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*Research Article*

# NAF, OAF, or Noncooperation: Which Protocol to Choose?

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The two main Amplify and Forward cooperative protocols are the orthogonal (OAF) and the nonorthogonal one (NAF). In this paper, we consider a given source, $N$ relays, a destination, and a channel realization and we try to resolve the following problem: what is the best way to communicate: without cooperation or using one of the two cooperative protocols? This is equivalent to a power-sharing problem on the cooperation frame between source and relays aiming to the short-term channel capacity maximization. The obtained solution shows that cooperative protocol choice depends only on the available power at the relays. However the decision to cooperate depends on the channel conditions. We show that our power allocation scheme with relay selection improves the outage probability compared to the selective OAF and the NAF protocols and has a significant capacity gain.

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## 1. INTRODUCTION

Communications on wireless channels are limited by multiple impairment sources (multipath fading, shadowing, and path loss). Many diversity techniques have been developed to fight the fast fading such as multiple antennas for the spatial diversity, coding for the time diversity. Recently, cooperative diversity technique has attracted much attention because it is able to combat not only the fast fading but also the shadowing and the path loss [1, 2]. It considers a source, a destination, and several relay nodes distributed throughout the network. The relay set forms a virtual antenna array and by using cooperation protocols they can exploit the diversity as a multiple-in multiple-out (MIMO) system [3]. One can distinguish three main classes of cooperative strategies [2]: amplify-and-forward (AF), decode-and-forward (DF) and compress-and-forward (CF).

A cooperation protocol is in general composed of two phases. In the first one, the source transmits the information to the relays and to the destination. In the second, when only the relays are authorized to transmit, the protocol is considered as orthogonal. In this case, the receiver processing is simple. However, when the source continues to transmit leading to a throughput increasing [4] the protocol is not orthogonal. The AF protocols have been more studied than others because of their simplicity. Indeed, the relay stations have to only amplify and forward to the destination the signal received from the source by respecting a power constraint.

A way to prolong the different network nodes lifetime and to optimize the system performance is to make a power allocation. The adaptive power allocation for wireless networks has been mainly addressed for orthogonal protocols. In [5–9], the ODF protocol ergodic capacity or the outage capacity was optimized. In [5, 10–12], the OAF protocol power allocation was optimized by considering the signal-to-noise ratio or the outage probability. They respect in general the source and relay maximum power constraints and a per frame power budget. Solutions are optimal power allocations to the source at the first cooperation slot and to the relay at the second one. In [13], only one relay is considered and the NAF protocol power allocation was obtained for downlink using iterative procedure considering separately the source and relay maximum power constraints. For some channel conditions, zero power was allocated to the relay leading to a direct transmission. Hence, selective cooperative protocols are obtained.

Previous works studied separately these NAF and OAF protocols but the problem of the best protocol choice was rarely addressed. In our paper, we fix the sum power per slot over all transmitters and we consider a general problem of power sharing between the source and the relays under maximum power constraints at the relays. The power repartition per slot is chosen to make fair the comparison with no cooperation case and to limit the interference level in the network. The considered criterion to be optimized is the instantaneous mutual information between the source and the destination. When the individual power constraint at the relay surpasses the transmitting one, the optimal solution is that the source and the relay should not share the power in the second slot: either the source or the relay should transmit and the choice is dictated by the channel conditions. However, when the cooperation is chosen and the relay has not sufficient power to achieve the allowed transmission level per slot, the remaining power is reallocated to the source to transmit in second slot. This is equivalent to the selective NAF protocol use.

This paper is organized as follows. In Section 2, we describe the system model. The problem formulation is addressed in Section 3. In Section 4, we point out the best protocol to use with its optimal power allocation respecting the considered constraints. Section 5 gives simulation results that compare the outage behavior and the capacity of the proposed solutions compared to the selective OAF [2] and the NAF [4] protocols. In Section 6, we give conclusions.

## 2. SYSTEM MODEL

We consider a network with $N + 2$ nodes uniformly distributed. It consists of a source ($s$), a destination ($d$) and the remaining $N$ nodes can serve as potential relay nodes ($r_i$). The cooperation frame for the $N$ relays is shown in Figure 1 and is composed of $N$ subframes. Each one is divided into two slots. In the sequel $h$, $f_i$, and $g_i$ denote respectively the instantaneous channel gains between source and destination, source and node $i$, and node $i$ and destination. $w_i$ and $n_{ik}$ denote, respectively, the additive noises at the $i$th relay node and at the destination during the $i$th cooperation subframe and the $k$th time slot. The channel gains are assumed to be independent, zero-mean complex gaussian distributed random variables with variances $\sigma_h, \sigma_{f_i}$, and $\sigma_{g_i}$. The additive noises at the relay nodes and at the destination are assumed to be independent, zero-mean gaussian distributed random variables with variance $N_0$. We consider the NAF protocol proposed in [4] which is a general cooperative protocol representation since the OAF one and the direct transmission correspond to particular power allocations per slot. The source ($s$) transmits during the $i$th cooperation subframe duration to the destination ($d$), the relay ($r_i$) retransmits to the destination ($d$) by amplifying what it has received from the source ($s$) during the first time slot. The system can be characterized as follows:

$$\begin{aligned} y_{i1}^d &= h\sqrt{P_1}x_{i1} + n_{i1}, \\ y_{i2}^d &= h\sqrt{P_2}x_{i2} + g_i\beta_i y_i^r + n_{i2}, \\ y_i^r &= f_i\sqrt{P_1}x_{i1} + w_i, \end{aligned} \tag{1}$$

where $x_{i1}$ and $x_{i2}$ are, respectively, the first and the second symbols transmitted by the source during the $i$th cooperation subframe. $y_{i1}^d$ and $y_{i2}^d$ are the first and the second symbols received at the destination during the $i$th cooperation subframe. $y_i^r$ is the symbol received by the $i$th relay node from the source, and $\beta_i$ is the scale factor of the $i$th relay node with

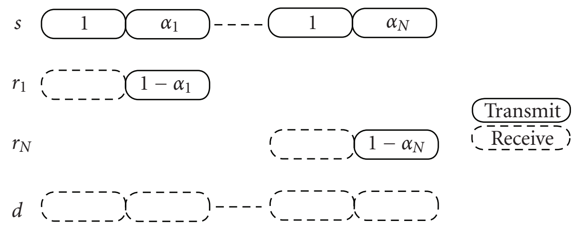


Figure 1: General cooperative frame for $N$ relays.

$$\beta_i \leq \sqrt{\frac{P_{r_i}}{P_1|f_i|^2 + N_0}}, \tag{2}$$

where $P_{r_i}$ is the relay $i$ transmitting power that should satisfy the constrain

$$P_{r_i} \leq P_{r_i}^{\max} \tag{3}$$

and $P_1$ and $P_2$ are, respectively, the transmitting power of the source at the first and the second slots. After vectorization, the received frame can be written as

$$\mathbf{y}^d = \begin{pmatrix} \mathbf{H}_1 & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{H}_2 & \mathbf{0} & \vdots \\ \vdots & \mathbf{0} & \ddots & \mathbf{0} \\ \mathbf{0} & \dots & \mathbf{0} & \mathbf{H}_N \end{pmatrix} \mathbf{x} + \mathbf{w}, \tag{4}$$

where $\mathbf{y}^d = [\mathbf{y}_1^d, \dots, \mathbf{y}_N^d]^t$ with $\mathbf{y}_i^d = [y_{i1}^d, y_{i2}^d]$, $\mathbf{x} = [\mathbf{x}_1, \dots, \mathbf{x}_N]^t$ with $\mathbf{x}_i = [x_{i1}, x_{i2}]$,

$$\mathbf{H}_i = \begin{pmatrix} h\sqrt{\dfrac{P_1}{N_0}} & 0 \\ \dfrac{f_i g_i \sqrt{P_1}\beta_i}{\sqrt{N_0(1+\beta_i^2|g_i|^2)}} & h\sqrt{\dfrac{P_2}{N_0(1+\beta_i^2|g_i|^2)}} \end{pmatrix} \tag{5}$$

is the normalized channel matrix and $\mathbf{w}$ is the noise vector with $\mathbf{w} \sim \mathcal{N}(0, I)$.

## 3. PROBLEM FORMULATION

We propose to determine the best protocol to use for a given channel realization $h$, $g_i$, $f_i$, a fixed sum over all transmitters power budget $P_1$ per slot and relays power constraints $P_{r_i}^{\max}$. For this purpose, we consider the NAF protocol with the general power allocation presented in Figure 1. At each second slot per subframe $i$, the power $P_1$ is divided into a part $P_2 = \alpha_i P_1$ allocated to the source and $P_{r_i} = (1-\alpha_i)P_1$ to the relay with $0 \leq \alpha_i \leq 1$. The power allocation is chosen to maximize the mutual information between the source and the destination

$$\{\alpha_1, \dots, \alpha_N\} = \arg\ \max \mathcal{I}(\mathbf{x}, \mathbf{y}^d) \quad \text{with } (1-\alpha_i)P_1 \leq P_{r_i}^{\max}. \tag{6}$$

Using (4) the mutual information is

$$\mathcal{I}(\mathbf{x},\mathbf{y}^d) = \log_2\left(\det\left(\mathbf{I}_{2N}+\mathbf{H}\mathbf{H}^H\right)\right) = \sum_{i=1}^{N}\mathcal{I}(\mathbf{x}_i,\mathbf{y}_i^d) \le N\max_i\{\mathcal{I}(\mathbf{x}_i,\mathbf{y}_i^d)\}, \tag{7}$$

where

$$\mathcal{I}(\mathbf{x}_i,\mathbf{y}_i^d) = \log_2\Bigg(1+\frac{|h|^2P_1}{N_0}+\frac{|\beta_i|^2|f_i|^2|g_i|^2P_1+|h|^2P_2}{N_0(1+|\beta_i|^2|g_i|^2)} + \frac{|h|^4P_1P_2}{N_0^2(1+|\beta_i|^2|g_i|^2)}\Bigg). \tag{8}$$

Replacing $\beta_i$ by its maximum value (2), we obtain

$$\mathcal{I}(\mathbf{x}_i,\mathbf{y}_i^d) = \log_2\Bigg(1+\frac{|h|^2P_1}{N_0}+\frac{|f_i|^2P_1|g_i|^2P_{r_i}+|h|^2P_2(N_0+|f_i|^2P_1)}{N_0(N_0+|f_i|^2P_1+|g_i|^2P_{r_i})} + \frac{|h|^4P_1P_2(N_0+|f_i|^2P_1)}{N_0^2(N_0+|f_i|^2P_1+|g_i|^2P_{r_i})}\Bigg). \tag{9}$$

Now, let $a_0 = |h|^2P_1/N_0$, $a_i = |f_i|^2P_1/N_0$ and $b_i = |g_i|^2P_1/N_0$. By replacing $P_2$ and $P_{r_i}$ by their values it is easy to obtain that

$$\mathcal{I}(\mathbf{x}_i,\mathbf{y}_i^d) = \log_2\Bigg(1+a_0+\frac{a_ib_i(1-\alpha_i)+a_0\alpha_i(1+a_i)}{1+a_i+b_i(1-\alpha_i)} + \frac{a_0^2\alpha_i(1+a_i)}{1+a_i+b_i(1-\alpha_i)}\Bigg). \tag{10}$$

Hence, resolving problem (6) is equivalent to find for every $i$ the $\alpha_i$ that maximizes (10) under the constraint that $(1-\alpha_i)P_1 \le P_{r_i}^{\max}$. Once the optimal values $\alpha_i^{\text{opt}}$ are obtained, the upper bound (7) is achieved by communicating with only one relay $i_0$ that satisfies $\mathcal{I}(\mathbf{x}_{i_0},\mathbf{y}_{i_0}^d) = \max_i\{\mathcal{I}(\mathbf{x}_i,\mathbf{y}_i^d)\}$. This selection leads to a short-term cooperation protocol choice which could be NAF, OAF, or direct transmission.

## 4. PROTOCOL SELECTION

Resolving the formulated problem allows to find a method that selects the best protocol based on the power available at the relay nodes and the channel realizations.

Let $A_i = a_0(1+a_0)(1+a_i) - b_i(1+a_0+a_i)$, and $B_i = a_ib_i + (1+a_0)(1+a_i+b_i)$ and $C_i = -b_i$ and $D_i = 1+a_i+b_i$. Equation (10) is equivalent to

$$\mathcal{I}(\mathbf{x}_i,\mathbf{y}_i^d) = \log_2\left(\frac{A_i\alpha_i+B_i}{C_i\alpha_i+D_i}\right). \tag{11}$$

The behavior of (11) is reflected by its first derivative sign. We show in the Appendix that for a fixed channels realization $h$, $f_i$, and $g_i$ the $\partial\mathcal{I}(\mathbf{x}_i,\mathbf{y}_i^d)/\partial\alpha_i$ has a constant sign and hence $\mathcal{I}(\mathbf{x}_i,\mathbf{y}_i^d)$ is a monotonous function of $\alpha_i \in [0,1]$. This is an important result, indeed

(i) when function (11) is increasing, we have $\alpha_i^{\text{opt}} = 1$ which means that the relay $i$ should not cooperate;

(ii) when function (11) is decreasing, we have $\alpha_i^{\text{opt}} = \max(1-P_{r_i}^{\max}/P_1, 0)$. Hence, the relay $i$ can cooperate,

(iii) for each subframe, we should allocated all power to the relay (i.e., $\alpha_i^{\text{opt}} = 0$) leading to an OAF protocol choice. However, when this power exceeds the individual power constrain, it is reallocated to the source (i.e., $\alpha_i = 1 - P_{r_i}^{\max}/P_1$) which means that the NAF protocol is selected;

(iv) the optimal $\alpha_i^{\text{opt}}$ is expected to have infinite possibilities depending on $h$, $f_i$, and $g_i$, however there is only two possible values which make the feedback very simple (only one bit per relay is needed).

In the sequel, the optimal power allocation per subframe $i$ respecting the power constraints on the relay $i$ is detailed. Without loss of generality, we distinguish two cases depending on the available power for all relays: if it is higher than the source one or not.

### 4.1. Case $P_{r_i}^{\max} \ge P_1$

In this case, the constraint $(1-\alpha_i)P_1 \le P_{r_i}^{\max}$ is always met meaning that the relay $i$ could transmit with power $P_1$. The power allocation scheme used to maximize the system capacity is given by

$$\alpha_i^{\text{opt}} = \begin{cases} 1 & \text{if } A_iD_i - B_iC_i \ge 0, \\ 0 & \text{if } A_iD_i - B_iC_i < 0, \end{cases} \tag{12}$$

where $A_iD_i - B_iC_i$ is the term determining the derivation sign of (11) (see the Appendix). We recommend hence either the OAF protocol (i.e., $\alpha_i^{\text{opt}} = 1$) presented in [2] or not to cooperate with the relay $i$ (i.e., $\alpha_i^{\text{opt}} = 0$). A relay station $i$ will serve during the subframe $i$ if the global channel capacity when relaying the source's signal to the destination is enhanced.

If there are more than one relay station and all of them have the same power constraint, one can select for the global cooperative frame the one that maximizes the following expression:

$$\gamma_i = \frac{a_ib_i}{1+a_i+b_i}. \tag{13}$$

In fact, since $\alpha_i^{\text{opt}} = 0$, it is easy to show that (13) maximizes (7) and the upper bound is achieved. The cooperative frame will be reduced to only one subframe.

Our best relay selection leads to a selective OAF protocol that we call OAFPA (OAF with power allocation) protocol. We remind that selective OAF (S-OAF) protocol was addressed in [2] where the selection is based on the outage probability: The cooperation is used only when the direct link is in outage. However in our protocol, the cooperation can be used even if the direct link is not in outage since we select the transmission method that maximizes the instantaneous capacity. Performance comparison between the two protocols is done and discussed in Section 5.

### 4.2. Case $P_{r_i}^{\max} \le P_1$

Here, the NAF protocol should be used, since the constraint $(1-\alpha_i)P_1 \le P_{r_i}^{\max}$ is met if and only if $\alpha_i \ge (1 - P_{r_i}^{\max}/P_1)$.

The power allocation scheme used to maximize the system's capacity is given by

$$\alpha_i^{\text{opt}} = \begin{cases} 1 & \text{if } A_iD_i - B_iC_i \geq 0, \\ 1 - \dfrac{P_{r_i}^{\max}}{P_1} & \text{if } A_iD_i - B_iC_i < 0. \end{cases} \quad (14)$$

As previously stated, a relay station will only serve if it performs better than the direct transmission. Unlike [4], there is a relay selection depending on the channel realizations.

If there are more than one relay and all of them have the same power constraint, using (7) and (11) one can easily show that to achieve the upper bound of ($\alpha_i \neq 0$), it suffices to select only the relay that maximizes

$$\gamma_i = \frac{A_i\alpha_i^{\text{opt}} + B_i}{C_i\alpha_i^{\text{opt}} + D_i}, \quad (15)$$

since $\alpha_i^{\text{opt}} \neq 0$.

## 5. SIMULATION RESULTS

We consider a symmetric network with equal channel variances $\sigma_h = \sigma_{f_i} = \sigma_{g_i} = 1$. The relay number $N$ is fixed to one or three. The analyzed performance is the outage probability and the capacity. The proposed protocols based on the optimal power allocation with relay selection are compared with the following.

(i) The S-OAF protocol proposed in [2] and reminded in Section 4.1. Our proposed protocol is OAFPA (OAF with power allocation).

(ii) The NAF protocol proposed in [4] since there is any selective NAF yet known. We remind that the power is equally divided between the source and the relay at the second slot for every subframe [4]. Our proposed protocol is called NAFPA.

For simplicity, we assume that all the relays have the same maximum power $P_{r_i}^{\max}$. As previously, we distinguish hence the following two cases.

### 5.1. Case $P_{r_i}^{\max} \geq P_1$

In order to satisfy the power constraint at the relay, the $P_{r_i}^{\max}$ is fixed to $P_1$.

*(1) Outage probability*

Figure 2 compares the outage probability for $N = 1$ and different transmitting rates $R$ (bits per channel use). The proposed solution and the S-OAF protocol have the same performance because they have the same outage criterion. However, for $N = 3$ the OAFPA protocol gives the best performance as shown in Figure 3. Indeed, it selects from the three relays the one that maximizes the system capacity corresponding to the upper bound of (7) when it is higher than the noncooperation ones. On the other hand, S-OAF tests first if the direct link is in outage. If it is the case, it uses the three relays to evaluate the capacity which is in general lower than the upper bound of (7).

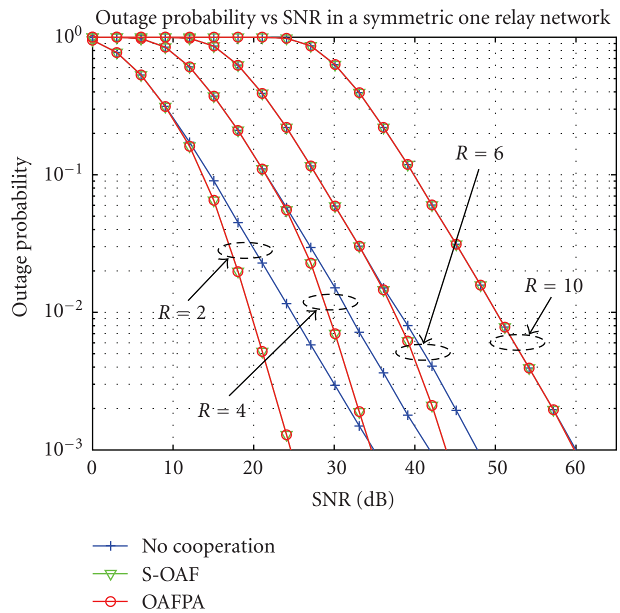


Figure 2: Outage probability comparison for orthogonal protocols, $N = 1$.

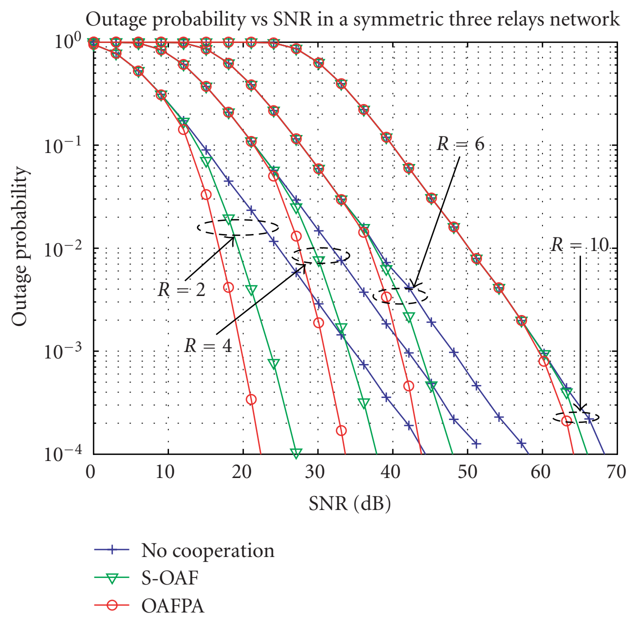


Figure 3: Outage probability comparison for orthogonal protocols, $N = 3$.

*(2) Ergodic capacity*

Unlike the outage probability, Figure 4 shows that even with one relay the OAFPA protocol capacity outperforms the S-OAF ones. The ergodic capacity depends on $R$ because this parameter is used for the relay selection criterion in the S-OAF protocol. The fact that it decides not to cooperate when the direct link is not in outage, without considering if the capacity when relaying is better, degrades the performance. This is amplified for high spectral efficiencies $R$. The OAFPA

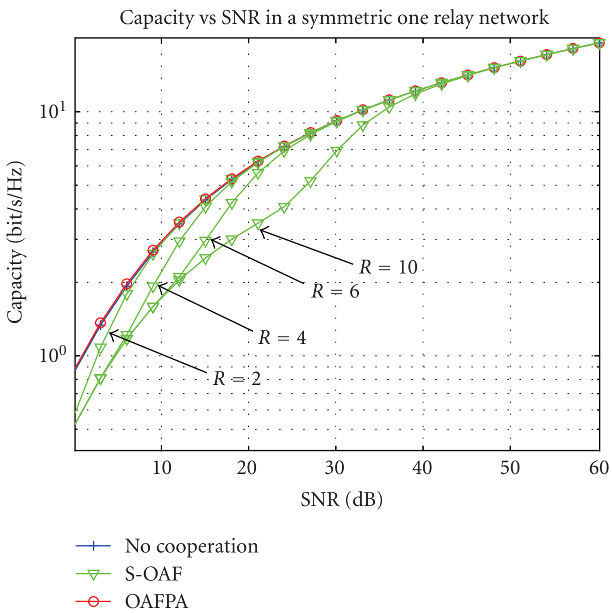


FIGURE 4: Ergodic capacity comparison for orthogonal protocols, $N = 1$ and minimum transmitting rate $R$.

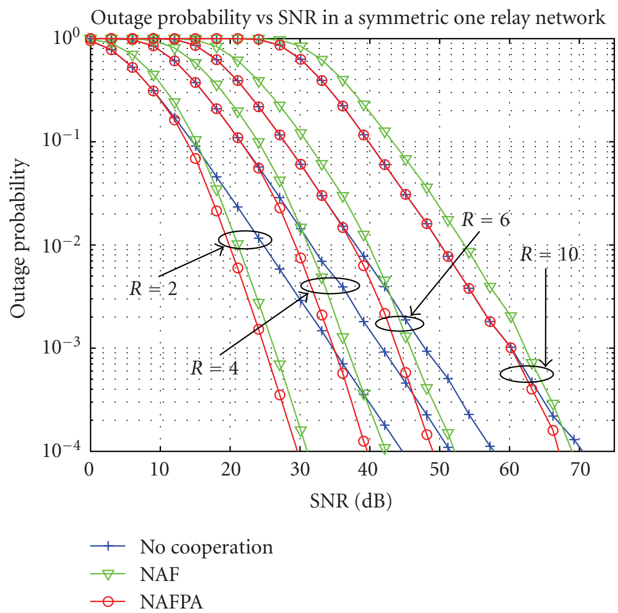


FIGURE 5: Outage probability comparison for nonorthogonal protocols, $N = 1$.

protocol capacity is very close to the noncooperation ones since the cooperation is not frequently decided for a symmetric network. Finally at very high SNR, the three protocols have the same capacity since the direct transmission is always selected.

### 5.2. Case $P_{r_i}^{\max} < P_1$

We assume that $P_{r_i}^{\max} = 3P_1/4$ and hence $P_1/4$ is used by the source in the second slot.

#### *(1) Outage probability*

The outage probabilities with different spectral efficiencies $R$ are presented for $N = 1$ and $N = 3$, respectively in Figures 5 and 6. The NAFPA protocol outperforms the NAF protocol for all cases thanks to the power allocation and the optimal relay selection. The NAF protocol performance suffers from the selection absence at low SNR.

#### *(2) Ergodic capacity*

In Figure 7, the NAFPA protocol capacity outperforms the NAF one for all SNR for $N = 1$. Indeed, this is due to the selection of the best way to communicate that maximizes the capacity. Both protocols have the same instantaneous capacity only when the cooperation is decided.

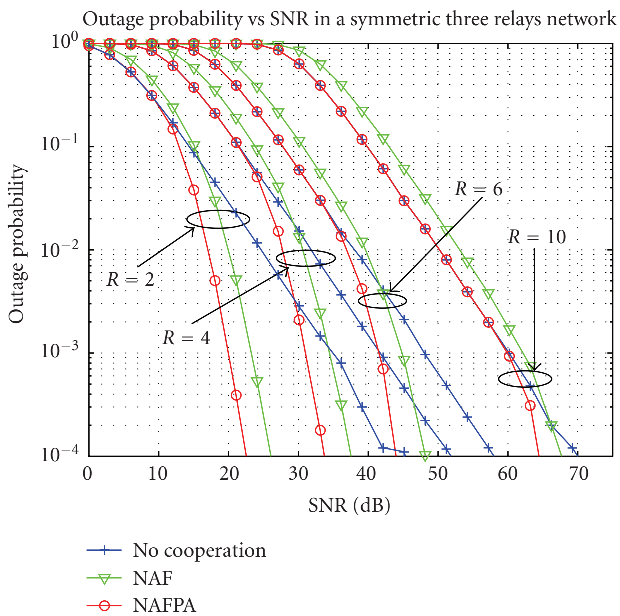


FIGURE 6: Outage probability comparison for nonorthogonal protocols, $N = 3$.

## 6. CONCLUSION

In this work, we have proposed to find the best way to communicate under power constraints per slot at the relays. The NAF, OAF, or noncooperation protocols choice is equivalent to a power allocation problem to maximize the system capacity. The solution showed that the cooperation mode per subframe (OAF or NAF) depends only on the power constraints at the relays. We gave simple conditions needed to decide to cooperate or not. The obtained optimization leads to new proposed cooperation protocols that combines power allocation with relay selection (OAFPA and NAFPA protocols) respecting the per slot constraints.

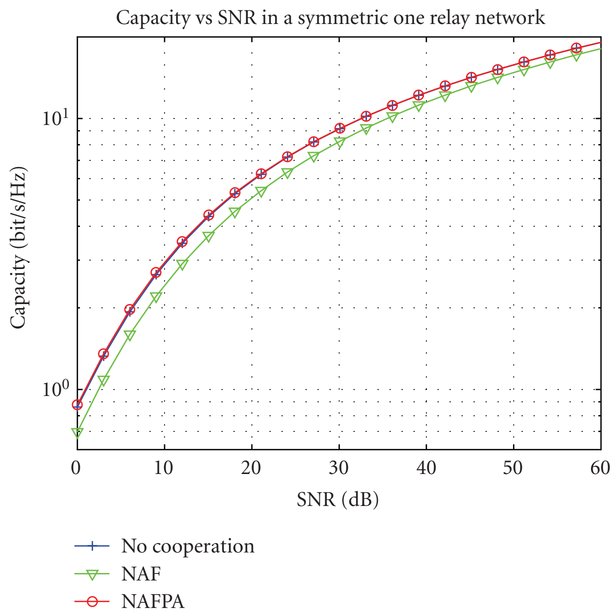


Figure 7: Ergodic capacity comparison for one relay and $P_{r_i}^{\max} = 3P_1/4$.

## APPENDIX

The first derivative of $\mathcal{I}(\mathbf{x}_i, \mathbf{y}_i^d)$ is

$$\frac{\partial \mathcal{I}(\mathbf{x}_i, \mathbf{y}_i^d)}{\partial \alpha_i} = \frac{1}{\ln 2} \frac{A_i D_i - B_i C_i}{(A_i\alpha_i + B_i)(C_i\alpha_i + D_i)}. \quad (A.1)$$

From the expressions of $A_i, B_i, C_i$, and $D_i$ given previously, the following signs determination is obvious

$$\begin{aligned} B_i &> 0 \\ D_i &> 0 \\ C_i &< 0 \\ \frac{-D_i}{C_i} &= \frac{1 + a_i + b_i}{b_i} > 1 \end{aligned} \quad (A.2)$$

and hence

$$(C_i\alpha_i + D_i) \geq 0, \quad \forall \alpha_i \in [0, 1]. \quad (A.3)$$

The derivative sign analysis lies on the sign of $A_i$. For this purpose, two cases are distinguished.

*Case A* ($A_i \geq 0$)

The numerator of (A.1) is hence positive and the sign depends only on the denominator one. This latter is the product of two linear functions of $\alpha_i$ with $\alpha_i \in [0, 1]$. The sign of each one has to be determined and to make a product afterwards.

Since $A_i \geq 0$, the ratio $-B_i/A_i \leq 0$ and the function $(A_i\alpha_i + B_i)$ are positive for all $\alpha_i \in [0, 1]$. The positiveness of the denominator lies on the function $(C_i\alpha_i + D_i)$ one which is the case as shown in (A.3). We then deduce that

$$\frac{\partial \mathcal{I}(\mathbf{x}_i, \mathbf{y}_i^d)}{\partial \alpha_i} \geq 0 \quad (A.4)$$

for $\alpha_i \in [0, 1]$ and the optimal choice is $\alpha_i^{\text{opt}} = 1$.

*Case B* ($A_i < 0$)

Now, to obtain the sign of (A.1) two subcases need to be considered: $A_iD_i - B_iC_i \geq 0$ and $A_iD_i - B_iC_i < 0$.

*(1) Case* $A_iD_i - B_iC_i \geq 0$

The mutual information derivative's numerator is assumed to be positive. First, we rewrite this numerator as

$$A_iD_i - B_iC_i = A_iC_i\left(\frac{D_i}{C_i} - \frac{B_i}{A_i}\right). \quad (A.5)$$

Since $A_i < 0$ and knowing that $C_i$ is always negative, (A.5) is positive if and only if $(D_i/C_i - B_i/A_i) > 0$. That means that

$$\frac{-D_i}{C_i} < \frac{-B_i}{A_i}. \quad (A.6)$$

The derivative sign depends only on the denominator $(A_i\alpha_i + B_i)(C_i\alpha_i + D_i)$ ones. But using (A.3), it only depends on the function $A_i\alpha_i + B_i$ sign. It is easy to see that this latter is always positive for all $\alpha_i \leq -B_i/A_i$. On the other hand, from (A.2) and (A.6) we have $-B_i/A_i > 1$ and consequently,

$$\frac{\partial \mathcal{I}(\mathbf{x}_i, \mathbf{y}_i^d)}{\partial \alpha_i} \geq 0 \quad (A.7)$$

for $\alpha_i \in [0, 1]$ and as previously the optimal choice is $\alpha_i^{\text{opt}} = 1$.

*(2) Case* $A_iD_i - B_iC_i < 0$

Similarly to the previous subcase, since $A_i < 0$ and knowing that $C_i \leq 0$, the expression (A.5) is negative if and only if $(D_i/C_i - B_i/A_i) < 0$. That means that

$$\frac{-D_i}{C_i} > \frac{-B_i}{A_i}. \quad (A.8)$$

Unlike the previous subcase, (A.8) does not show if the denominator zero is greater than 1.

Anyway, it is easy to see that the denominator is positive for all $\alpha_i \leq -B_i/A_i$. Knowing that the numerator is negative, the derivative is negative for all $\alpha_i \leq -B_i/A_i$. Moreover, before saying that the derivative is negative for $\alpha_i \in [0, 1]$, we ensure that $-B_i/A_i \geq 1$ which is equivalent to $A_i + B_i \geq 0$. By using

$$A_i + B_i = 1 + a_0 + a_i + a_0a_i + a_0(1 + a_0)(1 + a_i), \quad (A.9)$$

we have that $A_i + B_i$ is a sum of positive quantities and the sum is always positive. We can now write

$$\frac{\partial \mathcal{I}(\mathbf{x}_i, \mathbf{y}_i^d)}{\partial \alpha_i} \leq 0 \quad (A.10)$$

for $\alpha_i \in [0, 1]$, however the power constraints at the relay impose that the optimal choice is $\alpha_i^{\text{opt}} = \max(0, 1 - P_{r_i}^{\max}/P_1)$.

Using (A.4), (A.7), and (A.10) it is shown that (11) is a monotonous function.

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