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Research Article

Joint Angle and Frequency Estimation Using Multiple-Delay Output Based on ESPRIT

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This paper presents a novel ESPRIT algorithm-based joint angle and frequency estimation using multiple-delay output (MDJAFE). The algorithm can estimate the joint angles and frequencies, since the use of multiple output makes the estimation accuracy greatly improved when compared with a conventional algorithm. The useful behavior of the proposed algorithm is verified by simulations.

1. Introduction

Antenna array has been used in many fields such as radar, sonar, electron reconnaissance and seismic data processing. The direction-of-arrival- (DOA-) estimation of signals impinging on an array of sensors is a fundamental problem in array processing [1–5]. Angle estimation and frequency estimation [6, 7] are two key problems in the signal processing field. The problem of joint DOA and frequency estimation arises in the applications of radar, wireless communications and electron reconnaissance. For example, these parameters can be applied to locate the radars and to locate pilot tones in electron reconnaissance systems [8]. Furthermore, a precise estimation of these parameters is helpful to attain a better pulse descriptor word (PDW) and thus enhances the system performance. Optimal techniques based on maximum likelihood [9] are often applicable but might be computationally prohibitive. Some ESPRIT-based joint angle and frequency estimation methods have been proposed in [10–14]. Zoltowski and Mathew [10] discuss this problem in the context of radar applications. Pro-ESPRIT is proposed to estimate angle and frequency. Haardt and Nossek [11] discuss the problem in the context of mobile communications for space division multiple access applications. Their method is based on Unitary-ESPRIT, which involves a certain transformation of the data to real valued matrices. Multi resolution ESPRIT is used for joint angle frequency estimation in [12]. ESPRIT method is used for frequency and angle estimation under uniform circular array in [13, 14]. References [15, 16] proposed the trilinear decomposition method for joint angle and frequency estimation method. The other joint angle and frequency estimation method is proposed in [17–24].

This paper uses multiple-delay output, so as to achieve the purpose of improving estimation accuracy. This algorithm has the improved performance compared with conventional method. The proposed algorithm is applicable to uniform linear array.

Note 1. We denote by $(\cdot)^T$ the matrix transpose, and by $(\cdot)^H$ the matrix conjugate transpose. The notation $(\cdot)^+$ refers to the Moore-Penrose inverse (pseudoinverse).

2. The Data Model

There are K sources to reach uniform linear array with M elements. Suppose that the ith source has a carrier frequency f_i. The signal received at the mth antenna is

$$x_m(t) = \sum_{i=1}^{K} e^{j2\pi(m-1)df_i \sin(\theta_i)/c} s_i(t), \tag{1}$$

where θ_i is direction of arrival (DOA) of the ith signal, and d is array spacing. $s_i(t)$ is the narrow-band signal of the ith source. In order to estimate frequency, we add $P-1$ delayed

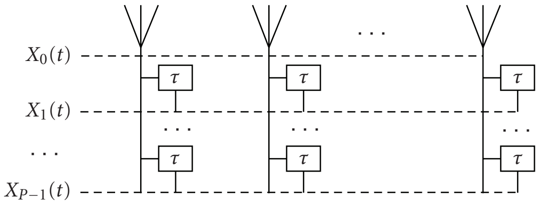


FIGURE 1: The received signal with delayed output.

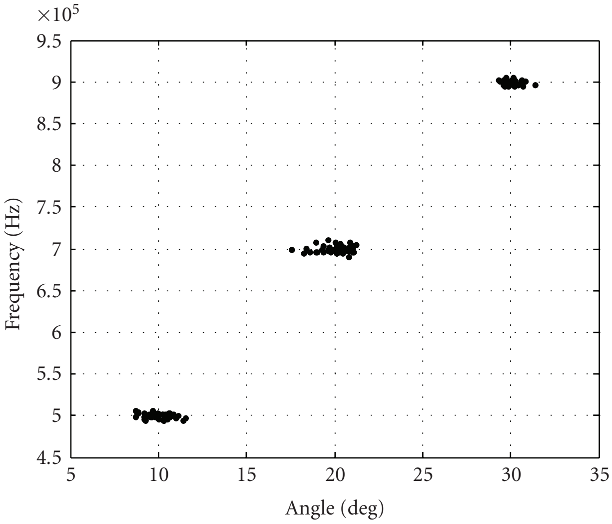


FIGURE 2: Angle-frequency scatter, SNR = 15 dB.

outputs for the received signal of array antenna, as shown in Figure 1. We suppose that $0 < (P-1)\tau < 1/\max(f_i)$.

The delayed signal for (1) with delay τ is

$$\begin{aligned} x_m(t-\tau) &= \sum_{i=1}^{K} \mathrm{e}^{j2\pi(m-1)df_i\sin(\theta_i)/c}s_i(t-\tau) \\ &= \sum_{i=1}^{K} \mathrm{e}^{j2\pi(m-1)df_i\sin(\theta_i)/c}s_i(t)\,\mathrm{e}^{-j2\pi f_i\tau}, \end{aligned} \tag{2}$$

where c is velocity of light. We assume that channel state information is constant for N symbols. The received signal of array antennas without delay can be denoted as

$$X_0 = AS, \tag{3}$$

where the source matrix $\mathbf{S}$ and the direction matrix $\mathbf{A}$ are shown as follows

$$\mathbf{S} = \begin{bmatrix} \mathbf{s}_1 & \mathbf{s}_2 & \cdots & \mathbf{s}_K \end{bmatrix}^T \in \mathbb{C}^{K\times N}, \tag{4}$$

$$A = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ \mathrm{e}^{-j\alpha_1} & \mathrm{e}^{-j\alpha_2} & \cdots & \mathrm{e}^{-j\alpha_K} \\ \vdots & \vdots & \ddots & \vdots \\ \mathrm{e}^{-j(M-1)\alpha_1} & \mathrm{e}^{-j(M-1)\alpha_2} & \cdots & \mathrm{e}^{-j(M-1)\alpha_K} \end{bmatrix}, \tag{5}$$

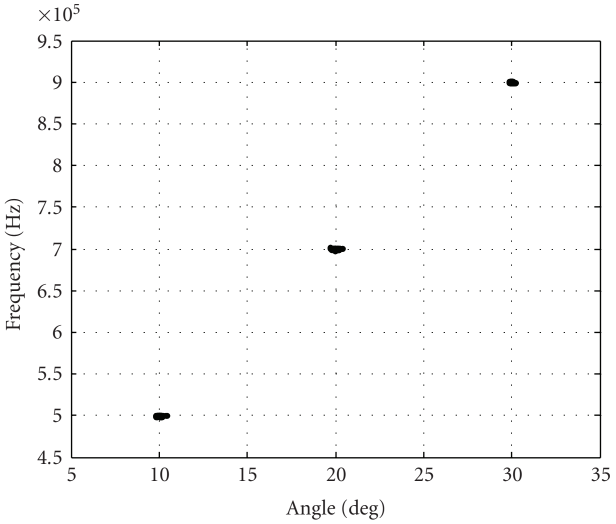


FIGURE 3: Angle-frequency scatter, SNR = 30 dB.

where $\alpha_k = 2\pi df_k\sin(\theta_k)/c$, $k = 1,2,\ldots,K$. According to (5), we define

$$A = \begin{bmatrix} A_1 \\ a_M \end{bmatrix} = \begin{bmatrix} a_1 \\ A_2 \end{bmatrix}, \tag{6}$$

where A_1 is the first $M-1$ rows of A, a_M is the last row of A. A_2 is the second to the Mth row of A, a_1 is the first row of A. The delayed signal for (2) with τ can be denoted as

$$X_1 = A\Phi S, \tag{7}$$

where

$$\mathbf{\Phi} = \operatorname{diag}\left\{e^{-j\beta_1}, e^{-j\beta_2}, \ldots, e^{-j\beta_K}\right\}, \tag{8}$$

where $\beta_k = 2\pi f_k\tau$, $k = 1,2,\ldots,K$.

The delayed signal for (2) with $p\tau$ can be denoted as

$$X_p = A\Phi^p S, \quad p = 0,1,\ldots,P-1. \tag{9}$$

According to (3), (7), and (9), we define

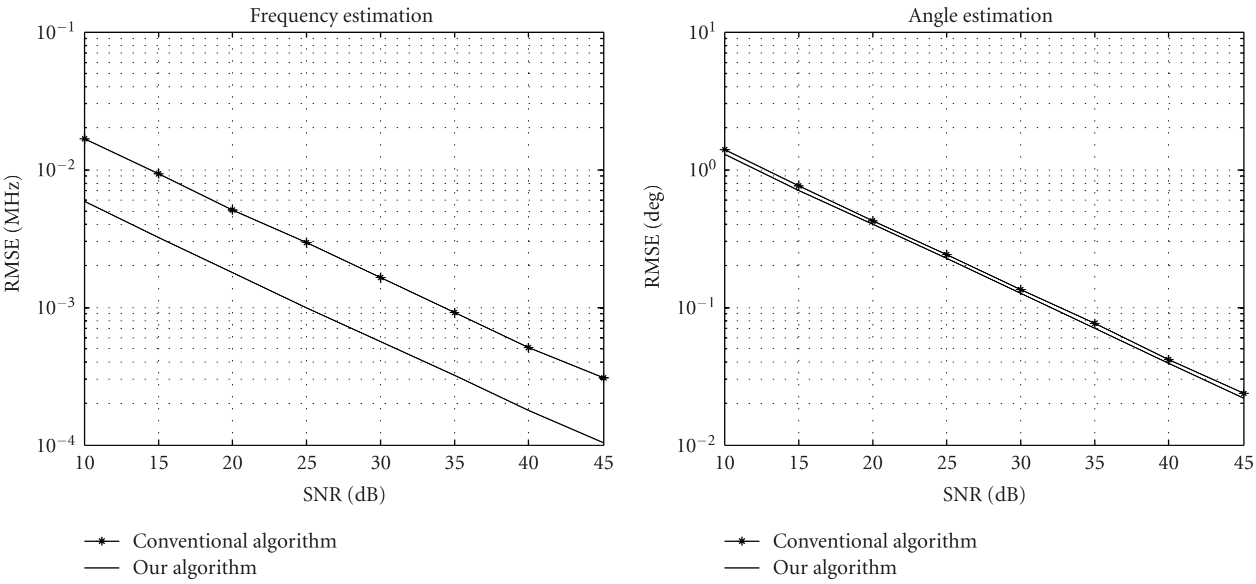
$$X = \begin{bmatrix} X_0 \\ X_1 \\ X_{P-1} \end{bmatrix} = \begin{bmatrix} A \\ A\Phi \\ A\Phi^{P-1} \end{bmatrix} S. \tag{10}$$

3. Joint Angle and Frequency Estimation

We can use received signal to attain the direction matrix $\mathbf{A}$ and the delay matrix $\mathbf{\Phi}$, and then estimate angle and frequency. The covariance matrix of the received signal can be reconstructed via $R_x = XX^H$. Using eigenvalue decomposition of R_x, we can get the signal subspace E_s. In the free-noise case, E_s can be denoted as

$$E_s = \begin{bmatrix} A \\ A\Phi \\ A\Phi^{P-1} \end{bmatrix} T, \tag{11}$$

where $\mathbf{T}$ is a $\mathbf{K}\times\mathbf{K}$ full-rank matrix.



(a)

(b)

FIGURE 4: Angle-frequency estimation performance comparison.

3.1. Frequency Estimation. According to (11), we define E_1 and E_2

$$E_1 = \begin{bmatrix} A \\ A\Phi \\ A\Phi^{P-2} \end{bmatrix} T, \qquad E_2 = \begin{bmatrix} A\Phi \\ A\Phi^{2} \\ A\Phi^{P-1} \end{bmatrix} T. \tag{12}$$

According to (12),

$$E_2 = \begin{bmatrix} A\Phi \\ A\Phi^{2} \\ A\Phi^{P-1} \end{bmatrix} T = \begin{bmatrix} A \\ A\Phi \\ A\Phi^{P-2} \end{bmatrix} TT^{-1}\Phi T = E_1 T^{-1}\Phi T. \tag{13}$$

Let $\Psi = T^{-1}\Phi T$, so $\Psi = E_1{}^{+}E_2$. Because $\mathbf{\Psi}$ has the same eigenvalues as $\mathbf{\Phi}$, we use eigenvalue decomposition on $\mathbf{\Psi}$ to get β_k, $k = 1,2,\ldots,K$ and then estimate frequency f_k, $k = 1,2,\ldots,K$. Using eigenvalue decomposition of Ψ, we can get the eigenvalues λ_k, $k = 1,2,\ldots,K$.

$$\widehat{f}_k = \frac{1}{2\pi\tau}\operatorname{angle}(\lambda_k), \tag{14}$$

where angle $(\cdot)$ denotes taking the phase angles.

3.2. Angle Estimation. According to (11), take first to $(M-1)$th row of E_s to get E_3, which is shown as follows

$$E_3 = A_1 T \in \mathbb{C}^{(M-1)\times K}. \tag{15}$$

Take second to Mth row of E_s to get E_4,

$$E_4 = A_2 T \in \mathbb{C}^{(M-1)\times K}. \tag{16}$$

We can get

$$E_4 = E_3 T^{-1}\phi T, \tag{17}$$

where

$$\varphi = \operatorname{diag}\left\{e^{-j\alpha_1}, e^{-j\alpha_2}, \ldots, e^{-j\alpha_K}\right\} \in \mathbb{C}^{K\times K}, \tag{18}$$

where $\alpha_k = 2\pi d f_k \sin\theta_k / c$, $k = 1,2,\ldots,K$.

Let $\Omega = T^{-1}\varphi T$, so $\Omega = E_3{}^{+}E_4$. Because Ω has the same eigenvalues as φ, we use eigenvalue decomposition on Ω to get α_k, $k = 1,2,\ldots,K$. And then estimate θ_k, $k = 1,2,\ldots,K$. Using eigenvalue decomposition of Ω, we can get the eigenvalues ξ_k, $k = 1,2,\ldots,K$.

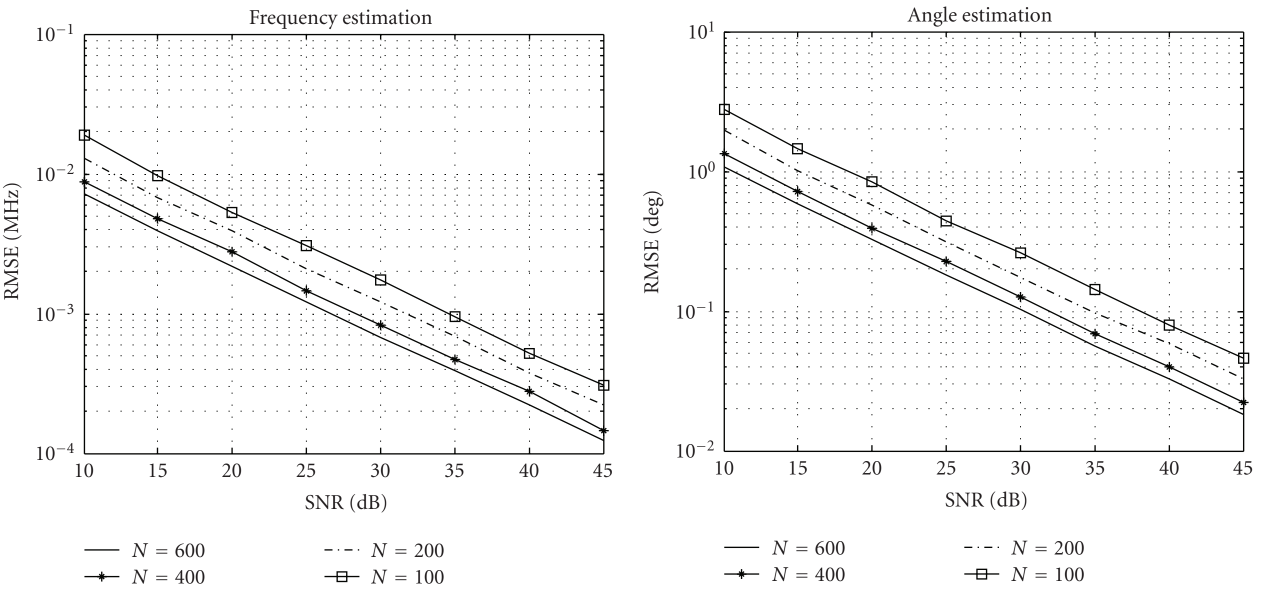
$$\widehat{\theta}_k = \arcsin\left(\frac{c}{2\pi f_k d}\operatorname{angle}(\xi_k)\right). \tag{19}$$

In contrast to ESPRIT algorithm [13], this algorithm has a high computational load, which is usually dominated by formation of the covariance matrix, matrix inversion and calculation of EVD. The major computational complexity of this algorithm is $O(M^2P^2N + M^3P^3 + K^3)$, while ESPRIT requires $O(M^2N + M^3 + K^3)$, where M, P, N, and K are the number of antennas, delays, snapshots, and sources.

4. Simulation Results

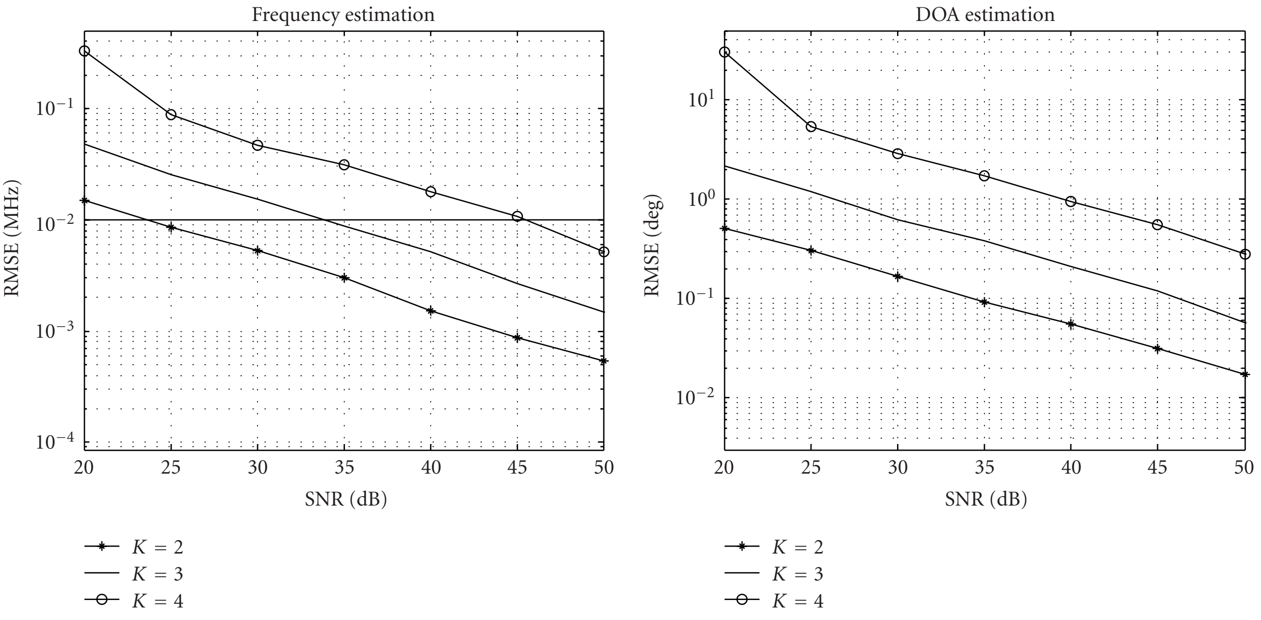
We present Monte Carlo simulations that are used to assess the angle and frequency estimation performance of MDJAFE algorithm. The number of Monte Carlo trials is 1000. Note that M is the number of antennas; P is the number of the delays; N is the number of snapshots; K is the number of the sources.

Define RMSE $= \sqrt{(1/1000)\sum_{m=1}^{1000}[a_m - a_0]^2}$, where a_m is the estimated angle/frequency, and a_0 is the perfect angle/frequency.



(a) (b)

Figure 5: Angle-frequency estimation with different snapshot N.



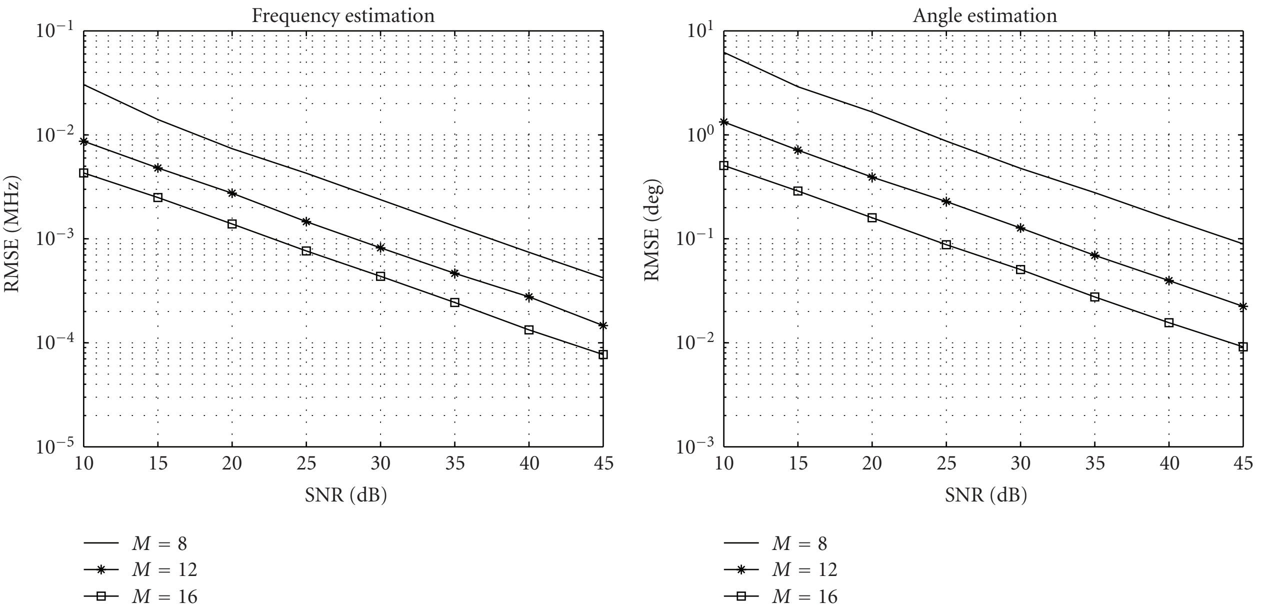
(a) (b)

Figure 6: Angle-frequency estimation with different sources.

Simulation 1. The performance of this proposed algorithm is investigated. $M = 12$, $K = 3$, $P = 3$, and $N = 400$ in this simulation. Their DOAs are 10°, 20° and 30°), and their carrier frequencies are 500 kHZ, 700 kHZ and 900 kHZ. Figure 2 shows the performance of this proposed algorithm with SNR = 15 dB, 30 dB. From Figures 2 and 3 we find that this proposed algorithm works well.

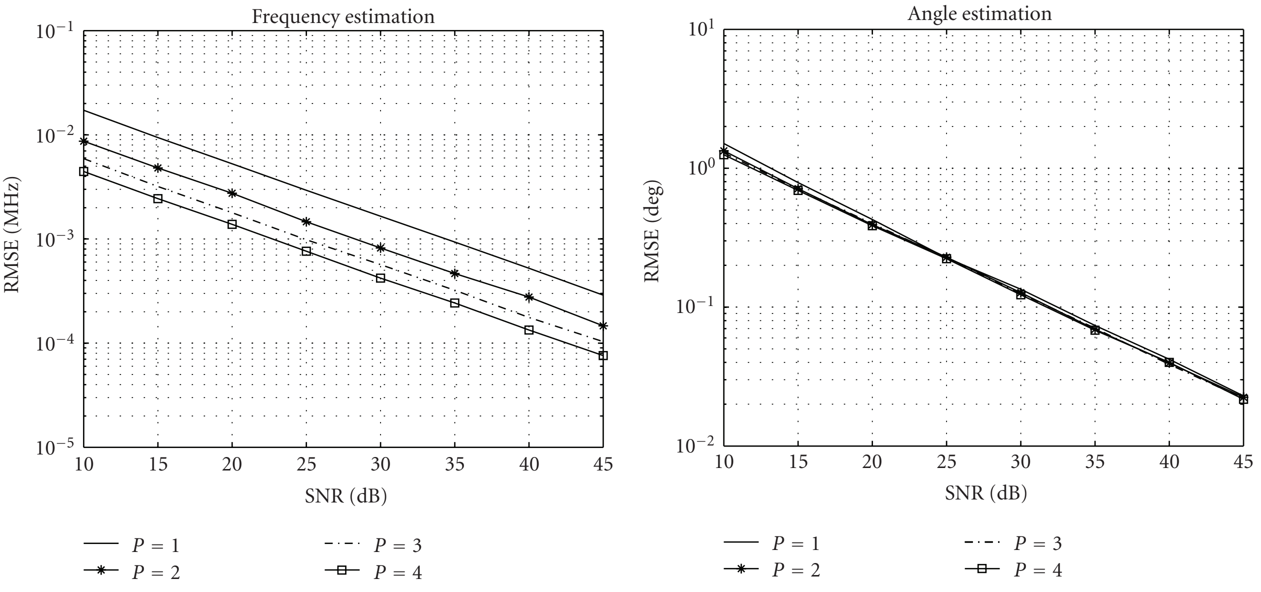
Simulation 2. We compare this proposed algorithm with conventional method [13] which is without delay. $M = 12$, $K = 3$, $P = 3$, and $N = 400$ in this simulation. From Figure 4 we find that this proposed algorithm has better angle-frequency estimation performance than conventional method.

Simulation 3. MDJAFE algorithm performance under different snapshots N is investigated in this simulation. $M = 12$, $K = 3$, and $P = 3$ in this simulation. Figure 5 shows the angle-frequency estimation performance under different N.



(a) (b)

Figure 7: Angle-frequency estimation with different antennas.



(a) (b)

Figure 8: Angle-frequency estimation with different delay number.

We find that the angle-frequency estimation performance of MDJAFE algorithm is improved with N increasing.

Simulation 4. The performance of this algorithm under different source number K is investigated in the simulation. $M = 12$, $P = 3$, and $N = 400$ in this simulation. The source number K is set to 2, 3, and 4. MDJAFE algorithm has different performance under different source numbers, as shown in Figure 6. From Figure 6, we find that angle and frequency estimation performance of MDJAFE algorithm degrades with the increase of the source number K.

Simulation 5. The performance of this algorithm under different antenna number M is investigated in the simulation. $K = 3$, $P = 3$, and $N = 400$ in this simulation. The antenna number M is set to 8, 12, and 16. MDJAFE algorithm has different performance under different antenna number, as shown in Figure 7. From Figure 7, we find that angle and frequency estimation performance of MDJAFE algorithm is improved with M increasing.

Simulation 6. The performance of this algorithm under different delay number P is investigated in the simulation. $M = 12$, $K = 3$, and $N = 400$ in this simulation. The delay number P is set to 1, 2, 3, and 4. MDJAFE algorithm has different performance under different delay numbers, as shown in Figure 8. From Figure 8, we find that angle and frequency estimation performance of MDJAFE algorithm is improved with P increasing.

5. Conclusion

This work presents a new ESPRIT algorithm-based joint angle and frequency estimation using multiple-delay output. The advantage of this proposed algorithm using the multiple-delay output over the conventional algorithm is that the estimation accuracy has been greatly improved.

Acknowledgments

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