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Research Article

Code Combination for Blind Channel Estimation in General MIMO-STBC Systems

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The problem of blind channel estimation under space-time block coded (STBC) transmissions is addressed. Firstly, a blind channel estimation criterion that generalizes previous works is proposed. The technique is solely based on second-order statistics (SOS) and if the channel is identifiable, the estimate is obtained as the main eigenvector of a generalized eigenvalue problem (GEV). Secondly, a new transmission technique is proposed to solve the indeterminacies associated to the blind channel estimation problem. The technique is based on the combination of different STBCs, and it can be reduced to a nonredundant precoding consisting in the rotation or permutation of the transmit antennas. Unlike other previous approaches, the proposed technique does not imply a penalty in the transmission rate or capacity of the STBC system, while it is able to avoid the ambiguities in many practical cases, which is illustrated by means of some simulation examples.

1. Introduction

In the last ten years, since the well-known work of Alamouti [1] and the later generalization by Tarokh et al. [2], several families of space-time block codes (STBCs) have been proposed to exploit the spatial diversity in multiple-input multiple-output (MIMO) systems. Some examples are as follows.

(i) Orthogonal STBCs (OSTBCs) [2], which achieve full diversity with a low decoding complexity at expense of a loss in the transmission rate for more than two transmit antennas [3].

(ii) Quasiorthogonal STBCs (QOSTBCs) [3–5], which can achieve full diversity and rate one with a slight increase in the complexity of the maximum likelihood (ML) decoder.

(iii) Trace-orthogonal codes (TOSTBCs) [6, 7], which are information lossless codes with full rate and diversity and also provide minimum bit error rate (BER) when the suboptimal linear minimum mean square error (LMMSE) decoder is employed.

(iv) Perfect STBCs [8], which are full-rate and full-diversity codes and also ensure energy efficiency and a nonzero lower bound in the coding gain.

A common assumption for most of the STBCs is that perfect channel state information (CSI) is available at the receiver, which has motivated an increasing interest on blind channel estimation algorithms [9–15]. The main advantage of blind techniques resides in their ability to avoid the penalty in bandwidth efficiency or signal-to-noise ratio (SNR) associated, respectively, to training-based approaches [16, 17], or differential techniques [18–20]. Among blind channel estimation techniques, those solely based on second-order statistics (SOS) [12, 14, 15, 21] are specially appealing due to their reduced computational complexity and their independence of the specific signal constellation.

The literature on blind and semiblind channel estimation under STBC transmissions is abundant, ranging from some techniques for the specific OSTBC case [10–14], to extensions for more general STBCs or multiuser settings [9, 15, 21], and even to equalization and estimation of frequency selective channels [22–25]. However, excluding the OSTBC case [12, 13] and some specific low-rate codes

[21], the existing approaches fail to extract the channel in a completely blind manner, which is due to two different reasons. On one hand, the problem of blind STBC channel estimation is usually affected by a set of ambiguities provoked by the special structure of the codes. On the other hand, most of the proposed methods do not exploit the impropery (or noncircularity) of the STBC data model [26–28], which creates additional indeterminacies to those associated to the blind channel estimation problem. Several techniques for solving these ambiguities have been proposed in literature, including the transmission of a short training sequence [9, 15, 21, 24, 25], linear precoding approaches [12, 22, 23, 29], and schemes based on a slight reduction of the transmission rate [30–32]. Nevertheless, most of these techniques are specific for OSTBCs, and all of them incur in a penalty in terms of transmission rate or capacity of the overall MIMO-STBC system.

The main contributions of this paper are twofold. Firstly, in Section 3 we propose a new blind channel estimation algorithm for a general class of STBCs. The proposed technique is inspired by the relaxed blind ML decoder and it is solely based on the SOS of the observations. The computational complexity of the proposed technique reduces to the extraction of the principal eigenvector of a generalized eigenvalue problem (GEV). Furthermore, it exploits the improperty induced by the STBC data model and does not introduce additional ambiguities to those inherent to the problem.

Secondly, in Section 5 we propose a new transmission technique to avoid the indeterminacies associated to the blind channel estimation problem. The technique is based on combining different STBCs, but it can be reduced to a nonredundant precoding consisting of a single rotation or permutation of the transmit antennas, which comes at virtually no computational expense at the transmitter. Finally, we show by means of some numerical examples that this technique allows the unambiguous channel estimation in most of the practical situations.

The structure of the paper is as follows. The STBC data model is introduced in Section 2, and the blind channel estimation technique is presented in Section 3. In Section 4 we present a brief analysis of the indeterminacies associated to the blind channel estimation problem, which is used in Section 5 to introduce a transmission technique to avoid the ambiguity problems. Finally, the performance of the proposed technique is evaluated by means of some numerical examples in Section 6, and the concluding remarks are pointed out in Section 7.

2. Notation and Space-Time Block Coding Data Model

Throughout this paper we will use bold-faced uppercase letters to denote matrices, bold-faced lowercase letters for column vector, and light-faced lowercase letters for scalar quantities. Superscript $(\hat{\cdot})$ denotes estimated matrices, vectors or scalars, and superscripts $(\cdot)^T$, $(\cdot)^H$ denote transpose and Hermitian, respectively. $\mathbf{I}_p$ is the identity matrix of dimension p, $\mathbf{0}$ will denote the zero matrix of the required dimensions, and $E[\cdot]$ will denote the expectation operator. Finally, the trace, range (or column space), and Frobenius norm of a matrix $\mathbf{A}$ will be denoted as $\mathrm{Tr}(\mathbf{A})$, $\mathrm{range}(\mathbf{A})$, and $\|\mathbf{A}\|$, respectively.

In this paper we consider a linear space-time block code (STBC) transmitting M symbols during L channel uses with n_T antennas at the transmitter side. The transmission rate is defined as $R = M/L$, and the number of real symbols M' transmitted in each block is

$$M' = \begin{cases} M & \text{for real codes,} \\ 2M & \text{for complex codes.} \end{cases} \tag{1}$$

Let us assume an $n_T \times n_R$ flat fading MIMO channel $\mathbf{H}$ and additive white Gaussian noise with variance σ^2 at the receiver. Thus, using the notation in [29, 31], the received signal associated to the nth data block can be written as

$$\tilde{\mathbf{y}}[n] = \widetilde{\mathbf{W}}(\tilde{\mathbf{h}})\mathbf{s}[n] + \tilde{\mathbf{n}}[n], \tag{2}$$

where $\mathbf{s}[n]$ is a real vector with the M' real information symbols, $\tilde{\mathbf{y}}[n] \in \mathbb{R}^{2Ln_R \times 1}$ contains the real and imaginary parts of the observations, $\tilde{\mathbf{n}}[n] \in \mathbb{R}^{2Ln_R \times 1}$ is the real i.i.d. Gaussian noise with variance $\sigma^2/2$, and

$$\widetilde{\mathbf{W}}(\tilde{\mathbf{h}}) = \underbrace{\left[\widetilde{\mathbf{D}}_1\tilde{\mathbf{h}} \ \cdots \ \widetilde{\mathbf{D}}_{M'}\tilde{\mathbf{h}}\right]}_{2Ln_R \times M'}, \tag{3}$$

is the equivalent channel matrix, which represents the effect of both the STBC and the MIMO channel. Finally, the matrices $\widetilde{\mathbf{D}}_k \in \mathbb{R}^{2Ln_R \times 2n_T n_R}$ are easily obtained from the STBC code matrices [29, 31], and $\tilde{\mathbf{h}} \in \mathbb{R}^{2n_T n_R \times 1}$ represents the real and vectorized version of $\mathbf{H}$.

3. Proposed Blind Channel Estimation Method

In this section we propose a general blind channel estimation technique inspired by the relaxed (or unconstrained) blind ML receiver. Let us start by introducing the main assumptions of the proposed technique.

3.1. Main Assumptions

Assumption 1 (Number of available blocks). The MIMO channel is flat fading and constant during a period of $N \geq M'$ transmission blocks.

Assumption 2 (Input signals). The input is persistently exciting, that is, the matrix $[\mathbf{s}[0] \cdots \mathbf{s}[N-1]]$ is full row-rank.

Assumption 3 (Equivalent channel). The equivalent channel $\widetilde{\mathbf{W}}(\tilde{\mathbf{h}})$ is full column-rank.

Assumption 1 establishes a mild condition on the coherence time of the channel. For independent information vectors $\mathbf{s}[n]$, Assumption 2 is asymptotically (for $N \to \infty$) equivalent to the condition of having a nonsingular correlation matrix $E[\mathbf{s}[n]\mathbf{s}^T[n]]$. On the other hand, if $\widetilde{\mathbf{W}}(\widetilde{\mathbf{h}})$ is not full column-rank, any information vector $\mathbf{s}[n] + \mathbf{z}[n]$, with $\mathbf{z}[n]$ belonging to the null subspace of $\widetilde{\mathbf{W}}(\widetilde{\mathbf{h}})$, provides the same observations $\tilde{\mathbf{y}}[n]$ as $\mathbf{s}[n]$. Therefore, Assumption 3 is a desirable property for any STBC and, for the most common codes, the MIMO channels providing rank deficient matrices $\widetilde{\mathbf{W}}(\widetilde{\mathbf{h}})$ form a set of measure zero.

3.2. Proposed Criterion. The proposed criterion is inspired by the blind maximum likelihood (BML) receiver, which is formulated as

$$\{\hat{\tilde{\mathbf{h}}}_{\mathrm{BML}}, \hat{\mathbf{s}}_{\mathrm{BML}}[n]\} = \arg\min_{\tilde{\mathbf{h}}, \mathbf{s}[n]\in\mathcal{S}} \sum_{n=0}^{N-1} \left|\left|\tilde{\mathbf{y}}[n] - \widetilde{\mathbf{W}}(\tilde{\mathbf{h}})\mathbf{s}[n]\right|\right|^2. \quad (4)$$

Unfortunately, this is a fairly difficult problem due to the constraint that the elements of $\mathbf{s}[n]$ belong to the symbol constellation $\mathcal{S}$. A direct simplification is obtained by relaxing the finite alphabet constraint

$$\{\hat{\tilde{\mathbf{h}}}_{\mathrm{RML}}, \hat{\mathbf{s}}_{\mathrm{RML}}[n]\} = \arg\min_{\tilde{\mathbf{h}}, \mathbf{s}[n]} \sum_{n=0}^{N-1} \left|\left|\tilde{\mathbf{y}}[n] - \widetilde{\mathbf{W}}(\tilde{\mathbf{h}})\mathbf{s}[n]\right|\right|^2. \quad (5)$$

Now, considering the singular value decomposition (SVD) of the equivalent channel $\widetilde{\mathbf{W}}(\tilde{\mathbf{h}}) = \widetilde{\mathbf{U}}(\tilde{\mathbf{h}})\widetilde{\mathbf{\Lambda}}(\tilde{\mathbf{h}})\widetilde{\mathbf{V}}^T(\tilde{\mathbf{h}})$, the solution of (5) with respect to $\mathbf{s}[n]$ yields

$$\hat{\mathbf{s}}_{\mathrm{RML}}[n] = \widetilde{\mathbf{V}}(\tilde{\mathbf{h}})\widetilde{\mathbf{\Lambda}}^{-1}(\tilde{\mathbf{h}})\widetilde{\mathbf{U}}^T(\tilde{\mathbf{h}})\tilde{\mathbf{y}}[n], \quad (6)$$

where $\widetilde{\mathbf{U}}(\tilde{\mathbf{h}}) \in \mathbb{R}^{2Ln_R\times M'}$ and $\widetilde{\mathbf{V}}(\tilde{\mathbf{h}}) \in \mathbb{R}^{M'\times M'}$ are orthogonal (i.e., real and unitary) matrices, and $\widetilde{\mathbf{\Lambda}}(\tilde{\mathbf{h}})$ is a diagonal matrix containing the singular values. Thus, the criterion in (5) can be rewritten in terms of $\tilde{\mathbf{h}}$ as

$$\hat{\tilde{\mathbf{h}}}_{\mathrm{RML}} = \arg\max_{\tilde{\mathbf{h}}} \mathrm{Tr}\left(\widetilde{\mathbf{U}}^T(\tilde{\mathbf{h}})\hat{\mathbf{R}}_{\tilde{\mathbf{y}}}\widetilde{\mathbf{U}}(\tilde{\mathbf{h}})\right), \quad (7)$$

where

$$\hat{\mathbf{R}}_{\tilde{\mathbf{y}}} = \frac{1}{N}\sum_{n=0}^{N-1} \tilde{\mathbf{y}}[n]\tilde{\mathbf{y}}^T[n] \quad (8)$$

is an estimate of the correlation matrix of the observations.

Unfortunately, in a general situation the dependence of $\widetilde{\mathbf{U}}(\tilde{\mathbf{h}})$ with $\tilde{\mathbf{h}}$ is not trivial, and (7) cannot be easily solved. (An exception is the OSTBC case, for which $\widetilde{\mathbf{W}}(\tilde{\mathbf{h}}) = \|\tilde{\mathbf{h}}\|\widetilde{\mathbf{U}}(\tilde{\mathbf{h}})$ and (7) can be directly solved [12].) However, (7) can be interpreted as the problem of finding a channel $\tilde{\mathbf{h}}$ maximizing the projection of the observations $\tilde{\mathbf{y}}[n]$ onto the signal subspace defined by the equivalent channel $\widetilde{\mathbf{W}}(\tilde{\mathbf{h}})$. Thus, we propose the following alternative criterion:

$$\hat{\tilde{\mathbf{h}}} = \arg\max_{\tilde{\mathbf{h}}} \mathrm{Tr}\left(\widetilde{\mathbf{W}}^T(\tilde{\mathbf{h}})\mathbf{\Phi}_{\tilde{\mathbf{y}}}\widetilde{\mathbf{W}}(\tilde{\mathbf{h}})\right), \quad \text{subject to } \|\widetilde{\mathbf{W}}(\tilde{\mathbf{h}})\|^2 = 1, \quad (9)$$

where $\mathbf{\Phi}_{\tilde{\mathbf{y}}} = \widetilde{\mathbf{U}}_{\tilde{\mathbf{y}}}\widetilde{\mathbf{U}}_{\tilde{\mathbf{y}}}^T$ is the projection matrix onto the signal subspace of $\hat{\mathbf{R}}_{\tilde{\mathbf{y}}}$, $\widetilde{\mathbf{U}}_{\tilde{\mathbf{y}}} \in \mathbb{R}^{2Ln_R\times M'}$ is a matrix containing the M' principal eigenvectors of $\hat{\mathbf{R}}_{\tilde{\mathbf{y}}}$, and the constraint $\|\widetilde{\mathbf{W}}(\tilde{\mathbf{h}})\|^2 = 1$ is introduced in order to avoid trivial solutions. In other words, we propose to maximize the correlation between the equivalent channel $\widetilde{\mathbf{W}}(\tilde{\mathbf{h}})$ and the whitened and rank-reduced version of the observations $\tilde{\mathbf{y}}[n]$.

3.3. Algorithm Implementation. Unlike $\widetilde{\mathbf{U}}(\tilde{\mathbf{h}})$, the equivalent channel $\widetilde{\mathbf{W}}(\tilde{\mathbf{h}})$ can be easily obtained from $\tilde{\mathbf{h}}$ and the matrices $\widetilde{\mathbf{D}}_k$, $k = 1, \ldots, M'$. This allows us to rewrite

$$\mathrm{Tr}\left(\widetilde{\mathbf{W}}^T(\tilde{\mathbf{h}})\mathbf{\Phi}_{\tilde{\mathbf{y}}}\widetilde{\mathbf{W}}(\tilde{\mathbf{h}})\right) = \tilde{\mathbf{h}}^T\mathbf{\Theta}\tilde{\mathbf{h}} \quad (10)$$

and $\|\widetilde{\mathbf{W}}(\tilde{\mathbf{h}})\|^2 = \tilde{\mathbf{h}}^T\mathbf{\Psi}\tilde{\mathbf{h}}$, where

$$\mathbf{\Theta} = \sum_{k=1}^{M'} \widetilde{\mathbf{D}}_k^T\mathbf{\Phi}_{\tilde{\mathbf{y}}}\widetilde{\mathbf{D}}_k \quad (11)$$

can be seen as a modified correlation matrix, and

$$\mathbf{\Psi} = \sum_{k=1}^{M'} \widetilde{\mathbf{D}}_k^T\widetilde{\mathbf{D}}_k. \quad (12)$$

Finally, (9) can be rewritten as

$$\hat{\tilde{\mathbf{h}}} = \arg\max_{\tilde{\mathbf{h}}} \tilde{\mathbf{h}}^T\mathbf{\Theta}\tilde{\mathbf{h}}, \quad \text{subject to } \tilde{\mathbf{h}}^T\mathbf{\Psi}\tilde{\mathbf{h}} = 1, \quad (13)$$

and the solution $\hat{\tilde{\mathbf{h}}}$ is given by the eigenvector associated to the largest eigenvalue β of the following GEV:

$$\mathbf{\Theta}\hat{\tilde{\mathbf{h}}} = \beta\mathbf{\Psi}\hat{\tilde{\mathbf{h}}}. \quad (14)$$

3.4. Main Properties and Further Discussion. As we have shown, the solution of the proposed criterion can be obtained in closed-form by means of (14). However, we must note that the proposed method is a suboptimal approximation to the blind ML decoder.

(i) Firstly, the relaxation of the finite alphabet constraint translates into less accurate estimates than that of the optimal blind receiver. Additionally, this relaxation introduces a real scalar ambiguity in the channel estimate. However, the scale factor is a minor problem that can be easily solved in a latter step and, as it will be shown in the simulations section, in the absence of additional indeterminacies, the performance of the proposed method is very close to that of the receiver with perfect channel knowledge. (In the OSTBC case, the identifiability properties before and after this relaxation can be found in [32] and [31], resp.)

(ii) The second approximation consists in the substitution of (7) by (9). Although this leads to different criteria, the following lemma states the equivalence, in the asymptotic cases of $\sigma \to 0$ or $N \to \infty$, between the proposed method and the relaxed blind ML decoder.

Lemma 1. *In the absence of noise, or in the asymptotic case of $N \to \infty$, the solutions of the proposed criterion are those of the relaxed blind ML decoder.*

Proof. Let us start by pointing out that, for $\sigma \to 0$ or $N \to \infty$, the value of $\mathbf{\Phi}_{\tilde{\mathbf{y}}}$ is

$$\mathbf{\Phi}_{\tilde{\mathbf{y}}} = \widetilde{\mathbf{U}}_{\tilde{\mathbf{y}}}\widetilde{\mathbf{U}}_{\tilde{\mathbf{y}}}^T = \widetilde{\mathbf{U}}(\tilde{\mathbf{h}})\widetilde{\mathbf{U}}^T(\tilde{\mathbf{h}}), \tag{15}$$

which is the projection matrix onto the subspace spanned by the columns of $\widetilde{\mathbf{W}}(\tilde{\mathbf{h}})$. Thus, taking into account the assumptions in Section 3.1, the theoretical value of the term $\mathrm{Tr}(\widetilde{\mathbf{W}}^T(\hat{\tilde{\mathbf{h}}})\mathbf{\Phi}_{\tilde{\mathbf{y}}}\widetilde{\mathbf{W}}(\hat{\tilde{\mathbf{h}}}))$ in (9) can be upper-bounded by

$$\mathrm{Tr}(\widetilde{\mathbf{W}}^T(\hat{\tilde{\mathbf{h}}})\mathbf{\Phi}_{\tilde{\mathbf{y}}}\widetilde{\mathbf{W}}(\hat{\tilde{\mathbf{h}}})) \le \|\widetilde{\mathbf{W}}(\hat{\tilde{\mathbf{h}}})\|^2, \tag{16}$$

where the equality is satisfied if and only if

$$\mathrm{range}(\widetilde{\mathbf{W}}(\hat{\tilde{\mathbf{h}}})) = \mathrm{range}(\widetilde{\mathbf{W}}(\tilde{\mathbf{h}})). \tag{17}$$

Finally, the above equation is equivalent to the following ambiguity condition:

$$\widetilde{\mathbf{W}}(\tilde{\mathbf{h}})\mathbf{s}[n] = \widetilde{\mathbf{W}}(\hat{\tilde{\mathbf{h}}})\hat{\mathbf{s}}[n] \Longleftrightarrow \mathbf{S}(\mathbf{s}[n])\mathbf{H} = \mathbf{S}(\hat{\mathbf{s}}[n])\widehat{\mathbf{H}}, \quad \forall \mathbf{s}[n], \tag{18}$$

where $\mathbf{S}(\mathbf{s}[n]) \in \mathbb{C}^{L\times n_T}$ is the transmission matrix for the information vector $\mathbf{s}[n]$ [29, 31]. In other words, the proposed method does not introduce additional ambiguities to those associated to the problem of blind channel estimation from SOS, that is, the estimated channel $\hat{\tilde{\mathbf{h}}}$ and signal $\hat{\mathbf{s}}[n] = \widetilde{\mathbf{V}}(\hat{\tilde{\mathbf{h}}})\widetilde{\mathbf{\Lambda}}^{-1}(\hat{\tilde{\mathbf{h}}})\widetilde{\mathbf{U}}^T(\hat{\tilde{\mathbf{h}}})\tilde{\mathbf{y}}[n]$ are congruent with the data model in (2). □

As a direct consequence of the above lemma, we can see that the proposed technique is a deterministic (or self-noise-free) estimator, that is, in the absence of noise it exactly recovers the channel, up to a real scalar, within a finite number of observations. Furthermore, it can be interpreted as a subspace method based on maximizing the energy of the projection of $\widetilde{\mathbf{W}}(\hat{\tilde{\mathbf{h}}})$ onto the signal subspace of $\widehat{\mathbf{R}}_{\tilde{\mathbf{y}}}$. Finally, we must point out that the upper-bound in (16) justifies the use of the constraint $\|\widetilde{\mathbf{W}}(\tilde{\mathbf{h}})\|^2 = 1$ instead of the more common $\|\tilde{\mathbf{h}}\|^2 = 1$ [12, 15, 21, 22].

4. Identifiability Analysis

As previously pointed out, the problem of blind STBC channel estimation is usually affected by a set of indeterminacies which preclude the completely blind recovery of the channel. In general, these ambiguities can be due to two different reasons. On one hand, they can be the result of a lack of redundancy in the STBC. On the other hand, even in the case of STBCs introducing enough redundancy in the transmitted signals, the indeterminacies can appear as the result of the special structure, or symmetries, of the specific STBC. Although the exhaustive study of the identifiability conditions for a wide class of STBCs is a difficult task beyond the scope of this paper, in this section we introduce an intuitive necessary identifiability condition, and the effect of the ambiguity problems is illustrated by means of some practical examples.

4.1. Necessary Identifiability Condition. In this subsection we show that, in order to blindly identify the MIMO channel from SOS, the following necessary condition must be satisfied.

Condition 1 (Rate and number of antennas). Consider a STBC with transmission rate $R = M/L$, and a $n_T \times n_R$ MIMO channel. If the system parameters do not satisfy

$$\min(n_T, n_R) > \begin{cases} R & \text{for complex codes,} \\ \dfrac{R}{2} & \text{for real codes,} \end{cases} \tag{19}$$

then, the channel cannot be unambiguously identified from SOS.

This condition can be seen as the combination of two different requirements. Firstly, taking (17) into account, it is clear that the unambiguous channel recovery is only possible if there exists a nonempty noise subspace, that is, the equivalent channel matrix $\widetilde{\mathbf{W}}(\tilde{\mathbf{h}}) \in \mathbb{R}^{2Ln_R\times M'}$ must be tall, or equivalently

$$n_R > \frac{M'}{2L} = \begin{cases} R & \text{for complex codes,} \\ \dfrac{R}{2} & \text{for real codes.} \end{cases} \tag{20}$$

Additionally, when $M' = 2Ln_R$, we have $\mathbf{\Phi}_{\tilde{\mathbf{y}}} = \mathbf{I}_{2Ln_R}$, which implies $\mathbf{\Theta} = \mathbf{\Psi}$, and the multiplicity of the largest eigenvalue of (14) is $P = 2n_Tn_R$, that is, all the MIMO channels are congruent with the data model.

Secondly, let us consider that the ambiguity condition

$$\mathbf{S}(\mathbf{s}[n])\mathbf{H} = \mathbf{S}(\hat{\mathbf{s}}[n])\widehat{\mathbf{H}}, \quad \forall \mathbf{s}[n], \tag{21}$$

with $\widehat{\mathbf{H}} \neq c\mathbf{H}$ ($c \in \mathbb{R}$), is satisfied for some full row-rank MIMO channel $\mathbf{H}$. Then, defining $\mathbf{A} = \widehat{\mathbf{H}}\mathbf{H}^H(\mathbf{H}\mathbf{H}^H)^{-1}$, the above equation can be rewritten as

$$\mathbf{S}(\mathbf{s}[n]) = \mathbf{S}(\hat{\mathbf{s}}[n])\mathbf{A}, \quad \forall \mathbf{s}[n], \tag{22}$$

which ensures the nonidentifiability of any MIMO channel regardless of the number of receive antennas. In other words, from an identifiability point of view and considering a full-rank MIMO channel with $n_R = n_T$, the addition of more receive antennas does not provide any additional information about the channel. As a consequence, a necessary identifiability condition states that all the full-rank MIMO channels with $n_R = n_T$ must be identifiable.

Finally, the combination of the two previous conditions implies that (20) must be satisfied for $n_R = n_T$, which yields Condition 1. As a direct consequence, this result ensures the nonidentifiability, from SOS, of full-rate ($R = n_T$) STBCs such as the trace-orthogonal STBCs (TOSTBCs) [6, 7] or the recently proposed perfect STBCs [8]. (In this paper we refer to STBCs with $R = n_T$ as full-rate codes. The reader must note the difference with most of the OSTBC literature, where the term full-rate is used for OSTBCs with $R = 1$.)

4.2. Practical Examples. Unfortunately, the identifiability condition presented in the previous subsection is not sufficient for the unambiguous channel estimation. From a practical point of view, the existence of indeterminacies translates into a multiplicity $P > 1$ of the largest eigenvalue of the GEV problem in (14).

In this subsection the ambiguity problems are illustrated by means of some numerical examples. In particular, all the examples have been repeated for random Rayleigh distributed channels $\mathbf{H}$ and, as pointed out in [12], we have found that the multiplicity P of the largest eigenvalue is an almost-deterministic parameter, that is, in all the experiments with the same STBC and number of receive antennas n_R, the multiplicity P was the same. Although it is clear that the multiplicity also depends on the specific channel realization (consider, e.g., $\mathbf{H} = \mathbf{0}$), this suggests that the channels inducing different multiplicities form a set of measure zero.

The first set of examples is illustrated in Table 1, which is a partial reproduction of the table presented in [12]. Here, we can see that, for most of the common OSTBCs and $n_R > 1$, the multiplicity of the largest eigenvalue of (14) is $P = 1$, that is, the MIMO channel can be recovered, up to a real scalar, by means of the proposed method. An identifiability analysis for OSTBCs, which sheds some light into these results, can be found in [31].

In the second set of examples, the rate one QOSTBCs introduced in [3–5] have been analyzed. The codes have been recursively obtained using the Alamouti code [1] as a basic block (see [3]), which provides designs for a number of transmit antennas n_T power of two. For different n_T values, the codes have been obtained by removing some of the transmit antennas from the design for the smallest power of two greater than n_T. Table 2 shows the multiplicity P in this case, which in general decreases with n_R up to a certain value (the rightmost multiplicity shown in the table). As can be seen, the QOSTBCs do not allow the unambiguous recovery of the channel, and as predicted in the previous subsection, in the case of $n_R = R = 1$ the multiplicity is $P = 2n_T$.

To summarize, a direct application of the method proposed so far is restricted to those OSTBCs with $P = 1$ in Table 1. Obviously, this is a very small subset of all the STBCs. In the next section we propose a simple technique that notably enlarges the set of STBCs that can be blindly identified from SOS.

5. Code Combination

In this section, a transmission technique for the resolution of the ambiguities is presented. The proposed method is based on the combination of different STBCs, and it can be reduced to a nonredundant precoding consisting of a single rotation or permutation of the transmit antennas. Unlike previously proposed approaches [9, 10, 12, 29, 31], the transmission technique is able to resolve many of the ambiguity problems without reducing neither the transmission rate nor the capacity of the MIMO-STBC system. The aim of this section is presenting the new ideas behind the proposed transmission technique, whose performance is illustrated by means of numerical examples in Section 6. The theoretical analysis of the identifiability conditions associated to the proposed scheme is beyond the scope of this paper, and it will be a topic for future research.

Let us consider the noise-free case and define the matrix $\mathbf{G}(\mathbf{H}, \mathcal{C}) \in \mathbb{R}^{2n_T n_R \times P(\mathcal{C})}$ as a basis for the subspace spanned by the $P(\mathcal{C})$ eigenvectors associated to the largest eigenvalue of the GEV in (14), where we have explicitly included the dependency on the channel $\mathbf{H}$ and the code $\mathcal{C}$. Then, it is clear that the true MIMO channel $\widetilde{\mathbf{h}}$ belongs to that subspace, that is,

$$\widetilde{\mathbf{h}} \in \operatorname{range}(\mathbf{G}(\mathbf{H}, \mathcal{C})). \tag{23}$$

Considering now K different codes $\mathcal{C}_k$, $k = 1, \dots, K$, with n_T transmit antennas and transmitting $M(\mathcal{C}_k)$ information symbols in $L(\mathcal{C}_k)$ uses of the channel, we can ensure

$$\widetilde{\mathbf{h}} \in \{\operatorname{range}(\mathbf{G}(\mathbf{H}, \mathcal{C}_1)) \cap \cdots \cap \operatorname{range}(\mathbf{G}(\mathbf{H}, \mathcal{C}_K))\}, \tag{24}$$

that is, the true channel belongs to the intersection of the K different subspaces, of size $P(\mathcal{C}_k)$, defined by the matrices $\mathbf{G}(\mathbf{H}, \mathcal{C}_k)$. However, in a general case, there is no reason to think that the rank of such intersection will be larger than 1, that is, the spurious solutions to the blind channel estimation problem for code $\mathcal{C}_k$ do not necessarily maximize the criterion (9) when a different code $\mathcal{C}_l$ $(l \neq k)$ is used.

The proposed technique is based on this idea. Assuming that the MIMO channel remains constant during a large enough interval, the first $M(\mathcal{C}_1)$ information symbols are transmitted during the first $L(\mathcal{C}_1)$ time slots using $\mathcal{C}_1$. In the following $L(\mathcal{C}_2)$ channel uses, $M(\mathcal{C}_2)$ new information symbols are transmitted by means of $\mathcal{C}_2$, and the same procedure is used with the K STBCs. Thus, with obvious definitions of $\widetilde{\mathbf{W}}(\widetilde{\mathbf{h}}, \mathcal{C}_k)$, $\widetilde{\mathbf{y}}[n,k]$, $\widetilde{\mathbf{n}}[n,k]$, and $\mathbf{s}[n,k]$, the data model in (2) remains valid, where now $\widetilde{\mathbf{y}}[n] = [\widetilde{\mathbf{y}}^T[n,1], \dots, \widetilde{\mathbf{y}}^T[n,K]]^T$, $\widetilde{\mathbf{n}}[n] = [\widetilde{\mathbf{n}}^T[n,1], \dots, \widetilde{\mathbf{n}}^T[n,K]]^T$, $\mathbf{s}[n] = [\mathbf{s}^T[n,1], \dots, \mathbf{s}^T[n,K]]^T$, and

$$\widetilde{\mathbf{W}}(\widetilde{\mathbf{h}}) = \begin{bmatrix} \widetilde{\mathbf{W}}(\widetilde{\mathbf{h}}, \mathcal{C}_1) & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \widetilde{\mathbf{W}}(\widetilde{\mathbf{h}}, \mathcal{C}_K) \end{bmatrix}. \tag{25}$$

Following a similar derivation to that of Section 3, the final channel estimation criterion amounts to maximize

$$\hat{\widetilde{\mathbf{h}}} = \arg\max_{\widetilde{\mathbf{h}}} \sum_{k=1}^{K} \widetilde{\mathbf{h}}^T \boldsymbol{\Theta}(\mathcal{C}_k) \widetilde{\mathbf{h}}, \quad \text{subject to } \sum_{k=1}^{K} \widetilde{\mathbf{h}}^T \boldsymbol{\Psi}(\mathcal{C}_k) \widetilde{\mathbf{h}}, \tag{26}$$

Table 1: Identifiability characteristics for some of the most common OSTBCs.

Constellation	n_T	M	L	$R = M/L$	Design	$P_{n_R=1}$	$P_{n_R>1}$
Real	2	2	2	1	Alamouti	2	2
Real	3	4	4	1	Gen. ort	2	1
Real	4	4	4	1	Gen. ort	4	4
Real	5	8	8	1	Gen. ort	2	1
Real	6	8	8	1	Gen. ort	2	1
Real	7	8	8	1	Gen. ort	2	1
Real	8	8	8	1	Gen. ort	2	1
Complex	2	2	2	1	Alamouti	4	4
Complex	3	4	8	1/2	Gen. ort	2	1
Complex	4	4	8	1/2	Gen. ort	4	4
Complex	5	8	16	1/2	Gen. ort	2	1
Complex	6	8	16	1/2	Gen. ort	2	1
Complex	7	8	16	1/2	Gen. ort	2	1
Complex	8	8	16	1/2	Gen. ort	2	1
Complex	3	3	4	3/4	Amicable	2	1
Complex	4	3	4	3/4	Amicable	2	1
Complex	5	4	8	1/2	Amicable	1	1
Complex	6	4	8	1/2	Amicable	1	1
Complex	7	4	8	1/2	Amicable	1	1
Complex	8	4	8	1/2	Amicable	1	1

Table 2: Identifiability characteristics for QOSTBCs. Empty spaces indicate that the rightmost multiplicity in the row is repeated.

n_T	$P_{n_R=1}$	$P_{n_R=2}$	$P_{n_R=3}$	$P_{n_R=4}$	$P_{n_R=5}$	$P_{n_R=6}$	$P_{n_R=7}$	$\cdots$	$P_{n_R=15}$
2	4								
3	6	4	2						
4	8								
5	10	4	2						
6	12	8	4						
7	14	12	10	8	6	4	2		
8	16								
9	18	4	2						
10	20	8	4						
11	22	12	2						
12	24	16	8						
13	26	20	14	8	2				
14	28	24	20	16	12	8	4		
15	30	28	26	24	22	20	18	$32 - 2n_R$	2
16	32								

where

$$\begin{aligned} \boldsymbol{\Theta}(\mathcal{C}_k) &= \sum_{l=1}^{M'(\mathcal{C}_k)} \widetilde{\mathbf{D}}_l^T(\mathcal{C}_k)\boldsymbol{\Phi}_{\tilde{\mathbf{y}}}(\mathcal{C}_k)\widetilde{\mathbf{D}}_l(\mathcal{C}_k), \\ \boldsymbol{\Psi}(\mathcal{C}_k) &= \sum_{l=1}^{M'(\mathcal{C}_k)} \widetilde{\mathbf{D}}_l^T(\mathcal{C}_k)\widetilde{\mathbf{D}}_l(\mathcal{C}_k), \end{aligned} \tag{27}$$

$\boldsymbol{\Phi}_{\tilde{\mathbf{y}}}(\mathcal{C}_k)$ is the rank-$M'(\mathcal{C}_k)$ and whitened version of

$$\hat{\mathbf{R}}_{\tilde{\mathbf{y}}}(\mathcal{C}_k) = \sum_{n=0}^{N/K-1} \tilde{\mathbf{y}}[n,k]\tilde{\mathbf{y}}^T[n,k], \tag{28}$$

and $\widetilde{\mathbf{D}}_l(\mathcal{C}_k)$ are the code-dependent matrices for $\mathcal{C}_k$.

From (26), we can conclude that the channel estimate $\hat{\hat{\mathbf{h}}}$ can be obtained as the eigenvector associated to the largest eigenvalue β of the following GEV:

$$\boldsymbol{\Theta}\hat{\hat{\mathbf{h}}} = \beta\boldsymbol{\Psi}\hat{\hat{\mathbf{h}}}, \tag{29}$$

where

$$\boldsymbol{\Theta} = \sum_{k=1}^{K}\boldsymbol{\Theta}(\mathcal{C}_k), \qquad \boldsymbol{\Psi} = \sum_{k=1}^{K}\boldsymbol{\Psi}(\mathcal{C}_k). \tag{30}$$

Furthermore, since the true channel $\tilde{\mathbf{h}}$ maximizes the K factors $\tilde{\mathbf{h}}^T\mathbf{\Theta}(\mathcal{C}_k)\tilde{\mathbf{h}}$ simultaneously, it is clear that all the solutions to (26) belong to the intersection of the subspaces spanned by $\mathbf{G}(\mathbf{H},\mathcal{C}_k)$, for $k = 1,\dots,K$.

Interestingly, the set of K code matrices can be interpreted as a composite STBC with a larger delay $L = \sum_{k=1}^{K} L(\mathcal{C}_k)$, which resembles the idea of designing codes with $L \gg n_T$ in order to improve the bit error rate (BER) [33, 34]. However, we must note the following.

(i) Although the consecutive STBC blocks can be seen as a longer code, the decoding of the K blocks is still uncoupled. Therefore, the complexity of the proposed scheme is lower than that of a hypothetical longer STBC with the desired identifiability properties.

(ii) Analogously, if we consider the set of K blocks as a longer code, the direct application of the method proposed in Section 3 would require the eigenvalue decomposition of the overall matrix $\hat{\mathbf{R}}_{\tilde{\mathbf{y}}}$ of size $2Ln_R$. On the other hand, (26) is based on the eigenvalue decomposition of the smaller matrices $\hat{\mathbf{R}}_{\tilde{\mathbf{y}}}(\mathcal{C}_k)$ $(k = 1,\dots,K)$, which translates into a lower computational complexity and more accurate channel estimates.

Finally, we must take into account that the total number of time slots used in the blind channel estimation process has to be distributed among the K different STBCs. This implies a reduction of the effective number of blocks for the estimation of the K correlation matrices $\hat{\mathbf{R}}_{\tilde{\mathbf{y}}}(\mathcal{C}_k)$. Therefore, there exists a tradeoff between the identifiability properties of the composite code, which are improved by increasing K, and the accuracy of the estimates $\hat{\mathbf{R}}_{\tilde{\mathbf{y}}}(\mathcal{C}_k)$, which is degraded with increasing K.

5.1. A Particular Solution: Nonredundant Precoding. The proposed technique for the solution of the ambiguities raises a great number of questions related to the analysis of the identifiability conditions, or the best combination of codes. Here, we propose a particular code combination strategy, which uses a single STBC and a nonredundant precoding consisting in the rotation of the transmit antennas.

Let us consider K different unitary matrices $\mathbf{Q}_k$, $k = 1,\dots,K$, and one STBC code $\mathcal{C}$ with transmission matrix $\mathbf{S}(\mathbf{s}[n],\mathcal{C}) \in \mathbb{C}^{L\times n_T}$. Then, we can introduce the following transmission matrices:

$$\mathbf{S}(\mathbf{s}[n],\mathcal{C}_k) = \mathbf{S}(\mathbf{s}[n],\mathcal{C})\mathbf{Q}_k, \quad k = 1,\dots,K, \tag{31}$$

which define K virtually different codes $\mathcal{C}_k$. Thus, the code combination is obtained by rotating the transmission matrix of one STBC and if the matrices $\mathbf{Q}_k$ are chosen as permutation matrices, the proposed technique reduces to a virtual permutation of the transmit antennas, which does not increase the complexity of the transmitter and preserves the power properties associated to each transmit antenna. Furthermore, since the effect of the rotations can be considered as part of the channel, the proposed strategy preserves the code properties.

Table 3: Application of the nonredundant precoding technique. QOSTBC codes and $K = 4$ random rotations.

n_T	$P_{n_R=1}$	$P_{n_R>1}$
2	4	4
3	6	1
4	8	1
5	10	1
6	12	1
7	14	1
8	16	1
9	18	1
10	20	1
11	22	1
12	24	1
13	26	1
14	28	1
15	30	1
16	32	1

Finally, it is interesting to point out that the rotation of the transmitted signals has been previously proposed in other contexts. On one hand, the rotation or randomization of a given STBC has also been employed for distributed space-time block coding in relay networks [35, 36]. On the other hand, in [37–39] the authors have proven that the rotation of the transmitted signals improves the frame error rate and increases the outage capacity of the MIMO system. Therefore, the proposed nonredundant precoding technique is not only able to solve the blind channel estimation indeterminacies in many practical cases, but also to increase the outage capacity and helps to reduce the frame error rate.

5.2. Practical Examples. Let us illustrate the performance of the proposed nonredundant precoding technique by means of a numerical example. Table 3 shows the identifiability properties for QOSTBCs when the nonredundant precoding technique is applied. The results have been obtained by random generations of $K = 4$ unitary matrices, and Rayleigh MIMO channels. As can be seen, most of the ambiguities in Table 2 have been resolved, with the only exceptions of the MISO cases ($n_R = 1$) and the Alamouti code ($n_T = 2$) [1]. The ambiguity in the MISO cases is explained by the result in Section 4.1, that is, the condition $n_R > R$ is still violated and then the composite code does not allow the blind channel recovery from SOS. On the other hand, the nonidentifiability of the Alamouti code has been previously reported in [9, 32].

As previously pointed out, the indeterminacies in the blind extraction of the channel can be due to the special structure or symmetries introduced by the STBC, or to the lack of redundancy. The empirical results obtained in this section suggest that the nonredundant precoding technique is able to *break* the symmetries and if the code introduces enough redundancy, it permits the unambiguous recovery of the MIMO channel from SOS.

6. Simulation Results

In this section, the performance of the proposed technique is illustrated by means of some simulation examples. All the results have been obtained by averaging 5000 independent experiments, where the MIMO channel $\mathbf{H}$ has been generated as a Rayleigh channel with unit-variance elements. The i.i.d information symbols belong to a quadrature phase shift keying (QPSK) constellation and the receivers have been designed based on the linear minimum mean square error (LMMSE) criterion and a hard decision decoder. In the case of OSTBC transmissions this is equivalent to the ML receiver.

In order to avoid the ambiguity problems, the nonredundant precoding technique with $K = 4$ permutations has been applied. Specifically, the permutations of the transmit antennas are based on the following matrices:

$$\mathbf{Q}_1^{(4)} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \qquad \mathbf{Q}_2^{(4)} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix},$$
$$\mathbf{Q}_3^{(4)} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}, \qquad \mathbf{Q}_4^{(4)} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}. \tag{32}$$

Finally, the proposed blind channel estimation technique has been compared with the following schemes.

(i) A receiver with perfect channel state information (CSI), which we refer to as coherent receiver.

(ii) A training-based approach consisting of the transmission, over $N_{\text{tr}}L$ channel uses, of n_T orthogonal training signals, which are used in the receiver to obtain the channel estimate by means of the least squares (LS) technique.

(iii) The subspace blind method proposed in [12] and its extension to nonorthogonal codes [15]. In this case, we have considered a transmission scheme without nonredundant precoding (referred to as SS), as well as transmissions with nonredundant precoding and a receiver considering the K consecutive blocks as a longer STBC (referred to as SS + NRP).

In the first set of examples, the real OSTBC design with $n_T = M = L = 4$ [2] and $n_R = 4$ receive antennas has been evaluated. As shown in Table 1 (see also [12]), this code does not allow the direct blind channel recovery for any number of receive antennas. The nonredundant precoding technique has been applied with the permutation matrices $\mathbf{Q}_1^{(4)}, \ldots, \mathbf{Q}_4^{(4)}$, that is, the precoding technique is limited to a single set of permutations of the transmit antennas, which comes at virtually no computational expense at the transmitter. Figures 1 and 2 show the MSE in the channel estimate and the bit error rate (BER) after decoding for different numbers N of available blocks at the receiver. As can be seen, the SS method is not able to unambiguously recover the channel, whereas the SS + NRP technique is not as accurate as the proposed technique, whose performance loss with respect to the coherent receiver is much lower than the 3-dB loss associated to the differential receivers [18–20].

A comparison with a training-based method in terms of BER can be seen in Figure 3, where the two channel estimation techniques and the coherent receiver have been evaluated for three different SNR values. As can be seen,

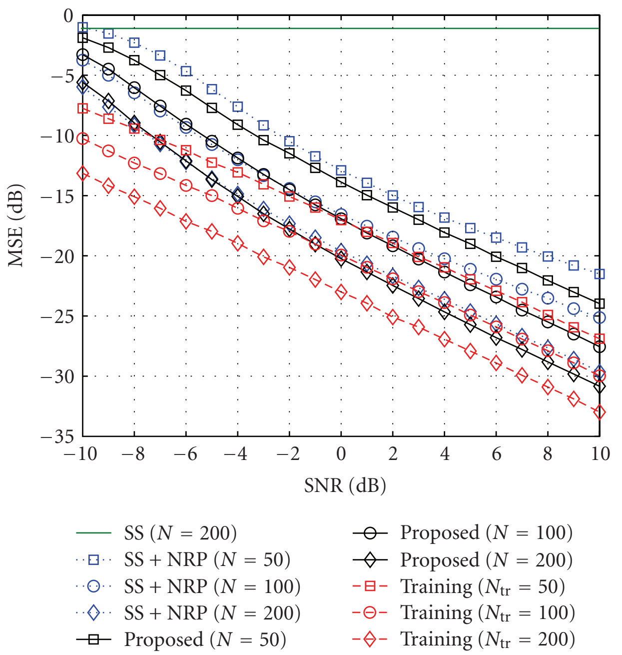


Figure 1: MSE in the channel estimate. $K = 4$ code permutations. Real OSTBC design with $n_T = n_R = M = L = 4$.

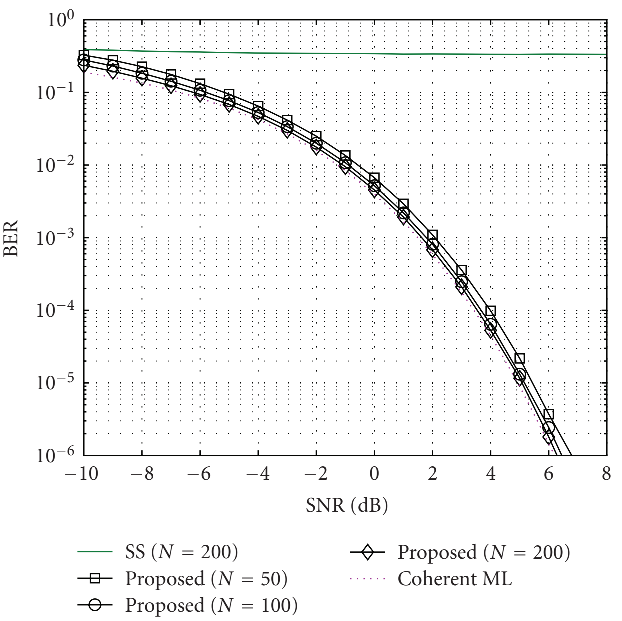
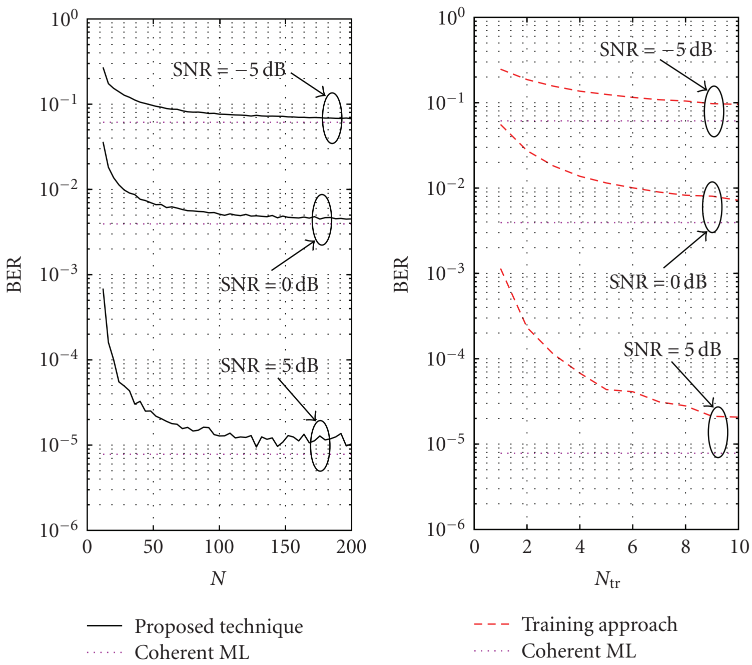


Figure 2: Bit error rate (BER) for the ML receiver with perfect and estimated CSI. Real OSTBC design with $n_T = n_R = M = L = 4$. $K = 4$ code permutations.



(a) (b)

Figure 3: (a) Bit error rate (BER) versus number N of observation blocks for the proposed method or (b) number N_{tr} of training blocks. Real OSTBC design with $n_T = n_R = M = L = 4$. $K = 4$ code permutations.

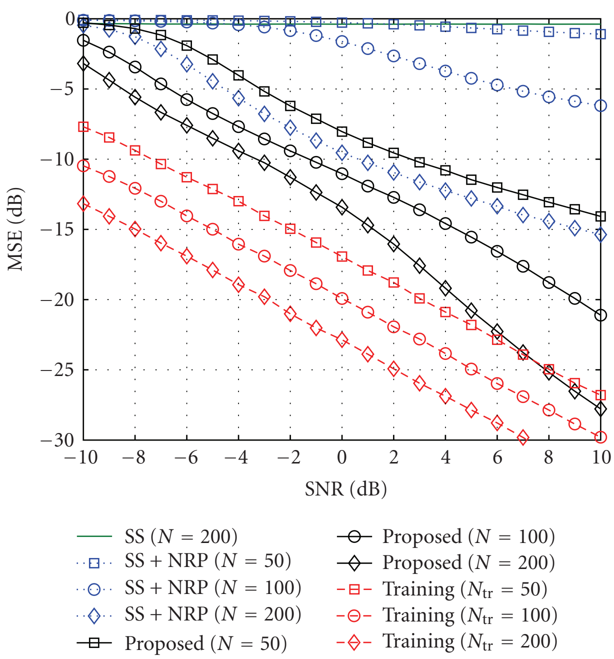


Figure 4: MSE in the channel estimate. $K = 4$ code permutations. QOSTBC with $n_T = M = L = 8$ and $n_R = 4$.

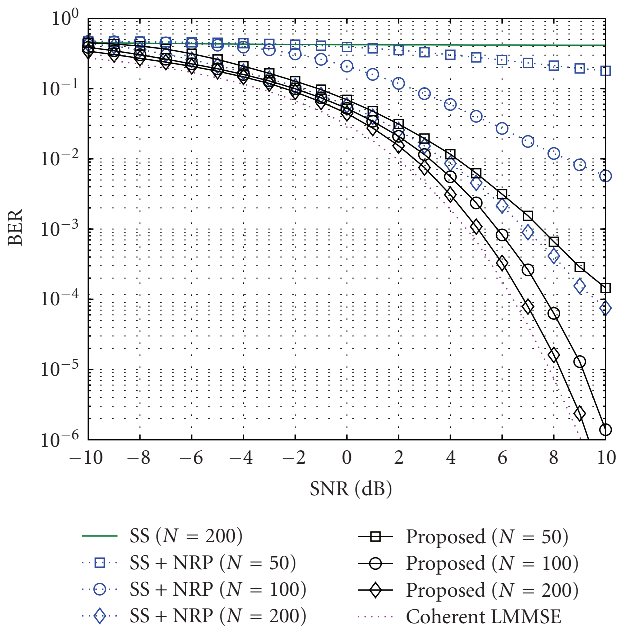


Figure 5: Bit error rate (BER) for the LMMSE receiver with perfect and estimated CSI. QOSTBC with $n_T = M = L = 8$ and $n_R = 4$. $K = 4$ code permutations.

the BER performance obtained by the proposed method is equivalent to that obtained with a training-based approach and some number $N_{tr} < N$ of training blocks. However, the blind channel estimation technique avoids the loss in bandwidth efficiency. Furthermore, we can see that for $N = 100$ available blocks at the receiver, the proposed method outperforms the training-based approach with $N_{tr} = 10$, not only in bandwidth efficiency, but also in terms of BER.

In the second set of examples, the QOSTBC design for $n_T = M = L = 8$ [3] and $n_R = 4$ has been evaluated.

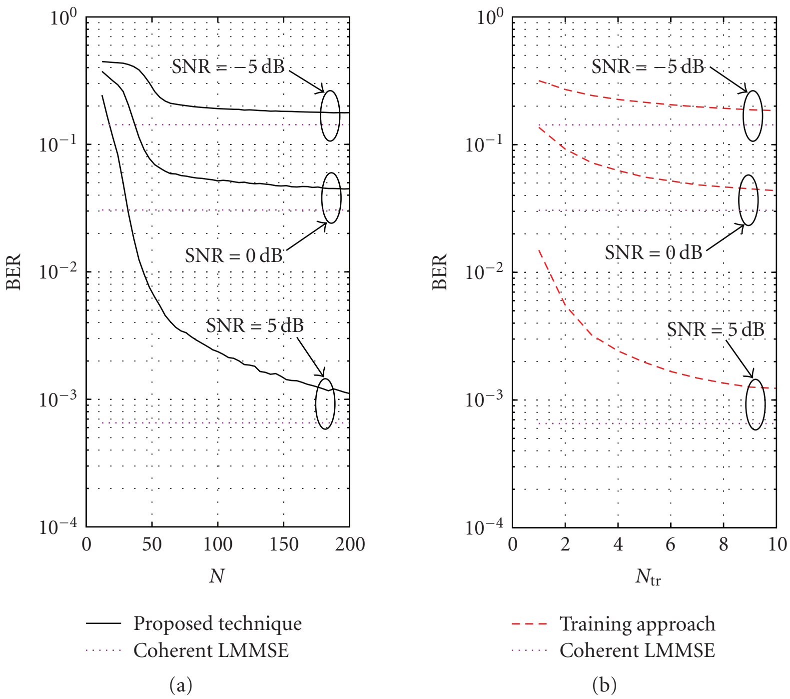


Figure 6: (a) Bit error rate (BER) versus number N of observation blocks for the proposed method or (b) number N_{tr} of training blocks. QOSTBC with $n_T = M = L = 8$ and $n_R = 4$. $K = 4$ code permutations.

Analogously to the previous example, this code does not allow the direct blind channel identification based only on SOS (see Table 2). The $K = 4$ permutation matrices have been defined as

$$\mathbf{Q}_1^{(8)} = \begin{bmatrix} \mathbf{Q}_1^{(4)} & \mathbf{0} \\ \mathbf{0} & \mathbf{Q}_2^{(4)} \end{bmatrix}, \qquad \mathbf{Q}_2^{(8)} = \begin{bmatrix} \mathbf{Q}_3^{(4)} & \mathbf{0} \\ \mathbf{0} & \mathbf{Q}_4^{(4)} \end{bmatrix},$$
$$\mathbf{Q}_3^{(8)} = \begin{bmatrix} \mathbf{Q}_1^{(4)} & \mathbf{0} \\ \mathbf{0} & \mathbf{Q}_3^{(4)} \end{bmatrix}, \qquad \mathbf{Q}_4^{(8)} = \begin{bmatrix} \mathbf{Q}_2^{(4)} & \mathbf{0} \\ \mathbf{0} & \mathbf{Q}_4^{(4)} \end{bmatrix}, \tag{33}$$

and the simulation results are shown in Figures 4, 5, and 6. As can be seen, the results are not as accurate as in the OSTBC examples, which is due to the fact that we are trying to solve a more complicated problem (nonorthogonal transmissions, higher n_T and transmission rate), that is, with a lower degree of redundancy. However, for a sufficiently large N, the proposed technique outperforms the pilot-based approach, and its performance degradation with respect to the coherent receiver is lower than the minimal loss (3-dB) associated to the QOSTBC differential technique proposed in [20]. Furthermore, we can see that in this case, the proposed method clearly outperforms the SS + NRP method. This is due to the fact that the SS + NRP is based on the eigenvalue decomposition of a matrix $\hat{\mathbf{R}}_{\tilde{\mathbf{y}}}$ of size $2Ln_RK = 256$, whereas the proposed method requires $K = 4$ eigenvalue decompositions of size $2Ln_R = 64$, which translates into a lower computational complexity and an increased robustness to noise.

7. Conclusions

In this paper, a blind channel estimation technique for multiple-input multiple-output (MIMO) space-time block coded (STBC) systems has been proposed. The technique is solely based on second-order statistics (SOS), and its computational complexity reduces to the extraction of the principal eigenvector of a generalized eigenvalue problem (GEV). In the absence of noise it exactly recovers the channel, up to a real scalar, within a finite number of observations, that is, it is a deterministic technique. Additionally, it has been shown that the ambiguity problems associated to certain STBCs are due to the code structure, and not to the proposed channel estimation algorithm. Furthermore, we have proposed a general method to avoid the ambiguities, which is based on the idea of code combination. As a particular case, this technique can be reduced to a nonredundant precoding of the transmitted signals, consisting of a single rotation or permutation of the transmit antennas. Finally, the proposed technique has been evaluated by means of numerical examples, showing that, for a sufficiently large number of observations, its performance is close to that of the coherent receivers.

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