

Research Article

Introducing Switching Ordered Statistic CFAR Type I in Different Radar Environments

Saeed Erfanian and Vahid Tabataba Vakili

Electrical Engineering Department, Iran University of Science & Technology (IUST), Narmak, Tehran 16846, Iran

Correspondence should be addressed to Saeed Erfanian, s.erfanian@iust.ac.ir

Received 22 September 2008; Revised 3 December 2008; Accepted 17 February 2009

Recommended by M. Greco

In this paper, a new CFAR detector based on a switching algorithm and OS-CFAR for nonhomogeneous background environments is introduced. The new detector is named Switching Ordered Statistic CFAR type I (SOS CFAR I). The SOS CFAR I selects a set of suitable cells and then with the help of the ordering method, estimates the unknown background noise level. The proposed detector does not require any prior information about the background environment and uses cells with similar statistical specifications to estimate the background noise. The performance of SOS CFAR I is evaluated and compared with other detectors such as CA-CFAR, GO-CFAR, SO-CFAR, and OS-CFAR for the Swerling I target model in homogeneous and nonhomogeneous noise environments such as those with multiple interference and clutter edges. The results show that SOS CFAR I detectors considerably reduce the problem of excessive false alarm probability near clutter edges while maintaining good performance in other environments. Also, simulation results confirm the achievement of an optimum detection threshold in homogenous and nonhomogeneous radar environments by the mentioned processor.

Copyright © 2009 S. Erfanian and V. Tabataba Vakili. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

1. Introduction

A common routine test in any detection system is to compare the received signal level with a predefined threshold value. If the threshold is crossed, the presence of the signal of interest is declared. In modern radar detection, the decision on target presence or absence is often performed automatically, that is, without the visual intervention of the radar operator. When the threshold is a fixed value, the false alarm rate will increase intolerably (i.e., beyond a level that the computer of an automatic detector can handle) as the interference power varies. In this situation, a *constant false alarm rate* (CFAR) algorithm with an adaptive threshold is required to keep the false alarm rate constant.

In a radar receiver, after amplitude detection, the backscattered signal is sampled in range and/or Doppler and a one- or two-dimensional reference window is formed. The detection in radar means existence or nonexistence of a target in the middle cell of a reference window or a cell under test (CUT). The noise and clutter background is estimated by processing the output from neighbouring cells. A well-

known group for noise estimation is mean-level detectors such as cell averaging CFAR (CA) [1]. Unfortunately because of differences in environmental conditions such as change in clutter edge, multiple targets, or jamming the target detection will be corrupted. As solutions for these problems, various CFAR schemes are proposed. A few examples are the greatest of CFAR (GO-CFAR), the smallest of CFAR (SO-CFAR), order statistics CFAR (OS-CFAR), the excision cell-averaging CFAR (EXCA-CFAR), and the excision of the greatest of CFAR (EXGO-CFAR) [2, 3]. Each of these schemes has advantages and disadvantages but none of them shows considerably good performance in all types of environments. However, the processors which use ordering have better performance than mean levels.

Following up on the results in [4, 5] we focus on a new type of switching processor which we call Switching Ordered Statistic CFAR type I (SOS CFAR I). SOS I can be used in many instances, especially in nonhomogeneous environments, and in this paper its performance will be analysed in comparison with conventional CFAR processors in the presence of clutter edge and multiple targets. The suggested

detector is based on comparing cells with scaled CUT to set the cells with the same statistical specifications in two groups. By counting the number of cells in each group and finding the group with more cells, estimation of background noise will be performed [4]. In this paper after describing the SOS CFAR I algorithm in Section 2, mathematical and related probabilities of detection and false alarms are presented in Section 3. In Section 4 the performance and simulation of the SOS I processor in homogeneous and nonhomogeneous environments will be analysed, and in the last section the results are presented.

2. Description of SOS CFAR I Method

In this paper, it is assumed that the CFAR processor's inputs are range samples (range cells) which are received from the square law detector and are saved into a tapped delay line of length $2N + 1$. The $2N$ samples correspond to reference cells X_i surrounding the test cell X_0 . The SOS I detector block diagram is provided in Figure 1. Here, single pulse detection and a Rayleigh fading model are assumed for fluctuating targets corresponding to Swerling I in single pulse processing. For a homogeneous noise pulse clutter level, the in-phase I and quadrature Q input signals are assumed independent and identically distributed (iid) Gaussian random variables with zero mean. Consequently, the output samples of the square law detector are also iid R.V.s with an exponential distribution [6, 7]. Thus, the probability density function (PDF) of the i th cell is

$$f_{X_i}(x_i) = \frac{1}{\lambda} e^{-x_i/\lambda}, \quad x_i \geq 0, \lambda \geq 0, 1 \leq i \leq 2N, \quad (1)$$

in which X_i s are $2N$ window samples (excluding the CUT), and λ is the total background clutter-plus-thermal noise power. If a cell contains thermal noise then $\lambda = \lambda_0 = 2\eta$, and if it consists of clutter then $\lambda = \lambda_c = 2\eta(1 + \sigma_c)$. If a cell consists of multiple (not primary) targets then in (2) $\lambda = \lambda_I = 2\eta(1 + \sigma_I)$. Also σ_c is the ratio of clutter's power to the noise power, and σ_I is the ratio of multiple targets' power to the noise.

Target detection in CUT is carried out by estimating the $2N$ reference window cells that surround it. The PDF of CUT is the same as (1) in the case of thermal noise with $\lambda = \lambda_0 = 2\eta$, and in the case of primary (main) target it will be in the form of (2) with $\lambda = \lambda_s = 2\eta(1 + \sigma_s)$ while σ_s is the ratio of the signal power to the noise power:

$$f_{X_0}(x_0) = \frac{1}{\lambda} e^{-x_0/\lambda}, \quad x_0 \geq 0, \lambda \geq 0. \quad (2)$$

In Figure 1 the SOS I detector first divides reference samples into two S_1 and S_0 groups by comparing them with scaled X_0 with $\alpha < 1$. This is a criterion for finding samples with the same specification. In other words, it implies collecting samples with the same amplitude in one group. Next, estimation of the background noise will be done based on S_0 group or $2N$ samples of reference window. The manner of selection is based on comparing the number of S_0 group samples (n_0) with an integer threshold N_T . If n_0 is more

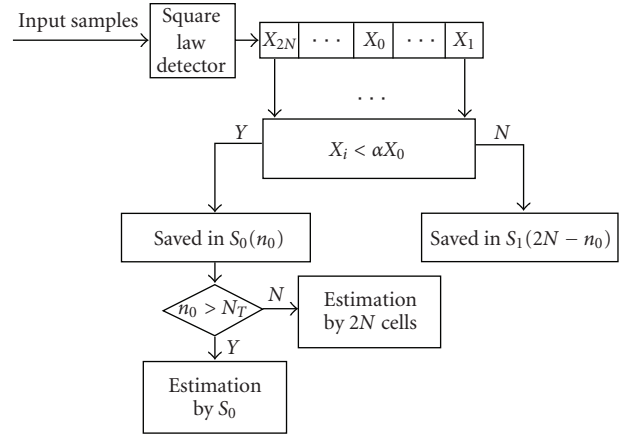


FIGURE 1: Block diagram of SOS CFAR I.

than N_T , background noise will be estimated only with the S_0 samples, but if n_0 is less than N_T , similar to SCFAR [8, 9], all the samples of the reference window are selected. In both cases, background noise estimation is obtained from one of the ordered samples of S_0 or the whole reference window. The range samples in both groups are first ordered according to their magnitudes, and the estimation is taken to be the k th largest sample. Also, N_T is selected based on detector requirements and environment conditions. This process gives the detector an ability to suppress the masking effect caused by interfering targets and clutter edge. Thus, the algorithm will be carried out in the following two steps.

(i) $2N$ cells in the reference window will be compared with the scaled CUT by α ($\alpha < 1$). If a cell value is less than αX_0 it will be saved in group S_0 , otherwise it will be saved in S_1 as in

$$X_i \begin{cases} \geq & S_1 \\ & \alpha X_0 \\ \leq & S_0 \end{cases}, \quad i = 1, 2, \dots, 2N. \quad (3)$$

(ii) If the number of samples saved in group S_0 is n_0 , then the target will exist in CUT according to the following conditions:

$$\text{If } X_0 > \beta_0 X_{(k_0)} = \beta_0 Z_0, \quad \text{when } n_0 > N_T, \quad (4)$$

or

$$\text{If } X_0 > \beta_1 X_{(k_1)} = \beta_1 Z_1, \quad \text{when } n_0 \leq N_T, \quad (5)$$

where β_0 and β_1 are constants for achieving the desired false alarm probability, and N_T is the threshold integer. Also, in (4) and (5) k_0 and k_1 are rounded $(g_0 \times n_0)$ and $(g_1 \times 2N)$ where g_0 and g_1 are parameters between 0 and 1. By adjusting g_0 and g_1 , the order of each selected group is achieved.

Inequalities (4) and (5) mean that SOS CFAR I switches between the sample set S_0 and whole reference, depending on the value of n_0 . For example, if the number of samples which have a value lower than the scaled CUT (and are saved in S_0) is more than the preset threshold N_T , noise level estimation is carried out by ordering the homogeneous saved samples in

S_0 and selecting k_0 th of them; but if the number of samples which have a value lower than the scaled CUT is less than the considered threshold, the noise level estimation is carried out by ordering homogeneous saved samples in the whole reference window and selecting k_1 th of them. This type of processing by the SOS CFAR I processor means selecting an optimized threshold of detection in homogeneous and nonhomogeneous environments.

3. Mathematical Analysis of SOS CFAR I

Switching Ordered Statistics type I CFAR detector incorporates a switching method to estimate the total noise power. Such a detector is specifically tailored to provide good estimates of the noise power with an exponential PDF. In this section we analyse the performance of the SOS CFAR I processor in a homogeneous background as well as in regions of clutter transitions and in multiple target environments. We obtain closed-form performance expressions in each case.

3.1. Homogeneous Environment. Considering the algorithm described in Section 2 and considering the existence of n_0 samples in S_0 and $2N - n_0$ samples in S_1 , and by referring to (4) and (5), the detection probability of SOS CFAR I is

$$P_d = P_{d_0} + P_{d_1}, \quad (6)$$

in which P_{d_0} is the probability of detection when S_0 is selected, and P_{d_1} is the probability of detection when the whole reference window is selected. If we assume presence of a target, H_1 , we have

$$P_d = P(\text{when } S_0 \text{ is selected} \mid H_1) + P(\text{when the whole reference window is selected} \mid H_1), \quad (7)$$

which is equal to

$$P_d = \begin{aligned} & \text{Probability of saving } (N_T + 1 \leq n_0 \leq 2N) \text{ samples in } S_0 \\ & \times \text{Probability that } X_0 \text{ is more than } (\beta_0 Z_0) \\ & + \text{Probability of saving } (0 \leq n_0 \leq N_T) \text{ samples in } S_0 \\ & \times \text{Probability that } X_0 \text{ is more than } (\beta_1 Z_1). \end{aligned} \quad (8)$$

Therefore, the probability of detection when the whole reference window is selected is

$$P_{d_1} = \sum_{n_0=0}^{N_T} P_s \times P(X_0 > \beta_1 Z_1 \mid H_1), \quad (9)$$

where P_s is the probability that there are (based on (8)) exactly n_0 noise samples in S_0 and is equal to

$$P_s = E_{X_0} \left\{ \binom{2N}{n_0} P_0^{n_0}(x_0) (1 - P_0(x_0))^{2N-n_0} \right\}. \quad (10)$$

$P_0(x_0)$ is the probability that a noise sample belongs to S_0 , which is computed as

$$\begin{aligned} P_0(x_0) &= P(X_i < \alpha X_0) \\ &= \int_{x_i=0}^{\alpha x_0} f_{X_i}(x_i) dx_i \\ &= \int_{x_i=0}^{\alpha x_0} \frac{1}{\lambda} e^{-x_i/\lambda} dx_i \\ &= 1 - e^{-(\alpha/\lambda)x_0}. \end{aligned} \quad (11)$$

In (11), it is assumed that the samples are independent, the window samples contain thermal noise and the CUT contains signal [10]. By using (A.1) in Appendix A, (10) becomes

$$P_s = \binom{2N}{n_0} \sum_{i=0}^{n_0} \binom{n_0}{i} \frac{(-1)^i}{\alpha(2N - n_0 + i)(1 + \sigma_s) + 1}. \quad (12)$$

In (9), Z_1 is the random variable obtained from the ordering of $2N$ noise samples in the reference window and selecting k_1 of them as an estimation of noise in the case of $n_0 < N_T$. Now, if as (5) we consider $Z_1 = X_{(k_1)}$, then the PDF of Z_1 is given by [11]

$$f_{k_1}(z_1) = K_1 \binom{2N}{K_1} (e^{-z_1/\lambda})^{2N-K_1} (1 - e^{-z_1/\lambda})^{K_1-1} \frac{1}{\lambda} e^{-z_1/\lambda}. \quad (13)$$

Therefore by referring to Appendix B,

$$\begin{aligned} P(X_0 > \beta_1 Z_1 \mid H_1) &= \frac{(2N)!}{(2N - k_1)!} \sum_{m=0}^{k_1-1} \frac{(-1)^m}{m!(k_1 - m - 1)!} \frac{1}{2N - k_1 + m + \beta_1/(1 + \sigma_s)}. \end{aligned} \quad (14)$$

For calculating the probability of detection when only the samples in S_0 are selected, similar to (9), one has

$$P_{d_0} = \sum_{n_0=N_T+1}^{2N} P_s \times P(X_0 > \beta_0 Z_0 \mid H_1), \quad (15)$$

where P_s was calculated in (10). Therefore same as (14),

$$\begin{aligned} P(X_0 > \beta_0 Z_0 \mid H_1) &= \frac{n_0!}{(n_0 - k_0)!} \sum_{m=0}^{k_0-1} \frac{(-1)^m}{m!(k_0 - m - 1)!} \frac{1}{n_0 - k_0 + m + \beta_0/(1 + \sigma_s)}. \end{aligned} \quad (16)$$

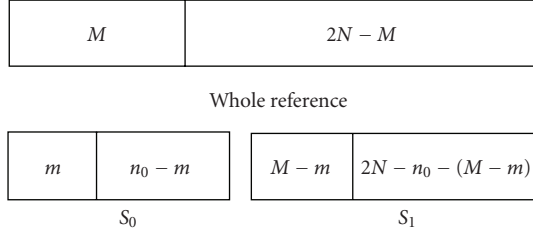


FIGURE 2: Position of noise samples in nonhomogeneous environment.

By using (9) and (15), the result will be

$$\begin{aligned}
& P_d^{\text{SOS I}}(N_T, \alpha, \beta_0, \beta_1, g_0, g_1) \\
&= \sum_{n_0=0}^{N_T} \binom{2N}{n_0} \sum_{i=0}^{n_0} \binom{n_0}{i} \frac{(-1)^i}{\alpha(2N - n_0 + i)(1 + \sigma_s) + 1} \frac{(2N)!}{(2N - k_1)!} \\
&\times \sum_{m=0}^{k_1-1} \frac{(-1)^m}{m!(k_1 - m - 1)!} \frac{1}{2N - k_1 + m + \beta_1 / (1 + \sigma_s)} \\
&+ \sum_{n_0=N_T+1}^{2N} \binom{2N}{n_0} \sum_{i=0}^{n_0} \binom{n_0}{i} \frac{(-1)^i}{\alpha(2N - n_0 + i)(1 + \sigma_s) + 1} \frac{n_0!}{(n_0 - k_0)!} \\
&\times \sum_{m=0}^{k_0-1} \frac{(-1)^m}{m!(k_0 - m - 1)!} \frac{1}{n_0 - k_0 + m + \beta_0 / (1 + \sigma_s)}. \tag{17}
\end{aligned}$$

Also, using (17) and setting σ_s equal to zero, the probability of occurrence of a false alarm can be determined as

$$\begin{aligned}
& P_{\text{fs}}^{\text{SOS I}}(N_T, \alpha, \beta_0, \beta_1, g_0, g_1) \\
&= \sum_{n_0=0}^{N_T} \binom{2N}{n_0} \sum_{i=0}^{n_0} \binom{n_0}{i} \frac{(-1)^i}{\alpha(2N - n_0 + i) + 1} \frac{(2N)!}{(2N - k_1)!} \\
&\times \sum_{m=0}^{k_1-1} \frac{(-1)^m}{m!(k_1 - m - 1)!} \frac{1}{2N - k_1 + m + \beta_1} \\
&+ \sum_{n_0=N_T+1}^{2N} \binom{2N}{n_0} \sum_{i=0}^{n_0} \binom{n_0}{i} \frac{(-1)^i}{\alpha(2N - n_0 + i) + 1} \frac{n_0!}{(n_0 - k_0)!} \\
&\times \sum_{m=0}^{k_0-1} \frac{(-1)^m}{m!(k_0 - m - 1)!} \frac{1}{n_0 - k_0 + m + \beta_0}. \tag{18}
\end{aligned}$$

3.2. Nonhomogeneous Environment. Considering the algorithm described in Section 2, it is assumed that in a reference window with a size equal to $2N$, there are M interfering samples and $2N - M$ thermal noise samples, and S_0 contains m interfering samples and $n_0 - m$ thermal noise samples as illustrated in Figure 2. Assuming that there are n_0 samples in S_0 and $2N - n_0$ samples in S_1 and by referring to (4) and (5), the detection probability in SOS I will be as follows.

For M interfering samples that appear in the CFAR window, the probability that exactly m of them are stored in S_0 is

$$q_{1s} = \binom{M}{m} (P'_0(X_0))^m (1 - P'_0(X_0))^{M-m}. \tag{19}$$

For $2N - M$ thermal noise samples that appear in the CFAR window, the probability that exactly $n_0 - m$ of them are stored in S_0 is [8]

$$q_{0s} = \binom{2N - M}{n_0 - m} (P_0(X_0))^{n_0 - m} (1 - P_0(X_0))^{2N - M - (n_0 - m)}. \tag{20}$$

Therefore, the probability that there are exactly m interfering samples and $n_0 - m$ thermal noise samples in S_0 is

$$\begin{aligned}
Q_s &= q_{0s} \times q_{1s} \\
&= E_{X_0} \left\{ \sum_{m=m_1}^{\min(M, n_0)} \binom{2N - M}{n_0 - m} \binom{M}{m} (P_0(X_0))^{n_0 - m} \right. \\
&\quad \times (1 - P_0(X_0))^{2N - M - (n_0 - m)} \\
&\quad \left. \times P_0^m(X_0) (1 - P'_0(X_0))^{M-m} \right\}. \tag{21}
\end{aligned}$$

Here, the probability of existence of a sample with thermal noise in the S_0 group is determined by (11). Also, the probability of existence of a sample with interference noise in S_0 group according to (3) is

$$\begin{aligned}
P'_0(X_0) &= P(X_i < \alpha X_0) \\
&= \int_{x_i=0}^{\alpha x_0} f_{X_i}(X_i) d_{x_i} \\
&= \int_{x_i=0}^{\alpha x_0} \frac{1}{\lambda_1} e^{-x_i/\lambda_1} d_{x_i} \\
&= 1 - e^{-(\alpha/\lambda_1)x_0}. \tag{22}
\end{aligned}$$

Therefore, with the help of (11) and (22) and referring to Appendix C, (21) will be

$$\begin{aligned}
Q_s &= \sum_{m=m_1}^{\min(M, n_0)} \binom{2N - M}{n_0 - m} \binom{M}{m} \\
&\times \sum_{t=0}^{n_0 - m} \sum_{q=0}^m \frac{(-1)^{t+q} \binom{n_0 - m}{t} \binom{m}{q}}{1 + \mathfrak{N}(1 + \sigma_s)\alpha + (M - m + q)((1 + \sigma_s)/(1 + \sigma_1))\alpha}, \tag{23}
\end{aligned}$$

where \mathfrak{N} denotes $(2N - M - (n_0 - m) + t)$. In the equation above, m_1 is equal to $\max(0, n_0 - 2N + M)$. Now, similar to (6),

the probability of detection in the case of interfering targets will be

$$\begin{aligned}
 P_d^{\text{SOS I}}(N_T, \alpha, \beta_0, \beta_1, g_0, g_1) &= \sum_{n_0=0}^{N_T} Q_s \times P(X_0 > \beta_1 Z_1 | H_1) \\
 &+ \sum_{n_0=N_T+1}^{2N} Q_s \times P(X_0 > \beta_0 Z_0 | H_1). \quad (24)
 \end{aligned}$$

Referring to Appendix D, in the case of a nonhomogeneous environment, one has

$$\begin{aligned}
 P(X_0 > \beta_1 Z_1 | H_1) &= \frac{\beta_1}{(1 + \sigma_s)} \sum_{i=k_1}^{2N} \sum_{L=p_1}^{p_2} \binom{2N-M}{L} \binom{M}{i-L} \\
 &\times \sum_{j_1=0}^L \sum_{j_2=0}^{i-L} \frac{\binom{L}{j_1} \binom{i-L}{j_2} (-1)^{j_1+j_2}}{2N-M-L+\beta_1/(1+\sigma_s)+j_1+\Omega/(1+\sigma_I)}, \quad (25)
 \end{aligned}$$

where Ω denotes $(j_2+M-i+L)$. And with the same procedure in the S_0 group,

$$\begin{aligned}
 P(X_0 > \beta_0 Z_0 | H_1) &= \frac{\beta_0}{(1 + \sigma_s)} \sum_{i=k_0}^{2N} \sum_{L=t_1}^{t_2} \binom{n_0-m}{L} \binom{m}{i-L} \\
 &\times \sum_{j_1=0}^L \sum_{j_2=0}^{i-L} \frac{\binom{L}{j_1} \binom{i-L}{j_2} (-1)^{j_1+j_2}}{n_0-m-L+\beta_0/(1+\sigma_s)+j_1+\mathfrak{C}/(1+\sigma_I)}, \quad (26)
 \end{aligned}$$

where \mathfrak{C} denotes $(j_2+m-i+L)$ and t_1 and t_2 are equal to $\max(0, i-m)$ and $\min(i, n_0-m)$.

Now we investigate the performance of the SOS I processor when the reference window contains a clutter edge. First, consider the special case where CUT is not from the clutter region. Also similar to the multiple targets case, it is assumed that in a reference window with a size equal to $2N$, there are M samples from clutter and $2N-M$ thermal noise samples, and S_0 contains m samples from clutter and n_0-m thermal noise samples. First, the probability of existence of a sample with thermal noise in the S_0 group according to (3) was calculated in (11). Also, the probability of existence of a sample with clutter noise in the S_0 group according to (22) is

$$\begin{aligned}
 P'_0(X_0) &= P(X_i < \alpha X_0) \\
 &= \int_{x_i=0}^{\alpha x_0} f_{X_i}(x_i) dx_i \\
 &= \int_{x_i=0}^{\alpha x_0} \frac{1}{\lambda_C} e^{-x_i/\lambda_C} dx_i \\
 &= 1 - e^{-(\alpha/\lambda_C)x_0}. \quad (27)
 \end{aligned}$$

Here with the help of (23), (24), (25), and (26) and considering $\sigma_s \rightarrow 0$ and $\sigma_I \rightarrow \sigma_C$, the $P_{fa}^{\text{SOS I}}$ will be

$$\begin{aligned}
 P_{fa}^{\text{SOS I}}(N_T, \alpha, \beta_0, \beta_1, g_0, g_1) &= \sum_{n_0=0}^{N_T} Q_s \times P(X_0 > \beta_1 Z_1 | H_0) \\
 &+ \sum_{n_0=N_T+1}^{2N} Q_s \times P(X_0 > \beta_0 Z_0 | H_0), \quad (28)
 \end{aligned}$$

where

$$\begin{aligned}
 P(X_0 > \beta_1 Z_1 | H_0) &= \beta_1 \sum_{i=k_1}^{2N} \sum_{L=p_1}^{p_2} \binom{2N-M}{L} \binom{M}{i-L} \\
 &\times \sum_{j_1=0}^L \sum_{j_2=0}^{i-L} \frac{\binom{L}{j_1} \binom{i-L}{j_2} (-1)^{j_1+j_2}}{2N-M-L+\beta_1+j_1+\Omega/(1+\sigma_C)}, \quad (29)
 \end{aligned}$$

$$\begin{aligned}
 P(X_0 > \beta_0 Z_0 | H_0) &= \beta_0 \sum_{i=k_0}^{2N} \sum_{L=t_1}^{t_2} \binom{n_0-m}{L} \binom{m}{i-L} \\
 &\times \sum_{j_1=0}^L \sum_{j_2=0}^{i-L} \frac{\binom{L}{j_1} \binom{i-L}{j_2} (-1)^{j_1+j_2}}{n_0-m-L+\beta_0+j_1+\mathfrak{C}/(1+\sigma_C)}, \quad (30)
 \end{aligned}$$

where

$$\begin{aligned}
 Q_s &= \sum_{m=m_1}^{\min(M, n_0)} \binom{2N-M}{n_0-m} \binom{M}{m} \\
 &\times \sum_{t=0}^{n_0-m} \sum_{q=0}^m \frac{(-1)^{t+q} \binom{n_0-m}{t} \binom{m}{q}}{1 + \mathfrak{N}\alpha + (M-m+q)/(1+\sigma_C)} \alpha. \quad (31)
 \end{aligned}$$

Now if CUT is from the clutter region, after substituting $\sigma_I = \sigma_s$, β_0 , β_1 , and α by σ_C , $\beta_0/(1+\sigma_C)$, $\beta_1/(1+\sigma_C)$, and $\alpha(1+\sigma_C)$ in (23), (24), (25), (26), and also

$$\begin{aligned}
 P(X_0 > \beta_1 Z_1 | H_0) &= \frac{\beta_1}{1 + \sigma_C} \sum_{i=k_1}^{2N} \sum_{L=p_1}^{p_2} \binom{2N-M}{L} \binom{M}{i-L} \\
 &\times \sum_{j_1=0}^L \sum_{j_2=0}^{i-L} \frac{\binom{L}{j_1} \binom{i-L}{j_2} (-1)^{j_1+j_2}}{2N-M-L+\beta_1/(1+\sigma_C)+j_1+\Omega/(1+\sigma_C)}. \quad (32)
 \end{aligned}$$

$$\begin{aligned}
 P(X_0 > \beta_0 Z_0 | H_0) &= \frac{\beta_0}{1 + \sigma_C} \sum_{i=k_0}^{2N} \sum_{L=t_1}^{t_2} \binom{n_0-m}{L} \binom{m}{i-L} \\
 &\times \sum_{j_1=0}^L \sum_{j_2=0}^{i-L} \frac{\binom{L}{j_1} \binom{i-L}{j_2} (-1)^{j_1+j_2}}{n_0-m-L+\beta_0/(1+\sigma_C)+j_1+\mathfrak{C}/(1+\sigma_C)}, \quad (33)
 \end{aligned}$$

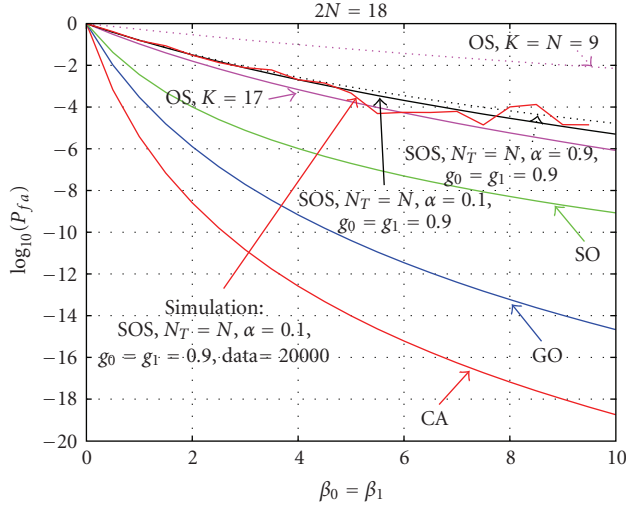


FIGURE 3: False alarm probability of the CA, GO, SO, OS ($k = N = 9$ and $k = 17$), and SOS CFAR I processors for $2N = 18$.

where

$$Q_s = \sum_{m=m_1}^{\min(M, n_0)} \binom{2N - M}{n_0 - m} \binom{M}{m} \times \sum_{t=0}^{n_0 - m} \sum_{q=0}^m \frac{(-1)^{t+q} \binom{n_0 - m}{t} \binom{m}{q}}{1 + \Re(1 + \sigma_C)\alpha + (M - m + q)\alpha}. \quad (34)$$

4. Studying SOS CFAR I in Different Conditions

The performance of the Switching Ordered Statistic CFAR I processor algorithm, according to (11), is a function of β_0 , β_1 , N_T , α , g_0 , and g_1 . These parameters should be tuned such that the SOS CFAR I processor has minimum CFAR loss when operating in a homogeneous environment. Provided that there are only noise samples within the CFAR window, almost all reference samples will be stored to S_0 if the test cell contains a target return signal with substantial SNR. The SOS CFAR I processor then tends to switch to S_0 , and the threshold multiplier β_0 and g_0 are employed with high probability. In order to minimise the CFAR loss in this situation, g_0 (k_0) should be set as close as possible to the order of OS CFAR (corresponding to the false alarm probability of interest). If the test cell contains no target signal, far fewer reference samples are sorted to S_0 . The whole CFAR window and ordering g_1 (k_1) are then employed with high probability. In order to maintain the false alarm rate as that of the OS-CFAR, g_1 (k_1) should also be set as close as possible to k of the OS. Therefore, a reasonable choice is $g_0 = g_1$. Also, for preventing complexity, $\beta_0 = \beta_1$ is considered, although different values for β_0 and β_1 could be considered for the future. The setting of the SOS CFAR I parameters for the case $g_0 = g_1$ and $\beta_0 = \beta_1$ are briefly discussed in this section [8].

In order to detect targets near a clutter edge, the threshold integer should be approximately equal to or smaller than

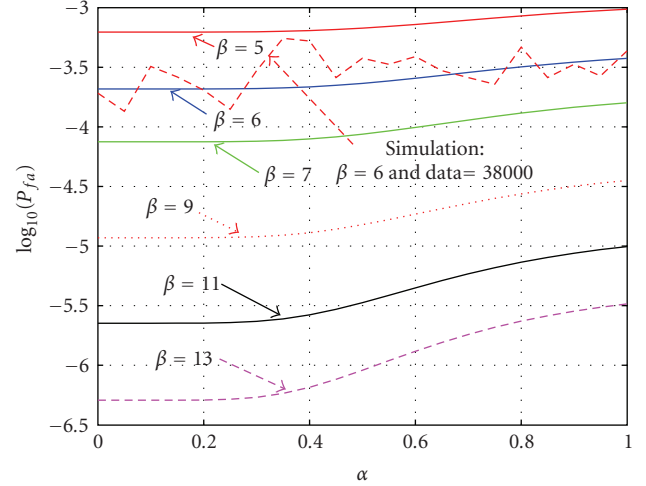


FIGURE 4: False alarm probability of the SOS CFAR I processor for different $\beta_0 = \beta_1$ in terms of α .

the half window size, that is, $N_T \leq N$. An SOS CFAR I processor with a smaller N_T can tolerate a greater number of interference samples, but in a homogeneous environment it suffers from more CFAR loss compared to the CA-CFAR. Here CFAR loss, based on [12, 13], is defined as the additional SNR the CFAR processor requires in order to achieve the same detection probability at a given false alarm rate.

The curves in Figure 3 are the false alarm probabilities (P_{fa}) for a reference window with the size $2N = 18$ for SOS I, with $N_T = N = 9$, $\beta_0 = \beta_1$, $g_0 = g_1 = 0.9$, $\alpha = 0.1$, and $\alpha = 0.9$, CA, GO, SO, and OS (with $k = N = 9$ and $k = 17$) in a homogeneous environment. It is seen that in a homogeneous environment, only P_{fa} of OS with $k = N = 9$ is worse than SOS I. For verifying the theoretical results, the performance of SOS I with $N_T = N = 9$, $\beta_0 = \beta_1$, $g_0 = g_1 = 0.9$, and $\alpha = 0.1$ has been simulated by the Monte Carlo method for about 20 000 data for each point. As it is shown, this curve is compatible with the analytical curve.

In Figure 4, P_{fa} of the performance of SOS I processor in a homogeneous environment with $2N = 18$, $N_T = N$, and $g_0 = g_1 = 0.9$ have been plotted for α and different values of $\beta_0 = \beta_1$. It is clear that by increasing $\beta_0 = \beta_1$, P_{fa} is decreasing. Also, it is seen that in all cases with α larger than 0.4 and with $\beta_0 = \beta_1$, P_{fa} is increasing. Also in Figure 4 the curve for $\beta = 6$ has been simulated with the Monte Carlo method for about 38000 data for each point which is compatible with the analytical curve with the same parameters.

In Figure 5, P_{fa} of the SOS I processor in a homogeneous environment with $2N = 18$, $\beta_0 = \beta_1 = 9$, and $\alpha = 0.5$ and for different N_T values have been plotted for $g_0 = g_1$. It is clear that by increasing N_T , P_{fa} is decreasing.

The probability of the occurrence of a false alarm by the SOS I detector in a homogeneous environment based on different N_T and α values have been plotted for $g_0 = g_1$ in Figure 6; here $\beta_0 = \beta_1 = 9$. It is seen that by decreasing N_T

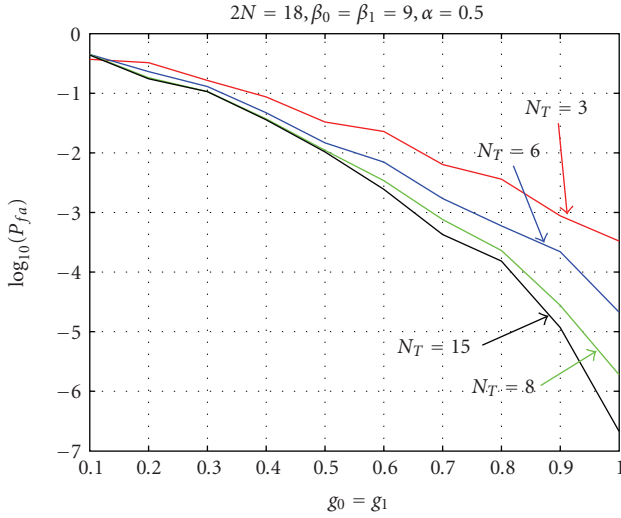


FIGURE 5: Comparison of P_{fa} in SOS I with different N_T in terms of $g_0 = g_1$ in homogeneous environment with $2N = 18$.

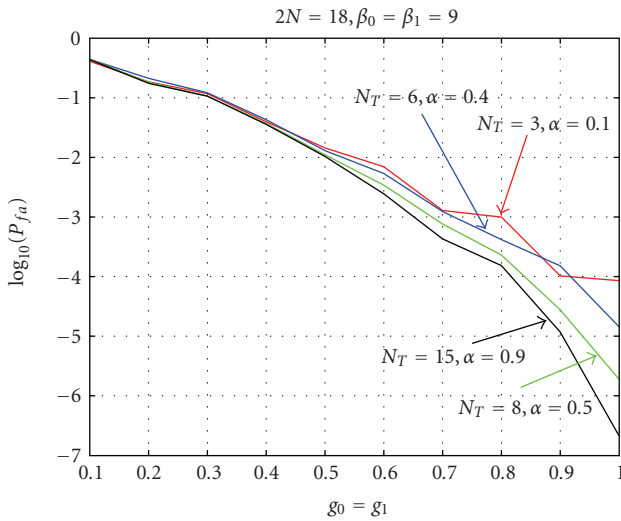


FIGURE 6: Comparison of P_{fa} for SOS I processor in homogeneous environment in terms of $g_0 = g_1$ and some different N_T and α with $2N = 18$.

and increasing α , P_{fa} is decreasing. Also it is clear that by increasing both N_T and α parameters, P_{fa} is again decreasing.

Now, in Figure 7, the detection probability of the SOS I detector in a homogeneous environment in comparison with the optimum detector, CA, GO, SO, and OS (with $k = N = 9$ and $k = 17$) and for $P_{fa} = 10^{-5}$, has been drawn. The optimum detector sets a fixed threshold to determine the presence of a target under the assumption that the total homogeneous noise power is known a priori [10]. Considering the loss detection, it is seen that the SOS I processor with $N_T = N$, $\alpha = 0.1$, $\beta_0 = \beta_1 = 9$, and $g_0 = g_1 = 0.9$ has inherent detection loss in the homogeneous environment which is more than CA and GO but is less than SO and OS ($k = N = 9$). Also, this figure shows that by increasing the order of OS to $k = 17$, its detection loss will be

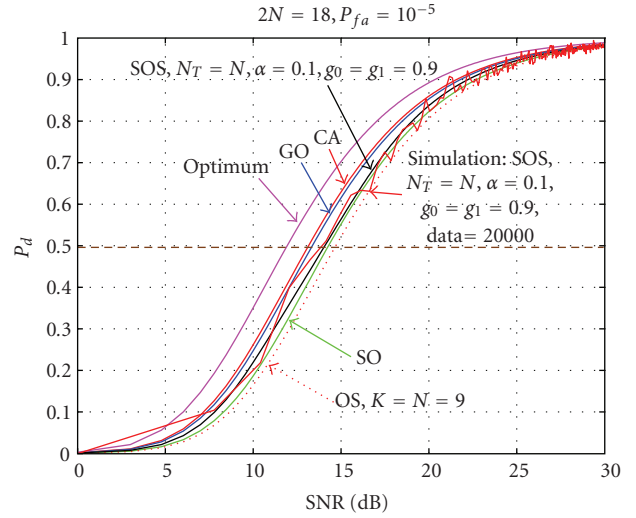


FIGURE 7: Comparison of P_d for CA, GO, SO, OS ($k = N = 9$ and $k = 17$), and SOS I processors ($2N = 18$ and $P_{fa} = 10^{-5}$).

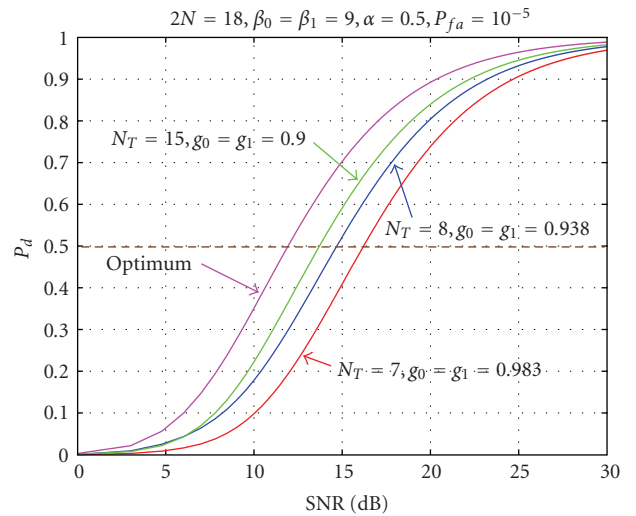


FIGURE 8: Comparison of P_d for SOS I with different N_T and $g_0 = g_1$ ($2N = 18$ and $P_{fa} = 10^{-5}$).

less than the SOS I detector with the mentioned parameters. In fact, with the help of Figure 3, increasing k in OS causes less detection loss but a higher probability of false alarm. For better comparison, the P_d of SOS I is achieved by the Monte Carlo simulation with 20000 data for each point. As Figure 7 shows, the result of the Monte Carlo simulation is the same as the analysis result of Section 3.

In Figure 8 the detection probability of the SOS I detector in a homogeneous environment with different values of N_T , $g_0 = g_1$ and for $P_{fa} = 10^{-5}$, $\beta_0 = \beta_1 = 9$, and $\alpha = 0.5$ has been plotted. The result shows that with greater N_T and smaller $g_0 = g_1$, it has less detection loss.

Next, the performance of the SOS I in presence of clutter edge is analysed. The result in Figure 9 is achieved in the presence of clutter edge with clutter to noise ratio (CNR)

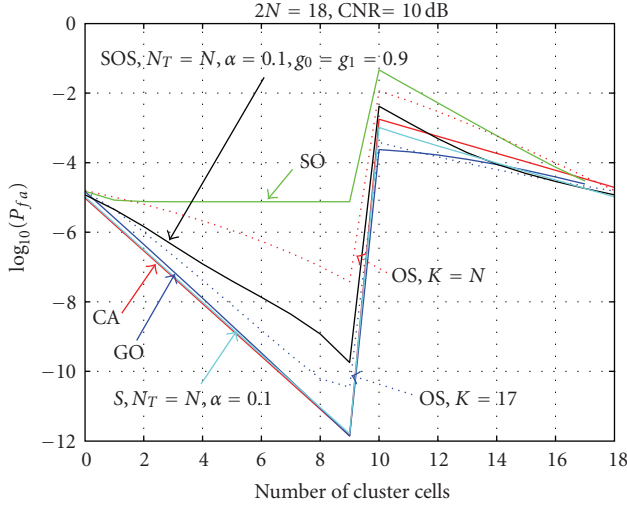


FIGURE 9: Comparison of P_{fa} for CA, GO, SO, OS ($k = N = 9$), and SOS I processors ($2N = 18$ and $CNR = 10$ dB).

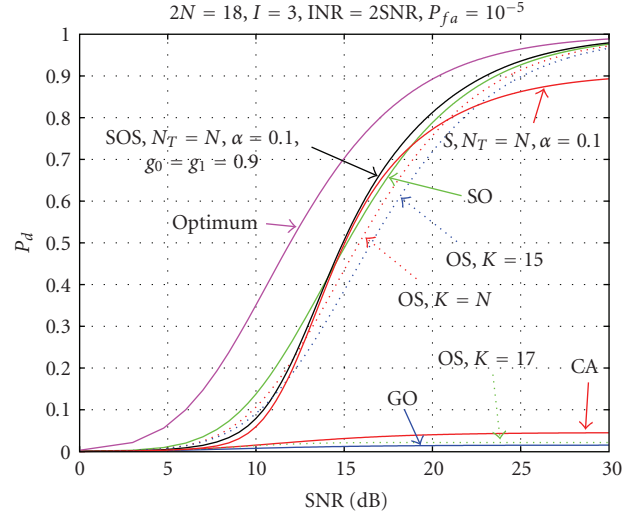


FIGURE 11: Comparison of P_d of SOS I by CA, GO, SO, and OS ($k = N = 9$, $k = 15$, and $k = 17$) in the case of three multiple targets ($INR = 2SNR$) for $P_{fa} = 10^{-5}$ and $2N = 18$.

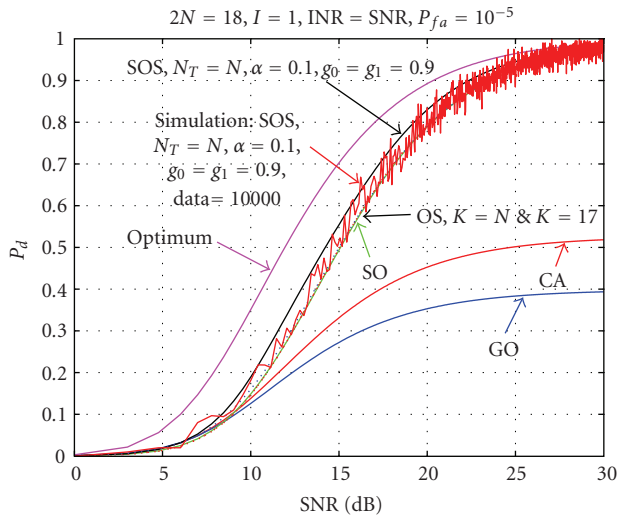


FIGURE 10: Comparison of P_d of SOS I by CA, GO, SO, and OS ($k = N$ and $k = 17$) in the case of one multiple targets ($INR = SNR$) for $P_{fa} = 10^{-5}$ and $2N = 18$.

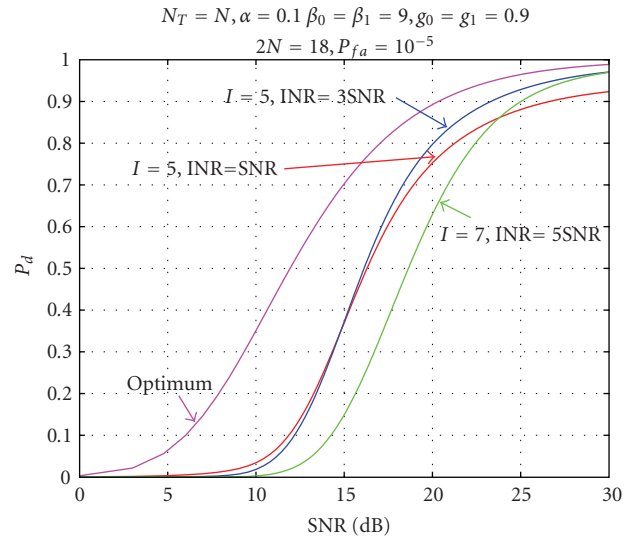


FIGURE 12: Comparison of P_d of SOS I with $N_T = N, \alpha = 0.1, \beta_0 = \beta_1 = 9, g_0 = g_1 = 0.9$ in multiple targets environment for $2N = 18$ and $P_{fa} = 10^{-6}$.

equal to 10 dB, $P_{fa} = 10^{-5}$, and $2N = 18$. It is known that in the presence of clutter edge, the GO processor has the lowest probability of false alarm and is followed by CA, S with $N_T = N, \alpha = 0.1$, OS ($k = 17$), and SOS I with $N_T = N, \alpha = 0.1$, and $g_0 = g_1 = 0.9$. It is clear from Figure 9 that OS ($k = N = 9$) and SO are after it, and also GO has the best performance among all the CFAR processors in the presence of clutter.

The presence of multiple targets is another case in studying the SOS I processor. In Figure 10 one interfering target with interference to noise ratio (INR) equal to SNR and the size of reference window $2N = 18$ for CA, GO, SO, OS ($k = N = 9$ and $k = 17$), and SOS I processors

with considered parameter in this figure and for $P_{fa} = 10^{-5}$ are considered. As the result shows, SOS I has the best performance. By increasing the order of OS, its performance will become constant and will be equal to SO which have less P_d in terms of SOS I with considered parameters. The result of the Monte Carlo simulation for 10000 data for each point is also confirmed by the result of theoretical analysis. It is noticeable that if data numbers for each point increase, the Monte Carlo simulation will have better compatibility with theoretical results.

If there is more than one multiple target, for example, 3 targets with $INR = 2SNR$, Figure 11 can be considered. The other conditions of this figure are the same as the previous

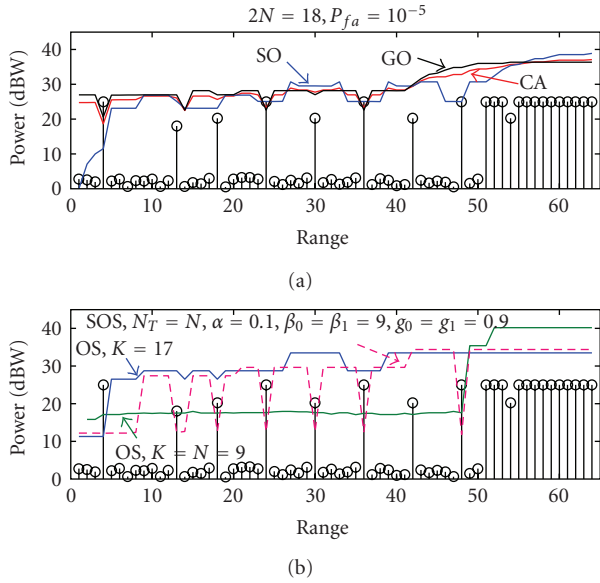


FIGURE 13: Detection thresholds of CA, GO, SO, OS ($k = N = 9$ and 17), and SOS I ($N_T = N$, $\alpha = 0.1$, $\beta_0 = \beta_1 = 9$, and $g_0 = g_1 = 0.9$) in worst case ($2N = 18$ and $P_{fa} = 10^{-5}$).

figure. The results show that SOS I has the best performance and following it, there is the SO detector and then OS with $k = N = 9$. If k increases (e.g., $k = 15$ and $k = 17$ are considered), its performance decreases and even will be worse than the CA detector. Also, Figure 11 shows that the P_d of S with $N_T = N$ and $\alpha = 0.1$ in the case of three multiple targets and $INR = 2SNR$ will be reduced, and its performance is less than SOS I and SO with on SNR of more than 16 dB.

In Figure 12, P_d for the case of five and seven multiple targets and different INR values is shown. The results show that P_d of this detector for $I = 5$ and $INR = 3SNR$ is the highest. If in this case INR decreases, then P_d for an SNR higher than 16 dB will decrease. $I = 7$ and $INR = 5SNR$ have the lowest P_d . In this figure, $2N = 18$, $N_T = N$, $\alpha = 0.5$, $\beta_0 = \beta_1 = 9$, $g_0 = g_1 = 0.9$, and $P_{fa} = 10^{-5}$.

The detection threshold simulation is carried out using Matlab software in the presence of clutter and multiple targets. In Figures 13(a)-13(b), there are 8 targets in ranges 4, 13, 18, 24, 30, 36, 42, and 48 with the SNR values mentioned in the figure. Considering the cases with the reference window's sizes equal to $2N = 18$ and $P_{fa} = 10^{-5}$ and from Figure 13(a), the CA processor can only detect the first target while GO can detect the 1st, 4th, and 6th targets and SO can detect the 1st, 4th, 6th, and 8th targets. From Figure 13(b), OS ($k = N = 9$) can detect all the targets, OS ($k = 17$) detects only the 1st, and SOS with the mentioned parameters detects all the targets except the 7th target.

In Figure 14 the effect of changing SOS I parameters on its detection threshold has been analysed. As seen, SOS I with $\beta_0 = \beta_1 = 149$ has the worst detection level and misses many targets. In general, as Figure 14 shows, if α decreases, the processor has a better estimation level.

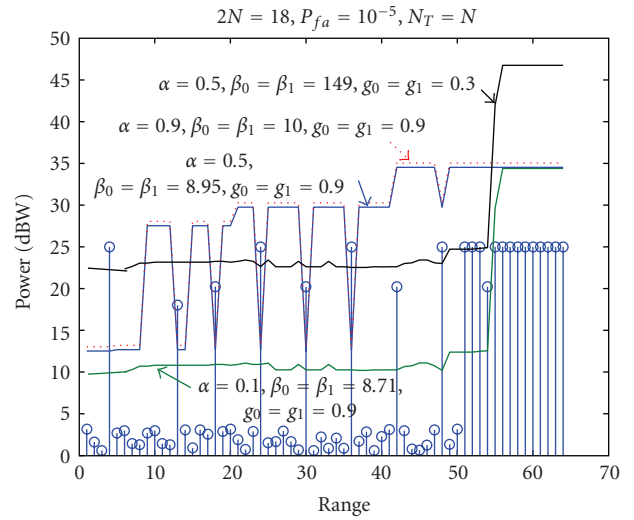


FIGURE 14: Detection thresholds of SOS I with different parameters in worst case for $2N = 18$ and $P_{fa} = 10^{-5}$.

Considering the result of this section, we see that selecting larger $\beta_0 = \beta_1$ and $g_0 = g_1$ versus smaller α and $N_T = N$ causes less P_{fa} in a homogeneous environment and better performance for P_d in homogeneous and nonhomogeneous environments, but results in worse P_{fa} in the presence of clutter edge. Therefore, to have the best performance in various radar environments, suitable parameters, as discussed, should be selected to achieve optimal performance. Also, as the results above and equations in Section 3 show, increasing N_T causes the performance of SOS I to become similar to OS; since, based on (17) and (18), in this case all the samples in the reference window are chosen for background noise estimation (which is based on an Ordered Statistic process), therefore its performance will be near to OS.

5. Conclusions

Considering the results of Section 4 and the comparison with other mean-level processors, the SOS CFAR I processor has a better performance in different radar environments. Also these results show that the SOS CFAR I processor has a good performance with less detection loss not only in homogeneous environments but also in nonhomogeneous ones such as multiple targets and especially in clutter edge. In addition, simulation results confirm that the acquired detection threshold of SOS CFAR I will be optimised if the number of interfering targets is less than the size of the reference window, and it will be a processor which can detect all the targets. However, implementation of the SOS CFAR I is complex compared to conventional processors. Also, these results show that by adjusting the SOS CFAR I parameters, it has better performance than OS in the presence of clutter region and multiple targets and better performance than S-CFAR in the presence of multiple targets with INR more than SNR. It means with using ordered statistic method in S-CFAR, SOS CFAR I will have better performance in the presence of multiple targets environment.

Appendices

A.

By using (11), P_S in (10) can be calculated as follows:

$$\begin{aligned}
P_S &= E_{X_0} \left\{ \binom{2N}{n_0} P_0^{n_0}(x_0) (1 - P_0(x_0))^{2N-n_0} \right\} \\
&= \int_{x_0=0}^{\infty} \binom{2N}{n_0} P_0^{n_0}(x_0) (1 - P_0(x_0))^{2N-n_0} f_{X_0}(x_0) dx_0 \\
&= \int_{x_0=0}^{\infty} \binom{2N}{n_0} (1 - e^{-(\alpha/\lambda)x_0})^{n_0} (e^{-(\alpha/\lambda)x_0})^{2N-n_0} \frac{1}{\lambda_S} e^{-x_0/\lambda_S} dx_0 \\
&= \frac{1}{\lambda_S} \binom{2N}{n_0} \int_{x_0=0}^{\infty} e^{-(\alpha/\lambda)(2N-n_0)+1/\lambda_S)x_0} \\
&\quad \times \sum_{i=0}^{n_0} \binom{n_0}{i} (-e^{-(\alpha/\lambda)x_0})^i dx_0 \\
&= \frac{1}{\lambda_S} \binom{2N}{n_0} \sum_{i=0}^{n_0} \binom{n_0}{i} (-1)^i \int_{x_0=0}^{\infty} e^{-(\alpha/\lambda)(2N-n_0+i)+1/\lambda_S)x_0} dx_0 \\
&= \frac{1}{\lambda_S} \binom{2N}{n_0} \sum_{i=0}^{n_0} \binom{n_0}{i} (-1)^i \frac{1}{(\alpha/\lambda)(2N-n_0+i)+1/\lambda_S} \\
&= \binom{2N}{n_0} \sum_{i=0}^{n_0} \binom{n_0}{i} \frac{(-1)^i}{\alpha(2N-n_0+i)(1+\sigma_s)+1}.
\end{aligned} \tag{A.1}$$

B.

By employing (12), for calculating $P(X_0 > \beta_1 Z_1 \mid H_1)$, one has [8]

$$\begin{aligned}
P(x_0 > \beta_1 Z_1 \mid H_1) &= \int_{z_1=0}^{\infty} \int_{x_0=\beta_1 z_1}^{\infty} \frac{1}{\lambda(1+\sigma_s)} e^{-x_0/\lambda(1+\sigma_s)} dx_0 f_{K_1}(z_1) dz_1 \\
&= \int_{z_1=0}^{\infty} e^{-(\beta_1/\lambda)(1+\sigma_s)z_1} f_{K_1}(z_1) dz_1 \\
&= M_{Z_1} \left(\frac{\beta_1}{\lambda(1+\sigma_s)} \right),
\end{aligned} \tag{B.1}$$

where $M_{Z_1}(u)$ is the moment generating function of Z_1 and gives

$$\begin{aligned}
M_{Z_1}(u) &= \int_{z_1=0}^{\infty} e^{-uz_1} K_1 \binom{2N}{K_1} (e^{-z_1/\lambda})^{2N-K_1} \\
&\quad \times (1 - e^{-z_1/\lambda})^{K_1-1} \frac{1}{\lambda} e^{-z_1/\lambda} dz \\
&= K_1 \binom{2N}{K_1} \frac{1}{\lambda} \int_{z_1=0}^{\infty} (1 - e^{-z_1/\lambda})^{K_1-1} \\
&\quad \times e^{-(u+(2N-K_1+1)/\lambda)z_1} dz_1
\end{aligned}$$

$$\begin{aligned}
&= K_1 \binom{2N}{K_1} \frac{1}{\lambda} \int_{z_1=0}^{\infty} \sum_{m=0}^{K_1-1} \binom{K_1-1}{m} (-e^{-z_1/\lambda})^m \\
&\quad \times e^{-(u+(2N-K_1+1)/\lambda)z_1} dz_1 \\
&= K_1 \binom{2N}{K_1} \frac{1}{\lambda} \sum_{m=0}^{K_1-1} \binom{K_1-1}{m} (-1)^m \\
&\quad \times \int_{z_1=0}^{\infty} e^{-(u+(2N-K_1+1)/\lambda+m/\lambda)z_1} dz_1 \\
&= K_1 \binom{2N}{K_1} \frac{1}{\lambda} \sum_{m=0}^{K_1-1} \binom{K_1-1}{m} (-1)^m \\
&\quad \times \frac{1}{u + (2N - K_1 + 1)/\lambda + m/\lambda}.
\end{aligned} \tag{B.2}$$

Therefore by setting $u = \beta_1/\lambda(1 + \sigma_s)$ in (B.2),

$$\begin{aligned}
P(X_0 > \beta_1 Z_1 \mid H_1) &= \frac{(2N)!}{(2N - K_1)!} \\
&\quad \times \sum_{m=0}^{K_1-1} \frac{(-1)^m}{m!(K_1 - m - 1)!} \frac{1}{\beta_1/(1 + \sigma_s) + 2N - K_1 + 1 + m}.
\end{aligned} \tag{B.3}$$

C.

Referring to (11) and (22), Q_S in (21) can be calculated in the following manner:

$$\begin{aligned}
Q_S &= E_{X_0} \left\{ \sum_{m=m_1}^{\min(M, n_0)} \binom{2N-M}{n_0-m} \binom{M}{m} (P_0(X_0))^{n_0-m} \right. \\
&\quad \times (1 - P_0(X_0))^{2N-M-(n_0-m)} \\
&\quad \times (P'_0(X_0))^{n_0-m} (1 - P'_0(X_0))^{M-m} \left. \right\} \\
&= \int_{X_0=0}^{\infty} \sum_{m=m_1}^{\min(M, n_0)} \binom{2N-M}{n_0-m} \binom{M}{m} (P_0(X_0))^{n_0-m} \\
&\quad \times (1 - P_0(X_0))^{2N-M-(n_0-m)} \\
&\quad \times (P'_0(X_0))^m (1 - P'_0(X_0))^{M-m} f_{X_0}(x_0) dx_0 \\
&= \sum_{m=m_1}^{\min(M, n_0)} \binom{2N-M}{n_0-m} \binom{M}{m} \\
&\quad \times \int_{x_0=0}^{\infty} (1 - e^{-(\alpha/\lambda)x_0})^{n_0-m} (e^{-(\alpha/\lambda)x_0})^{2N-M-(n_0-m)} \\
&\quad \times (1 - ve^{-(\alpha/\lambda_1)x_0})^m (e^{-(\alpha/\lambda_1)x_0})^{M-m} \frac{1}{\lambda_S} e^{-x_0/\lambda_S} dx_0
\end{aligned}$$

$$\begin{aligned}
 &= \sum_{m=m_1}^{\min(M, n_0)} \binom{2N-M}{n_0-m} \binom{M}{m} \\
 &\quad \times \int_{x_0=0}^{\infty} \frac{1}{\lambda_S} e^{-((2N-M-(n_0-m))/\lambda)\alpha + ((M-m)/\lambda_i)\alpha + 1/\lambda_S)x_0} \\
 &\quad \times \sum_{t=0}^{n_0-m} \binom{n_0-m}{t} (-1)^t e^{-((\alpha/\lambda)t)x_0} \\
 &\quad \times \sum_{q=0}^m \binom{m}{q} (-1)^q e^{-((\alpha/\lambda_i)q)x_0} dx_0 \\
 &= \sum_{m=m_1}^{\min(M, n_0)} \binom{2N-M}{n_0-m} \binom{M}{m} \\
 &\quad \times \frac{1}{\lambda_S} \sum_{t=0}^{n_0-m} \sum_{q=0}^m \binom{n_0-m}{t} \binom{m}{q} (-1)^{t+q} \\
 &\quad \times \int_{x_0=0}^{\infty} e^{-((2N-M-(n_0-m))/\lambda)\alpha + ((M-m)/\lambda_i)\alpha + 1/\lambda_S + (t/\lambda)\alpha + (q/\lambda_i)\alpha)x_0} dx_0 \\
 &= \sum_{m=m_1}^{\min(M, n_0)} \binom{2N-M}{n_0-m} \binom{M}{m} \\
 &\quad \times \sum_{t=0}^{n_0-m} \sum_{q=0}^m \frac{(-1)^{t+q} \binom{n_0-m}{t} \binom{m}{q}}{1 + \mathfrak{N}(1+\sigma_s)\alpha + (M-m+q)((1+\sigma_s)/(1+\sigma_i))\alpha}.
 \end{aligned} \tag{C.1}$$

D.

For calculating $P(X_0 > \beta_1 Z_1 | H_1)$, one has

$$\begin{aligned}
 &P(X_0 > \beta_1 Z_1 | H_1) \\
 &= \int_{z_1=0}^{\infty} \int_{x_0=\beta_1 z_1}^{\infty} \frac{1}{\lambda(1+\sigma_s)} e^{-x_0/\lambda(1+\sigma_s)} dx_0 f_{K_1}(z_1) dz_1 \\
 &= \int_{z_1=0}^{\infty} e^{-(\beta_1/\lambda(1+\sigma_s))z_1} f_{K_1}(z_1) dz_1 \\
 &= \frac{\beta_1}{\lambda(1+\sigma_s)} \int_{z_1=0}^{\infty} F_{K_1}(z_1) e^{-(\beta_1/\lambda(1+\sigma_s))z_1} dz_1.
 \end{aligned} \tag{D.1}$$

Here, $F_{K_1}(z_1)$ is the CDF of $f_{K_1}(z_1)$ in the case of M interfering samples in the reference window and is equal to [10]

$$\begin{aligned}
 F_{K_1}(z_1) &= \sum_{i=k_1}^{2N} \sum_{L=\max(0, i-M)}^{\min(i, 2N-M)} \binom{2N-M}{L} \binom{M}{i-L} \\
 &\quad \times e^{-(2N-M-L)z_1} (1 - e^{-z_1})^L \\
 &\quad \times e^{-(M-i+L)(z_1/(1+\sigma_i))} (1 - e^{-z_1/(1+\sigma_i)})^{i-L}.
 \end{aligned} \tag{D.2}$$

Therefore, (D.1) will be

$$\begin{aligned}
 &P(X_0 > \beta_1 Z_1 | H_1) \\
 &= \frac{\beta_1}{(1+\sigma_s)} \sum_{i=k_1}^{2N} \sum_{L=p_1}^{p_2} \binom{2N-M}{L} \binom{M}{i-L} \\
 &\quad \times \sum_{j_1=0}^L \sum_{j_2=0}^{i-L} \frac{\binom{L}{j_1} \binom{i-L}{j_2} (-1)^{j_1+j_2}}{2N-M-L+\beta_1/(1+\sigma_s)+j_1+\mathfrak{N}/(1+\sigma_i)},
 \end{aligned} \tag{D.3}$$

where p_1 and p_2 are $\max(0, i-M)$ and $\min(i, 2N-M)$.

References

- [1] H. Rohling, "Some radar topics: waveform design, range CFAR and target recognition," in *Advances in Sensing with Security Applications*, vol. 2 of NATO Security through Science Series, pp. 293–322, Springer, Amsterdam, The Netherlands, 2006.
- [2] Y. I. Han and T. Kim, "Performance of excision GO-CFAR detectors in nonhomogeneous environments," *IEEE Proceedings: Radar, Sonar and Navigation*, vol. 143, no. 2, pp. 105–111, 1996.
- [3] H. Goldman, "Performance of the excision CFAR detector in the presence of interferers," *IEEE Proceedings, Part F: Radar and Signal Processing*, vol. 137, no. 3, pp. 163–171, 1990.
- [4] S. Erfanian and V. T. Vakili, "Analysis of improved switching CFAR in the presence of clutter and multiple targets," in *Proceedings of the 50th International Symposium ELMAR-2008*, vol. 1, pp. 257–260, Zadar, Croatia, September 2008.
- [5] S. Erfanian and V. T. Vakili, "Optimum detection of multiple targets by improved switching CFAR processor," in *Proceedings of the 14th Asia-Pacific Conference on Communications (APCC '08)*, pp. 1–5, Tokyo, Japan, October 2008.
- [6] M. Barkat, *Signal Detection and Estimation*, Artech House, Boston, Mass, USA, 2005.
- [7] S. Erfanian and S. Faramarzi, "Performance of excision switching-CFAR in K distributed sea clutter," in *Proceedings of the 14th Asia-Pacific Conference on Communications (APCC '08)*, pp. 1–4, Tokyo, Japan, October 2008.
- [8] T.-T. Van Cao, "A CFAR thresholding approach based on test cell statistics," in *Proceedings of IEEE National Radar Conference*, pp. 349–354, Philadelphia, Pa, USA, April 2004.
- [9] T.-T. Van Cao, "A CFAR algorithm for radar detection under severe interference," in *Proceedings of the Intelligent Sensors, Sensor Networks and Information Processing Conference (ISSNIP '04)*, pp. 167–172, Melbourne, Canada, December 2004.
- [10] P. P. Gandhi and S. A. Kassam, "Analysis of CFAR processors in homogeneous background," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 24, no. 4, pp. 427–445, 1988.
- [11] R. Peihong, D. Qingfen, and C. Yuanhen, "The research on the detection performance of OS-CFAR and its modified methods," in *Proceedings of CIE International Conference of Radar (ICR '96)*, pp. 422–425, Beijing, China, October 1996.
- [12] H. Rohling, "Radar CFAR thresholding in clutter and multiple target situations," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 19, no. 4, pp. 608–621, 1983.
- [13] V. G. Hansen and J. H. Sawyers, "Detectability loss due to "greatest of" selection in a cell-averaging CFAR," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 16, no. 1, pp. 115–118, 1980.