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Research Article

Speech Enhancement via EMD

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In this study, two new approaches for speech signal noise reduction based on the empirical mode decomposition (EMD) recently introduced by Huang et al. (1998) are proposed. Based on the EMD, both reduction schemes are fully data-driven approaches. Noisy signal is decomposed adaptively into oscillatory components called intrinsic mode functions (IMFs), using a temporal decomposition called sifting process. Two strategies for noise reduction are proposed: filtering and thresholding. The basic principle of these two methods is the signal reconstruction with IMFs previously filtered, using the minimum mean-squared error (MMSE) filter introduced by I. Y. Soon et al. (1998), or thresholded using a shrinkage function. The performance of these methods is analyzed and compared with those of the MMSE filter and wavelet shrinkage. The study is limited to signals corrupted by additive white Gaussian noise. The obtained results show that the proposed denoising schemes perform better than the MMSE filter and wavelet approach.

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1. INTRODUCTION

Speech enhancement is a classical problem in signal processing, particularly in the case of additive white Gaussian noise where different noise reduction methods have been proposed [1-4]. When noise estimation is available, then filtering gives accurate results. However, these methods are not so effective when noise is difficult to estimate. Linear methods such as Wiener filtering [5] are used because linear filters are easy to implement and design. These linear methods are not so effective for signals presenting sharp edges or impulses of short duration. Furthermore, real signals are often nonstationary. In order to overcome these shortcomings, nonlinear methods have been proposed and especially those based on wavelets thresholding [6, 7]. The idea of wavelet thresholding relies on the assumption that signal magnitudes dominate the magnitudes of noise in a wavelet representation so that wavelet coefficients can be set to zero if their magnitudes are less than a predetermined threshold [7]. A limit of the wavelet approach is that basis functions are fixed, and, thus, do not necessarily match all real signals. To avoid this problem, time-frequency atomic signal decomposition can be used [8, 9]. As for wavelet packets, if the dictionary is very large and rich with a collection of atomic waveforms which are located on a much finer grid in time-frequency space than wavelet and cosine packet tables, then it should be possible to represent a large class of real signals; but, in spite of this, the basis functions must be specified (Gabor functions, damped sinusoids,...).

Recently, a new data-driven technique, referred to as empirical mode decomposition (EMD) has been introduced by Huang et al. [10] for analyzing data from nonstationary and nonlinear processes. The EMD has received more attention in terms of applications [11-23], interpretation [24, 25], and improvement [26, 27]. The major advantage of the EMD is that basis functions are derived from the signal itself. Hence, the analysis is adaptive in contrast to the traditional methods where basis functions are fixed. The EMD is based on the sequential extraction of energy associated with various intrinsic time scales of the signal, called intrinsic mode functions (IMFs), starting from finer temporal scales (high-frequency IMFs) to coarser ones (lowfrequency IMFs). The total sum of the IMFs matches the signal very well and therefore ensures completeness [10]. We have shown that the EMD can be used for signals denoising [14, 15] or filtering [17]. The denoising method reconstructs the signal with all the IMFs previously thresholded as in wavelet analysis or filtered [14, 15]. The filtering scheme

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relies on the basic idea that most structures of the signal are often concentrated on lower-frequency components (last IMFs), and decrease toward high-frequency modes (first IMFs) [17]. Thus, the recovered signal is reconstructed with only few IMFs that are signal dominated using an energy criterion. Thus, compared to the approach introduced in [14, 15], no thresholding or filtered is required. The proposed filtering method is a fully data approach [17].

In this paper, we show how the idea of thresholding IMFs using hard or soft shrinkage introduced in [14, 15] can be extended and adapted to speech signal for enhancement purpose. According to if the noise level can be correctly estimated or not, two noise reduction methods are proposed. The first strategy combines the EMD and the minimum mean-squared error (MMSE) filter [1], and the second one associates the EMD with hard shrinkage [14, 15]. The two methods are applied to speech signals corrupted with different noise levels, and the results are compared to the MMSE filter and the wavelet approach.

2. EMD ALGORITHM

The EMD decomposes a signal x(t) into a series of IMFs through an iterative process called sifting; each one, with distinct time scale [10]. The decomposition is based on the local time scale of x(t) and yields adaptive basis functions. The EMD can be seen as a type of wavelet decomposition whose subbands are built up as needed to separate the different components of x(t). Each IMF replaces the signals detail, at a certain scale or frequency band [24]. The EMD picks out the highest-frequency oscillation that remains in x(t). By definition, an IMF satisfies two conditions:

- (1) the number of extrema and the number of zeros crossings may differ by no more than one;
- (2) the average value of the envelope defined by the local maxima and the envelope defined by the local minima is zero.

Thus, locally, each IMF contains lower-frequency oscillations than the just-extracted one. To be successfully decomposed into IMFs, x(t) must have at least two extrema; one minimum and one maximum. The sifting involves the following steps:

Step 1. fix the threshold ϵ and set $i \leftarrow 1$ (jth IMF);

Step 2. $r_{i-1}(t) \leftarrow x(t)$ (residual);

Step 3. extract the *j*th IMF:

- (a) $h_{i,i-1}(t) \leftarrow r_{i-1}(t)$, $i \leftarrow 1$ (*i* number of sifts),
- (b) extract local maxima/minima of $h_{i,i-1}(t)$,
- (c) compute upper and lower envelopes $U_{j,i-1}(t)$ and $L_{j,i-1}(t)$ by interpolating, using cubic spline, respectively, local maxima and minima of $h_{j,i-1}(t)$,
- (d) compute the mean of the envelopes: $\mu_{j,i-1}(t) = (U_{j,i-1}(t) + L_{j,i-1}(t))/2$,
- (e) update: $h_{j,i}(t) := h_{j,i-1}(t) \mu_{j,i-1}(t)$, i := i + 1,

- (f) calculate the following stopping criterion: SD(i) = $\sum_{t=1}^{T} (|h_{j,i-1}(t) h_{j,i}(t)|^2 / (h_{j,i-1}(t))^2),$
- (g) repeat Steps (b)–(f) until SD(i) < ϵ and then put IMF_i(t) $\leftarrow h_{i,i}(t)$ (jth IMF);

Step 4. update residual: $r_i(t) := r_{i-1}(t) - IMF_i(t)$;

Step 5. repeat Step 3 with j := j + 1 until the number of extrema in $r_i(t)$ is ≤ 2 ;

where T is x(t) time duration. The sifting is repeated several times (i) in order to get h true IMF that fulfills the conditions (1) and (2). The result of the sifting is that x(t) will be decomposed into a sum of C IMFs and a residual $r_C(t)$ such that

$$x(t) = \sum_{j=1}^{C} \text{IMF}_{j}(t) + r_{C}(t),$$
 (1)

C value is determined automatically using SD (Step 3(f)). The sifting has two effects: (a) it eliminates riding waves and (b) to smoothen uneven amplitudes. To guarantee that IMF components retain enough physical sense of both amplitude and frequency modulation, we have to determine SD value for the sifting. This is accomplished by limiting the size of the standard deviation SD computed from the two consecutive sifting results. Usually, SD (or ϵ) is set between 0.2 to 0.3 [10].

3. DENOISING PRINCIPLE

Let a clean speech signal x(t) be corrupted by an additive white Gaussian noise b(t) as follows:

$$y(t) = x(t) + b(t). \tag{2}$$

The noisy signal is decomposed into a sum of IMFs by the EMD, such that

$$y(t) = \sum_{j=1}^{C} \text{IMF}_{j}(t) + r_{C}(t),$$
 (3)

where IMF_i is a noisy version of the data f_i :

$$IMF_i(t) = f_i(t) + b_i(t). \tag{4}$$

An estimation $\widetilde{f}_j(t)$ of $f_j(t)$ based on the noisy observation $IMF_j(t)$ is given by

$$\widetilde{f}_i(t) = \Gamma[\text{IMF}_i(t); \tau_i],$$
 (5)

where $\Gamma[\mathrm{IMF}_j(t); \tau_j]$ is a preprocessing function, defined by a set of parameters τ_j , applied to signal IMF $_j$ [14, 15]. The function Γ is chosen according to if noise level can be estimated or not. When this estimation is possible, Γ is reduced to the MMSE filter [1]. However, when the estimation is not easy, the preprocessing can be a thresholding [14, 15]. The function Γ is a shrinkage, and τ is a threshold parameter. Finally, the denoised signal, $\widetilde{x}(t)$, is given by

$$\widetilde{x}(t) = \sum_{i=1}^{C} \widetilde{f}_{i}(t) + r_{C}(t). \tag{6}$$

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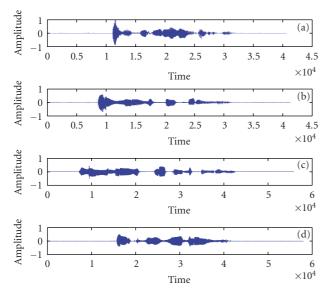


Figure 1: The original signals "a", "b", "c", and "d".

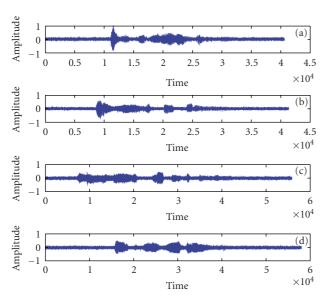


FIGURE 2: The noisy version of signals "a", "b", "c", and "d". (SNR = 5 dB).

3.1. EMD-MMSE

Generally, speech noise estimation is performed using the Boll's method [28]. Accordingly, the silence periods of the signal are detected, and then power spectra noise estimation is performed by considering the average of the power spectra of the noisy signal on the M first temporal frames which are considered as being moments of silence, following the relation

$$|\hat{\mathbf{B}}(\mathbf{fe}, m)|^2 = \frac{1}{M} \sum_{i=0}^{M-1} |\mathbf{B}(\mathbf{fe}, i)|^2,$$
 (7)

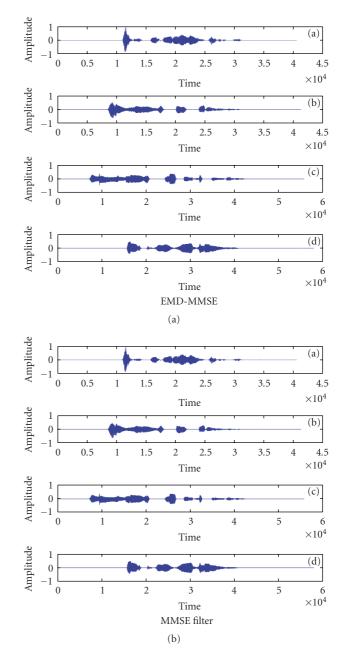


FIGURE 3: Denoising results of signals "a", "b", "c", and "d" by the EMD-MMSE and the MMSE filter.

where |B(fe, i)| is power spectra value at the discrete frequency fe of frame i. This method gives a correct estimation of the noise [28].

Extensive simulations have shown that when a speech signal with a silence sequence is decomposed by EMD, its first IMF corresponds to that silence sequence. Thus, the first IMF can be used to correctly estimate the noise level. According to [24], the noise level of the modes following the first IMF (k = 1) is estimated via

$$\widetilde{\sigma}_k = \frac{\widetilde{\sigma}_1}{\sqrt{2}^{k-1}} \quad \text{with } k \ge 2,$$
 (8)

where $\tilde{\sigma}_1$ is the noise level of first IMF.

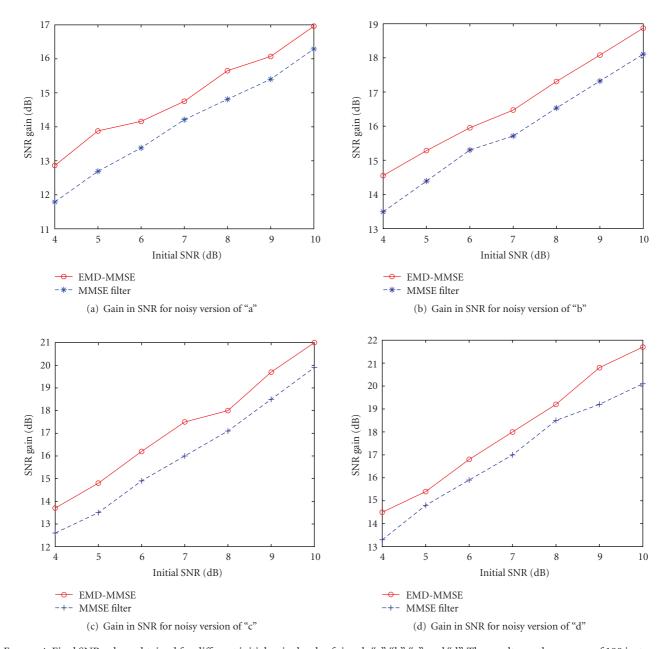


FIGURE 4: Final SNR values obtained for different initial noise levels of signals "a", "b", "c", and "d". The results are the average of 100 instances signal. It is reported for EMD-MMSE and the MMSE filter.

The combination of the EMD and the MMSE filter [1] is called EMD-MMSE strategy. Thus, each IMF is filtered by the MMSE filter as follows:

$$\widetilde{F}_{j}(fe, m) = H(fe, m)IMF_{j}(fe, m),$$
 (9)

where $F_j(fe, m)$ and $\widetilde{F}_j(fe, m)$ are the spectral noisy IMF and the spectral estimated IMF, respectively, observed at the discrete frequency fe on the frame m. H(fe, m) is described as follows [1]:

$$H(fe, m) = \frac{SNR_{prio}(fe, m)}{1 + SNR_{prio}(fe, m)}.$$
 (10)

The signal-to-noise ratio, SNR_{prio}, is estimated according to the method of Ephraim and Malah [2] which is based on the

estimated $\widetilde{F}(fe, m-1)$ from the previous frame and on a local estimation of SNR_{inst}:

$$SNR_{prio}(fe, m)$$

$$= \alpha \frac{\widetilde{F}^{2}(fe, m-1)}{B^{2}(fe, m-1)} + (1 - \alpha) \max(SNR_{inst}(fe, m), 0),$$
(11)

where α is a weighting factor (equal to 0.98) and SNR_{inst} indicates the instantaneous SNR, defined as the local estimation of SNR_{prio}:

$$SNR_{inst} = \frac{IMF^{2}(fe, m)}{B^{2}(fe, m)}.$$
 (12)

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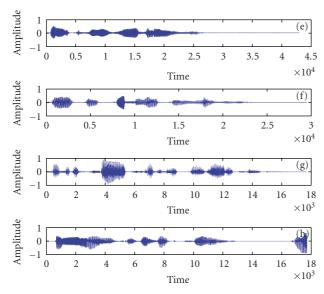


FIGURE 5: The original signals "e", "f", "g", and "h".

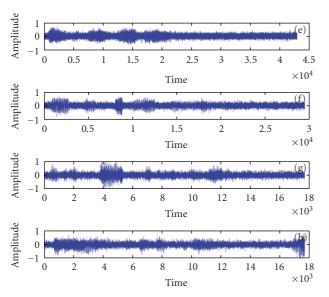


FIGURE 6: The noisy version of signals "e", "f", "g", and "h" (SNR = -1 dB).

3.2. EMD-shrinkage

A smooth version of the input signal can be obtained by thresholding the IMFs before signal reconstruction [14, 15]. In this case, the threshold parameter is estimated by the following expression [6, 14, 15, 29, 30]:

$$\tau = \sqrt{2\log(T)}\sigma,\tag{13}$$

where *T* is the signal length and σ is the estimated noise level (scale level). The $\tilde{\sigma}_1$ is given by [14, 15, 31]

$$\widetilde{\sigma}_1 = 1.4826 \times \text{Median}\{|\text{IMF}_1(t) - \text{Median}\{\text{IMF}_1(t)\}|\}.$$
(14)

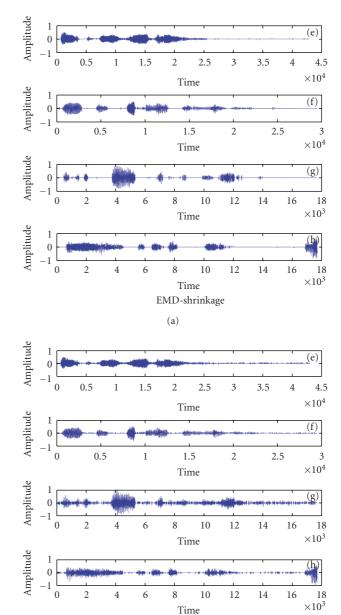


FIGURE 7: Denoising results of signals "e", "f", "g", and "h" by the EMD-shrinkage and the wavelet approach (Daubechies 4).

(b)

Wavelet-shrinkage (Daubechies 4)

According to [24, 32], using $\tilde{\sigma}_1$, the noise level $\tilde{\sigma}_k$ of the IMFs can be estimated using (8).

There are different nonlinear shrinkage functions [33]. In the present work, we use the hard shrinkage which has given interesting denoising results for speech enhancement:

$$\widetilde{f}_{j} = \begin{cases} \text{IMF}_{j}(t) & \text{if } |\text{IMF}_{j}(t)| > \tau_{j}, \\ 0 & \text{if } |\text{IMF}_{j}(t)| \leq \tau_{j}. \end{cases}$$
(15)

The association of the EMD and the hard shrinkage is called EMD-shrinkage method.

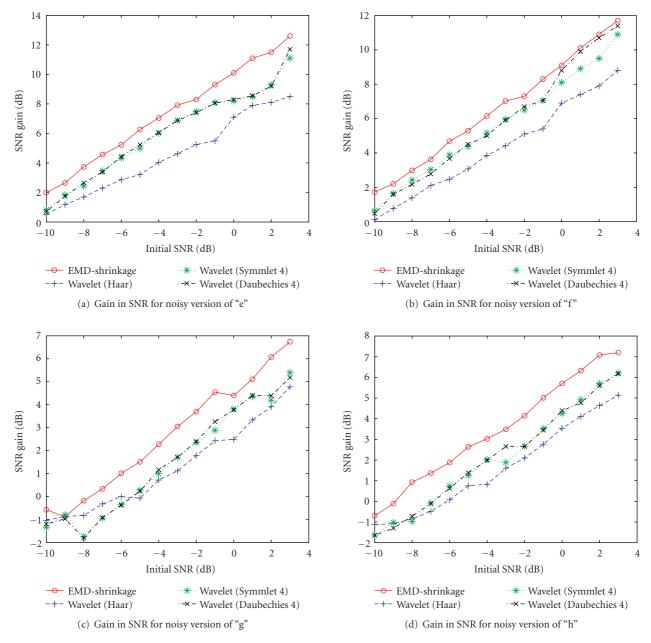


FIGURE 8: Final SNR values obtained for different initial noise levels of signals "e", "f", "g", and "h". The results are the average of 100 instances signal. It's reported for EMD-shrinkage and for three different wavelets (Haar, Symmlet 4, Daubechies 4).

4. RESULTS

The two proposed noise reduction methods are tested on speech signals corrupted by additive white Gaussian noise with different SNRs. The results are compared to the MMSE filter and the wavelet approach (Haar, Symmlet 4, Daubechies 4). As indicated, the EMD denoising schemes depend on the noise estimation. So, if the prespeech period of the noisy signal is detected, then the EMD-MMSE is used. Otherwise, the EMD-shrinkage is used. The SNR is used as an objective measure to evaluate the denoising methods performance. More precisely, the SNR is used to compare the

EMD-MMSE to the MMSE filter and the wavelet approach to the EMD-shrinkage. The SNR is defined by

SNR =
$$10 \log_{10} \frac{\sum_{i=1}^{T} (x(t_i))^2}{\sum_{i=1}^{T} (x(t_i) - \widetilde{x}(t_i))^2}$$
, (16)

where $x(t_i)$ and $\tilde{x}(t_i)$ are the original signal and the reconstructed one, respectively.

The EMD-MMSE denoising scheme is applied to four clean speech signals "a", "b", "c", and "d" (Figures 1(a)–1(d)) corrupted by additive white Gaussian noise with SNR values ranging from 4 dB to 10 dB. Noisy versions of the

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original signals corresponding to SNR = 5 dB are shown in Figure 2. We carried out numerical simulations where for each SNR value, 100 independent noise simulations are generated and averaged values calculated. Figure 3 shows the denoising result obtained by the EMD-MMSE and the MMSE filter. From this figure and compared to the respective clean signals of Figure 1, one can conclude that the EMD-MMSE performs better in terms of noise reduction than the MMSE filter. This fact is confirmed by the results shown in Figure 4 where interesting improvement in SNR are given by the EMD-MMSE compared to the MMSE filter. Indeed, the EMD-MMSE's SNR improvement is about 1 dB greater than the MMSE filter for all the four considered signals "a", "b", "c", and "d".

The EMD-shrinkage is applied to four clean speech signals "e", "f", "g", and "h" (Figure 5), corrupted by additive white Gaussian noise with SNR values ranging from – 10 dB to 3 dB. Noisy versions of the original signals corresponding to $SNR = -1 \, dB$ are shown in Figure 6. Denoising results of the EMD-shrinkage (hard thresholding) and the wavelet method (Daubechies 4) are shown in Figure 7. A careful examination of the signals of Figures 5 and 7 shows that the EMD-shrinkage performs better than the wavelet method in terms of noise reduction. Furthermore, signals structures or features are globally better preserved with the EMDshrinkage than the wavelet method. Figure 8 shows the improvement in SNR values obtained for different noise levels of the signals "e", "f", "g", and "h" for the EMDshrinkage and three-type wavelet method (Haar, Symmlet 4, Daubechies 4). This figure demonstrates that for noise SNR values from $-10 \, dB$ to $3 \, dB$, the improvement in SNR provided by the EMD-shrinkage varies from $-0.7 \, dB$ to 11.5 dB. In addition, the gain in SNR of the EMD-shrinkage is much better than the one obtained by the other method for the three wavelets. When listening to the enhanced speeches, both the EMD-MMSE and the EMD-shrinkage are found to produce lower residual noise and, noticeably, less speech distortion for all the signals compared to the MMSE or the wavelet method.

5. CONCLUSION

This paper presents two new speech denoising methods. Both schemes are based on the EMD and thus are simple and fully data-driven methods. The methods do not use any pre- or postprocessing and do not require any use of parameters setting (except the threshold ϵ). The study is limited to signals corrupted by additive white Gaussian noise. Obtained results for clean speech signals corrupted with additive Gaussian noise with different SNR values ranging from – 10 dB to 10 dB show that the proposed EMDdenoising methods, associated with the MMSE filter or the shrinkage strategy, perform better than the MMSE filter and the wavelet approach, respectively. These results show that the EMD-denoising methods are effective for noise removal and confirm our previous findings [14, 15]. The EMDshrinkage is very attractive, especially in the case where the noise estimation is not easy. Even in the case when the noise level estimation is possible, the EMD improves the denoising result with the classical MMSE filter. The obtained results also show that it is more efficient to apply thresholding or filtering to the different components (IMFs) of the signal than to the signal itself. To confirm the obtained results and the effectiveness of the EMD-denoising approaches, the schemes must be evaluated with a large class of speech signals and in different experimental conditions, such as sampling rates, sample sizes, multiplicative noise, or the type of noise.

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