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Research Article

Time-Division Multiuser MIMO with Statistical Feedback

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This paper investigates a time-division multiuser multiple-input multiple-output (MIMO) antenna system in K-block flat fading where users are given individual outage rate probability constraints and only one user accesses the channel at any given time slot (or block). Assuming a downlink channel and that the transmitter knows only the statistical information about the channel, our aim is to minimize the overall transmit power for achieving the users' outage constraint by jointly optimizing the power allocation and the time-sharing (i.e., the number of time slots) of the users. This paper first derives the so-called minimum power equation (MPE) to solve for the minimum transmit power required for attaining a given outage rate probability of a single-user MIMO block-fading channel if the number of blocks is predetermined. We then construct a *convex* optimization problem, which can mimic the original problem structure and permits to jointly consider the power consumption and the probability constraints of the users, to give a suboptimal multiuser time-sharing solution. This is finally combined with the MPE to provide a joint power allocation and time-sharing solution for the time-division multiuser MIMO system. Numerical results demonstrate that the proposed scheme performs nearly the same as the global optimum with inappreciable difference.

1. INTRODUCTION

Due to the instability nature of wireless channels, there has long been the challenge of communicating reliably and efficiently (in terms of both power and bandwidth) over wireless channels [1], and the subject of providing diversity transmissions and receptions is still a very hot ongoing research area today. An attractive means to obtain diversity is through the use of multiple antennas (or widely known as multiple-input multiple-output (MIMO) antenna systems), which gain diversity benefits without the need for any bandwidth expansion and increase in transmit power (e.g., see [2–8]).

In the past, most efforts focused on which rate a particular wireless channel can support. In particular, in an additive white Gaussian noise (AWGN) channel, practical coding techniques with finite (but long) code length are available to approach the Shannon capacity within a fraction of decibel [9, 10]. Later in [11], Goldsmith and Varaiya derived the ergodic capacity of a fading channel and showed that ergodic capacity can be achieved without knowing the channel state information at the transmitter (CSIT) if a very long codeword is permitted. Similar conclusion has also been drawn to MIMO channels [2, 3], which offer a capacity increase by a factor determined by the rank of the channel. Results of this sort are undoubtedly important to system optimization if the aim is to maximize the rate over a wireless channel.

However, for delay-sensitive applications, the rate is usually preset and the preferred aim would be to minimize the transmission cost for a given outage probability constraint (i.e., the probability that the target rate is not reached) [12–18]. To model this, it is customary to consider a K-block fading channel in which the fade is assumed to occur identically and independently from one block to another, but it remains static (or time-invariant) within a block (A packet of information data for communications may be regarded as a block. In the context of this paper, the terminologies such as block, packet and time slot will be used interchangeably.) of symbols [19]. In light of this, a delay constraint can be described as the probability of the outage event, which allows to include the target rate, the time-delay in the number of blocks, and the outage tolerance in probability as a single constraint [17, 18].

Recently, there have been some profound contributions in delay-limited channels assuming the use of causal CSIT.

In [14], Negi and Cioffi investigated the optimal power control for minimizing the outage probability using a dynamic programming (DP) approach with certain power constraints. Similar methodology was also proposed in [15] for a two-user downlink channel for expected capacity maximization with a short-term power constraint. Furthermore, in [16], Berry and Gallager looked into the delay-constrained problem taking into account the size of the buffer. Most recently in [17], an algorithm that finds the optimal power allocation over the blocks to minimize the overall transmit power while constraining an upper bound of the outage probability constraint was proposed. Unfortunately, the assumption of having perfect CSIT is questionable, and the required amount of channel feedback may not justify the diversity gain obtained from the intelligent power control.

The scope of this paper is fundamentally different from the previous works in that field. Only the receiver has perfect channel state information (CSIR), but the transmitter knows only the channel statistics (CST). Moreover, a time-division multiuser MIMO system in the downlink is considered. (Note that the works in [12–17] are all limited to single-user (or two-user) single-antenna channels.) In this setup, each user is given an individual outage rate probability constraint and only one user is allowed to access the channel for each block. Our goal is to optimize the power allocation among the users and to schedule the users smartly so that the overall transmit power is minimized while the outage probability constraints of the users are satisfied. Assuming that all users are subjected to a delay tolerance of K-blocks, (The result of this paper is extendable to the case where users have different K. However, this assumption greatly simplifies the presentation of this paper and makes it more accessible to the readers.) the exact order of how the users are scheduled within the blocks is irrelevant. As a consequence, our aim boils down to finding the optimal power allocation and the optimal time-sharing (i.e., the number of blocks/time slots assigned) among the users. The problem under investigation is specially crucial if the target rates of the users are predetermined and the cost of transmission is to be minimized with only statistical channel feedback. Note that this paper can be thought of as an extension of [18] to MIMO channels.

Our proposed approach is based on two major contributions: (1) the minimum power equation (MPE), and (2) a convexization of the original multiuser joint power allocation and time-sharing problem by upper bound formulation and relaxation. The solution of the MPE gives the minimum transmit power required for ensuring a given outage rate probability for a single-user MIMO n-block fading channel, while the *convex* problem enables to find a sensible time-sharing solution for a time-division multiuser MIMO channel by taking into account both users' potential power consumption and their likelihood of being in an outage. An algorithm that intelligently combines the MPE and convex problem is presented to obtain a suboptimal joint multiuser time-sharing and power allocation solution, which will be shown by numerical results to yield near optimal performance with inappreciable difference.

The remainder of the paper is structured as follows. In Section 2, we present the block-fading channel model for a time-division multiuser MIMO antenna system, and formulate the joint multiuser time-sharing and power allocation problem. Section 3 derives the MPE for a single-user MIMO block-fading channel. Section 4 proposes a convex problem to obtain a suboptimal multiuser time-sharing solution. In Section 5, an algorithm which finds a joint time-sharing and power allocation solution is presented. Numerical results will be provided in Section 6. Finally, we have some concluding remarks in Section 7.

2. SYSTEM MODEL AND PROBLEM FORMULATION

2.1. Single-user MIMO block-fading channel

Let us first assume a block flat-fading noisy channel as in [14, 17, 19]. Every set of information symbols T_0 is encoded as a single codeword and transmitted as one block (in a time slot). Data are required to arrive at the receiver in at most K-blocks of symbols. The channel is assumed to fade identically and independently from one block to another, but the fade can be considered static within a block of T_0 symbols. (In this paper, the exact value of T_0 is not important but it is assumed to be large enough so that noise can be averaged out from the information-theoretic perspective and the classical Shannon capacity formula is permitted.) We will use c_k to denote the channel power gain in block k and assume that the channel amplitude $\sqrt{c_k}$ is in Rayleigh fading so that c_k has the following probability density function (pdf):

$$\mathcal{F}(c_k) = \begin{cases} e^{-c_k}, & c_k \geq 0, \\ 0, & c_k < 0. \end{cases} \tag{1}$$

For a given block, say k, the Gaussian codebook is used with an assigned power of Q/K per block (i.e., with total power of Q), and the rate can be expressed in bps/Hz as

$$r_k = \log_2\left(1 + \frac{C_0 d^{-\gamma} Q c_k}{K N_0}\right), \tag{2}$$

where N_0 is the noise power, d denotes the distance between the transmitter and the receiver, γ is the power loss exponent, and C_0 is the distance-independent mean channel power gain. An outage is said to occur if $\sum_{k=1}^{K} r_k \leq R$ for some target rate R.

Our assumption is that the transmitter knows (1) and the channel statistical parameter $C_0 d^{-\gamma}$ (i.e., CST), but the receiver knows $\{c_n\}_{n\leq k}$ at time slot k (i.e., CSIR) so that maximum-likelihood decoding can be used to realize the rate in (2).

The above single-antenna model can be extended easily to a channel with MIMO antennas. This extension can be done by replacing the scalar channel $\sqrt{c_k}$ by a matrix channel, $\mathbf{H}_k = [h_{i,j}^{(k)}] \in \mathbb{C}^{n_r \times n_t}$, where n_t and n_r antennas are, respectively, located at the transmitter and the receiver. The amplitude square of each element, $|h_{i,j}^{(k)}|^2$, has the pdf of (1) as that of c_k, and the elements of $\mathbf{H}_k$ are independent and identically distributed (i.i.d.) for different k and antenna pairs. The rate achieved for block k can be written in bps/Hz as [3]

$$r_k = \log_2 \det\left[I + \left(\frac{C_0 d^{-\gamma} Q}{n_t K}\right)\frac{\mathbf{H}_k \mathbf{H}_k^{\dagger}}{N_0}\right], \tag{3}$$

where $\det(\cdot)$ denotes the determinant of a matrix, and the superscript $\dagger$ is the conjugate transposition. In (3), we have used the fact that the transmit covariance matrix at time k is $Q\mathbf{I}/n_tK$ because the transmitter does not have the instantaneous channel state information, and thus it transmits the same power across the antennas. By transmitting power of Q/n_tK at each antenna, the transmit power at each block is kept as Q/K. For conciseness, in the sequel, we will assume that $n_t \geq n_r$ and that the matrix $\mathbf{H}_k$ is always of full rank. The case of $n_t < n_r$ can be treated in a similar way and thus omitted.

2.2. Time-division multiuser MIMO system

In a time-division multiuser system, each block (or time slot) will be given to one of the users. If CSIT is available, it will be possible to gain multiuser diversity by assigning the time slot to a user with a strong channel. In that case, scheduling of users will be specific to the instantaneous CSIT. In this paper, however, only CST is known to the transmitter, and multiuser diversity of such kind is not obtainable. In what follows, the exact order of how the users are scheduled for transmission within the K-blocks is unimportant, and the only thing that matters is the amount of channel resources (such as the number of time slots) allocated to the users.

As a result, for a U-user system where w_u time slots are allocated to user u (note that $\sum_u w_u \leq K$), we can now assume, without loss of generality, that user u accesses the channels in time slots (or blocks) k such that

$$k \in \mathcal{D}_u \equiv \left\{\forall k \in \mathbb{Z} : \sum_{j=1}^{u-1} w_j + 1 \leq k \leq \sum_{j=1}^{u} w_j\right\}. \qquad (4)$$

Following the model described previously, the sum-rate attained for user u is given in bps/Hz by

$$\sum_{k\in\mathcal{D}_u} r_k = \sum_{k\in\mathcal{D}_u} \log_2 \det\left[\mathbf{I} + \left(\frac{C_0^{(u)}Q_u}{n_t w_u}\right)\frac{\mathbf{H}_k^{(u)}\mathbf{H}_k^{(u)\dagger}}{N_0}\right], \qquad (5)$$

where Q_u denotes the transmit power, $\mathbf{H}_k^{(u)}$ is the MIMO channel matrix from the transmitter to user u at slot k, and $C_0^{(u)} \triangleq C_0 d_u^{-\gamma}$ refers to the mean channel power gain between the transmitter and user u. The statistical property of the amplitude squared entries of $\mathbf{H}_k^{(u)}$ follows exactly (1).

Given a target rate R_u for user u in K-blocks, an outage will occur if $\sum_{k\in\mathcal{D}_u} r_k < R_u$, and the outage tolerance of a user can be characterized by the outage probability constraint

$$\mathcal{P}\left(\left\{\sum_{k\in\mathcal{D}_u} r_k < R_u\right\}\right) \leq \varepsilon_u, \qquad (6)$$

where $\mathcal{P}(\mathcal{A})$ denotes the probability of an event $\mathcal{A}$, and ε_u denotes the maximum allowable outage probability for user u. Note that (6) can be viewed as a probabilistic delay constraint which enables us to consider requirements such as target rate (R_u), outage tolerance (ε_u), and time delay in a number of time slots (K) altogether [17].

2.3. The joint multiuser time-sharing and power allocation problem

The problem of interest is to minimize the overall transmit power (i.e., $\sum_u Q_u$) while ensuring the users' individual outage probability constraints by jointly optimizing the time-sharing (i.e., the number of allocated time slots $\{w_u\}$) and the power allocation (i.e., $\{Q_u\}$) for the users. Mathematically, this is written as

$$\mathbb{M} \longmapsto \begin{cases} \min\limits_{\{Q_u\},\{w_u\}} \sum\limits_{u=1}^{U} Q_u \quad \text{s.t. } \mathcal{P}\left(\left\{\sum\limits_{k\in\mathcal{D}_u} r_k \leq R_u\right\}\right) \leq \varepsilon_u \ \forall u, \\ \qquad\qquad\qquad\qquad\qquad\qquad Q_u \geq 0 \quad \forall u, \\ \sum\limits_{u=1}^{U} w_u \leq K, \quad w_u \in \{1, 2, \ldots, K-U+1\} \ \forall u, \end{cases} \qquad (7)$$

where

(i) Q_u is the total power allocated to user u;
(ii) w_u is the number of blocks (or the amount of time) allocated to user u;
(iii) $\mathcal{D}_u$ is the set storing the indices of the channel assigned to user u;
(iv) U is the total number of users;
(v) K is the number of blocks;
(vi) R_u is the target rate for user u;
(vii) ε_u is the outage probability requirement for user u.

The challenge of $\mathbb{M}$ is that it is a mixed integer problem which has no known method of achieving the global optimum [20]. The rest of the paper will be devoted to solving (7). In particular, Section 3 will look into obtaining the optimal $\{Q_u\}$ for a given $\{w_u\}$. Section 4 will focus on finding the suboptimal time-sharing parameters $\{w_u\}$ using relaxation followed by convex optimization. Section 5 combines the two approaches to suboptimally solve (7). Numerical results in Section 6 will, however, show that the proposed suboptimal method performs nearly the same as the global optimum with inappreciable difference.

3. MINIMUM POWER EQUATION

In this section, we will derive an equation to determine the minimum power required for attaining a given outage rate probability if the number of blocks is fixed. In time-division systems, as each block is occupied by one user only, if $\{w_u\}$ are fixed, then the optimization for the users is completely uncoupled and will be equivalent to multiple individual users' power minimization. Therefore, it suffices to focus on a single-user system for a given number of blocks, n, or

$$\min_{Q\geq 0} Q \quad \text{s.t. } \mathcal{P}\left(\{r_1 + r_2 + \cdots + r_n \leq R\}\right) \leq \varepsilon, \qquad (8)$$

where the user index u is omitted for convenience.

To proceed further, we rewrite the outage probability as follows:

$$\begin{aligned}\mathcal{P}_{\text{out}} &\triangleq \mathcal{P}\left(\left\{\sum_{k=1}^{n}\log_2\det\left[\mathbf{I}+\left(\frac{C_0 d^{-\gamma}Q}{n_t n}\right)\frac{\mathbf{H}_k\mathbf{H}_k^\dagger}{N_0}\right]\le R\right\}\right)\\ &= \mathcal{P}\left(\left\{\sum_{k=1}^{n}\log_2\det\left(\mathbf{I}+\frac{C_0 d^{-\gamma}Q\mathbf{\Lambda}_k}{N_0 n_t n}\right)\le R\right\}\right),\end{aligned} \tag{9}$$

where $\mathbf{\Lambda}_k \triangleq \operatorname{diag}(\lambda_1^{(k)},\lambda_2^{(k)},\ldots,\lambda_{n_r}^{(k)})$ with $\lambda_1^{(k)}\ge\lambda_2^{(k)}\ge\cdots\ge\lambda_{n_r}^{(k)}>0$ standing for the ordered eigenvalues of $\mathbf{H}_k\mathbf{H}_k^\dagger$. Note also from our assumption that $n_r=\min\{n_t,n_r\}=\operatorname{rank}(\mathbf{H}_k)$ for all k. The random variables of the outage probability are the eigenvalues $\{\lambda_j^{(k)}\}$ whose joint pdf is [21]

$$\begin{aligned}\mathcal{F}(\mathbf{\Lambda}) &= \frac{\left(\prod_{i=1}^{n_r}\lambda_i\right)^{n_t-n_r}e^{-\sum_{i=1}^{n_r}\lambda_i}}{n_r!\prod_{i=1}^{n_r}(n_r-i)!(n_t-i)!}\\ &\quad\times\prod_{1\le i\le j\le n_r}(\lambda_i-\lambda_j)^2 \quad \text{for } \lambda_1,\ldots,\lambda_{n_r}>0,\end{aligned} \tag{10}$$

where the time index k is omitted for conciseness. Evaluation of the outage probability requires knowing the pdf of $\sum_k r_k$ with (10), and it has unfortunately been unknown so far. Recently, it was found in [22] that the pdf of r (r_k with the subscript k omitted) can be approximated as Gaussian (so does the sum-rate $\sum_k r_k$) with the mean $\mathrm{E}[r]$ and variance $\mathrm{VAR}[r]$ given, respectively, as [3, 22]

$$\begin{aligned}\mu(Q)\triangleq \mathrm{E}[r] &= \int_0^\infty \log_2\left(1+\frac{C_0 d^{-\gamma}Q\lambda}{n_t n N_0}\right)\\ &\quad\times\sum_{j=0}^{n_r-1}\frac{j!\lambda^{n_t-n_r}e^{-\lambda}}{(j+n_t-n_r)!}\left[L_j^{(n_t-n_r)}(\lambda)\right]^2 d\lambda,\end{aligned} \tag{11}$$

where $L_j^{(n_t-n_r)}(x)$ denotes the generalized Laguerre polynomial of order j, and

$$\begin{aligned}\sigma^2(Q) &\triangleq \mathrm{VAR}[r]\\ &= n_r\int_0^\infty \omega^2(Q,\lambda)p(\lambda)d\lambda\\ &\quad-\sum_{i=1}^{n_r}\sum_{j=1}^{n_r}\frac{(i-1)!(j-1)!}{(i-1+n_t-n_r)!(j-1+n_t-n_r)!}\\ &\quad\times\left[\int_0^\infty \lambda^{n_t-n_r}e^{-\lambda}L_{i-1}^{(n_t-n_r)}(\lambda)L_{j-1}^{(n_t-n_r)}(\lambda)\omega(Q,\lambda)d\lambda\right]^2\end{aligned} \tag{12}$$

in which $\omega(Q,\lambda)\triangleq\log_2(1+C_0 d^{-\gamma}Q\lambda/n_t n N_0)$ and

$$p(\lambda)\triangleq\frac{1}{n_r}\sum_{i=1}^{n_r}\frac{(i-1)!}{(i-1+n_t-n_r)!}\lambda^{n_t-n_r}e^{-\lambda}\left[L_{i-1}^{(n_t-n_r)}(\lambda)\right]^2. \tag{13}$$

In [22], it was revealed that using Gaussian approximation on the rate of a MIMO channel is accurate even with small number of antennas, and this claim will be substantiated in Section 4 where numerical results will be provided to verify its validity. In light of this, we will use Gaussian approximation on the sum-rate $\sum_{k=1}^{n} r_k$ (as this is a sum of independent random variables, clearly, the approximation will further improve if n increases). Consequently, the probability constraint can be expressed as

$$\frac{1}{2}\left\{1+\operatorname{erf}\left[\frac{R-n\mu(Q)}{\sigma(Q)\sqrt{2n}}\right]\right\}\le\varepsilon, \tag{14}$$

where $\operatorname{erf}(x)\triangleq(2/\sqrt{\pi})\int_0^x e^{-t^2}dt$. This probability constraint can further be simplified as

$$g(Q)\triangleq n\mu(Q)-\left[\sqrt{2n}\operatorname{erf}^{-1}(1-2\varepsilon)\right]\sigma(Q)-R\ge 0. \tag{15}$$

Accordingly, (8) can be re-expressed as

$$\mathbb{S}_n \longmapsto \begin{cases}\min\limits_{Q\ge 0} Q\\ \text{s.t. } g(Q)\ge 0.\end{cases} \tag{16}$$

Intuitively, g should be a strictly increasing function of Q because more transmit power leads to less chance of being in an outage. As a result, the minimum value of Q occurs when the equality of (15) holds, or the constraint becomes active, that is, $g(Q_{\min})=0$. Throughout this paper, we will refer to this equation as the minimum power equation (MPE). Because of the monotonicity of g, the solution of MPE is unique, and solving the MPE numerically can be done very efficiently using methods such as "fzero" in MATLAB. The challenge, however, remains to derive the closed-form expressions for the mean (11) and the variance (12).

In Appendix A, we have derived that

$$\begin{aligned}\mu(\Gamma_0) &= \frac{1}{\ln 2}\sum_{\ell=0}^{n_r-1}\sum_{m=0}^{\ell}\frac{\ell!}{(\ell+\delta)!}\left[\frac{1}{m!}\binom{\ell+\delta}{m+\delta}\right]^2\\ &\quad\times\int_0^\infty \ln(1+\Gamma_0\lambda)\lambda^{\delta+2m}e^{-\lambda}d\lambda\\ &\quad+\frac{2}{\ln 2}\sum_{\ell=1}^{n_r-1}\sum_{i=0}^{\ell-1}\sum_{j=i+1}^{\ell}\frac{\ell!}{(\ell+\delta)!}\frac{(-1)^{i+j}}{i!j!}\\ &\quad\times\binom{\ell+\delta}{i+\delta}\binom{\ell+\delta}{j+\delta}\int_0^\infty \ln(1+\Gamma_0\lambda)\lambda^{\delta+i+j}e^{-\lambda}d\lambda,\end{aligned}$$

$$\begin{aligned}\sigma^2(\Gamma_0) &= \frac{1}{\ln^2 2}\sum_{\ell=0}^{n_r-1}\sum_{m=0}^{\ell}\frac{\ell!}{(\ell+\delta)!}\left[\frac{1}{m!}\binom{\ell+\delta}{m+\delta}\right]^2\\ &\quad\times\int_0^\infty \ln^2(1+\Gamma_0\lambda)\lambda^{\delta+2m}e^{-\lambda}d\lambda\\ &\quad+\frac{2}{\ln^2 2}\sum_{\ell=1}^{n_r-1}\sum_{i=0}^{\ell-1}\sum_{j=i+1}^{\ell}\frac{\ell!}{(\ell+\delta)!}\frac{(-1)^{i+j}}{i!j!}\\ &\quad\times\binom{\ell+\delta}{i+\delta}\binom{\ell+\delta}{j+\delta}\int_0^\infty \ln^2(1+\Gamma_0\lambda)\lambda^{\delta+i+j}e^{-\lambda}d\lambda\end{aligned}$$

$$- \frac{1}{\ln^2 2} \sum_{i=0}^{n_r-1} \sum_{j=0}^{n_r-1} \frac{i!j!}{(i+\delta)!(j+\delta)!} \left[\sum_{m=0}^{i} \sum_{\ell=0}^{j} \frac{(-1)^{m+\ell}}{m!\ell!} \binom{i+\delta}{m+\delta} \right.$$
$$\left. \times \binom{j+\delta}{\ell+\delta} \int_0^\infty \lambda^{\delta+m+\ell} e^{-\lambda} \ln(1+\Gamma_0\lambda) d\lambda \right]^2, \tag{17}$$

where $\Gamma_0 \triangleq C_0 d^{-\gamma} Q / n_t n N_0$ and $\delta \triangleq n_t - n_r$. Further, the integrals of the forms $\int_0^\infty \lambda^j e^{-\lambda} \ln(1+\Gamma_0\lambda) d\lambda$ and $\int_0^\infty \lambda^j e^{-\lambda} \ln^2(1+\Gamma_0\lambda) d\lambda$ are, respectively, given by

$$\begin{aligned}
&\int_0^\infty \lambda^j e^{-\lambda} \ln(1+\Gamma_0\lambda) d\lambda \\
&\quad = \frac{e^{1/\Gamma_0}}{\Gamma_0^{j-1}} \sum_{\ell=0}^{j} (-1)^\ell \binom{j}{\ell} \frac{(j-\ell)!}{(1/\ell)^{j-\ell}} E_1\left(\frac{1}{\Gamma_0}\right) \\
&\qquad + \frac{1}{\Gamma_0^j} \sum_{\ell=0}^{j-1} \sum_{p=1}^{j-\ell} (-1)^\ell \binom{j}{\ell} \frac{(j-\ell)!}{(1/\ell)^{j-\ell}} \cdot \frac{1}{j-\ell+1-p} \\
&\qquad + \frac{1}{\Gamma_0^{j+1}} \sum_{\ell=0}^{j-2} \sum_{p=2}^{j-\ell} \sum_{q=1}^{p-1} (-1)^\ell \\
&\qquad \times \binom{j}{\ell} \frac{(j-\ell)!}{(j-\ell-p+1)!} \frac{\Gamma_0^p}{j-\ell-q+1}
\end{aligned} \tag{18}$$

and

$$\begin{aligned}
&\int_0^\infty [\ln(1+\Gamma_0\lambda)]^2 \lambda^j e^{-\lambda} d\lambda \\
&\quad = \frac{e^{1/\Gamma_0}}{\Gamma_0^{j+1}} \sum_{\ell=0}^{j} (-1)^\ell \binom{j}{\ell} \frac{(j-\ell)!}{(1/\Gamma_0)^{j-\ell}} \\
&\qquad \times \left\{ \Gamma_0 \left[\left(\ln \frac{1}{\Gamma_0} - \gamma_{\mathrm{EM}} \right)^2 + \frac{\pi^2}{6} \right] \right. \\
&\qquad\qquad \left. - 2\,{}_3F_3\left([1,1,1]; [2,2,2]; -\frac{1}{\Gamma_0} \right) \right\} \\
&\qquad + \frac{2e^{1/\Gamma_0}}{\Gamma_0^j} \sum_{\ell=0}^{j-1} \sum_{p=1}^{j-\ell} (-1)^\ell \binom{j}{\ell} \frac{(j-\ell)!}{(1/\Gamma_0)^{j-\ell}} \\
&\qquad \cdot \frac{1}{j-\ell+1-p} E_1\left(\frac{1}{\Gamma_0}\right) \\
&\qquad + \frac{2}{\Gamma_0^j} \sum_{\ell=0}^{j-2} \sum_{p=1}^{j-\ell-1} \sum_{q=p+1}^{j-\ell} (-1)^\ell \binom{j}{\ell} \frac{(j-\ell)!}{(1/\Gamma_0)^{j-\ell}} \\
&\qquad \cdot \frac{1}{(j-\ell+1-p)(j-\ell+1-q)} \\
&\qquad + \frac{2}{\Gamma_0^{j+1}} \sum_{\ell=0}^{j-3} \sum_{t=3}^{j-\ell} \sum_{p=1}^{t-2} \sum_{q=p+1}^{t-1} (-1)^\ell \binom{j}{\ell} \frac{(j-\ell)!}{(j-\ell-t+1)!} \\
&\qquad \cdot \frac{\Gamma_0^t}{(j-\ell+1-p)(j-\ell+1-q)}
\end{aligned} \tag{19}$$

in which $E_1(\cdot)$ stands for the exponential integral, ${}_pF_q$ is the generalized hypergeometric function, and γ_{EM} is the Euler-Mascheroni constant [23].

To summarize, we now have the MPE to determine the minimum required transmit power for achieving a given information outage probability in an n-block MIMO fading channel. Presumably, if the time-sharing parameters (i.e., $\{w_u\}$) of a time-division multiuser system are known, then the corresponding optimal power allocation for the users can be found from the MPEs. And, the optimal solution of (7) could be found using the MPE by an exhaustive search over the space of $\{w_u\}$ (see Section 6.1 for details). However, this searching approach will be too complex to be done even if the number of users or blocks is moderate. To address this, in the next section, we will focus on how a sensible solution of $\{w_u\}$ can be found suboptimally.

4. MULTIUSER TIME-SHARING FROM CONVEX OPTIMIZATION

In this section, our aim is to optimize the time-sharing parameters $\{w_u\}$ by *joint* consideration of the power consumption and the probability constraints of the users. Ideally, it requires to solve $\mathbb{M}$, that is, (7), which is unfortunately not known. Here, we propose to mimic $\mathbb{M}$ by considering a simpler problem with the probability constraints replaced by some upper bounds, that is,

$$\begin{aligned}
&\mathcal{P}\left(\left\{ \sum_{k \in \mathcal{D}_u} r_k \le R_u \right\} \right) \\
&\quad \le \mathcal{P}\left(\left\{ \sum_{k \in \mathcal{D}_u} \log_2\left(1 + \frac{C_0^{(u)} Q_u}{n_t w_u} \cdot \frac{\lambda_{\max}^{(u,k)}}{N_0} \right) \le R_u \right\} \right) \\
&\quad < \mathcal{P}\left(\left\{ \rho \triangleq \prod_{k \in \mathcal{D}_u} \lambda_{\max}^{(u,k)} \le \left(\frac{w_u n_t N_0}{C_0^{(u)} Q_u} \right)^{w_u} 2^{R_u} \right\} \right) \triangleq \mathcal{P}_{\mathrm{UB}}^{(u)},
\end{aligned} \tag{20}$$

where $\lambda_{\max}^{(u,k)}$ denotes the maximum eigenvalue of the channel for user u at time slot k. The first inequality in (20) comes from ignoring the rates contributed by the smaller spatial subchannels, while the second inequality removes the unity inside the log expression (which may be regarded as a high signal-to-noise ratio (SNR) approximation). The pdf of $\lambda_{\max}^{(u,k)}$ is given by [24, 25]

$$\mathcal{F}(\lambda) = \sum_{i=1}^{n_r} \sum_{j=\delta}^{(n_t+n_r)i-2i^2} \left(d_{i,j} \cdot \frac{i^{j+1}}{j!} \right) \lambda^j e^{-i\lambda}, \quad \lambda > 0, \tag{21}$$

where the coefficients $\{d_{i,j}\}$ are independent of λ. In [25], the values of $d_{i,j}$ for a large number of MIMO settings have been enumerated.

The original outage rate probability constraint in (7) can therefore be ascertained by constraining the upper bound of the outage probability $\{\mathcal{P}_{\mathrm{UB}}^{(u)} \le \varepsilon_u\}$. The advantage by doing so is substantial. First of all, the optimizing variable Q_u can be separated from the random variable, and secondly, the distribution of $\ln\rho$ can be approximated as Gaussian, which permits to evaluate $\mathcal{P}_{\mathrm{UB}}^{(u)}$ as

$$\mathcal{P}_{\mathrm{UB}}^{(u)} = \frac{1}{2} + \frac{1}{2}\operatorname{erf}\left(\frac{\ln\left[\left(w_u n_t N_0/C_0^{(u)} Q_u\right)^{w_u} 2^{R_u}\right] - w_u\widetilde{\mu}}{\sqrt{2w_u}\widetilde{\sigma}}\right), \tag{22}$$

where $\widetilde{\mu}$ and $\widetilde{\sigma}$ are derived in Appendix B as

$$\begin{aligned}
\widetilde{\mu} &= \mathrm{E}[\ln\lambda] = \sum_{i=1}^{n_r}\sum_{j=\delta}^{(n_t+n_r)i-2i^2} d_{i,j}\left(H_j - \gamma_{\mathrm{EM}} - \ln i\right),\\
\widetilde{\sigma}^2 &= \mathrm{VAR}[\ln\lambda]\\
&= \sum_{i=1}^{n_r}\sum_{j=\delta}^{(n_t+n_r)i-2i^2} d_{i,j}\Bigg[\gamma_{\mathrm{EM}}^2 + 2\left(\ln i - H_j\right)\gamma_{\mathrm{EM}}\\
&\qquad + \frac{\pi^2}{6} - 2H_j\ln i + (\ln i)^2 + 2\sum_{t=1}^{j-1}\frac{H_t}{t+1}\Bigg] - \widetilde{\mu}^2,
\end{aligned} \tag{23}$$

where H_ℓ is the harmonic number defined as $\sum_{m=1}^{\ell}(1/m)$.

The constraint $\{\mathcal{P}_{\mathrm{UB}}^{(u)} \leq \varepsilon_u\}$ can therefore be simplified to

$$Q_u \geq \frac{n_t N_0}{C_0^{(u)} e^{\widetilde{\mu}}}\cdot\frac{w_u\left(2^{R_u}\right)^{1/w_u}}{\left[e^{-\sqrt{2}\widetilde{\sigma}\operatorname{erf}^{-1}(1-2\varepsilon_u)}\right]^{1/\sqrt{w_u}}}. \tag{24}$$

Using the upper bound constraints in the multiuser problem (7), we then have

$$\widetilde{\mathbb{M}} \longmapsto \begin{cases} \displaystyle\min_{\{Q_u\},\{w_u\}}\sum_{u=1}^{U} Q_u \\ \text{s.t. } Q_u \geq \dfrac{n_t N_0}{C_0^{(u)} e^{\widetilde{\mu}}}\cdot\dfrac{w_u\left(2^{R_u}\right)^{1/w_u}}{\left[e^{-\sqrt{2}\widetilde{\sigma}\operatorname{erf}^{-1}(1-2\varepsilon_u)}\right]^{1/\sqrt{w_u}}} \quad \forall u, \\ \displaystyle\sum_{u=1}^{U} w_u \leq K, \quad w_u \in \{1,2,\ldots,K-U+1\} \ \forall u, \end{cases} \tag{25}$$

where the constraints are now written in closed forms.

It is anticipated that the power allocation from the modified problem (25) may be quite conservative, that is, $Q_{\mathrm{opt}}|_{\widetilde{\mathbb{M}}} \gg Q_{\mathrm{opt}}|_{\mathbb{M}}$, because the upper bound may be loose. However, our conjecture is that the problem structure of $\mathbb{M}$ on $\{w_u\}$ would be accurately imitated by $\widetilde{\mathbb{M}}$. Accordingly, we may be able to obtain near optimal solution for $\{w_u\}$ by solving $\widetilde{\mathbb{M}}$, though accurate power consumption cannot be estimated from $\widetilde{\mathbb{M}}$. Following the same argument, the exact tightness of the upper bound and also how accurate the Gaussian approximation is in evaluating the upper bound probability are not important, as long as $\widetilde{\mathbb{M}}$ preserves the structure to balance the users' channel occupancy and power consumption to meet the outage probability requirements.

One remaining difficulty of solving $\widetilde{\mathbb{M}}$ is that the optimization is mixed with combinatorial search over the space of $\{w_u\}$ because they are integer-valued [20]. To tackle this, we relax $\{w_u\}$ to positive real numbers $\{x_u^2\}$ so that $\widetilde{\mathbb{M}}$ can be rewritten as

$$\widetilde{\mathbb{M}}_r \longmapsto \begin{cases} \displaystyle\min_{\{x_u\}} \frac{n_t N_0}{e^{\widetilde{\mu}}}\sum_{u=1}^{U}\frac{1}{C_0^{(u)}}\cdot\frac{x_u^2\left(a_u\right)^{1/x_u^2}}{\left(b_u\right)^{1/x_u}} \\ \text{s.t. } \displaystyle\sum_{u=1}^{U} x_u^2 \leq K,\ 1 \leq x_u \leq \sqrt{K-U+1}, \end{cases} \tag{26}$$

where $a_u \triangleq 2^{R_u}$ and $b_u \triangleq e^{-\sqrt{2}\widetilde{\sigma}\operatorname{erf}^{-1}(1-2\varepsilon_u)}$. Apparently, both constraints in (26) are convex, and if the cost is also convex, the problem can be solved using known convex programming routines [20].

Now, let us turn our attention to a function of the form

$$f(x) = x^2\cdot\frac{a^{1/x^2}}{b^{1/x}} \equiv x^2 h(x) \quad \text{for } a,b,x > 0, \tag{27}$$

where $h(x) \triangleq a^{1/x^2}/b^{1/x}$. Our interest is to examine if $f(x)$ is convex, or equivalently whether $f''(x) > 0$. To show this, we first obtain

$$\begin{aligned}
h'(x) &= h(x)\left(\frac{-2\ln a}{x^3} + \frac{\ln b}{x^2}\right),\\
h''(x) &= h(x)\left(\frac{-2\ln a}{x^3} + \frac{\ln b}{x^2}\right)^2 + h(x)\left(\frac{6\ln a}{x^4} - \frac{2\ln b}{x^3}\right).
\end{aligned} \tag{28}$$

Applying these results, $f''(x)$ can be found as

$$\frac{f''(x)}{h(x)} = \left(\frac{-2\ln a}{x^2} + \frac{\ln b}{x}\right)^2 + \left(\frac{-2\ln a}{x^2} + \frac{2\ln b}{x}\right) + 2. \tag{29}$$

Letting $\alpha = 2\ln a/x^2$ and $\beta = -\ln b/x$, we have

$$\begin{aligned}
\frac{f''(x)}{h(x)} &= (\alpha+\beta)^2 - \alpha - 2\beta + 2\\
&= \left(\alpha - \frac{1}{2}\right)^2 + (\beta-1)^2 + \frac{3}{4} + 2\alpha\beta > 0
\end{aligned} \tag{30}$$

since $\alpha, \beta > 0$, which can be seen from the definition of (a,b) that $\alpha > 0$ and $\beta > 0$ for $\varepsilon_u < 0.5$. (It should be emphasized that the convexity of f is subjected to the condition that $\varepsilon_u < 0.5$. However, in practice, it would not make sense to have a system operating with outage probability greater than 50%.) Together with the fact that $h(x) > 0$ for all $x > 0$, we can conclude that $f''(x) > 0$, and therefore $f(x)$ is convex. As the cost function in (26) is a summation of the functions of the form $f(x)$, it is convex, hence the problem (26) or $\widetilde{\mathbb{M}}_r$.

With $\widetilde{\mathbb{M}}_r$ being convex, we can find the globally optimal $\{x_u\}_{\mathrm{opt}}$ at polynomial time complexity. In particular, the complexity grows like $\mathcal{O}(U^3)$, which is scalable with the number of users [20]. The remaining task, however, is to derive the integer-valued $\{w_u\}$ from $\{x_u\}$. Simply, setting $w_u = x_u^2$ would result in noninteger solutions, while rounding them off could lead to violation of the outage rate probability constraints. In this paper, a greedy approach will be presented to obtain a feasible solution of $\{w_u\}$ from $\{x_u\}$, which will be described in the next section.

5. THE PROPOSED ALGORITHM

Thus, so far we have presented two main approaches: one that determines the optimal transmit power $\{Q_u\}$ based on MPE (see Section 3) and another one that finds the suboptimal (relaxed) time-sharing parameters $\{w_u\}$ by constraining the upper bound probability (see Section 4). In this section, we will devise an algorithm that combines the two approaches to jointly optimize the power allocation and time-sharing of the users. Our idea is to first map the optimal solution $\{x_u\}_{\text{opt}}$ from $\widetilde{\mathbb{M}}_r$ to a proper $\{w_u\}$ in $\mathbb{M}$ by rounding the results to the nearest positive integers, and then to step by step allocate one more block to the user who can minimize the overall required power using MPE. The proposed algorithm is described as follows.

(1) Solve $\{x_u\}$ in $\widetilde{\mathbb{M}}_r$ (see (26)) using convex optimization routines such as interior-point method [20].

(2) Initialize $w_u = \lfloor x_u^2 \rfloor$ for all u, where y returns the greatest integer that is smaller than y. Notice that at this point, $\{w_u\}$ and $\{Q_u\}$ from $\widetilde{\mathbb{M}}_r$ may not give a feasible solution to $\mathbb{M}$, and some outage rate probability constraints may not be satisfied.

(3) For each user u, compute the minimal required power to ensure the outage rate probability constraint by solving MPE:

$$Q_u = \arg\{g_u(Q \mid w_u) = 0\} = \arg\min_{Q\geq 0} |g_u(Q \mid w_u)|, \tag{31}$$

where the function $g_u(Q \mid w_u)$ is defined similarly as in (15). The notation $(Q \mid w_u)$ is used to emphasize the fact that w_u is given as a fixed constant.

(4) Then, initialize $m = K - \sum_{u=1}^{U} w_u$.

(5) Compute the power reduction metrics

$$\triangle Q_u = Q_u - \arg\min_{Q\geq 0} |g_u(Q \mid w_u + 1)| \quad \forall u. \tag{32}$$

(6) Find $u^* = \arg\max_u \triangle Q_u$ and update

$$\begin{aligned} w_{u^*} &:= w_{u^*} + 1, \\ Q_{u^*} &:= Q_{u^*} - \triangle Q_{u^*}, \\ m &:= m - 1. \end{aligned} \tag{33}$$

If $m \geq 1$, go back to step (5). Otherwise, go to step (7).

(7) Optimization is completed and the solutions for both $\{w_u\}$ and $\{Q_u\}$ have been found.

A first look at the algorithm reveals that the required complexity of the proposed algorithm is

$$\begin{aligned} \mathcal{C}_{\text{proposed}} &= \mathcal{O}(U^3) + mU\mathcal{C}_{\text{fzero}} \\ &\lesssim \mathcal{O}(U^3) + U^2\mathcal{C}_{\text{fzero}}, \end{aligned} \tag{34}$$

where $\mathcal{C}_{\text{fzero}}$ denotes the required complexity for finding a zero of $g(Q)$. The actual complexity for finding the root depends on the method used (e.g., bisection, secant, Brent's, etc.) and the required precision of the solution. For more details, we refer the interested readers to the classical paper [26] if Brent's method is used (note that fzero in MATLAB also implements Brent's method).

6. NUMERICAL RESULTS

6.1. Setup and benchmarks

Computer simulations are conducted to evaluate the performance of the proposed algorithm for the power-minimization problem with outage rate probability constraints. Only CST has been assumed, and capacity-achieving codec is used so that the expression $\log_2(1 + \text{SNR})$ can be used to express the rate achieved for each block. The total transmit SNR, defined as $((1/U)\sum_{u=1}^{U} C_0^{(u)})(\sum_{u=1}^{U} Q_u)(1/N_0)$, is considered as the performance metric. To compute the required SNR for a given set of simulation parameters such as the numbers of users and blocks, the users' target rates, and outage probabilities, the algorithm presented in Section 5, which iteratively solves the MPE, is used. Note that the MPE itself has already taken into account the randomness of the channel for outage evaluation.

Results for the proposed algorithm will be compared with the following benchmarks.

(1) *Global optimum*: with MPE presented in Section 3, it is possible to find the global optimal solution of time and power allocations for the users by solving $\mathbb{M}$ over the space of $\{w_u\}$ at the expense of much greater complexity, that is,

$$\mathbb{M} \longmapsto \begin{cases} \min\limits_{\{w_u\}} \sum\limits_{u=1}^{U} \arg\{g_u(Q_u \mid w_u) = 0\} \\ \text{s.t.} \sum\limits_{u=1}^{U} w_u \leq K, \quad w_u \in \{1, 2, \ldots, K-U+1\} \ \forall u. \end{cases} \tag{35}$$

The required complexity is given by

$$\mathcal{C}_{\text{optimum}} = \binom{K}{U} U\mathcal{C}_{\text{fzero}} \approx \binom{K}{U}\left[\frac{\mathcal{C}_{\text{proposed}} - \mathcal{O}(U^3)}{U}\right]. \tag{36}$$

For large K, we have

$$\frac{\mathcal{C}_{\text{optimum}}}{\mathcal{C}_{\text{proposed}}} \approx \frac{1}{U}\binom{K}{U} \tag{37}$$

and the complexity saving by the proposed scheme will be enormous. For instance, if $U = 4$ and $K = 30$, the ratio is approximately 6851.

(2) *Equal-time with optimized power*: an interesting benchmark is the system where each user is allocated more or less an equal number of blocks (i.e., $w_u \approx \lfloor K/U \rfloor$ for all u with $\sum_u w_u = K$), while the power allocation for each user is optimized by solving MPE. Obviously, if the system has homogeneous users (e.g., users with the same channel statistics and outage requirements), then equal-time allocation should be optimal. This system can show how important time-sharing optimization is if the system has highly heterogeneous users.

(3) *Equal-time with suboptimal power (see* (24)): a suboptimal power allocation to achieve a given outage probability can be found by (24) based on the upper bound probability without relying on the MPE. This system enables us to see how important the MPE is.

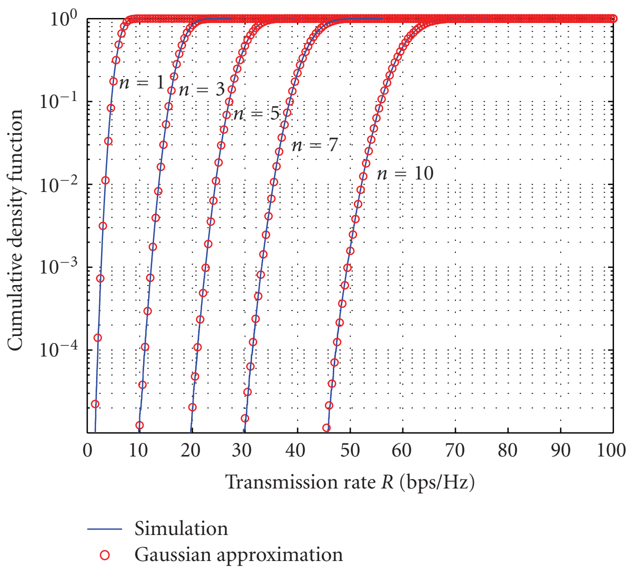

Figure 1: Comparison between the actual and approximated distributions for a (3,2) MIMO system with SNR per block of 10 dB.

6.2. *Results*

The cumulative distribution functions (cdfs) of the actual sum-rate and Gaussian approximation for a (3,2) system with 10 dB of SNR per block for various numbers of blocks n and target rates are compared in Figure 1. As we can see, for a wide range of outage probabilities (e.g., $\varepsilon \geq 10^{-5}$), they have inappreciable difference even if n is as small as 1. This shows that using a Gaussian cdf to evaluate the outage probability for a block-fading MIMO channel is accurate and reliable.

Results in Figure 2 are provided for the transmit SNR against the outage probability requirements for a 3-user system with 20 blocks (i.e., $K = 20$). The users are considered to have target rates $(R_1, R_2, R_3) = (8, 12, 16)$ bps/Hz, channel power gains $(C_0^{(1)}, C_0^{(2)}, C_0^{(3)}) = (0.8, 1, 1.2)$, multiple receive antennas $(n_r^{(1)}, n_r^{(2)}, n_r^{(3)}) = (2, 3, 2)$, and the same outage probability requirements (ε). The number of transmit antennas at the base station is set to be 4 (i.e., $n_t = 4$). Results in this figure show that the total transmit SNR of the proposed scheme decreases if the required outage probability increases. For example, there is about 2 dB power reduction when ε increases from 10^{-5} to 10^{-1}. Results also illustrate that the proposed method performs nearly the same as the global optimum. However, compared with the equal-time method with optimum power solution, there is only about 0.2 dB reduction in SNR by the proposed method. This is because the optimal strategy tends to allocate similar number of blocks to the users, which can be observed from configuration 1 of Table 1. In addition, as can be seen, the transmit SNR of the equal-time method with suboptimal power is much greater than that with optimum power, which shows that the MPE is very important in optimizing the power allocation. In particular, more than 3 dB of SNR is required when compared with the equal-time method with optimal power solution.

The SNR results against the target rate for a 3-user system with total numbers of blocks $K = 15$, $(\varepsilon_1, \varepsilon_2, \varepsilon_3) = (10^{-4}, 10^{-3}, 10^{-2})$, and $(C_0^{(1)}, C_0^{(2)}, C_0^{(3)}) = (0.5, 1, 1.5)$ are plotted in Figure 3. Also, 3 transmit antennas and 2 receive antennas per users are considered, and all the users have the same target rate R. Results indicate that the total transmit SNR increases dramatically with R (e.g., 10 dB increase from 8 bps/Hz to 32 bps/Hz for the proposed method). As can be observed, the increase in SNR is almost linear with R. In addition, the proposed method consistently performs nearly as

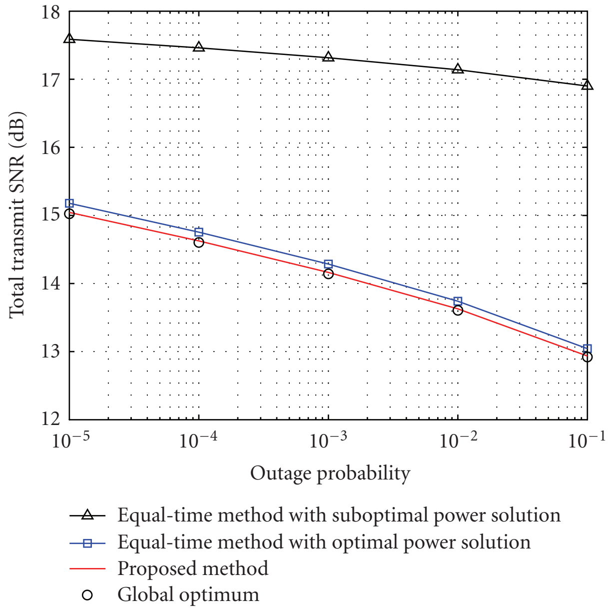

Figure 2: Results of the transmit SNR against the outage probability when $U = 3$, $K = 20$, $(R_1, R_2, R_3) = (8, 12, 16)$ bps/Hz, $(C_0^{(1)}, C_0^{(2)}, C_0^{(3)}) = (0.8, 1, 1.2)$, $n_t = 4$, and $(n_r^{(1)}, n_r^{(2)}, n_r^{(3)}) = (2, 3, 2)$.

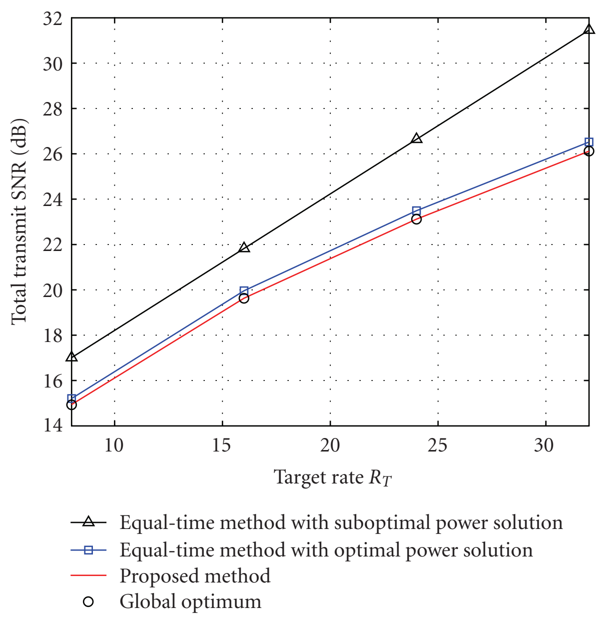

Figure 3: Results of the transmit SNR against the target rate when $U = 3$, $K = 15$, $(\varepsilon_1, \varepsilon_2, \varepsilon_3) = (10^{-4}, 10^{-3}, 10^{-2})$, $(C_0^{(1)}, C_0^{(2)}, C_0^{(3)}) = (0.5, 1, 1.5)$, $n_t = 3$, and $(n_r^{(1)}, n_r^{(2)}, n_r^{(3)}) = (2, 2, 2)$.

TABLE 1: Various configurations tested from Figures 2–5. The superscript ⋆ highlights the solution that is not the same as the optimum.

Configuration	u	R_u (bps/Hz)	ε_u	$C_0^{(u)}$	$n_r^{(u)}$	$(w_u)_{\text{opt}}$	$(w_u)_{\text{proposed}}$	$(w_u)_{\text{equal-time}}$
1 (n_t = 4 and K = 20)	1	8	10^{-3}	0.8	2	6	6	6
	2	12	10^{-3}	1	3	5	$6^\star$	$7^\star$
	3	16	10^{-3}	1.2	2	9	$8^\star$	$7^\star$
2 (n_t = 3 and K = 15)	1	16	10^{-4}	0.5	2	6	6	$5^\star$
	2	16	10^{-3}	1	2	5	5	5
	3	16	10^{-2}	1.5	2	4	4	$5^\star$
3 (n_t = 4 and K = 12)	1	16	10^{-2}	1.5	4	2	2	$4^\star$
	2	20	10^{-3}	1	3	3	3	$4^\star$
	3	24	10^{-4}	0.5	2	7	7	$4^\star$
4 (n_t = 4 and K = 20)	1	8	10^{-1}	1.5	3	2	$3^\star$	$6^\star$
	2	16	10^{-3}	1	3	6	$7^\star$	$7^\star$
	3	24	10^{-4}	0.5	3	12	$10^\star$	$7^\star$

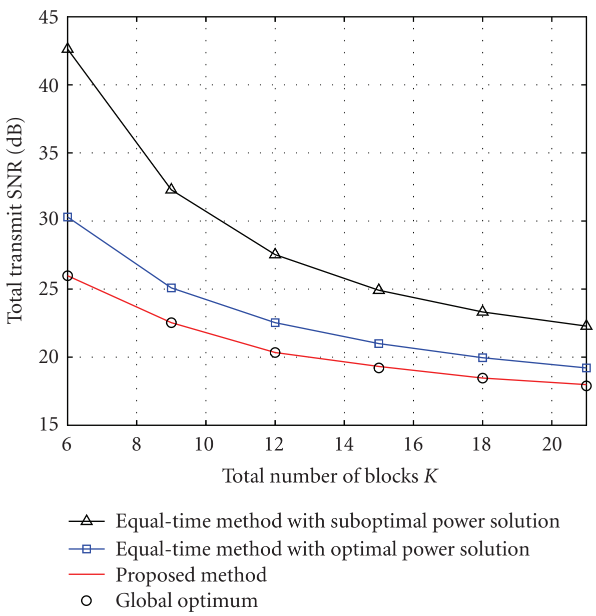

FIGURE 4: Results of the transmit SNR against the number of blocks when $U = 3$, $(\varepsilon_1,\varepsilon_2,\varepsilon_3) = (10^{-2},10^{-3},10^{-4})$, $(R_1,R_2,R_3) = (16,20,24)$ bps/Hz, $(C_0^{(1)},C_0^{(2)},C_0^{(3)}) = (1.5,1,0.5)$, $n_t = 4$, and $(n_r^{(1)},n_r^{(2)},n_r^{(3)}) = (4,3,2)$.

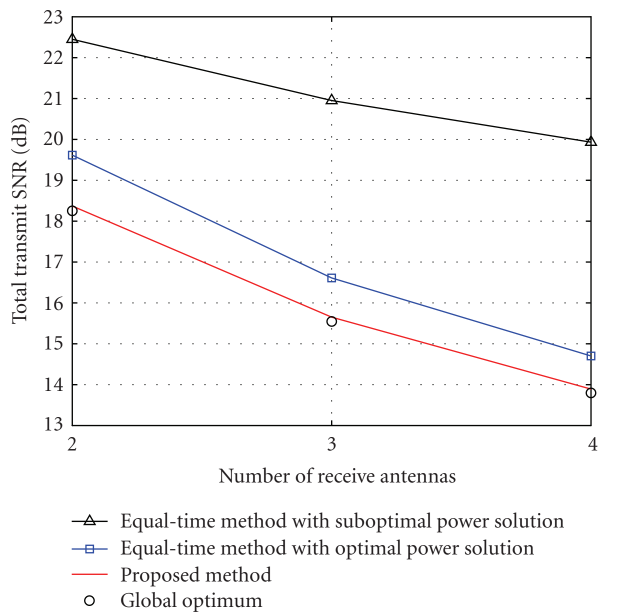

FIGURE 5: Results of the transmit SNR against the receive antennas when $U = 3$, $K = 20$, $(\varepsilon_1,\varepsilon_2,\varepsilon_3) = (10^{-1},10^{-3},10^{-4})$, $(R_1,R_2,R_3) = (8,16,24)$ bps/Hz, $(C_0^{(1)},C_0^{(2)},C_0^{(3)}) = (1.5,1,0.5)$, and $n_t = 4$.

the global optimum although the gap between the proposed method and the equal-time method with optimal power solution is not very obvious.

In Figure 4, we have the results for the transmit SNR against the total number of blocks K for a 3-user system with $(\varepsilon_1,\varepsilon_2,\varepsilon_3) = (10^{-2},10^{-3},10^{-4})$, $(R_1,R_2,R_3) = (16,20,24)$ bps/Hz, and $(C_0^{(1)},C_0^{(2)},C_0^{(3)}) = (1.5,1,0.5)$. The number of transmit antennas is 4 while users' numbers of receive antennas are $(n_r^{(1)},n_r^{(2)},n_r^{(3)}) = (4,3,2)$. Note that in this case, we have set the conditions for different users, such as users' requirements and channel conditions, to be quite different from each other to see how the proposed scheme performs. As we can see, the total transmit SNR decreases as K increases. In particular, the SNR for the proposed method decreases by 8 dB when K increases from 6 to 18. Again, results show that the performance of the proposed scheme is nearly optimal, while this time the gap between the proposed method and the equal-time methods becomes more obvious (about 5 dB for $K = 6$ and 2 dB for $K = 18$). This is because the optimal strategy tends to allocate more blocks to high-demand poor-channel-condition users (the numbers of allocated blocks for the users for different methods with $K = 12$ are shown in configuration 3 of Table 1).

Figure 5 plots the SNR results against the number of receive antennas for a 3-user system with $K = 20$, $(\varepsilon_1,\varepsilon_2,\varepsilon_3) = (10^{-1}, 10^{-3}, 10^{-4})$, $(R_1,R_2,R_3) = (8,16,24)$ bps/Hz, $(C_0^{(1)}, C_0^{(2)}, C_0^{(3)}) = (1.5, 1, 0.5)$, and $n_t = 4$. As expected, the required transmit SNR decreases with the number of receive antennas. This can be explained by the fact that the transmission rate mainly depends on the rank of the MIMO system, which is limited by the number of receive antennas (n_r). The actual number of block allocation for various methods is provided in configuration 4 of Table 1.

7. CONCLUSION

This paper has addressed the optimization problem of power allocation and scheduling for a time-division multiuser MIMO system in Rayleigh block-fading channels when the transmitter has only the channel statistics of the users, and the users are given individual outage rate probability constraints. By Gaussian approximation, we have derived the so-called MPE to determine the minimum power for attaining a given outage rate probability constraint if the number of blocks for a user is fixed. On the other hand, we have proposed a convex programming approach to find the suboptimal number of blocks allocated to the users. The two main techniques have been then combined to obtain a joint solution for both power and time allocations for the users. Results have demonstrated that the proposed method achieves near optimal performance.

APPENDICES

A. DERIVATION OF $\mu = \mathbf{E}[r]$ AND $\sigma^2 = \mathbf{VAR}[\, r\,]$

A.1. Main derivation

Before we proceed, we find the following expansion of the generalized Laguerre polynomial useful:

$$L_\ell^\delta(\lambda) = \sum_{m=0}^{\ell} (-1)^m \frac{(\ell+\delta)!}{(\ell-m)!(\delta+m)!m!} \cdot \lambda^m. \tag{A.1}$$

To make our notation succinct, we define $\Gamma_0 \triangleq C_0 d^{-\gamma} Q / n_t n N_0$ and

$$b_m(\ell,\delta) \triangleq (-1)^m \frac{(\ell+\delta)!}{(\ell-m)!(\delta+m)!m!} = \frac{(-1)^m}{m!}\binom{\ell+\delta}{m+\delta} \tag{A.2}$$

so that

$$L_\ell^\delta(\lambda) = \sum_{m=0}^{\ell} b_m(\ell,\delta)\lambda^m. \tag{A.3}$$

Also, in the following derivation, we will treat $\delta = n_t - n_r$ for convenience. As a result, the mean μ can be derived as follows:

$$\begin{aligned}
u &= \int_0^\infty \log_2(1+\Gamma_0\lambda) \sum_{\ell=0}^{n_r-1} \frac{\ell!\lambda^\delta e^{-\lambda}}{(\ell+\delta)!}\left[L_\ell^\delta(\lambda)\right]^2 d\lambda \\
&= \sum_{\ell=0}^{n_r-1} \frac{\ell!}{(\ell+\delta)!} \int_0^\infty \log_2(1+\Gamma_0\lambda)\lambda^\delta e^{-\lambda}\left[L_\ell^\delta(\lambda)\right]^2 d\lambda \\
&= \sum_{\ell=0}^{n_r-1} \frac{\ell!}{(\ell+\delta)!} \int_0^\infty \log_2(1+\Gamma_0\lambda)\lambda^\delta e^{-\lambda}\left[\sum_{m=0}^{\ell} b_m(\ell,\delta)\lambda^m\right]^2 d\lambda \\
&= \sum_{\ell=0}^{n_r-1} \frac{\ell!}{(\ell+\delta)!} \int_0^\infty \log_2(1+\Gamma_0\lambda)\lambda^\delta e^{-\lambda} \\
&\quad \times \left[\sum_{m=0}^{\ell} b_m^2(\ell,\delta)\lambda^{2m} + 2\sum_{i=0}^{\ell-1}\sum_{j=i+1}^{\ell} b_i(\ell,\delta)b_j(\ell,\delta)\lambda^{i+j}\right] d\lambda \\
&= \sum_{\ell=0}^{n_r-1}\sum_{m=0}^{\ell} \frac{\ell!}{(\ell+\delta)!} b_m^2(\ell,\delta) \int_0^\infty \log_2(1+\Gamma_0\lambda)\lambda^\delta e^{-\lambda}\lambda^{2m} d\lambda \\
&\quad + 2\sum_{\ell=1}^{n_r-1}\sum_{i=0}^{\ell-1}\sum_{j=i+1}^{\ell} \frac{\ell!}{(\ell+\delta)!} b_i(\ell,\delta)b_j(\ell,\delta) \\
&\quad \times \int_0^\infty \log_2(1+\Gamma_0\lambda)\lambda^\delta e^{-\lambda}\lambda^{i+j} d\lambda \\
&= \frac{1}{\ln 2}\sum_{\ell=0}^{n_r-1}\sum_{m=0}^{\ell} \frac{\ell!}{(\ell+\delta)!}\left[\frac{1}{m!}\binom{\ell+\delta}{m+\delta}\right]^2 \\
&\quad \times \int_0^\infty \ln(1+\Gamma_0\lambda)\lambda^{\delta+2m} e^{-\lambda} d\lambda \\
&\quad + \frac{2}{\ln 2}\sum_{\ell=1}^{n_r-1}\sum_{i=0}^{\ell-1}\sum_{j=i+1}^{\ell} \frac{\ell!}{(\ell+\delta)!}\frac{(-1)^{i+j}}{i!j!} \\
&\quad \times \binom{\ell+\delta}{i+\delta}\binom{\ell+\delta}{j+\delta} \int_0^\infty \ln(1+\Gamma_0\lambda)\lambda^{\delta+i+j} e^{-\lambda} d\lambda,
\end{aligned} \tag{A.4}$$

where the integral of the form $\int_0^\infty \lambda^j e^{-\lambda}\ln(1+\Gamma_0\lambda)d\lambda$ is given by (A.13) in Appendix A.2.

For the variance, we first express it using the standard result as

$$\begin{aligned}
\sigma^2 &= \int_0^\infty \log_2^2(1+\Gamma_0\lambda)\sum_{\ell=0}^{n_r-1}\frac{\ell!\lambda^\delta e^{-\lambda}}{(\ell+\delta)!}\left[L_\ell^\delta(\lambda)\right]^2 d\lambda \\
&\quad - \sum_{i=0}^{n_r-1}\sum_{j=0}^{n_r-1}\frac{i!j!}{(i+\delta)!(j+\delta)!} \\
&\quad \times \left[\int_0^\infty \lambda^\delta e^{-\lambda}L_i^\delta(\lambda)L_j^\delta(\lambda)\log_2(1+\Gamma_0\lambda)d\lambda\right]^2 \\
&\equiv \mathcal{I}_1 - \mathcal{I}_2,
\end{aligned} \tag{A.5}$$

which boils down to evaluating the integrals $\mathcal{I}_1$ and $\mathcal{I}_2$. After some manipulations, we have $\mathcal{I}_1$ as

$$\begin{aligned}
\mathcal{I}_1 &= \frac{1}{\ln^2 2}\sum_{\ell=0}^{n_r-1}\sum_{m=0}^{\ell}\frac{\ell!}{(\ell+\delta)!}\left(\frac{1}{m!}\binom{\ell+\delta}{m+\delta}\right)^2 \\
&\quad \times \int_0^\infty \ln^2(1+\Gamma_0\lambda)\lambda^{\delta+2m}e^{-\lambda}d\lambda \\
&\quad + \frac{2}{\ln^2 2}\sum_{\ell=1}^{n_r-1}\sum_{i=0}^{\ell-1}\sum_{j=i+1}^{\ell}\frac{\ell!}{(\ell+\delta)!}\frac{(-1)^{i+j}}{i!j!} \\
&\quad \times \binom{\ell+\delta}{i+\delta}\binom{\ell+\delta}{j+\delta}\int_0^\infty \ln^2(1+\Gamma_0\lambda)\lambda^{\delta+i+j}e^{-\lambda}d\lambda,
\end{aligned} \tag{A.6}$$

where $\int_0^\infty \lambda^j e^{-\lambda} \ln(1+\Gamma_0\lambda)^2 d\lambda$ is derived in Appendix A.2 as (A.17). On the other hand, $\mathcal{I}_2$ can be obtained as follows:

$$\begin{aligned}
&\int_0^\infty \lambda^\delta e^{-\lambda} L_i^\delta(\lambda) L_j^\delta(\lambda) \log_2(1+\Gamma_0\lambda)\, d\lambda \\
&\quad = \int_0^\infty \lambda^\delta e^{-\lambda} \log_2(1+\Gamma_0\lambda) \left(\sum_{m=0}^{i} b_m(i,\delta)\lambda^m\right) \qquad \text{(A.7)}\\
&\qquad \times \left(\sum_{\ell=0}^{j} b_m(j,\delta)\lambda^\ell\right) d\lambda \\
&\quad = \int_0^\infty \lambda^\delta e^{-\lambda} \log_2(1+\Gamma_0\lambda) \left(\sum_{m=0}^{i}\sum_{\ell=0}^{j} b_m(i,\delta) b_\ell(j,\delta)\lambda^{m+\ell}\right) d\lambda \qquad \text{(A.8)}\\
&\quad = \sum_{m=0}^{i}\sum_{\ell=0}^{j} b_m(i,\delta) b_\ell(j,\delta) \int_0^\infty \lambda^{\delta+m+\ell} e^{-\lambda} \log_2(1+\Gamma_0\lambda)\, d\lambda. \qquad \text{(A.9)}
\end{aligned}$$

Now, combining the results (A.5), (A.6), and (A.9), we have

$$\begin{aligned}
\sigma^2 = {}& \frac{1}{\ln^2 2} \sum_{\ell=0}^{n_r-1} \sum_{m=0}^{\ell} \frac{\ell!}{(\ell+\delta)!} \left[\frac{1}{m!} \binom{\ell+\delta}{m+\delta}\right]^2 \\
&\times \int_0^\infty \ln^2(1+\Gamma_0\lambda) \lambda^{\delta+2m} e^{-\lambda} d\lambda \\
&+ \frac{2}{\ln^2 2} \sum_{\ell=1}^{n_r-1} \sum_{i=0}^{\ell-1} \sum_{j=i+1}^{\ell} \frac{\ell!}{(\ell+\delta)!} \frac{(-1)^{i+j}}{i!j!} \\
&\times \binom{\ell+\delta}{i+\delta} \binom{\ell+\delta}{j+\delta} \int_0^\infty \ln^2(1+\Gamma_0\lambda) \lambda^{\delta+i+j} e^{-\lambda} d\lambda \\
&- \frac{1}{\ln^2 2} \sum_{i=0}^{n_r-1} \sum_{j=0}^{n_r-1} \frac{i!j!}{(i+\delta)!(j+\delta)!} \\
&\times \left[\sum_{m=0}^{i}\sum_{\ell=0}^{j} \frac{(-1)^{m+\ell}}{m!\ell!} \binom{i+\delta}{m+\delta}\right. \\
&\qquad \left. \times \binom{j+\delta}{\ell+\delta} \int_0^\infty \lambda^{\delta+m+\ell} e^{-\lambda} \ln(1+\Gamma_0\lambda)\, d\lambda\right]^2, \qquad \text{(A.10)}
\end{aligned}$$

where the integrals of the forms $\int_0^\infty \lambda^j e^{-\lambda} \ln(1+a\lambda) d\lambda$ and $\int_0^\infty \lambda^j e^{-\lambda} \ln^2(1+a\lambda) d\lambda$ are, respectively, given by (A.13) and (A.17).

A.2. Evaluation of $\int_0^\infty \ln(1+\Gamma_0\lambda)\lambda^j e^{-\lambda} d\lambda$

To begin, we rewrite the integral as

$$\begin{aligned}
&\int_0^\infty \lambda^j e^{-\lambda} \ln(1+\Gamma_0\lambda)\, d\lambda \\
&\qquad = \frac{e^{1/\Gamma_0}}{\Gamma_0^{j+1}} \int_1^\infty e^{-(1/\Gamma_0)t} (t-1)^j \ln t\, dt \qquad \text{(A.11)}\\
&\qquad = \frac{e^{1/\Gamma_0}}{\Gamma_0^{j+1}} \sum_{\ell=0}^{j} (-1)^\ell \binom{j}{\ell} \int_1^\infty e^{-(1/\Gamma_0)t} t^{j-\ell} \ln t\, dt,
\end{aligned}$$

where the integral of the last line of the right-hand side can be derived as follows:

$$\begin{aligned}
&\int_1^\infty e^{-(1/\Gamma_0)t} t^{j-\ell} \ln t\, dt \\
&= \Gamma_0 \int_1^\infty e^{-(1/\Gamma_0)t} [t^{j-\ell-1} + t^{j-\ell-1}(j-\ell)\ln t] dt \\
&= \Gamma_0 \Bigg(\Gamma_0 \Bigg\{ e^{-1/\Gamma_0} + \int_1^\infty e^{-(1/\Gamma_0)t} \\
&\qquad\qquad \times [(j-\ell-1)t^{j-\ell-2} + (j-\ell)t^{j-\ell-2} \\
&\qquad\qquad\quad + (j-\ell-1)(j-\ell)t^{j-\ell-2}\ln t] \Bigg\}\Bigg) \\
&\;\vdots \\
&= (\Gamma_0)^2 e^{-1/\Gamma_0} + (\Gamma_0)^3 [(j-\ell-1) + (j-\ell)] \\
&\quad + (\Gamma_0)^4 [(j-\ell)(j-\ell-2) \\
&\qquad\qquad + (j-\ell-1)(j-\ell-2) + (j-\ell-1)(j-\ell)] \\
&\quad + \cdots + \frac{(j-\ell)!}{(1/\Gamma_0)^{j-\ell}} \\
&\quad \times \left(\int_1^\infty e^{-(1/\Gamma_0)t} \ln t\, dt + \sum_{p=1}^{j-\ell} \frac{1}{j-\ell+1-p} \int_0^\infty e^{-(1/\Gamma_0)t} dt\right) \\
&= e^{-1/\Gamma_0} \sum_{p=2}^{j-\ell} (\Gamma_0)^p \frac{(j-\ell)!}{(j-\ell-p+1)!} \sum_{q=1}^{p-1} \frac{1}{j-\ell-q+1} + \frac{(j-\ell)!}{(1/\Gamma_0)^{j-\ell}} \\
&\quad \times \left(\int_1^\infty e^{-1/\Gamma_0} \ln t\, dt + \Gamma_0 e^{-1/\Gamma_0} \sum_{p=1}^{j-\ell} \frac{1}{j-\ell+1-p}\right). \qquad \text{(A.12)}
\end{aligned}$$

Substituting this result into (A.11), we can get

$$\begin{aligned}
&\int_0^\infty \lambda^j e^{-\lambda} \ln(1+\Gamma_0\lambda)\, d\lambda \\
&\quad = \frac{e^{1/\Gamma_0}}{\Gamma_0^j} \sum_{\ell=0}^{j} (-1)^\ell \binom{j}{\ell} \frac{(j-\ell)!}{(1/\ell)^{j-\ell}} \int_1^\infty e^{-(1/\Gamma_0)t} \ln t\, dt \\
&\quad + \frac{1}{\Gamma_0^j} \sum_{\ell=0}^{j-1} \sum_{p=1}^{j-\ell} (-1)^\ell \binom{j}{\ell} \frac{(j-\ell)!}{(1/\ell)^{j-\ell}} \cdot \frac{1}{j-\ell+1-p} \qquad \text{(A.13)}\\
&\quad + \frac{1}{\Gamma_0^{j+1}} \sum_{\ell=0}^{j-2} \sum_{p=2}^{j-\ell} \sum_{q=1}^{p-1} (-1)^\ell \\
&\quad \times \binom{j}{\ell} \frac{(j-\ell)!}{(j-\ell-p+1)!} (\Gamma_0)^p \frac{1}{j-\ell-q+1}.
\end{aligned}$$

In Appendix A.4, we will show that

$$\int_1^\infty e^{-(1/\Gamma_0)t} \ln t\, dt = \Gamma_0 E_1\left(\frac{1}{\Gamma_0}\right), \qquad \text{(A.14)}$$

where $E_1(\cdot)$ denotes the exponential integral.

A.3. Evaluation of $\int_0^\infty [\ln(1+\Gamma_0\lambda)]^2\lambda^j e^{-\lambda} d\lambda$

We start by writing

$$\begin{aligned}
&\int_0^\infty \left[\ln(1+\Gamma_0\lambda)\right]^2 \lambda^j e^{-\lambda} d\lambda \\
&\quad = \frac{e^{1/\Gamma_0}}{\Gamma_0^{j+1}} \sum_{\ell=0}^{j} (-1)^\ell \binom{j}{\ell} \int_1^\infty e^{-(1/\Gamma_0)t} (\ln t)^2 t^{j-\ell} dt,
\end{aligned} \tag{A.15}$$

where the integrand on the right can be evaluated as follows:

$$\begin{aligned}
&\int_1^\infty e^{-(1/\Gamma_0)t}(\ln t)^2 t^{j-\ell} dt \\
&= \Gamma_0 \int_1^\infty e^{-(1/\Gamma_0)t}\left[2t^{j-\ell-1}\ln t + t^{j-\ell-1}(j-\ell)(\ln t)^2\right] dt \\
&= (\Gamma_0)^2 \int_1^\infty e^{-(1/\Gamma_0)t} \\
&\quad \times \left[2t^{j-\ell-2} + 2t^{j-\ell-2}(j-\ell-1)\ln t + 2t^{j-\ell-2}(j-\ell)\ln t\right] dt \\
&\quad + (\Gamma_0)^2 \int_1^\infty e^{-(1/\Gamma_0)t} t^{j-\ell-2}(j-\ell-1)(j-\ell)(\ln t)^2 dt \\
&\ \vdots \\
&= 2e^{-1/\Gamma_0} \sum_{t=3}^{j-\ell} (\Gamma_0)^t \frac{(j-\ell)!}{(j-\ell-t+1)!} \\
&\quad \times \sum_{p=1}^{t-2} \sum_{q=p+1}^{t-1} \frac{1}{(j-\ell+1-p)(j-\ell+1-q)} \\
&\quad + \frac{(j-\ell)!}{(1/\Gamma_0)^{j-\ell}} \left(\int_1^\infty e^{-(1/\Gamma_0)t}(\ln t)^2 dt \right. \\
&\qquad\qquad \left. + 2\sum_{p=1}^{j-\ell} \frac{1}{j-\ell+1-p} \int_1^\infty e^{-(1/\Gamma_0)t} \ln t \, dt \right) \\
&\quad + \frac{(j-\ell)!}{(1/\Gamma_0)^{j-\ell}} \int_1^\infty 2e^{-(1/\Gamma_0)t} \\
&\qquad\qquad \times \sum_{p=1}^{j-\ell-1} \sum_{q=p+1}^{j-\ell} \frac{1}{(j-\ell+1-p)(j-\ell+1-q)} dt.
\end{aligned} \tag{A.16}$$

Using (A.16) into (A.15), we get

$$\begin{aligned}
&\int_0^\infty \left[\ln(1+\Gamma_0\lambda)\right]^2 \lambda^j e^{-\lambda} d\lambda \\
&\quad = \frac{e^{1/\Gamma_0}}{\Gamma_0^{j+1}} \sum_{\ell=0}^{j} (-1)^\ell \binom{j}{\ell} \frac{(j-\ell)!}{(1/\Gamma_0)^{j-\ell}} \int_1^\infty e^{-(1/\Gamma_0)t}(\ln t)^2 dt \\
&\quad + \frac{2e^{1/\Gamma_0}}{\Gamma_0^{j+1}} \int_1^\infty e^{-(1/\Gamma_0)t} \ln t \, dt \\
&\quad \times \sum_{\ell=0}^{j-1} \sum_{p=1}^{j-\ell} (-1)^\ell \binom{j}{\ell} \frac{(j-\ell)!}{(1/\Gamma_0)^{j-\ell}} \cdot \frac{1}{j-\ell+1-p} \\
&\quad + \frac{2}{\Gamma_0^{j}} \sum_{\ell=0}^{j-2} \sum_{p=1}^{j-\ell-1} \sum_{q=p+1}^{j-\ell} (-1)^\ell \binom{j}{\ell} \frac{(j-\ell)!}{(1/\Gamma_0)^{j-\ell}} \\
&\quad \cdot \frac{1}{(j-\ell+1-p)(j-\ell+1-q)} \\
&\quad + \frac{2}{\Gamma_0^{j+1}} \sum_{\ell=0}^{j-3} \sum_{t=3}^{j-\ell} \sum_{p=1}^{t-2} \sum_{q=p+1}^{t-1} (-1)^\ell \\
&\quad \times \binom{j}{\ell} (\Gamma_0)^t \frac{(j-\ell)!}{(j-\ell-t+1)!} \\
&\quad \cdot \frac{1}{(j-\ell+1-p)(j-\ell+1-q)}.
\end{aligned} \tag{A.17}$$

From [23, Section 4.331(2)], we can have

$$\int_1^\infty e^{-(1/\Gamma_0)t} \ln t \, dt = \Gamma_0 E_1\left(\frac{1}{\Gamma_0}\right). \tag{A.18}$$

Later in Appendix A.4, we will have

$$\begin{aligned}
&\int_1^\infty e^{-(1/\Gamma_0)t}(\ln t)^2 dt \\
&\quad = \Gamma_0\left[\left(\ln\frac{i}{\Gamma_0} - \gamma_{EM}\right)^2 + \frac{\pi^2}{6}\right] - 2\,{}_3F_3\left([1,1,1];[2,2,2];-\frac{1}{\Gamma_0}\right),
\end{aligned} \tag{A.19}$$

where ${}_pF_q$ denotes the generalized hypergeometric function.

A.4. Evaluation of $\int_1^\infty e^{-(1/\Gamma_0)t}(\ln t)^2 dt$

First of all, clearly, we have

$$\begin{aligned}
&\int_1^\infty e^{-(1/\Gamma_0)t}(\ln t)^2 dt \\
&\quad = \int_0^\infty e^{-(1/\Gamma_0)t}(\ln t)^2 dt - \int_0^1 e^{-(1/\Gamma_0)t}(\ln t)^2 dt.
\end{aligned} \tag{A.20}$$

The first integrand can be found from [23, Section 4.335(1)] as

$$\int_0^\infty e^{-(1/\Gamma_0)t}(\ln t)^2 dt = \Gamma_0\left[\left(\ln\frac{1}{\Gamma_0} - \gamma_{EM}\right)^2 + \frac{\pi^2}{6}\right], \tag{A.21}$$

where γ_{EM} refers to the Euler-Mascheroni constant.

Likewise, the second integrand can be evaluated as

$$\begin{aligned}
\int_0^1 e^{-(1/\Gamma_0)t}(\ln t)^2 dt &= \int_0^1 \sum_{\ell=0}^{\infty} \frac{\left(-(1/\Gamma_0)t\right)^\ell}{\ell!} (\ln t)^2 dt \\
&= \sum_{\ell=0}^{\infty} \frac{\left(-1/\Gamma_0\right)^\ell}{\ell!} \int_0^1 t^\ell (\ln t)^2 dt \\
&= \sum_{\ell=0}^{\infty} \frac{\left(-1/\Gamma_0\right)^\ell}{\ell!} \cdot \frac{(-1)^2 \cdot 2!}{(\ell+1)^3} \\
&= 2\sum_{\ell=0}^{\infty} \frac{\left(-1/\Gamma_0\right)^\ell}{\ell!} \cdot \frac{1}{(\ell+1)^3} \\
&= 2\,{}_3F_3\left([1,1,1];[2,2,2];-\frac{1}{\Gamma_0}\right)
\end{aligned} \tag{A.22}$$

in which ${}_pF_q$ denotes the generalized hypergeometric function.

Finally, we have

$$\int_1^{\infty} e^{-(1/\Gamma_0)t}(\ln t)^2 dt = \Gamma_0\left[\left(\ln\frac{1}{\Gamma_0}-\gamma_{\text{EM}}\right)^2+\frac{\pi^2}{6}\right] - 2{}_3F_3\left([1,1,1];[2,2,2];-\frac{1}{\Gamma_0}\right). \tag{A.23}$$

B. DERIVATION OF $\widetilde{\mu} = \mathbf{E}[\ln\lambda]$ AND $\widetilde{\sigma}^2 = \mathbf{VAR}[\ln\lambda]$

B.1. Main derivation

Given a random variable λ with pdf in (21), $\widetilde{\mu}$ can be found by

$$\widetilde{\mu} = \mathrm{E}[\ln\lambda] = \sum_{m=1}^{n_r}\sum_{\ell=\delta}^{(n_t+n_r)m-2m^2}\left(\frac{d_{m,\ell}\times m^{\ell+1}}{\ell!}\right) \times\int_0^{\infty}\lambda^{\ell}e^{-m\lambda}\ln\lambda d\lambda, \tag{B.1}$$

where $\int_0^{\infty}\lambda^{\ell}e^{-m\lambda}\ln\lambda d\lambda$ is calculated as [23, Section 4.352(1)]

$$\int_0^{\infty}\lambda^{\ell}e^{-m\lambda}\ln\lambda d\lambda = \frac{1}{m^{l+1}}\Gamma(l+1)[\psi(l+1)-\ln m], \tag{B.2}$$

where $\psi(l+1) = -\gamma_{\text{EM}}+\sum_{k=1}^{l}(1/k)$ is referred to [23, Section 8.365(4)].

To find $\widetilde{\sigma}^2$, we first obtain $\mathrm{E}[(\ln\lambda)^2]$ by

$$\begin{aligned}
\mathrm{E}[(\ln\lambda)^2] &= \sum_{m=1}^{n_r}\sum_{\ell=\delta}^{(n_t+n_r)m-2m^2}\left(\frac{d_{m,\ell}\times m^{\ell+1}}{\ell!}\right)\int_0^{\infty}\lambda^{\ell}e^{-m\lambda}(\ln\lambda)^2 d\lambda \\
&= \sum_{m=1}^{n_r}\sum_{\ell=\delta}^{(n_t+n_r)m-2m^2}\left(\frac{d_{m,\ell}\times m^{\ell+1}}{\ell!}\right)\int_0^{\infty}\left(\frac{t}{m}\right)^{\ell}e^{-t}(\ln t-\ln m)^2\frac{dt}{m} \\
&= \sum_{m=1}^{n_r}\sum_{\ell=\delta}^{(n_t+n_r)m-2m^2}\left(\frac{d_{m,\ell}}{\ell!}\right) \times\int_0^{\infty}t^{\ell}e^{-t}[(\ln t)^2-2(\ln t)(\ln m)+(\ln m)^2]dt \\
&= \sum_{m=1}^{n_r}\sum_{\ell=\delta}^{(n_t+n_r)m-2m^2}\left(\frac{d_{m,\ell}}{\ell!}\right)[W(\ell)-2(\ln m)S(\ell)+(\ln m)^2\ell!],
\end{aligned} \tag{B.3}$$

where $S(\ell) = \int_0^{\infty}t^{\ell}e^{-t}\ln t\,dt$ and $W(\ell) \triangleq \int_0^{\infty}t^{\ell}e^{-t}(\ln t)^2 dt$. Note that although the equation $\int_0^{\infty}\lambda^{\ell}e^{-m\lambda}(\ln\lambda)^2 d\lambda$ can be calculated as $(\Gamma(\ell+1)/m^{\ell+1})\{[\psi(\ell+1)-\ln m]^2+\zeta(2,l+1)\}$ from [23, Section 4.358(2)] and $\zeta(2,l+1)$ can be represented by an infinite series as $\sum_{n=0}^{\infty}(1/(l+1+n)^2)$ [23, Section 9.521(1)], it is practically undesirable. In Appendix B.2, instead, we provide the results of $W(\ell)$ as a finite series, which we give here as

$$W(\ell) = \ell!\left(\gamma_{\text{EM}}^2-2\gamma_{\text{EM}}+\frac{\pi^2}{6}\right)+2\ell!\sum_{j=1}^{\ell-1}\frac{H_j-\gamma_{\text{EM}}}{j+1}, \tag{B.4}$$

where H_j denotes the harmonic number. Using this result together with that for $S(\ell)$ from (B.2), we can express $\mathrm{E}[(\ln\lambda)^2]$ in closed form as

$$\begin{aligned}
\mathrm{E}[(\ln\lambda)^2] &= \sum_{m=1}^{n_r}\sum_{\ell=\delta}^{(n_t+n_r)m-2m^2} d_{m,\ell}\Bigg[\gamma_{\text{EM}}^2+2(\ln m-H_{\ell})\gamma_{\text{EM}} \\
&\quad +\frac{\pi^2}{6}-2(\ln m)H_{\ell}+(\ln m)^2+2\sum_{j=1}^{\ell-1}\frac{H_j}{j+1}\Bigg].
\end{aligned} \tag{B.5}$$

As such, the variance is given by

$$\begin{aligned}
\widetilde{\sigma}^2 &= \mathrm{E}[(\ln\lambda)^2]-(\mathrm{E}[\ln\lambda])^2 \\
&= \sum_{m=1}^{n_r}\sum_{\ell=\delta}^{(n_t+n_r)m-2m^2} d_{m,\ell}\Bigg[\gamma_{\text{EM}}^2+2(\ln m-H_{\ell})\gamma_{\text{EM}} \\
&\quad +\frac{\pi^2}{6}-2(\ln m)H_{\ell}+(\ln m)^2+2\sum_{j=1}^{\ell-1}\frac{H_j}{j+1}\Bigg] \\
&\quad -\left[\sum_{m=1}^{n_r}\sum_{\ell=\delta}^{(n_t+n_r)m-2m^2} d_{m,\ell}(H_{\ell}-\gamma_{\text{EM}}-\ln m)\right]^2.
\end{aligned} \tag{B.6}$$

B.2. Evaluation of $W(\ell) = \int_0^{\infty}t^{\ell}e^{-t}(\ln t)^2 dt$

Similar technique can be applied to derive $W(\ell)$, and the first is to have the recursive relation

$$\begin{aligned}
W(\ell) &= \int_0^{\infty}t^{\ell}e^{-t}(\ln t)^2 dt \\
&= -\int_0^{\infty}t^{\ell}(\ln t)^2 de^{-t} \\
&= 2\int_0^{\infty}t^{\ell-1}e^{-t}\ln t\,dt+\ell\int_0^{\infty}t^{\ell-1}e^{-t}(\ln t)^2 dt \\
&= 2S(\ell-1)+\ell W(\ell-1).
\end{aligned} \tag{B.7}$$

Applying this further, we can get

$$\begin{aligned}
W(\ell) &= 2S(\ell-1) + \ell W(\ell-1) \\
&= 2S(\ell-1) + 2\ell S(\ell-2) + \ell(\ell-1)W(\ell-2) \\
&= 2S(\ell-1) + 2\ell S(\ell-2) + 2\ell(\ell-1)S(\ell-3) \\
&\quad + \ell(\ell-1)(\ell-2)W(\ell-3) \\
&\ \vdots \\
&= 2S(\ell-1) + 2\ell S(\ell-2) + 2\ell(\ell-1)S(\ell-3) \\
&\quad + \cdots + 2\ell(\ell-1)(\ell-2)\cdots(2)S(0) \\
&\quad + \ell(\ell-1)(\ell-2)\cdots(2)(1)W(0) \\
&= \ell! W(0) + 2\left(\frac{\ell!}{1!}\right)S(0) + 2\left(\frac{\ell!}{2!}\right)S(1) \\
&\quad + 2\left(\frac{\ell!}{3!}\right)S(2) \\
&\quad + \cdots + 2\left(\frac{\ell!}{(\ell-1)!}\right)S(\ell-2) + 2\left(\frac{\ell!}{\ell!}\right)S(\ell-1) \\
&= \ell! W(0) + 2\ell! \sum_{j=0}^{\ell-1} \frac{S(j)}{(j+1)!}.
\end{aligned} \tag{B.8}$$

Again, note that

$$W(0) = \int_0^\infty e^{-t}(\ln t)^2 dt = \gamma_{\mathrm{EM}}^2 + \frac{\pi^2}{6} \tag{B.9}$$

and use the result just derived in Appendix B.2 for $S(j)$. We can find $W(\ell)$ as

$$W(\ell) = \ell!\left(\gamma_{\mathrm{EM}}^2 - 2\gamma_{\mathrm{EM}} + \frac{\pi^2}{6}\right) + 2\ell! \sum_{j=1}^{\ell-1} \frac{H_j - \gamma_{\mathrm{EM}}}{j+1}. \tag{B.10}$$

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