
\end{aligned}$$

where $K_{\zeta}(\cdot)$ denotes ζ th order modified Bessel function of second kind [21, Equation (9.6.2)], $[\cdot]_k$ is Pochhammer's symbol [21, Equation (6.1.22)], and the values of $m_{s,r}$ and $m_{r,d}$ can be noninteger with $m_{s,r}, m_{r,d} \geq 1/2$.

Proof. Theorem 1 can be proved with the help of results given in [22, Section III] and Gauss Hypergeometric series [21, Equation (15.1.1)]. \square

It can be seen from (17) and (18) that the SNR of the direct link from source to destination under DD modulation can be expressed as

$$\gamma_{s,d} \approx \frac{\gamma'_{s,d}}{2} - \frac{1}{4}, \quad (24)$$

where $\gamma'_{s,d}$ is the SNR of the link under single-differential modulation.

From the analogy between double- and single-differential modulations in Section 4.1, it follows that the approximate BER expressions of DD modulation can be obtained by replacing the SNR of the single-differential system by the SNR of the DD system. For a single-differential BPSK using two independent (but not identically) distributed channels, the BER conditioned on $\gamma = \gamma_{s,d} + \gamma_{s,r,d}$ is given by [23, Equation (12.1.13)] as

$$P_b(\gamma) = \frac{1}{8}(4 + \gamma)\exp(-\gamma). \quad (25)$$

Substituting the values of γ , $\gamma_{s,r,d}$, and $\gamma_{s,d}$ from (13), (20), and (24), respectively, in (25) we can have the BER for DDAAF system as

$$P_b(h_{s,d}, h_{s,r}, h_{r,d}) = \frac{1}{8} \left(\frac{7}{2} + \gamma'_{s,d} + \gamma'_{s,r,d} \right) e^{1/2} e^{-(\gamma'_{s,d} + \gamma'_{s,r,d})}. \quad (26)$$

It can be noticed from Sections 4.1 and 4.2 that for obtaining (26) we have taken the following approximations:

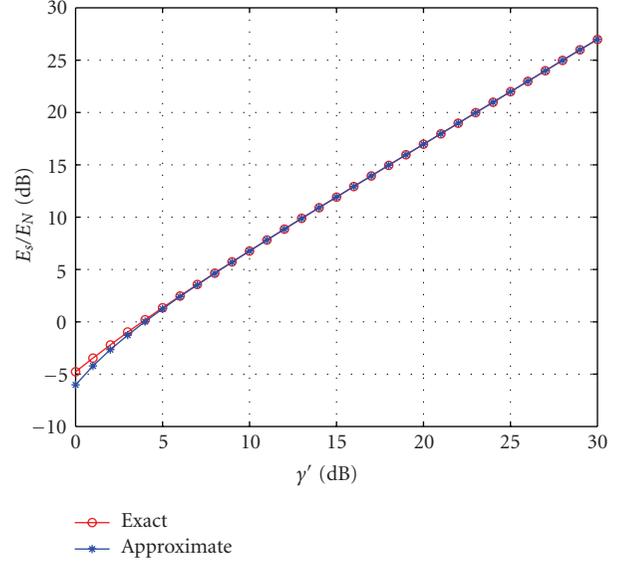


FIGURE 3: Comparison of approximate and exact SNRs. SNR of DD system is represented by E_s/E_N and shown over γ -axis and SNR of single-differential system is represented by γ' and shown over x -axis.

the instantaneous SNR of EMRC scheme is assumed to be equal to the instantaneous SNR of MRC scheme, the product of two noise terms in $X_{s,d}[n]$ and $X_{r,d}[n]$ is assumed to be zero-mean white circularly symmetric Gaussian, and the high SNR approximations for $\gamma_{s,r,d}$ and $\gamma_{s,d}$ given in (20) and (24), respectively, are assumed.

Theorem 2. The approximate BER of the DDAAF system with BPSK modulation averaged over all channels can be written as

$$P_b = \frac{\exp(1/2)}{16} \{7P_{b1}P_{b2} + P_{b2}P_{b3} + P_{b1}P_{b4}\}, \quad (27)$$

where

$$\begin{aligned}
P_{b1} &= \frac{1}{\Gamma(m_{r,d})} \left(\frac{m_{s,r}\sigma^2}{P_1\sigma_{s,r}^2} \right)^{m_{s,r}-1/2} \left(\frac{m_{r,d}\sigma^2}{P_2\sigma_{r,d}^2} \right)^{m_{r,d}-1/2} \\
&\times \exp\left(\frac{m_{s,r}m_{r,d}(P_1\sigma_{s,r}^2 + \sigma^2)\sigma^2}{P_2\sigma_{r,d}^2(2m_{s,r}\sigma^2 + P_1\sigma_{s,r}^2)}\right) \\
&\times \sum_{k=0}^{\infty} \left\{ (-1)^k \frac{[-m_{s,r}]_k}{k!} \left(\frac{m_{s,r}(\sigma^2 + P_1\sigma_{s,r}^2)P_2\sigma_{r,d}^2}{m_{r,d}P_1\sigma_{s,r}^2\sigma^2} \right)^{m_{r,d}-k/2} \right. \\
&\times \left(\frac{2m_{s,r}\sigma^2 + P_1\sigma_{s,r}^2}{2P_1\sigma_{s,r}^2} \right)^{(k+1-m_{r,d}-2m_{s,r})/2} \\
&\times \Gamma(m_{s,r} + m_{r,d} - k) \left(\frac{\sigma^2 + P_1\sigma_{s,r}^2}{\sigma^2} \right)^{k-1/2} \\
&\times \left. W_{(1+k-2m_{s,r}-m_{r,d})/2, m_{r,d}-k/2} \left(\frac{2m_{s,r}m_{r,d}(P_1\sigma_{s,r}^2 + \sigma^2)\sigma^2}{P_2\sigma_{r,d}^2(2m_{s,r}\sigma^2 + P_1\sigma_{s,r}^2)} \right) \right\},
\end{aligned}$$

$$\begin{aligned}
P_{b2} &= \left(\frac{2m_{s,d}\sigma^2}{2m_{s,d}\sigma^2 + P_1\sigma_{s,d}^2} \right)^{m_{s,d}}, \\
P_{b3} &= \frac{\Gamma(m_{s,r} + 1)}{\Gamma(m_{s,r})\Gamma(m_{r,d})} \left(\frac{m_{s,r}\sigma^2}{P_1\sigma_{s,r}^2} \right)^{m_{s,r}-1/2} \left(\frac{m_{r,d}\sigma^2}{P_2\sigma_{r,d}^2} \right)^{m_{r,d}-1/2} \\
&\quad \times \exp \left(\frac{m_{s,r}m_{r,d}(P_1\sigma_{s,r}^2 + \sigma^2)\sigma^2}{P_2\sigma_{r,d}^2(2m_{s,r}\sigma^2 + P_1\sigma_{s,r}^2)} \right) \\
&\quad \times \sum_{k=0}^{\infty} \left\{ \frac{(-1)^k \lfloor -m_{s,r} \rfloor_k}{k!} \right. \\
&\quad \times \left(\frac{m_{s,r}(\sigma^2 + P_1\sigma_{s,r}^2)P_2\sigma_{r,d}^2}{m_{r,d}P_1\sigma_{s,r}^2\sigma^2} \right)^{(m_{r,d}-k)/2} \\
&\quad \times \left(\frac{2m_{s,r}\sigma^2 + P_1\sigma_{s,r}^2}{2P_1\sigma_{s,r}^2} \right)^{(k-m_{r,d}-2m_{s,r}-1)/2} \\
&\quad \times \Gamma(m_{s,r} + m_{r,d} - k + 1) \left(\frac{\sigma^2 + P_1\sigma_{s,r}^2}{\sigma^2} \right)^{k-1/2} \\
&\quad \left. \times W_{(k-2m_{s,r}-m_{r,d}-1)/2, m_{r,d}-k/2} \left(\frac{2m_{s,r}m_{r,d}(P_1\sigma_{s,r}^2 + \sigma^2)\sigma^2}{P_2\sigma_{r,d}^2(2m_{s,r}\sigma^2 + P_1\sigma_{s,r}^2)} \right) \right\}, \\
P_{b4} &= \frac{\Gamma(m_{s,d} + 1)}{\Gamma(m_{s,d})} \frac{2(2m_{s,d}\sigma^2)^{m_{s,d}} P_1\sigma_{s,d}^2}{(2m_{s,d}\sigma^2 + P_1\sigma_{s,d}^2)^{m_{s,d}+1}}, \tag{28}
\end{aligned}$$

where $W_{\mu,\nu}(\cdot)$ represents Whittaker function [16, Equation (9.220.4)].

Proof. Equation (26) can be averaged over all the channels as follows:

$$\begin{aligned}
P_b &= \frac{e^{1/2}}{16} \left\{ 7 \int_0^{\infty} e^{-\alpha/2} f_{\gamma_{s,r,d}}(\alpha) d\alpha \int_0^{\infty} e^{-\beta/2} f_{\gamma_{s,d}}(\beta) d\beta \right. \\
&\quad + \int_0^{\infty} \alpha e^{-\alpha/2} f_{\gamma_{s,r,d}}(\alpha) d\alpha \int_0^{\infty} e^{-\beta/2} f_{\gamma_{s,d}}(\beta) d\beta \\
&\quad \left. + \int_0^{\infty} e^{-\alpha/2} f_{\gamma_{s,r,d}}(\alpha) d\alpha \int_0^{\infty} \beta e^{-\beta/2} f_{\gamma_{s,d}}(\beta) d\beta \right\}. \tag{29}
\end{aligned}$$

Each integral in (29) can be represented by P_{bi} , $i \in \{1, 2, 3, 4\}$, as shown by (27). These integrals can be solved by introducing another integration variable and [16, Equations (3.478.1) and (6.631.3)]. \square

Corollary 1. *The approximate BER of binary cooperative system with double-differential modulation over Rayleigh chan-*

nels is given as

$$\begin{aligned}
P_b &= \frac{\exp(\beta + 0.5)}{16} \\
&\quad \times \left\{ \frac{56\sigma^6 + 36P_1\sigma_{s,d}^2\sigma^4}{(2\sigma^2 + P_1\sigma_{s,d}^2)^2(2\sigma^2 + P_1\sigma_{s,r}^2)} W_{-1,0.5}(2\beta) \right. \\
&\quad + \frac{\sqrt{2}(\sigma^2 + P_1\sigma_{s,r}^2)^{1/2}(28\sigma^5 + 18P_1\sigma_{s,d}^2\sigma^3)}{\sqrt{P_2}\sigma_{r,d}(2\sigma^2 + P_1\sigma_{s,d}^2)^2(2\sigma^2 + P_1\sigma_{s,r}^2)^{1/2}} \\
&\quad \times W_{-0.5,0}(2\beta) \\
&\quad + \frac{16P_1\sigma_{s,r}^2\sigma^4}{(2\sigma^2 + P_1\sigma_{s,r}^2)^2(2\sigma^2 + P_1\sigma_{s,d}^2)} W_{-2,0.5}(2\beta) \\
&\quad + \frac{4\sqrt{2}P_1\sigma_{s,r}^2\sigma^4(\sigma^2 + P_1\sigma_{s,r}^2)^{1/2}}{\sqrt{P_2}\sigma_{r,d}(2\sigma^2 + P_1\sigma_{s,r}^2)^{3/2}(2\sigma^3 + P_1\sigma_{s,d}^2\sigma)} \\
&\quad \left. \times W_{-1.5,0}(2\beta) \right\}, \tag{30}
\end{aligned}$$

where

$$\beta = \frac{(\sigma^2 + P_1\sigma_{s,r}^2)\sigma^2}{P_2\sigma_{r,d}^2(2\sigma^2 + P_1\sigma_{s,r}^2)}. \tag{31}$$

Proof. Substituting $m_{s,d} = m_{s,r} = m_{r,d} = 1$ in (27) and after some manipulations we can obtain (30). \square

Let us consider a symmetric case when $P_1 = P_2 = P$, $\sigma_{s,r} = \sigma_{r,d} = \sigma_{s,d} = \sigma_s$, and the SNR of each link is $\gamma_s = P\sigma_s^2/\sigma^2$. From [21, Equation (13.1.33)], the Whittaker function can be expressed as

$$W_{\lambda,\mu}(z) = e^{-z} z^{\mu+0.5} U(0.5 + \mu - \lambda, 1 + 2\mu, z), \tag{32}$$

where $U(\cdot, \cdot, \cdot)$ is the confluent hypergeometric function of second kind. If $\gamma_s \rightarrow \infty$, (30) can be written using (32) as

$$\begin{aligned}
P_b &\approx \frac{\exp(1/2)}{16} [36\gamma_s^{-3} U(2, 2, 2\gamma_s^{-1}) + 18\sqrt{2}\gamma_s^{-2} U(1, 1, 2\gamma_s^{-1}) \\
&\quad + 16\gamma_s^{-3} U(3, 2, 2\gamma_s^{-1}) + 4\sqrt{2}\gamma_s^{-2} U(2, 1, 2\gamma_s^{-1})]. \tag{33}
\end{aligned}$$

At high SNR, the probability of error of the DDAAF system can be further approximated by using [21, Equations (13.5.7) and (13.5.9)] as

$$P_b \approx \epsilon \gamma_s^{-2}, \tag{34}$$

where ϵ is a positive constant which is independent of γ_s . It can be seen from (34) that $\lim_{\gamma_s \rightarrow \infty} \epsilon \gamma_s^{-2} = 0$, therefore, the DDAAF system achieves diversity of the order of two over the Rayleigh channel.

The approximate BER of DDAAF with M -PSK, $M > 2$, can also be found using the analogy we have developed in Section 4.1. With the help of [23, Equation (B.21)], valid for

single-differential modulation, the BER of DDAAF with M -PSK conditioned on the channels can be written as

$$P_b(\gamma) = \frac{1}{16\pi} \int_{-\pi}^{\pi} u(\psi) \exp(-v(\psi)\gamma) d\psi, \quad (35)$$

where

$$u(\psi) = \frac{(1 - \delta^2)[3 + \cos(2\psi) - (\delta + 1/\delta) \sin \psi]}{1 + 2\delta \sin(\psi) + \delta^2}, \quad (36)$$

$$v(\psi) = \frac{r^2(1 + 2\delta \sin(\psi) + \delta^2)}{2}, \quad (37)$$

and $\delta = q/r$. For QPSK constellation, $q = \sqrt{2 - \sqrt{2}}$ and $r = \sqrt{2 + \sqrt{2}}$. For other M -PSK constellations, the values of q and r can be obtained using the results in [23, Appendix B]. The approximate BER of the DDAAF system with M -PSK can be obtained by substituting the value of γ from (13) into (35) and then averaging (35) over the three channels as

$$P_b = \frac{1}{16\pi} \int_{-\pi}^{\pi} u(\psi) M_{\gamma_{s,d}}(v(\psi)) M_{\gamma_{s,r,d}}(v(\psi)) d\psi, \quad (38)$$

where $M_{\gamma}(\cdot)$ denotes the moment which is generating function (MGF) of γ .

Theorem 3. *The MGF of $\gamma_{s,r,d}$ is given as*

$$\begin{aligned} M_{\gamma_{s,r,d}}(v(\psi)) &= \frac{1}{\Gamma(m_{r,d})} \left(\frac{m_{s,r} \sigma^2}{P_1 \sigma_{s,r}^2} \right)^{m_{s,r}-1/2} \left(\frac{m_{r,d} \sigma^2}{P_2 \sigma_{r,d}^2} \right)^{m_{r,d}-1/2} \exp(\tau) \\ &\times \sum_{k=0}^{\infty} \left\{ (-1)^k \frac{[-m_{s,r}]_k}{k!} \right. \\ &\times \left(\frac{m_{s,r}(\sigma^2 + P_1 \sigma_{s,r}^2) P_2 \sigma_{r,d}^2}{m_{r,d} P_1 \sigma_{s,r}^2 \sigma^2} \right)^{(m_{r,d}-k)/2} \\ &\times \left(\frac{2m_{s,r} \sigma^2 + P_1 \sigma_{s,r}^2 v(\psi)}{2P_1 \sigma_{s,r}^2} \right)^{(k+1-m_{r,d}-2m_{s,r})/2} \\ &\times \Gamma(m_{s,r} + m_{r,d} - k) \left(\frac{\sigma^2 + P_1 \sigma_{s,r}^2}{\sigma^2} \right)^{k-1/2} \\ &\left. \times W_{(1+k-2m_{s,r}-m_{r,d})/2, m_{r,d}-k/2}(2\tau) \right\}, \end{aligned} \quad (39)$$

where

$$\begin{aligned} \tau &= \frac{P_1 \sigma_{s,r}^2 (4m_{s,r} m_{r,d} + P_2 \sigma_{s,r}^2 v^2(\psi))}{4P_2 \sigma_{r,d}^2 (2m_{s,r} \sigma^2 + P_1 \sigma_{s,r}^2 v(\psi))} \\ &+ \frac{2m_{s,r} \sigma^2 (2m_{r,d} \sigma^2 + P_2 \sigma_{r,d}^2 v(\psi))}{4P_2 \sigma_{r,d}^2 (2m_{s,r} \sigma^2 + P_1 \sigma_{s,r}^2 v(\psi))}. \end{aligned} \quad (40)$$

Proof. From the analogy between double- and single-differential systems in Section 4.1, it is clear that the MGF of $\gamma_{s,r,d}$ can be obtained by using the formulations of single-differential modulation. Hence, using (20) and (23), the MGF of $\gamma_{s,r,d}$ can be written as

$$M_{\gamma_{s,r,d}}(v(\psi)) = \int_0^{\infty} \exp\left(-\left(\frac{\gamma}{2} - \frac{1}{4}\right)v(\psi)\right) f_{\gamma_{s,r,d}}(\gamma) d\gamma. \quad (41)$$

The integral of (41) can be solved by introducing another integration variable and [16, Equations (3.478.1) and (6.631.3)]. \square

The MGF of $\gamma_{s,d}$ can be expressed as

$$M_{\gamma_{s,d}}(v(\psi)) = \left(\frac{2m_{s,d} \sigma^2 \exp(v(\psi)/4m_{s,d})}{2m_{s,d} \sigma^2 + P_1 \sigma_{s,d}^2} \right)^{m_{s,d}}. \quad (42)$$

A derivation of (42) is provided in Appendix A. It is very difficult to solve the integral in (38) and find a closed-form solution. However, we can obtain an upper bound of the BER of the DDAAF system with M -PSK constellation.

Corollary 2. *The approximate BER of the DDAAF system using M -PSK constellation can be upper bounded as*

$$P_b \leq \frac{1}{8} \tilde{u} M_{\gamma_{s,d}}(\tilde{v}) M_{\gamma_{s,r,d}}(\tilde{v}), \quad (43)$$

where $\tilde{u} = (1 + \delta)^3 / [\delta(1 - \delta)]$ and $\tilde{v} = r^2(1 - \delta)^2/2$.

See Appendix B for a proof of Corollary 2.

Let us summarize the analytical results found for DDAAF system in this section. Theorem 1 provides the pdf of the cooperative link from the source to the destination through the relay under general (integer and noninteger values of m) Nakagami- m fading. Theorem 2 suggests approximate analytical BER of DDAAF system with BPSK constellation under Nakagami- m fading. For finding BER expressions, we have assumed that higher-order noise terms in $X_{s,d}[n]$ and $X_{r,d}[n]$ are zero-mean white circularly symmetric Gaussian and made high SNR approximations for $\gamma_{s,r,d}$ and $\gamma_{s,d}$ given in (20) and (24), respectively. The BER of DDAAF system under Rayleigh fading with BPSK constellation is given by Corollary 1. The MGF of the cooperative link under Nakagami- m fading with M -PSK constellation is given by Theorem 3. Corollary 2 provides an upper bound of the BER of the DDAAF system under Nakagami- m fading with M -PSK signal constellation.

5. IMPLEMENTATION OF TRAINING-BASED COOPERATIVE SYSTEM

In this section, we will show how to implement a trained amplification-based cooperative system for comparison with the proposed DDAAF system. Let us assume that the trained decoder at the destination utilizes the *two* initialization symbols as training data, and estimates the carrier offsets and channels using the following maximum likelihood estimators [17, Equations (9.7.27) and (9.7.28)]:

$$\hat{\omega} = \arg \{x[1]x^*[0]\}, \quad \hat{h} = \frac{1}{2}(x[0] + \exp(-2\pi\hat{\omega})x[1]), \quad (44)$$

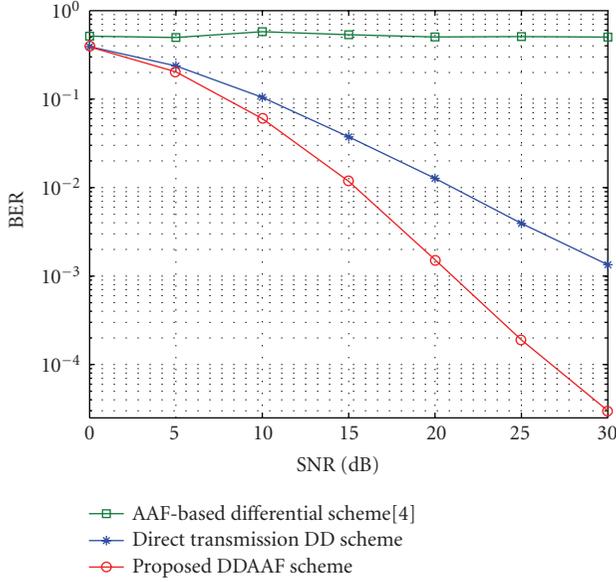


FIGURE 4: Comparison of BER versus SNR performance of DDAAF cooperative system with conventional schemes.

where $\arg\{\cdot\}$ provides angle of the complex scalar and $(\cdot)^*$ stands for the complex conjugate. The estimators of [17, Equations (9.7.27) and (9.7.28)] are proposed for an $n_t \times N$ space-time block code (STBC) in an MIMO system, where n_t is the number of transmit antennas and N is the time dimension. However, we are working with a cooperative system containing SISO links. Therefore, we use $n_t = N = 1$ in [17, Equations (9.7.27) and (9.7.28)] for obtaining (44). In the trained system, the symbols $z[n]$ are directly transmitted in the space without any differential encoding. Therefore, the received data equations for such a system can be obtained by replacing $v[n]$ by $z[n]$ in (5), (6), and (7). Let us also assume that $z[0] = z[1] = 1$. The receiver at the destination makes the following MRC-based decision variable [23]:

$$d[k] = \frac{\hat{h}_{s,d}^*}{\sigma^2} \exp(-2\pi\hat{\omega}_{s,d}) y_{s,d}[n] + \frac{\hat{h}_{s,r,d}^*}{E_N} \exp(-2\pi\hat{\omega}_{s,r,d}) y_{r,d}[m], \quad k = n = m, \quad (45)$$

where $h_{s,r,d}$ is the effective channel over the cooperative link (s, r, d), $\omega_{s,r,d}$ is the effective carrier offset introduced by the cooperative link, and $y_{s,d}[n]$ and $y_{r,d}[m]$ are the data received due to the direct transmission and relayed transmission, respectively, and E_N is the total noise power in $y_{r,d}[m]$, which is given by

$$E_N = \frac{(P_1\sigma_{s,r}^2 + P_2|h_{r,d}|^2 + \sigma^2)\sigma^2}{P_1\sigma_{s,r}^2 + \sigma^2}. \quad (46)$$

From (46), it can be seen that E_N contains $|h_{r,d}|^2$. However, it is difficult to estimate $h_{r,d}$ separately as it can be seen from (7) that the relay transmits an *amplified* version of the *received*

signal corresponding to the *training data* transmitted by the source. As the channel statistics vary far more slowly than the channel coefficients, we can assume that the destination has a perfect knowledge of $\sigma_{r,d}^2$. Therefore, the trained decoder can obtain the decision variable by replacing $|h_{r,d}|^2$ by $\sigma_{r,d}^2$ in (46). In addition, it can also be assumed that relay and destination has perfect knowledge about $\sigma_{s,r}^2, \sigma^2, P_1$, and P_2 .

In [24], channel estimation over a single cooperative link between the source and the destination through the relay using amplify-and-forward protocol is studied. It is assumed in [24] that there is no direct link between the source and the destination. However, the proposed training-based cooperative system is more general than [24], since we also consider a direct link between the source and the destination.

6. ANALYTICAL AND EXPERIMENTAL PERFORMANCE EVALUATIONS OF THE DDAAF SYSTEM

All the simulations are achieved by 10^6 channel realizations.

6.1. Comparisons of direct transmission and conventional differential cooperative system [4]

Figure 4 shows the comparison of the performance of the proposed DDAAF-based cooperative scheme, the DD direct transmission, and previously proposed *single differential* amplify-and-forward cooperative scheme [4] for Rayleigh fading channels, that is, $m_{s,d} = m_{s,r} = m_{r,d} = 1$ and BPSK constellation. All the links are assumed to be perturbed by different random carrier offsets uniformly distributed over $[-\pi, \pi)$. It is seen from Figure 4 that the proposed DDAAF scheme outperforms the direct DD transmission at all SNRs. It can be seen from Figure 4 that the proposed scheme has higher diversity as compared to the direct transmission scheme and a performance gain of more than 5 dB is observed at SER = 10^{-2} . It can also be observed that there is a collapse in the performance of the conventional differential scheme [4] because of the random carrier offsets.

6.2. Comparison of analytical and experimental performances

Figure 5 shows the analytical and experimental performances of the proposed DDDAF-based cooperative scheme with random carrier offsets. We have plotted the approximate analytical BER (26) for BPSK constellation, $P_1/P = P_2/P = 0.5$, $\sigma_{s,d}^2 = \sigma_{s,r}^2 = \sigma_{r,d}^2 = 1$, and $m_{s,d} = m_{s,r} = m_{r,d} \in \{1, 1.5, 2, 3\}$. The simulation results of the proposed DDAAF scheme are shown under the same conditions. From Figure 5, it is seen that the experimental data closely follows the analytical results from moderate to high SNR values. Hence, this justifies the assumption taken in (13) and (18).

6.3. Power allocation for DDAAF system

It can be seen from (27) that the BER of the DDAAF system depends nonlinearly upon P_1 and P_2 . Therefore, using the power constraint $P_1 + P_2 = P$, we can obtain the values

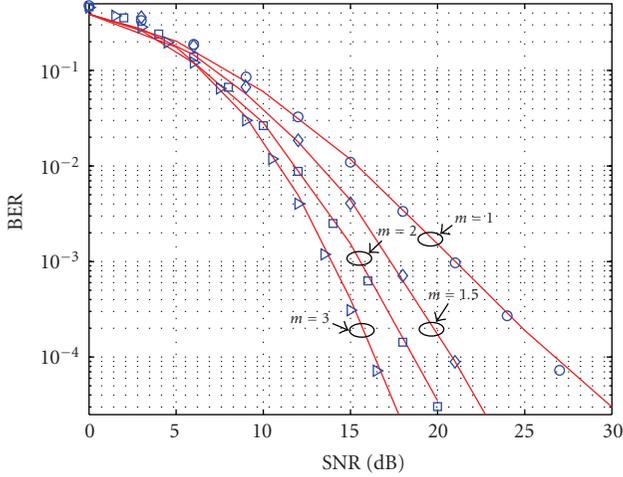


FIGURE 5: Analytical and experimental BERs versus SNR performance of DDAAF cooperative system over Nakagami- m channels with \circ $m = 1$, \diamond $m = 1.5$, \square $m = 2$, and Δ $m = 3$.

of P_1 and P_2 which minimize the BER. We have calculated the power distribution for SNR = 20 dB by numerically minimizing (27) subject to the power constraint $P = P_1 + P_2 = 2$, $\sigma_{s,d}^2 = \sigma_{s,r}^2 = 1$, and $\sigma_{r,d}^2 = 10$. Figure 6 shows the performance of the proposed DDAAF scheme using uniform and numerically calculated power allocation over Nakagami- m channels with $m \in \{1, 2\}$. It can be seen from Figure 6 that the DDAAF scheme with optimized power distribution outperforms the DDAAF scheme with uniform power distribution $P_1 = P_2 = 0.5P$.

6.4. Comparison of DDAAF with trained cooperative system

Figure 7 shows the comparison of the proposed DDAAF system with the trained cooperative system from Section 5. The simulations are performed using the QPSK constellation, $P_1 = P_2 = 1$, and Rayleigh fading channels with $\sigma_{s,d}^2 = \sigma_{s,r}^2 = \sigma_{r,d}^2 = 1$. It can be seen that the proposed DDAAF system outperforms the trained cooperative system of Section 5 for all SNR values. The upper bound of the BER of the proposed DDAAF system is calculated from (43) and is also plotted in Figure 7. We have also numerically calculated the power distribution $P_1 = 0.69P, P_2 = 0.31P$, which minimizes the upper bound of (43) at SNR = 20 dB. It can be seen that the DDAAF system with optimized power distribution performs better than the one with uniform power distribution. The performance of an AAF system with perfect channel state information (CSI) and perfect carrier offset knowledge (COK) at the relay and the destination is also shown in Figure 7. It can be seen from Figure 7 that the proposed DDAAF system performs approximately 7.5 dB poorer than the *ideal* AAF system at BER of 10^{-2} . However, with optimized power allocation, 1 dB improvement can be obtained at BER of 10^{-2} , as shown in Figure 7.

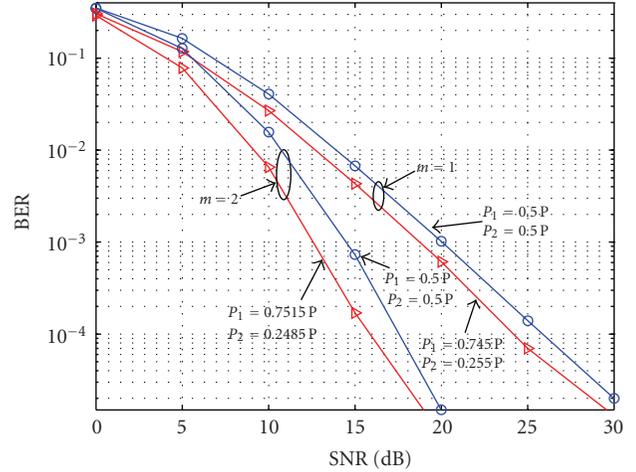


FIGURE 6: BER versus SNR performance of DDAAF cooperative system over Nakagami- m channels with uniform \circ and optimized Δ power distribution to minimize the BER, $\sigma_{s,d}^2 = \sigma_{s,r}^2 = 1$, and $\sigma_{r,d}^2 = 10$.

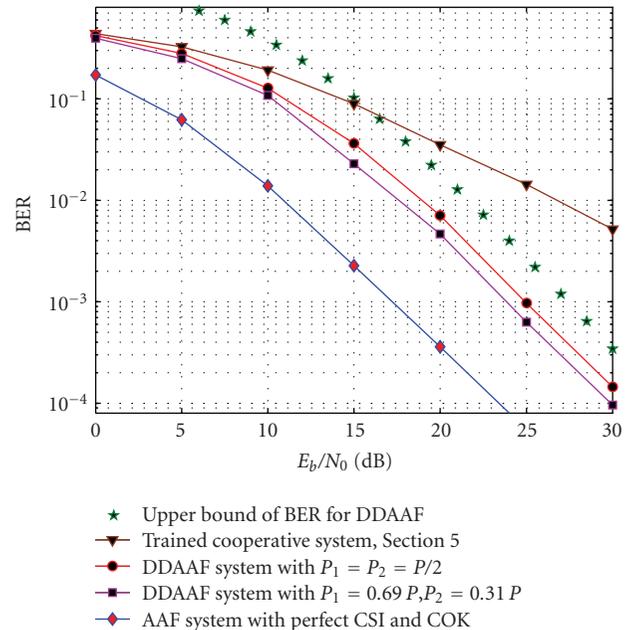


FIGURE 7: Comparison of DDAAF cooperative system with trained cooperative system.

7. CONCLUSIONS

We have implemented double-differential modulation in cooperative communication system with amplify-and-forward protocol. The proposed double-differentially modulated cooperative system can overcome the problem of carrier offsets in Nakagami- m fading channels. Our scheme performs well in the practical scenario, where the conventional differential modulation schemes fail. With our scheme, the users are still able to decode their data without knowing the channel gains or carrier offsets. We have also performed

the BER analysis to predict the behavior of the cooperative system. In addition, we have done a numerical power allocation based on this analysis to further improve the performance of the system. The proposed double-differential system also outperforms the similar rate trained cooperative system.

APPENDICES

A. PROOF OF (42)

From (1), (24), and using the analogy between double- and single-differential systems in Section 4.1, the MGF of $\gamma_{s,d}$ can be written as

$$M_{\gamma_{s,d}}(\nu(\psi)) = \int_0^\infty \exp\left(-\left(\frac{\gamma}{2} - \frac{1}{4}\right)\nu(\psi)\right) f_{\gamma_{s,d}}(\gamma) d\gamma. \quad (\text{A.1})$$

The integral of (A.1) can be solved with the help of [16, Equations (3.351.3)].

B. PROOF OF COROLLARY 2

In order to obtain an upper bound of BER, we need to maximize $u(\psi)$ and minimize $\nu(\psi)$ with respect to ψ . It can be observed from (37) that $\nu(\psi)$ has its minimum value at $\psi = -\pi/2$. To maximize $u(\psi)$, we need to find the first-order derivative of $u(\psi)$ with respect to ψ and equate it to zero. The first-order derivative of $u(\psi)$ can be written as

$$\begin{aligned} \frac{\partial u(\psi)}{\partial \psi} = \frac{(1 - \delta^2)}{a} & \left[-2(1 + \delta^2) \sin(2\psi) \right. \\ & - 4\delta \sin \psi \sin(2\psi) - \left(\frac{\delta + 1}{\delta}\right) \cos \psi \\ & + 2\delta \left(\frac{\delta + 1}{\delta - 1}\right) \sin \psi \cos \psi \\ & \left. - \delta(\delta^2 + 7) \cos \psi - 2\delta \cos \psi \cos(2\psi) \right], \end{aligned} \quad (\text{B.2})$$

where $a = (1 + 2\delta \sin \psi + \delta^2)^2$. It can be seen from (B.2) that $u(\psi)$ is maximized at $\psi = -\pi/2$. The maximization can be verified by plotting $u(\psi)$ versus ψ graph. Therefore, the upper bound over BER can be obtained at $\psi = -\pi/2$. By substituting $\psi = -\pi/2$ in (36) and (37), we obtain the values of \tilde{u} and $\tilde{\nu}$, respectively. Then, from (38), (39), and (42), we can obtain (43).

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