

# An Improved Overloading Scheme for Downlink CDMA

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An improved overloading scheme is presented for single-user detection in the downlink of multiple-access systems based on OCDMA/OCDMA (O/O). By displacing in time the orthogonal signatures of the two user sets that make up the overloaded system, the cross-correlation between the users of the two sets is reduced. For random O/O with square-root cosine rolloff chip pulses, the multiuser interference can be decreased by up to 50% (depending on the chip pulse bandwidth) as compared to quasiorthogonal sequences (QOS) that are presently part of the downlink standard of CDMA2000. This reduction of the multiuser interference gives rise to an increase of the achievable signal-to-interference-plus-noise ratio for a particular channel load.

**Keywords and phrases:** CDMA, quasiorthogonal sequences, downlink transmission, oversaturated channels, power control.

## 1. INTRODUCTION

In any synchronized multiple-access system based on code-division multiple access (CDMA), the maximum number of *orthogonal* users equals the spreading factor  $N$ . In order to be able to cope with overloading of a synchronized CDMA system (i.e., with a number of users  $K = N + M : N < K \leq 2N$ ), several schemes have been proposed in literature. Apart from the trivial random spread system (PN) [1], one can look for signature sets that are “as orthogonal as possible.” A popular measure for the quasiorthogonality of a signature set is the total squared correlation (TSC) [2]; signature sets that minimize TSC among all possible signature sets are called Welch bound equality (WBE) sequences [3, 4]. A third approach consists of the OCDMA/OCDMA (O/O) systems [5, 6, 7, 8, 9, 10, 11, 12], where a complete set of orthogonal signature sequences are assigned to  $N$  users (“set 1 users”), while the remaining  $M$  users (“set 2 users”) are assigned another set of orthogonal sequences. The motivation behind the latter proposal is that the interference levels of the users are decreased considerably as compared to other signature sequence sets (e.g., random spreading), since each user suffers from interference caused by the users of the other set only.

WBE sequences have some very interesting properties: they maximize both the sum capacity [3, 4] and achieve the network capacity [13] for synchronous systems based on CDMA. Unfortunately, two major drawbacks of WBE sequences seriously complicate their application to cellular

systems: (1) they give rise to an unscalable system,<sup>1</sup> and (2) the chips of the sequences can be taken binary only if  $K$  is a multiple of 4 [14, 15]. As a result, in spite of their superior performance, they are not considered for implementation in cellular systems and they have to be replaced by suboptimal signature sets that do not suffer from the above mentioned drawbacks, for example, the O/O system.

In [5, 6, 7, 8, 9, 10, 11], the potential of various O/O types with multiuser detection [16] was investigated, while [12] evaluates the downlink potential with single-user detection of a particular type of O/O: “quasiorthogonal sequences (QOS).” Especially the latter application is of practical interest since alignment of the different user signals is easy to achieve in the downlink (as opposed to the uplink), while single-user detection is the obvious choice for detection at the mobile stations. The QOS, discussed in [12], are obtained by assigning orthogonal Walsh-Hadamard sequences [17] to the set 1 users, while each set 2 user is assigned a Walsh-Hadamard sequence, overlaid by a common bent sequence with the window property [12]. These QOS minimize the maximum correlation between the set 1 and set 2 users, which was the incentive to add these QOS to the CDMA2000 standard, so that overloaded systems can be dealt with [18].

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<sup>1</sup>In a system with a fixed spreading gain  $N$ , a signature set is called “unscalable” if the users in the system have to update their signature each time the number of users  $K$  changes.

Up to now, the chip pulses of *all* users are perfectly aligned in time in all considered O/O systems. However, an additional degree of freedom between the set 1 and the set 2 users has been overlooked: one can actually displace the set 2 signatures with respect to the set 1 signatures, without destruction of the orthogonality within each set. In this contribution, we investigate the impact of this displacement on the cross-correlation between the set 1 and set 2 users, and the resulting favorable influence on the downlink performance. In Section 2, we present the system model, along with the conventional QOS system. In Section 3, we introduce a new type of O/O with the displaced signature sets, and we compute the cross-correlation among the user signals. In Section 4, we assess the downlink performance in terms of maximum achievable signal-to-interference-plus-noise ratio (SINR) as a function of channel overload. Finally in Section 5, conclusions are drawn and some topics for future research are identified.

## 2. CONVENTIONAL OCDMA/OCDMA: QOS

Consider the downlink of a perfectly synchronized single-cell CDMA system with spreading factor  $N$  and  $K$  users. Since all signals are generated and transmitted at the same base station, this signal alignment is easy to achieve, and the total transmitted signal  $S_1(t)$  is simply the sum of the signals  $s_k(t)$  of all users  $k$  ( $k = 1, \dots, K$ ):

$$\begin{aligned} S_1(t) &= \sum_{k=1}^K s_k(t) \\ &= \sum_{k=1}^K \left\{ \sum_{i=-\infty}^{+\infty} A_k(i) \cdot a_k(i) \right. \\ &\quad \left. \times \left[ \sum_{j=0}^{N-1} \beta_k^{(i)}(j) \cdot p_c(t - i \cdot NT_c - j \cdot T_c) \right] \right\}. \end{aligned} \quad (1)$$

In this expression,

- (i)  $\beta_k^{(i)} = (\beta_k^{(i)}(0) \dots \beta_k^{(i)}(N-1))$ ,  $A_k(i)$ , and  $a_k(i)$  are the signature sequence, the (real-valued) amplitude, and the data symbol of user  $k$  in the symbol interval  $i$ , respectively. We restrict our attention to BPSK modulation (i.e.,  $a_k(i) \in \{1, -1\}$ ) with normalized binary signature sequences  $\beta_k^{(i)} \in \{1/\sqrt{N}, -1/\sqrt{N}\}^N$ . The extension to QPSK modulation and complex-valued signatures is straightforward;
- (ii)  $p_c(t)$  is a real, unit-energy chip pulse. We restrict our attention to a square-root Nyquist pulse with bandwidth  $(1 + \alpha)/T_c$ , and chip period  $T_c$  [19]. The associated pulse, obtained after matched filtering of  $p_c(t)$ , is a Nyquist pulse  $\phi_c(t)$  with rolloff  $\alpha$ . Note that  $\phi_c(jT_c) = \delta_j$ .

We focus on a single-path channel with complex-valued gain  $\gamma_k(i)$  from the base station to user  $k$  in symbol interval  $i$ . In order to obtain a decision statistic  $z_k(i)$  for the detection of databit  $a_k(i)$ , the received signal is applied to a matched

filter  $p_c(-t)$ , followed by a sampling at the chip rate on the time instants  $(iN + j)T_c$  ( $j = 0, \dots, N-1$ ). The resulting samples are correlated with the signature sequence  $\beta_k^{(i)}$ , and finally a normalization is carried out by multiplying the result by  $\gamma_k^*(i)/|\gamma_k(i)|^2$ . Since  $\phi_c(t)$  is a Nyquist pulse, and due to the perfect alignment of the user signals, the observable  $z_k(i)$  is given by ( $k = 1, \dots, K$ )

$$z_k(i) = A_k(i) \cdot a_k(i) + \sum_{j \neq k} A_j(i) \cdot a_j(i) \cdot \rho_{k,j}(i) + n_k(i). \quad (2)$$

In (2),  $\rho_{k,j}(i) = (\beta_k^{(i)})^T \cdot \beta_j^{(i)}$  is the correlation between  $\beta_k^{(i)}$  and  $\beta_j^{(i)}$ , and  $n_k(i)$  is a real-valued noise sample with variance  $\sigma_k^2/|\gamma_k(i)|^2$ , where  $\sigma_k^2$  is the power spectral density of the noise at the receiver input of user  $k$ .

In order to assure that all users meet a predefined quality-of-service constraint, power control is applied in the downlink. Power control can be achieved by updating the amplitudes of the users once every  $L$  symbol intervals, based on (an estimate of) the variance of the sum of noise and interference over a time span of  $L$  symbol intervals, if  $LNT_c$  is smaller than the minimum coherence time of the channels between the base station and the users.<sup>2</sup> To meet the quality-of-service constraint for user  $k$ , it is necessary that this variance remains lower than the predefined threshold. Since the channel gain is essentially constant over these  $L$  symbol intervals, the variance of interest is given by ( $k = 1, \dots, K$ )

$$\bar{\mu}_k^2 = E[|z_k - \bar{A}_k a_k|^2] = \sum_{j \neq k} \bar{A}_j^2 \cdot \bar{R}_{k,j} + \bar{\delta}_k, \quad (3)$$

where  $\bar{x}$  denotes the (constant) value of  $x$  over the considered time span of  $L$  symbols that starts at time index  $i_1$ ,  $\bar{\delta}_k = \sigma_k^2/|\bar{\gamma}_k|^2$ , and

$$\bar{R}_{k,j} = \frac{1}{L} \sum_{l=1}^L |\rho_{k,j}(i_1 + l)|^2 \quad (4)$$

is the cross-interference between user  $k$  and user  $j$  over the considered time interval. The variance  $\bar{\mu}_k^2$  is related to the signal-to-interference-plus-noise ratio  $\text{SINR}_k$  of user  $k$  by  $\text{SINR}_k = \bar{A}_k^2/\bar{\mu}_k^2$ .

From expression (3), it is obvious that the squared cross-correlations between the signatures of the users should be as small as possible in order to restrict the multiuser interference (MUI). As long as  $K$  remains smaller than  $N$ , taking orthogonal signatures, as is done in IS-95 [22] and CDMA2000 [18], is optimum because they yield  $\rho_{k,j} = 0$  for  $k \neq j$ . If  $K$  exceeds  $N$  (i.e., an overloaded system), the O/O system tries to eliminate as much intracell interference as possible, by taking orthogonal signatures for  $N$  users (set 1 users),

<sup>2</sup>Most channels suffer from slow fading, so that  $L$  can take on quite large values, since the *minimum* coherence time is typically on the order of 5 milliseconds [20, 21]. In IS-95, for instance,  $N \cdot T_c$  is about 52 microseconds, and the power is updated every 1.25 millisecond ( $L = 24$ ) [22].

and by taking another set of orthogonal signatures for the remaining  $M$  users (set 2 users). This eliminates the interference of  $(N - 1)$  and  $(M - 1)$  users in the detection of the set 1 and set 2 users, respectively. Indexing the set 1 users as the first  $N$  users and the set 2 users as the last  $M$  users, (3) turns into

$$\tilde{\mu}_k^2 = \begin{cases} \sum_{j=N+1}^K \tilde{A}_j^2 \cdot \tilde{R}_{k,j} + \tilde{\delta}_k, & k = 1, \dots, N \\ & \text{(set 1 users),} \\ \sum_{j=1}^N \tilde{A}_j^2 \cdot \tilde{R}_{k,j} + \tilde{\delta}_k, & k = N + 1, \dots, K \\ & \text{(set 2 users).} \end{cases} \quad (5)$$

The signatures of the set 1 users span the complete vector space of dimension  $N$ , implying that

$$\sum_{j=1}^N |\rho_{k,j}(i)|^2 = 1, \quad k = N + 1, \dots, K. \quad (6)$$

From this expression, it is immediately seen that the maximal correlation between the set 1 and the set 2 users will be minimized if and only if all of these correlations are equal to  $1/N$ :

$$|\rho_{k,j}(i)|^2 = \frac{1}{N} \quad \forall (k, j), (j, k) \in \{1, \dots, N\} \times \{N + 1, \dots, K\}. \quad (7)$$

In CDMA2000 [18], condition (7) is met (approximately) by means of QOS, where the signatures of the users do not change from one symbol interval to the next: the signatures of the set 1 users are the Walsh-Hadamard sequences  $\mathbf{WH}_N^{(k)}$  ( $k = 1, \dots, N$ ) of order  $N$ , and the signatures of the set 2 users are obtained by overlaying the same Walsh-Hadamard sequences by means of a (quasi-)bent sequence  $\mathbf{Q} \in \{1, -1\}^N$  [12]:

$$\beta_k^{\text{QOS}} = \begin{cases} \mathbf{WH}_N^{(k)}, & k = 1, \dots, N, \\ \text{diag}\{Q_1, Q_2, \dots, Q_N\} \cdot \mathbf{WH}_N^{(k-N)}, & k = N+1, \dots, K. \end{cases} \quad (8)$$

For  $N = 2^{2n}$ , this signature set has the property that

$$|\rho_{k,l}^{\text{QOS}}|^2 = \frac{1}{N} \quad \forall (k, l) \in \{1, \dots, N\} \times \{N + 1, \dots, K\} \quad (9)$$

and (7) is met with equality. For  $N = 2^{2n+1}$ , however, (7) cannot be met with equality (for binary signature sequences), and the best one can do is to use a quasibent sequence as scrambling sequence, so that

$$|\rho_{k,l}^{\text{QOS}}|^2 \in \left\{ \frac{2}{N}, 0 \right\} \quad \forall (k, l) \in \{1, \dots, N\} \times \{N + 1, \dots, K\}. \quad (10)$$

### 3. IMPROVED OCDMA/OCDMA

In the conventional O/O systems, there is an additional degree of freedom that has not been exploited. Indeed, in order to make sure that the first  $N$  user signals are orthogonal, they have to be perfectly aligned in time, and the same is true for the set 2 user signals. However, the set 1 user signals do not need to be aligned with the set 2 user signals to provide this property. Hence, the displacement  $\tau$  ( $\tau \in [0, NT_c)$ ) of the set 2 users with respect to the set 1 users is an additional degree of freedom. Adopting the same notation  $s_k(t)$  as in (1) for the signal of user  $k$ , the total transmitted signal  $S_2(t)$  is now given as

$$S_2(t) = \sum_{i=1}^N s_i(t) + \sum_{i=N+1}^K s_i(t - \tau). \quad (11)$$

We focus on random O/O [8], where the signatures of the set 1 and set 2 users are obtained by overlaying the Walsh-Hadamard vectors in every symbol interval  $i$  with the respective scrambling sequences  $\mathbf{P}_1^{(i)}$  and  $\mathbf{P}_2^{(i)}$  that are chosen completely at random and independently out of  $\{1, -1\}^N$  in every symbol interval:

$$\beta_k^{(i)} = \begin{cases} \text{diag}\{P_1^{(i)}(0), \dots, P_1^{(i)}(N-1)\} \cdot \mathbf{WH}_N^{(k)}, & k = 1, \dots, N, \\ \text{diag}\{P_2^{(i)}(0), \dots, P_2^{(i)}(N-1)\} \cdot \mathbf{WH}_N^{(k-N)}, & k = N + 1, \dots, K. \end{cases} \quad (12)$$

The decision statistics  $z_k(i)$  for the detection of databit  $a_k(i)$  are obtained by applying the received signal to a matched filter  $p_c(-t)$ , sampling at the chip rate on the instants  $(iN + j)T_c$  (set 1 users) or  $(iN + j)T_c + \tau$  (set 2 users), followed by a correlation with the signature sequence  $\beta_k^{(i)}$  and a normalization. We consider the contribution  $z_k^j(i)$  of set-2 user  $j$  to the decision variable  $z_k(i)$  of set-1 user  $k$  in symbol interval  $i$ :

$$z_k^j(i) = \sum_{s=-\infty}^{+\infty} A_j(s) \cdot a_j(s) \cdot \rho_{k,j}^{(i,s)}(\tau), \quad (13)$$

where  $\rho_{k,j}^{(i,s)}(\tau)$  is defined as

$$\rho_{k,j}^{(i,s)}(\tau) = \sum_{m=0}^{N-1} \sum_{n=0}^{N-1} \beta_k^{(i)}(m) \cdot \beta_j^{(s)}(n) \cdot \phi_c[((i-s) \cdot N + (m-n))T_c - \tau]. \quad (14)$$

Hence, the cross-interference  $\tilde{R}_{k,j}$  between set-1 user  $k$  and set-2 user  $j$  is given by

$$\tilde{R}_{k,j} = \frac{1}{L} \sum_{l=1}^L \left\{ \sum_{s=-\infty}^{+\infty} |\rho_{k,j}^{(i_1+l,s)}(\tau)|^2 \right\} \quad (15)$$

and is an approximately Gaussian random variable. However, its expected value is multiplied by a factor  $\lambda \leq 1$  as compared

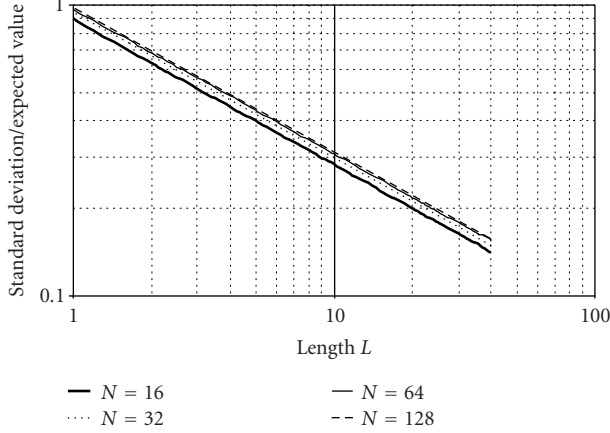


FIGURE 1: Dependence of  $\tilde{\psi}_{k,j}/E[\tilde{R}_{k,j}]$  on  $L$  for cosine rolloff chip pulse with  $\alpha = 0.25$  ( $\tau = (N+1)T_c/2$ ).

to QOS (see Appendix A), implying a reduction of the average cross-interference:

$$E[\tilde{R}_{k,j}] = \frac{\lambda(\alpha, \Delta)}{N} = \frac{1}{N} \sum_{s=-\infty}^{+\infty} \phi_c^2(s \cdot T_c - \Delta), \quad (16)$$

$$\Delta = \tau - \left\lfloor \frac{\tau}{T_c} \right\rfloor \cdot T_c.$$

The variance  $\tilde{\psi}_{k,j}^2 = E[(\tilde{R}_{k,j} - \lambda/N)^2]$  of  $\tilde{R}_{k,j}$  is dependent on the length  $L$  of the time interval. Figure 1 shows the relative spread  $\tilde{\psi}_{k,j}/E[\tilde{R}_{k,j}]$  obtained by simulations as a function of  $L$  for  $N = 16, 32, 64$ , and  $128$ , when  $p_c(t)$  is a square-root cosine rolloff pulse with rolloff  $\alpha = 0.25$ . A detailed observation of the plots of Figure 1 brings to light that the relative spread can be expressed as

$$\frac{\tilde{\psi}_{k,j}}{E[\tilde{R}_{k,j}]} = \frac{\Omega(N)}{\sqrt{L}}, \quad (17)$$

where  $\Omega(16)$ ,  $\Omega(32)$ ,  $\Omega(64)$ , and  $\Omega(128)$  are identified as 0.89, 0.94, 0.97, and 0.99, respectively.

So, as compared to the original QOS system, the expected value of the MUI of all set 1 and set 2 users is decreased by a factor  $1/\lambda$  that is dependent on the rolloff  $\alpha$  and on  $\Delta$ . For  $\Delta = 0$ , the chip pulses of all users are perfectly aligned, and  $\lambda = 1$ , whether the symbol boundaries of the set 1 and the set 2 users are aligned or not.

The function  $\lambda(\alpha, \Delta)$  is the interference function of the pulse  $p_c(t)$ . This interference function was introduced in another context (PN-spread asynchronous communication) in [23]. If the excess bandwidth of  $p_c(t)$  is less than 100%, it was shown in [23] that  $\lambda(\alpha, \Delta)$  can be written as a function of the Fourier transform  $P_c(f)$  of  $p_c(t)$ :

$$\lambda(\alpha, \Delta) = F(0) + [1 - F(0)] \cdot \cos\left(2\pi \cdot \frac{\Delta}{T_c}\right), \quad (18)$$

$$F(0) = \frac{1}{T_c} \int_{-\infty}^{+\infty} |P_c(f)|^4 df \geq 0.$$

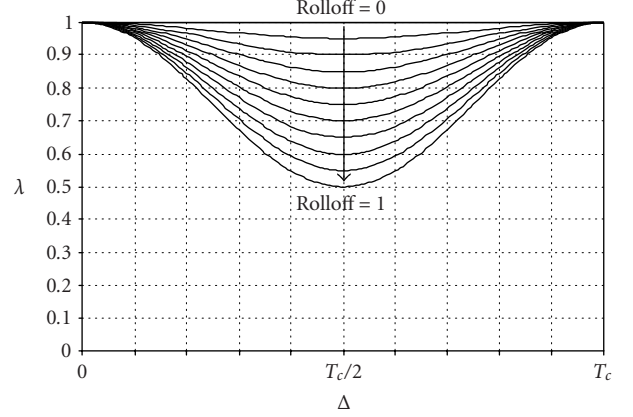


FIGURE 2:  $\lambda(\alpha, \Delta)$  for square-root cosine rolloff pulse with rolloff  $\alpha = 0, 0.1, \dots, 1$ .

According to this expression,  $\lambda$  is minimal for  $\Delta = T_c/2$ . This is illustrated in Figure 2 for a cosine rolloff chip pulse, where  $\lambda$  is plotted as a function of  $\Delta$ , for  $\alpha = 0, 0.1, 0.2, \dots, 1$ . In Appendix B, we derive the following relationship between the optimal value of  $\lambda$  and the rolloff of the square-root cosine rolloff chip pulse:

$$\lambda\left(\alpha, \frac{T_c}{2}\right) = 1 - \frac{\alpha}{2}. \quad (19)$$

So, it is obvious that we can obtain important decreases in MUI by displacing the chip pulses of the set 2 users by half a chip period as compared to the chip pulses of the set 1 users. This decrease can be up to 50% for  $\alpha = 1$ , and amounts to 12.5% for a practical rolloff value of 0.25.

#### 4. DOWNLINK PERFORMANCE

In order to assess the performance of the considered O/O signature sets in the downlink, we focus on a single-cell scenario, where each user suffers from intracell interference and white thermal noise. We assume that the cell is circular and that all users are within a range  $r \in [100 \text{ m}, 1500 \text{ m}]$  from the base station, scattered uniformly over this cell. It is assumed that all  $|\tilde{y}_k|^2$  ( $k = 1, \dots, K$ ) are independent, with probability density function (pdf) illustrated in Figure 3. It consists of three contributions [20, 21]:

$$|\tilde{y}_k|^2 = \frac{1}{r^n} \cdot \eta_{\log} \cdot \eta_{\text{Rayleigh}}, \quad (20)$$

where

- (i)  $n$  is the path loss exponent, which is taken here as  $n = 3$ ;
- (ii)  $\eta_{\log}$  is a lognormally distributed loss term that accounts for the large scale shadowing effects. We take the large-scale fading margin at 10 dB;
- (iii)  $\eta_{\text{Rayleigh}}$  is a loss term that accounts for the small scale Rayleigh fading.

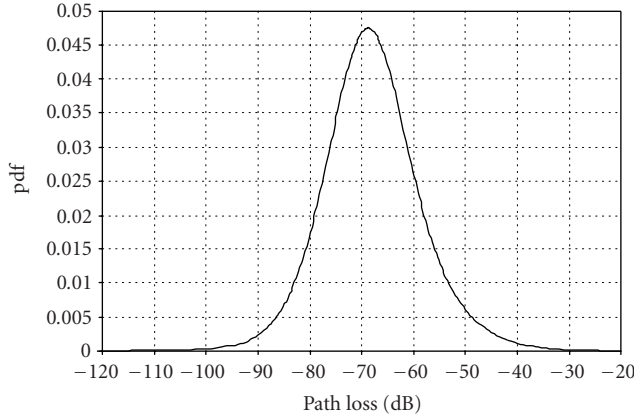


FIGURE 3: Probability density function of path loss  $|\bar{\gamma}_k|^2$ .

If we impose on all users a common quality-of-service constraint, so that the  $\text{SINR}_k$  for all users  $k$  has to be at least  $\kappa$ , then, as long as a solution exists, the minimum (optimum) power solution corresponds to the case where all  $\text{SINR}_k$  are exactly  $\kappa$  [24, 25]:

$$\text{SINR}_k = \frac{\tilde{A}_k^2}{\tilde{\mu}_k^2} = \kappa \quad \forall k \iff \mathbf{A} = \kappa[\mathbf{R} \cdot \mathbf{A} + \mathbf{d}] \quad (21)$$

with  $(\mathbf{A})_k = \tilde{A}_k^2$ ,  $(\mathbf{d})_k = \tilde{\delta}_k$  for  $k = 1, \dots, K$ , and  $(\mathbf{R})_{i,j} = \tilde{R}_{i,j}$  if  $i$  and  $j$  are from a different orthogonal set, while  $(\mathbf{R})_{i,j} = 0$  if  $i$  and  $j$  are from the same set. It is well known [24, 25, 26, 27] that (21) has a positive solution  $\mathbf{A}$ , if and only if the Perron-Frobenius eigenvalue of  $(\kappa \cdot \mathbf{R})$  is smaller than 1. Moreover, solving (21) by means of a Jacobi iteration converges monotonically to the optimal solution, if and only if a positive solution to (21) exists. As a consequence, it is sufficient to try to solve (21) by means of the Jacobi iteration ( $n = 0, \dots, +\infty$ )

$$\mathbf{A}^{(n+1)} = \kappa \cdot \mathbf{R} \cdot \mathbf{A}^{(n)} + \kappa \cdot \mathbf{d} \quad (22)$$

starting from any positive power vector  $\mathbf{A}^{(0)}$ . If the iterations converge to a solution, we obtain at the same time the optimum power vector. If the iterations diverge, the quality-of-service constraint  $\kappa$  cannot be met by any (positive) power vector  $\mathbf{A}$  for all users at the same time. Note, however, that for *any*  $\mathbf{R}$ , (21) always has a positive solution for a range of values  $\kappa \in (-\infty, \kappa_{\max}]$ , where  $\kappa_{\max}$  is the maximum achievable SINR for this setting. We replace  $\mathbf{R}$  in (21) by  $E[\mathbf{R}]$ , which implies ignoring the statistical fluctuation of  $\mathbf{R}$ . Taking (16) into account, the corresponding value of  $\kappa_{\max}$  is given by

$$\kappa_{\max} = \frac{1}{\lambda \sqrt{K/N} - 1} \quad (23)$$

and will be denoted as the achievable SINR for “static”  $\mathbf{R}$ .

Simulations have been performed for QOS and the improved O/O systems with square-root cosine rolloff chip

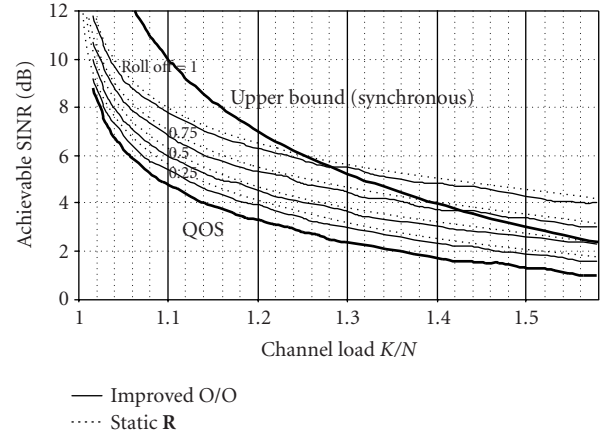


FIGURE 4:  $\kappa_{\max}^{(\min)}$  as a function of  $K$  for the considered O/O systems ( $N = 64$ ).

pulses with rolloff  $\alpha = 0.25, 0.5, 0.75$ , and 1, spreading factor  $N = 64$ , and a number of users  $K = 65, \dots, 101$ , in order to determine  $\kappa_{\max}$ . The time shift between the two orthogonal sets of the improved O/O system is taken as  $\tau = (N+1) \cdot T_c/2$ . The number of symbol intervals with fixed channel and amplitude characteristics is  $L = 20$ . The achievability of a particular SINR value  $\kappa$  for all users has been determined by means of the Jacobi iterations of (22), for a fixed value of  $\mathbf{d}$  and  $\mathbf{A}^{(0)}$ , with a maximum of 50 iterations. For improved O/O, the entries of  $\mathbf{R}$  are random variables, and we determined the minimum value  $\kappa_{\max}^{(\min)}(K)$  of  $\kappa_{\max}$  that was obtained over a wide range of random realizations of  $\mathbf{R}$  for  $K$  users. In Figure 4, we illustrate the obtained values of  $\kappa_{\max}^{(\min)}(K)$  for improved O/O and QOS. We also added the upper bound for synchronous CDMA systems (achieved for WBE sequences), along with the achievable SINR corresponding to static  $\mathbf{R}$ . It is immediately seen that  $\kappa_{\max}^{(\min)}$  is higher for any of the considered improved O/O systems as compared to QOS. For a practical rolloff value  $\alpha = 0.25$ , the maximum achievable SINR is about 0.6 dB higher than for QOS over the entire range of  $K$ . This gain in  $\kappa_{\max}$  rises to about 1.3 dB, 2 dB, and 3 dB for  $\alpha = 0.5, 0.75$ , and 1, respectively. In addition to this, improved O/O with  $\alpha = 1$  and 0.75 performs better than the upper bound for *synchronous* transmission for a number of users higher than 82 and 91, respectively. For  $\alpha = 0.5$ , improved O/O almost achieves the upper bound for  $K = 100$ . Further, we note that the achievable SINR is only slightly less than the value corresponding to static  $\mathbf{R}$ . This indicates that the statistical fluctuation of  $\mathbf{R}$  has only a minor effect on achievable SINR, which hence can be approximated by the simple expression (23). Alternatively, one can tabulate the maximum acceptable channel overload  $(K_{\max} - N)/N$  as a function of the required SINR for the considered O/O systems, as is done in Table 1. Once more, we notice the superiority of the improved O/O systems. Although we have presented numerical results for  $N = 64$  only, these results should be representative also for larger values of  $N$ , since the distribution of  $\tilde{R}_{i,j}$  is only slightly dependent on  $N$  (as illustrated in Figure 1).

TABLE 1: Maximum acceptable channel overload for a required SINR of the considered O/O systems.

	QOS	O/O ( $\alpha = 0.25$ )	O/O ( $\alpha = 0.5$ )	O/O ( $\alpha = 0.75$ )	O/O ( $\alpha = 1$ )
SINR = 5 dB	9%	13%	17%	23%	36%
SINR = 4 dB	14%	19%	25%	36%	56%
SINR = 3 dB	23%	30%	41%	59%	92%

## 5. CONCLUSION AND TOPICS FOR FUTURE RESEARCH

In this paper, we extended the idea of perfectly aligned oversaturated O/O systems to O/O systems where the set 1 and set 2 user signals are displaced in time. We found that such a displacement reduces the multiuser interference between the set 1 and set 2 users by up to 50% (depending on the chip pulse bandwidth) for randomized O/O signature sets with square-root cosine rolloff chip pulses. Hence, as compared to quasiorthogonal sequences that are presently applied in the downlink of the CDMA2000 system, one can achieve higher feasible SINR values for the improved O/O systems that even go beyond the upper bound for synchronous systems. We conclude that the improved O/O systems are a promising and superior alternative to QOS for channel overloading in the downlink of systems based on CDMA.

In this paper, the simulations focused on square-root cosine rolloff chip pulses, but one can expect to be able to decrease the MUI even further by a proper selection of the (band-limited) chip pulses. For instance, it was shown in [23] that the square-root brick-wall rolloff chip pulse<sup>3</sup> minimizes  $\lambda(\alpha, T_c/2)$  over all pulses with excess bandwidth  $\alpha/T_c$ . Another possibility is to add an extra degree of freedom to the system: one can actually try to optimize at the same time the chip pulses (possibly different for different user sets) and the time shifts for the users of the two sets. Both topics are left for future research.

## APPENDICES

### A. DETERMINATION OF $E(\tilde{R}_{k,j})$

In order to determine  $E(\tilde{R}_{k,j})$ , we take a closer look at

$$\begin{aligned}
 I(l) &= E \left[ \sum_{s=-\infty}^{+\infty} \left| \rho_{k,j}^{(i_1+l,s)}(\tau) \right|^2 \right] \\
 &= \sum_{s=-\infty}^{+\infty} E \left[ \left| \sum_{m=0}^{N-1} \sum_{n=0}^{N-1} \beta_k^{(i_1+l)}(m) \cdot \beta_j^{(s)}(n) \cdot \phi_c[(i_1+l-s) \right. \right. \\
 &\quad \left. \left. \cdot N + (m-n))T_c - \tau] \right|^2 \right]. \tag{A.1}
 \end{aligned}$$

Since the scrambling sequences of the different user sets are random, the chips of different users are uncorrelated, and the same is true for different chips of the same signature. This, combined with the fact that all chips belong to the set  $\{1/\sqrt{N}, -1/\sqrt{N}\}$ , reduces (A.1) to

$$\begin{aligned}
 I(l) &= \frac{1}{N^2} \sum_{s=-\infty}^{+\infty} \sum_{m=0}^{N-1} \sum_{n=0}^{N-1} \phi_c^2[(i_1+l-s) \\
 &\quad \cdot N + (m-n)) \cdot T_c - \tau] \\
 &= \frac{1}{N^2} \sum_{s=-\infty}^{+\infty} \sum_{m=0}^{N-1} \sum_{n=0}^{N-1} \phi_c^2[(s \cdot N + (m-n)) \cdot T_c - \tau]. \tag{A.2}
 \end{aligned}$$

An observation of the terms in the last expression for  $I(l)$  shows that each term  $\phi_c^2(k \cdot T_c - \Delta)$ , with  $\Delta = \tau - \lfloor \tau/T_c \rfloor \cdot T_c$ , occurs  $N$  times in the summation. As a consequence,

$$\begin{aligned}
 I(l) &= \frac{1}{N^2} \sum_{s=-\infty}^{+\infty} N \cdot \phi_c^2(s \cdot T_c - \Delta) = \frac{1}{N} \sum_{s=-\infty}^{+\infty} \phi_c^2(s \cdot T_c - \Delta), \\
 E(\tilde{R}_{k,j}) &= \frac{1}{L} \sum_{l=1}^L I(l) = \frac{1}{N} \sum_{s=-\infty}^{+\infty} \phi_c^2(s \cdot T_c - \Delta). \tag{A.3}
 \end{aligned}$$

### B. RELATION BETWEEN THE OPTIMAL VALUE OF $\lambda$ AND THE ROLLOFF OF THE SQUARE-ROOT COSINE ROLLOFF PULSE

For the square-root cosine rolloff pulse  $p_c(t)$  with rolloff  $\alpha$ , we have that [19]

$$|P_c(f)|^2 = \begin{cases} T_c, & |f| < \frac{1-\alpha}{2T_c}, \\ \frac{T_c}{2} \left[ 1 - \sin \left( \frac{\pi}{\alpha} \left( |f|T_c - \frac{1}{2} \right) \right) \right], & \frac{1-\alpha}{2T_c} < |f| < \frac{1+\alpha}{2T_c}, \\ 0, & \text{elsewhere.} \end{cases} \tag{B.1}$$

Bringing this into (18) for  $F(0)$ , we obtain

$$\begin{aligned}
 F(0) &= \frac{1}{T_c} \int_{-\infty}^{+\infty} |P_c(f)|^4 df \\
 &= (1-\alpha) + \frac{\alpha}{2\pi} \int_{-\pi/2}^{\pi/2} [1 - \sin(x)]^2 dx. \tag{B.2}
 \end{aligned}$$

<sup>3</sup>Unfortunately, this chip pulse is hard to realize in practice because of the sharp edges of the corresponding filter.

After some elementary integral calculations, we find  $F(0) = 1 - \alpha/4$ . As a result,

$$\lambda\left(\alpha, \frac{T_c}{2}\right) = 2F(0) - 1 = 1 - \frac{\alpha}{2}. \quad (\text{B.3})$$

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