EURASIP Journal on Applied Signal Processing 2005:5, 635–648

A Network of Kalman Filters for MAI and ISI Compensation in a Non-Gaussian Environment

Bessem Sayadi

Laboratoire des Signaux et Systèmes (LSS), Supélec CNRS, Plateau de Moulon, 3 rue Joliot Curie, 91192 Gif-sur-Yvette Cedex, France
Email: sayadi@lss.supelec.fr

Sylvie Marcos

Laboratoire des Signaux et Systèmes (LSS), Supélec CNRS, Plateau de Moulon, 3 rue Joliot Curie, 91192 Gif-sur-Yvette Cedex, France
Email: marcos@lss.supelec.fr

Received 4 September 2003; Revised 30 November 2004

This paper develops a new multiuser detector based on a network of kalman filters (NKF) dealing with multiple-access interference (MAI), intersymbol interference (ISI), and an impulsive observation noise. The two proposed schemes are based on the modeling of the DS-CDMA system by a discrete-time linear system that has non-Gaussian state and measurement noises. By approximating the non-Gaussian densities of the noises by a weighted sum of Gaussian terms and under the common MMSE estimation criterion, we first derive an NKF detector. This version is further optimized by introducing a feedback exploiting the ISI interference structure. The resulting scheme is an NKF detector based on a likelihood ratio test (LRT). Monte-Carlo simulations have shown that the NKF and the NKF based on LRT detectors significantly improve the efficiency and the performance of the classical Kalman algorithm.

Keywords and phrases: multiuser detection, Kalman filtering, Gaussian sum approximation, impulsive noise, likelihood ratio test.

1. INTRODUCTION

Direct-sequence code-division multiple access (DS-CDMA) is emerging as a popular multiple-access technology for personal, cellular, and satellite communication services [1, 2, 3] for its large capacity that results from several advantages [4], such as soft handoffs, a high-frequency reuse factor, and the efficient use of the voice activity. However, in the case of a multipath transmission channel, the signals received from different users cannot be kept orthogonal and multiple-access interference (MAI) arises. The need for an increased capacity in terms of the number of users per cell and a higher-bandwidth multimedia data communication constraints us to overcome the MAI limitation. One solution to this problem is multi-user detection, which is covered in [5] and the references within. In addition, high-speed data transmission over communication channels is subject to intersymbol interference (ISI). The ISI is usually the result of the restricted bandwidth allocated to the channel and/or the presence of multipath distortions in the medium through which the information is transmitted. This leads to a need for multiuser detection techniques that jointly suppress ISI as well as MAI, in order to obtain reliable estimates of the symbols transmitted by a particular user (or all the users).

A class of DS-CDMA receivers known as linear minimum mean-squared error (MMSE) detectors has been discussed in recent years. The Kalman filter is known to be the linear minimum variance state estimator. It is well known that the Kalman filter leads to the lowest mean-square error (MSE) among all the linear filters as it is shown in [6]. Motivated by this fact, some attention has been focused recently on Kalman-filter-based adaptive multiuser detection [6, 7, 8, 9]. This approach is based on a state-space expression of the DS-CDMA system. In this paper, we show that the DS-CDMA system can fit exactly the Kalman model in terms of a measurement equation and a state transition equation. The proposed model allows us to highlight the impact of ISI on the received signal and also to have an estimate of the user's data at the symbol rate.

Most of the work on multiuser detection, and especially the Kalman-filter-based techniques, assume that the ambient noise (observation noise) is Gaussian. However in many physical channels, the observation noise exhibits Gaussian as well as impulsive[1] characteristics. The source of impulsive noise may be either natural, such as lightnings, or man

[1]The term impulsive is used to indicate the probability of large interference levels.

made. It might come from relay contacts in switches, electromagnetic devices [10], transportation systems [11] such as underground trains and so forth. Recent measurements of outdoor and indoor mobile radio communications reveal the presence of a significant interference exceeding typical thermal noise levels [12, 13]. The empirical data indicate that the probability density function (pdf) of the impulsive noise processes exhibits a similarity to the Gaussian pdf, being bell-shaped, smooth, and symmetric but at the same time having significantly heavier tails. A variety of impulsive noise models have been proposed [14, 15, 16]. In this paper, we adopt the commonly used "ε-contaminated" model for the additive noise which is a tractable empirical model for impulsive environments and approximates a large variety of symmetric pdfs. The ε-contaminated model or the Gaussian mixture serves as an approximation to Middleton's canonical class A model which has been studied extensively over the past two decades [17, 18, 19].

The study of the impact of the impulsive noise on the performance of the Kalman-based detector presented in this paper shows the deterioration of the error rates. The same conclusion is outlined in [20, 21, 22]. The aim of this paper is to robustify the Kalman-based detector to a non-Gaussian observation noise in order to obtain a robust multiuser detector able to jointly cancel the MAI and ISI and take into account the impulsiveness of the observation noise. Our approach is original in the sense that it tries to correct the error induced by the presence of impulsive noise by introducing a feedback which exploits the ISI structure.

In fact, because of the numeric character of the state noise (related to the transmitted symbols) and the presence of outliers in the observation noise, the Kalman filtering approach is no longer optimal. Only when the state noise and the observation noise are both Gaussian distributed, the equation of the optimal detector reduces to the equation of the well-known Kalman algorithm [23]. In the other cases, a suboptimal or a robust Kalman filtering becomes necessary. Some Kalman-like filtering algorithms have been derived by Masreliez [24] and Alspach and Sorenson [25]. The first approach is based on strong assumptions (either the state or the measurement noise is Gaussian and the one step ahead prediction density function is also Gaussian). Its main idea is the characterization of the deviation of the non-Gaussian distribution from the Gaussian one by the so-called score function. However, a new problem that one has to handle is a rather difficult convolution operation involving the nonlinear score function. The approximate conditional mean (ACM) filter proposed in [26] for joint channel estimation and symbol detection exploits the Masreliez approximation.

In this paper, we adopt the second approach of Alspach and Sorenson [25] which considers the case where both the state and measurement noise sequences are non-Gaussian. In particular, we exploit the simplification introduced in [27] reducing the numerical complexity and keeping it constant over the iterations. The major idea is to approximate the non-Gaussian density function by a weighted sum of Gaussian density functions.

From an approximation of the *a posteriori* density functions of the data signals by a weighted sum of Gaussian density functions and by exploiting the mixture model of the observation noise, we propose here a new robust structure of a multiuser detector that is based on a network of kalman filters operating in parallel. Under the common MMSE estimation error criterion, the state vector (consisting of the last transmitted symbols of all users) is estimated from the received signal, where Kalman parameters are adjusted using one noise parameter (variance and contamination constant) and one Gaussian term in the *a posteriori* pdf approximation of the plant noise. This version is called *extended* NKF detector.

The resulting structure presents an internal mechanism for the localization of the impulses. So, in order to reduce the complexity of the proposed structure and to improve its performance, we propose, in the second part of this paper, to incorporate a likelihood ratio test allowing for the localization of the impulses in the received signal and to exploit the ISI structure introduced by the multipath channel. We suggest to incorporate a decision feedback in order to generate the required replicas of the corrupted symbols by operating on the adjacent state vectors which have been decided earlier (and assuming the decision to be correct). We, therefore, propose to reject the samples corrupted by the impulsive noise rather than to clip them as is done in many previous works [22, 28, 29, 30, 31, 32]. By adapting the transition equation to the number of successive corrupted samples, we can reestimate the corrupted symbols of the users by exploiting the proposed feedback. The algorithm proposed here exploits the diversity introduced by the intersymbol interference.

This paper is organized as follows. In Section 2, we introduce the state-space description of the CDMA system and the non-Gaussian noise model. We revisit the Kalman filter approach and we analyze the impact of the impulsive noise on its performance. In Section 3, we derive the proposed detector based on a network of Kalman filters operating in parallel: the *extended* NKF which takes into account the non-Gaussianity of the state and observation noises. Section 4 investigates the localization procedure based on the likelihood ratio test. Section 5 presents the resulting algorithm based on the introduced feedback. In Section 6, simulation results are provided supporting the analytical results. And finally, Section 7 draws our conclusions.

Throughout this paper, scalars, vectors and, matrices are lowercase, lowercase bold, and uppercase bold characters, respectively. $(\cdot)^T$, $(\cdot)^{-1}$ denote transposition and inversion, respectively. Moreover, $E(\cdot)$ denotes the expected value operator. $\lceil x \rceil$ denotes the smallest integer not less than x. Finally, $*$ denotes the convolution operator.

2. COMMUNICATION SYSTEM AND NON-GAUSSIAN NOISE MODEL

2.1. State-space model

We model here the uplink of the DS-CDMA communication system of K asynchronous users transmitting over K different frequency-selective channels. We denote by $d_i(m)$

the symbol of the ith user transmitted in the time interval $[mT_s, (m+1)T_s[$, where T_s represents the symbol period. We introduce $\mathbf{c}_i = [c_i(0), \dots, c_i(L-1)]^T$ as the spreading code of user i. L is the processing gain.

The transmitted signal due to the ith user can be written as $s_i(t) = \sum_n d_i(n)c_i(t-nT_s)$, where $c_i(t) = \sum_{q=0}^{L-1} c_i^q \psi(t-qT_c)$ and $1/T_c$ denotes the chip rate. $\{d_i(n)\}$ and $\{c_i^q\}$ denote the symbol stream and the spreading sequence, respectively. $\psi(t)$ is a normalized chip waveform of duration T_c. The baseband received signal containing the contribution of all the users over the frequency-selective channels denoted by $\widetilde{h}^{(i)}(t)$, $i = 1, \dots, K$, is given by

$$\begin{aligned} r(t) &= \sum_{i=1}^{K} \sum_{n} \widetilde{h}^{(i)}(t) * \left(d_i(n)c_i(t-nT_s)\right) + b(t) \\ &= \sum_{i=1}^{K} \sum_{n} \sum_{q=0}^{L-1} d_i(n)c_i^q \left(\widetilde{h}^{(i)} * \psi\right)(t-qT_c-nT_s) + b(t), \end{aligned} \tag{1}$$

where $b(t)$ is an additive noise.

The channel of the ith user is characterized by its impulse response $h^{(i)}(t)$:

$$h^{(i)}(t) = \widetilde{h}^{(i)} * \psi(t) \tag{2}$$

that includes equipment filtering (chip pulse waveform, transmitted filter and its matched filter in the receiver, etc.) and propagation effects (multipath, time delay).

The baseband received signal sampled at the chip rate $1/T_c$ leads to a chip-rate discrete-time model which can be written in $[kT_c, (k+1)T_c[$ as

$$r(k) = r(t = kT_c) = \sum_{i=1}^{K} \sum_{j} \widetilde{g}_i(k-jL)d_i(j) + b(k), \tag{3}$$

where $\widetilde{g}_i(k,l) = \sum_{q=0}^{L-1} c_i^q h^{(i)}(k,(l-q)T_c)$ is the global channel function including spreading and convolution by the channel. It is convenient to combine the signature modulation process with the effects of the channel in order to obtain an equivalent model in which the symbol streams of the individual users are time-division multiplexed before their transmission over a multiuser channel.

In this paper, we focus on a symbol-by-symbol multiuser detection scheme. For this reason, we let the observation interval be one symbol period. We concatenate the elements of $r(k)$ in a vector $\mathbf{r}(k)$. According to (3), we can write

$$\begin{aligned} \mathbf{r}(k) &= \left[r(kL), \dots, r(kL+L-1)\right]^T \\ &= \sum_{p} \mathbf{B}(k,p)\mathbf{x}(k-p) + \mathbf{b}(k), \end{aligned} \tag{4}$$

where the matrix $\mathbf{B}(k,p)$ is of size (L,K) and is obtained as follows:

$$\begin{aligned} \mathbf{B}(k,p) &= \left[\mathbf{g}_1(k,p), \dots, \mathbf{g}_K(k,p)\right], \\ \mathbf{g}_i(k,p) &= \left[\widetilde{g}_i(k,pL), \dots, \widetilde{g}_i(k,pL+L-1)\right]^T, \quad i = 1, \dots, K. \end{aligned} \tag{5}$$

$\mathbf{x}(k) = [d_1(k), \dots, d_K(k)]^T$ is a vector of size $(K,1)$ containing the symbols of K users and $\mathbf{b}(k) = [b(nL), \dots, b(nL+L-1)]^T$ is a vector of size $(L,1)$ containing the noise samples on a symbol period.

By denoting $\widetilde{k} = \lceil (P+L-1)/L \rceil$, where P represents the maximum delay introduced by the multipath channels, as the number of the symbols interfering in the transmission channel, the received signal can be expressed as a block transmission CDMA model:

$$\begin{aligned} \mathbf{r}(k) &= \mathbf{A}(k)_{L\times \widetilde{k}K}\mathbf{d}(k)_{\widetilde{k}K\times 1} + \mathbf{b}(k)_{\widetilde{k}K\times 1}, \\ \mathbf{A}(k) &= \left[\mathbf{B}(k,0), \dots, \mathbf{B}(k,\widetilde{k}-1)\right], \\ \mathbf{d}(k) &= \left[\mathbf{x}(k)^T, \dots, \mathbf{x}(k-\widetilde{k}+1)^T\right]^T. \end{aligned} \tag{6}$$

Matrix $\mathbf{A}(k)$ is of size $(L, \widetilde{k}K)$. We note that in the case of a time-invariant channel case, the observation matrix $\mathbf{A}(k)$ is a constant matrix $\mathbf{A}$. In this paper, we suppose that the convolution code-channel matrix is invariant on a slot duration. We also remark that the dimension of the observation matrix $\mathbf{A}(k)$ is $\widetilde{k}$ dependent since the $\widetilde{k}$ index is proportional to the ISI term. In fact, in the case of an AWGN channel, that is, $P = 0$, we have $\widetilde{k} = 1$, and the ISI term vanishes. However, in the case of a frequency-selective multipath channel and a low spreading factor, that is, $L \to 0$, the term $\widetilde{k}$ increases and causes a severe ISI term. So, (6) highlights the impact of ISI on the received signal.

Equation (6) represents the measurement equation required in the state-space model of the DS-CDMA system. $\mathbf{d}(k)$ represents the $(\widetilde{k}K \times 1)$ state vector containing all the symbols contributing to $\mathbf{r}(k)$. The state vector $\mathbf{d}(k)$ is time dependent and its first-order transition equation is described as follows:

$$\mathbf{d}(k+1) = \mathbf{F}\mathbf{d}(k) + \mathbf{G}\mathbf{x}(k+1), \tag{7}$$

where

$$\mathbf{F} = \begin{pmatrix} \mathbf{0}_{K\times K} & \mathbf{0}_{K\times K} & \cdots & \cdots & \mathbf{0}_{K\times K} \\ \mathbf{I}_{K\times K} & \mathbf{0}_{K\times K} & \ddots & \ddots & \vdots \\ \mathbf{0}_{K\times K} & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \mathbf{0}_{K\times K} \\ \mathbf{0}_{K\times K} & \cdots & \cdots & \mathbf{I}_{K\times K} & \mathbf{0}_{K\times K} \end{pmatrix}_{\widetilde{k}K\times\widetilde{k}K}, \qquad \mathbf{G} = \begin{pmatrix} \mathbf{I}_{K\times K} \\ \mathbf{0}_{K\times K} \\ \vdots \\ \mathbf{0}_{K\times K} \end{pmatrix}_{\widetilde{k}K\times K}. \tag{8}$$

$\mathbf{0}$ is the $(K \times K)$ null matrix and $\mathbf{I}$ is the $(K \times K)$ identity matrix. We assume that the users are uncorrelated and transmit white symbol streams, that is,

$$E\left[\mathbf{x}(k)\mathbf{x}(j)^T\right] = \sigma_d^2 \mathbf{I}_{K\times K}\delta(k-j), \tag{9}$$

where $\delta(\cdot)$ denotes the Kronecker symbol.

With (6) and (7), we have a state-space model for the DS-CDMA system. We note that this state-space model also applies when in addition there is multiple antenna at the receiver in the system. Although not explicitly developed in this paper, these extensions are obtained via considering a higher dimension for the state and/or the observation vectors.

The MMSE detection for the multiple-access system, described by (6) and (7) requires the construction of a linear MMSE estimate of the state. Based on the fact that the DS-CDMA system can be viewed as a linear dynamical system under the proposed state-space description, such estimate can be computed recursively and efficiently via the Kalman filtering algorithm. In fact it is well known that the Kalman filter is a good recursive state estimator for linear systems. The Kalman filter is a first-order recursive filter. It naturally processes all the information collected up to a given point in time. It produces state estimates that are optimal in the MMSE sense.

2.2. Problem setting

2.2.1. The Kalman filtering approach

In this section, we revisit the Kalman filtering approach. The measurement ($\mathbf{b}(k)$) and the state ($\mathbf{Gx}(k)$) noises are both white and mutually uncorrelated. Therefore, with the knowledge of the channel-code matrix $\mathbf{A}$ and the noise spectral density, the Kalman-filter-based detector can be implemented in a recursive form. The state vector $\mathbf{d}(k)$ is estimated from the observation of the DS-CDMA system output collected in $\mathbf{R}(k) = [\mathbf{r}(k), \mathbf{r}(k-1), \dots, \mathbf{r}(0)]$. In our case, the estimation of $\mathbf{x}(k)$ can be obtained at a delayed time $(k-r)$ where $0 \le r \le \widetilde{k} - 1$. The implementation involves the following steps in each iteration:

$$\begin{aligned}
\mathbf{d}(k|k-1) &= \mathbf{F}\mathbf{d}(k-1|k-1),\\
\mathbf{P}(k|k-1) &= \mathbf{F}\mathbf{P}(k-1|k-1)\mathbf{F}^T + \mathbf{G}\mathbf{G}^T,\\
\mathbf{K}(k) &= \mathbf{P}(k|k-1)\mathbf{A}\big(\mathbf{I}_L + \mathbf{A}\mathbf{P}(k|k-1)\mathbf{A}^T\big)^{-1},\\
\mathbf{d}(k|k) &= \mathbf{d}(k|k-1) + \mathbf{K}(k)\big(\mathbf{r}(k) - \mathbf{A}\mathbf{d}(k|k-1)\big),\\
\mathbf{P}(k|k) &= \Big(\mathbf{I}_{\widetilde{k}K\times\widetilde{k}K} - \mathbf{K}(k)\mathbf{A}\Big)\mathbf{P}(k|k-1).
\end{aligned} \tag{10}$$

In (10), $\mathbf{d}(k|k-1)$ and $\mathbf{d}(k-1|k-1)$ are the predicted and the estimated values of the state vector $\mathbf{d}(k)$ while $\mathbf{P}(k|k-1)$ and $\mathbf{P}(k-1|k-1)$ are the corresponding error covariance matrices. $\mathbf{K}(k)$ is the so-called Kalman gain [33].

2.2.2. Non-Gaussian state noise

Many works based on an approximate DS-CDMA state-space representation proposed the use of the Kalman algorithm as a multiuser detection for its recursive nature which is more suitable for a real-time implementation [6, 7, 34]. However, the derivation in (10) makes use of the Gaussian hypothesis of the signals, that is, the observation noise $\mathbf{b}(k)$ and the state noise $\mathbf{Gx}(k)$. This is not valid in our case for the plant noise ($\mathbf{Gx}(k)$) which is by definition formed by a set of discrete transmitted symbols. Its probability density function (pdf) will be a set of impulses centered on the possible states.

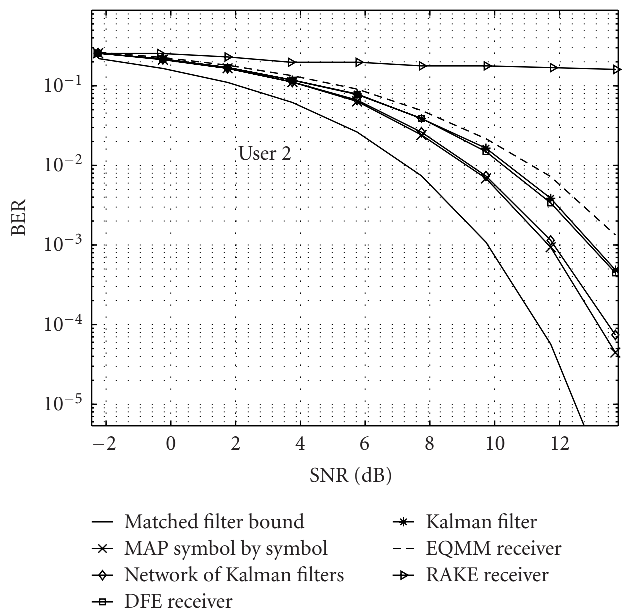

Figure 1: Performance of the proposed NKF detector compared to the RAKE, MMSE, DFE, Kalman, and MAP receivers: $K = 3$, $L = 7$ and $\widetilde{k} = 2$.

The Kalman filter approximates the first and the second orders of the exact pdf [27, 35]. The Kalman filter ignores the binary character of the state noise and loses its optimality.

In order to overcome this problem, and by supposing that the observation noise ($\mathbf{b}(k)$) is Gaussian, we presented in [36] a solution based on the approximation of the *a posteriori* probability of the state vector $p(\mathbf{d}(k)|\mathbf{R}(k))$ by a weighted sum of Gaussian terms (see the appendix) where each Gaussian term parameter adjusted using one Kalman filter. This approach was initially proposed in [25] and simplified [27] for linear channel equalization in a single-user communication system. A generalization to the asynchronous multiuser detection was first proposed in [36] where we show that the resulting structure is a network of Kalman filters operating in parallel.

From Figure 1, we notice that the NKF detector improves the performance in terms of bit error rate (BER) compared to the classical Kalman filter (see (10)) which ignores the digital character of the state noise, the RAKE receiver, the MMSE block receiver, and the DFE receiver [37]. The resulting performance is near the optimal maximum a posteriori (MAP) symbol-by-symbol detector [38]. The simulation is conducted by considering $K = 3$ users, $L = 7$ as a spreading factor, gold sequences, a multipath nonsymmetric channel ($H(z) = 0.802 + 0.535 \times z^{-1} + 0.267 \times z^{-2}$), and an access delay for each user equal to 0, 2, and 4 chips, respectively. In this case we have two interfering symbols: $\widetilde{k} = 2$. We incorporate a delay estimation equal to 1 symbol.

2.2.3. Impulsive channel model

In many communication channels, the observation noise exhibits Gaussian as well as impulsive characteristics. The source of impulsive noise may be either natural (e.g., lightnings) or man made. It might come from relay contacts, electromagnetic devices, transportation systems, and so forth. The empirical data indicate that the probability density functions (pdfs) of the impulsive noise processes exhibits a similarity to the Gaussian pdf, being bell-shaped, smooth, and symmetric but at the same time having significantly heavier tails.

In this paper, we adopt the commonly used Gaussian mixture model or ε-contaminated model for the additive noise samples $\{\mathbf{b}_j(k)\}$ which is a tractable empirical model for impulsive environments. The ε-contaminated model is frequently used to describe a noise environment that is nominally Gaussian with an additive impulsive noise component. Therefore, let the channel noise $b(k) = w(k) + v(k)$ where $w(k)$ is the background noise with zero mean and variance σ_w^2 and $v(k)$ is the impulsive component which is usually chosen to be more heavily tailed than the density of the background noise. Here, the impulse noise is modeled as in [39]:

$$v(k) = \gamma(k)N(k), \tag{11}$$

where $\{\gamma(k)\}$ stands for a Bernoulli process, a sequence of zeroes and ones with $p(\gamma = 1) = \epsilon$, where ϵ is the contamination constant or the probability that impulses occur. This parameter controls the contribution of the impulsive component in the observation noise. $N(k)$ is a white Gaussian noise with zero mean and variance σ_v^2 such that $\sigma_w^2 \ll \sigma_v^2$. In this paper, we will take $\sigma_v^2 = \kappa\sigma_w^2$ with $\kappa \gg 1$.

Under this model, the probability density of the observation channel noise $b(k) = w(k) + v(k)$ can be expressed as

$$p(b(k)) = (1-\epsilon)\mathcal{N}(0,\sigma_w^2) + \epsilon\mathcal{N}\left(0, (\underbrace{\kappa+1}_{\simeq\kappa})\sigma_w^2\right), \tag{12}$$

where $\mathcal{N}(m_x, \sigma_x^2)$ is the Gaussian density function with mean m_x and variance σ_x^2. $\{b(k)\}$ is called an "ε-contaminated" noise sequence. It serves as an approximation to the more fundamental Middleton class A noise model [14].

We propose to study the impact of the impulsive noise on the performance of some multiuser detectors. Especially, we focus on its impact on the performance of the Kalman-based detector and the NKF-based detector where both are optimized under a Gaussian observation noise hypothesis.

Figure 2 plots the BER versus SNR in dB defined as E_b/σ_w^2, where E_b denotes the bit energy. We consider the presence of $K = 3$ users with $L = 7$ as a spreading factor (gold codes). We consider for simplicity the downlink where we have a Rayleigh multipath channel described here by the standard deviation of its coefficients: $[0.227; 0.460; 0.688; 0.460; 0.227]$.

Comparing the impulsive non-Gaussian channel to the Gaussian one, the curves indicate a degradation in the BER performance. This is an expected result that has been observed in many previous studies for other multiuser detectors [31, 40].

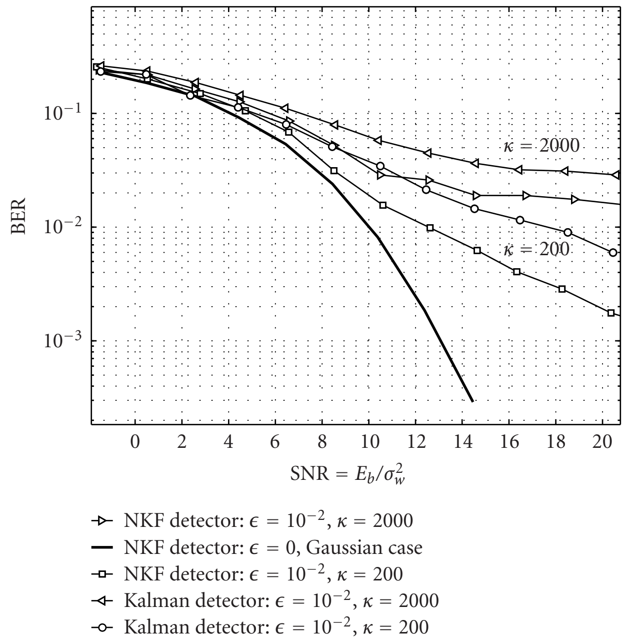

Figure 2: Performance of the NKF detector and the Kalman filter in the presence of an impulsive observation noise: $K = 3$, $N = 7$, $\epsilon = 10^{-2}$, $\kappa = 200$ and 2000.

In conclusion, the generalization of the classical multiuser detector initially optimized under a Gaussian framework is not immediate. The scope of this paper is to robustify the Kalman-filter-based detector to a general framework of non-Gaussian state and measurement noises. The proposed study yields to two novel algorithms which are able to correct the impulsive noise without clipping the received signal as is done in many previous works [29, 30, 31, 32, 41].

3. ROBUST RECURSIVE SYMBOL ESTIMATION BASED ON A NETWORK OF KALMAN FILTERS

The optimal detector computes recursively the *a posteriori* pdf $p(\mathbf{d}(k)|\mathbf{R}^k)$ of the state vector $\mathbf{d}(k)$ given all the observations $\mathbf{r}(k)$ collected up to the current time k, denoted here by $\mathbf{R}^k = [\mathbf{r}(k), \mathbf{r}(k-1), \dots, \mathbf{r}(0)]$. The recursion on $p(\mathbf{d}(k)|\mathbf{R}^k)$ is explicitly given by the following Bayes relations:

$$\begin{aligned} &p(\mathbf{d}(k) \mid \mathbf{R}^k) \\ &\quad = \theta_k p(\mathbf{d}(k) \mid \mathbf{R}^{k-1}) p(\mathbf{r}(k) \mid \mathbf{d}(k)), \end{aligned} \tag{13}$$

$$\begin{aligned} &p(\mathbf{d}(k) \mid \mathbf{R}^{k-1}) \\ &\quad = \int p(\mathbf{d}(k) \mid \mathbf{d}(k-1)) p(\mathbf{d}(k-1) \mid \mathbf{R}^{k-1}) d\mathbf{d}(k-1), \end{aligned} \tag{14}$$

where the normalizing constant θ_k is given by

$$\begin{aligned} \frac{1}{\theta_k} &= p(\mathbf{r}(k) \mid \mathbf{R}^{k-1}) \\ &= \int p(\mathbf{r}(k) \mid \mathbf{d}(k)) p(\mathbf{d}(k) \mid \mathbf{R}^{k-1}) d\mathbf{d}(k). \end{aligned} \tag{15}$$

The densities $p(\mathbf{r}(k)|\mathbf{d}(k))$ and $p(\mathbf{d}(k)|\mathbf{d}(k-1))$ are determined from (6) and (7) and the *a priori* distributions of $\mathbf{d}(k)$ and $\mathbf{b}(k)$. However, it is generally impossible to determine $p(\mathbf{d}(k)|\mathbf{R}^k)$ in a closed form using (13) and (14), except when the *a priori* distributions are Gaussian, in which case the Kalman filter is then the solution.

We propose here to approximate the *a posteriori* probability density function (pdf) of a sequence of delayed symbols by a WSG and to exploit the Gaussian mixture of the observation noise.

With knowledge of the channel-code matrix $\mathbf{A}$ and the parameters of the measurement noise (i.e., ϵ, κ, σ_w^2), the state vector $\mathbf{d}(k)$ is estimated from the observations collected in $\mathbf{R}^k$. The estimate of $\mathbf{x}(k)$ can be obtained at some delayed time $(k-r)$ where $0 \le r \le \widetilde{k}-1$. The development presented in this section considers, without loss of generality, a BPSK modulation.

We approximate the predicted pdf $p(\mathbf{d}(k)|\mathbf{R}^{k-1})$ by a WSG where the weights are denoted by α_i':

$$p(\mathbf{d}(k)\,|\,\mathbf{R}^{k-1}) = \sum_{i=1}^{\xi'(k)} \alpha_i'(k)\mathcal{N}(\mathbf{d}(k)-\mathbf{d}_i(k|k-1),\mathbf{P}_i(k|k-1)), \tag{16}$$

where $\{\mathbf{d}_i(k|k-1)\}_{i=1,\dots,\xi'(k)}$ and $\{\mathbf{P}_i(k|k-1)\}_{i=1,\dots,\xi'(k)}$ are vectors and matrices of dimensions $\widetilde{k}K \times 1$ and $\widetilde{k}K \times \widetilde{k}K$, respectively, and, where the matrices $\mathbf{P}_i(k|k-1)$ approach the zero matrix. Using the pdf of the noise (12), the likelihood of the observation $p(\mathbf{r}(k)|\mathbf{d}(k))$ can be written as a sum of two Gaussian terms:

$$\begin{aligned} p(\mathbf{r}(k)\,|\,\mathbf{d}(k)) \simeq{}& (1-\epsilon)\mathcal{N}(\mathbf{r}(k)-\mathbf{A}\mathbf{d}(k),\sigma_w^2\mathbf{I}_{L\times L}) \\ &+\epsilon\mathcal{N}(\mathbf{r}(k)-\mathbf{A}\mathbf{d}(k),\kappa\sigma_w^2\mathbf{I}_{L\times L}). \end{aligned} \tag{17}$$

By replacing (17), (16) in (13) and by denoting $\lambda_1 = 1-\epsilon$, $\lambda_2 = \epsilon$, $\sigma_1^2 = \sigma_w^2$, and $\sigma_2^2 = \kappa\sigma_w^2$, we get

$$p(\mathbf{d}(k)\,|\,\mathbf{R}^k) = \theta_k \sum_{i=1}^{\xi'(k)}\sum_{j=1}^{2}\lambda_j\alpha_i'(k)\Lambda_{i,j}, \tag{18}$$

where

$$\begin{aligned} \Lambda_{i,j} ={}& \mathcal{N}(\mathbf{d}(k)-\mathbf{d}_i(k|k-1),\mathbf{P}_i(k|k-1)) \\ &\times\mathcal{N}(\mathbf{r}(k)-\mathbf{A}\mathbf{d}(k),\sigma_j^2\mathbf{I}_{L\times L}), \end{aligned} \tag{19}$$

where $\times$ denotes the multiplication operator.

Based on the development done in [27], we define

$$\mathbf{P}_{i,j}(k|k) = \left[\mathbf{P}_i(k|k-1)^{-1} + \frac{\mathbf{A}^T\mathbf{A}}{\sigma_j^2}\right]^{-1}. \tag{20}$$

Remark 1. The indices i, j denote the dependence on both the ith Kalman filter parameters and the variance σ_j^2 (Gaussian or impulsive).

By applying the inversion matrix lemma on (20), we obtain

$$\begin{aligned} \mathbf{P}_{i,j}(k|k) &= \mathbf{P}_i(k|k-1) - \mathbf{K}_{i,j}(k)\mathbf{A}\mathbf{P}_i(k|k-1), \\ \mathbf{K}_{i,j}(k) &= \mathbf{P}_i(k|k-1)\mathbf{A}^T\left[\sigma_j^2\mathbf{I}_{L\times L}+\mathbf{A}\mathbf{P}_i(k|k-1)\mathbf{A}^T\right]^{-1}. \end{aligned} \tag{21}$$

We now introduce

$$\mathbf{d}_{i,j}(k|k) = \mathbf{d}_i(k|k-1)+\mathbf{K}_{i,j}(k)[\mathbf{r}(k)-\mathbf{A}\mathbf{d}_i(k|k-1)]. \tag{22}$$

By doing some rearrangements, we can show that $p(\mathbf{d}(k)|\mathbf{R}^k)$ can be written as a WSG:

$$p(\mathbf{d}(k)\,|\,\mathbf{R}^k) = \sum_{i=1}^{\xi'(k)}\sum_{j=1}^{2}\alpha_{i,j}(k)\mathcal{N}(\mathbf{d}(k)-\mathbf{d}_{i,j}(k|k),\mathbf{P}_{i,j}(k|k)) \tag{23}$$

with

$$\begin{aligned} \mathbf{d}_{i,j}(k|k) &= \mathbf{d}_i(k|k-1)+\mathbf{K}_{i,j}(k)[\mathbf{r}(k)-\mathbf{A}\mathbf{d}_i(k|k-1)], \\ \alpha_{i,j}(k) &= \frac{\lambda_j\alpha_i'(k)\beta_{i,j}(k)}{\sum_{i=1}^{\xi'(k)}\alpha_i'(k)\sum_{j=1}^{2}\lambda_j\beta_{i,j}(k)}, \\ \beta_{i,j}(k) &= \mathcal{N}(\mathbf{r}(k)-\mathbf{A}\mathbf{d}_i(k|k-1),\,\sigma_j^2\mathbf{I}_{L\times L}+\mathbf{A}\mathbf{P}_i(k|k-1)\mathbf{A}^T), \\ \mathbf{K}_{i,j}(k) &= \mathbf{P}_i(k|k-1)\mathbf{A}^T\left[\sigma_j^2\mathbf{I}_{L\times L}+\mathbf{A}\mathbf{P}_i(k|k-1)\mathbf{A}^T\right]^{-1}, \\ \mathbf{P}_{i,j}(k|k) &= \mathbf{P}_i(k|k-1) - \mathbf{K}_{i,j}(k)\mathbf{A}\mathbf{P}_i(k|k-1). \end{aligned} \tag{24}$$

For the next iteration, the predicted pdf $p(\mathbf{d}(k+1)|\mathbf{R}^k)$ is computed according to the Bayesian relation in (14):

$$p(\mathbf{d}(k+1)|\mathbf{R}^k) = \int p(\mathbf{d}(k)\,|\,\mathbf{R}^k)\,p(\mathbf{d}(k+1)\,|\,\mathbf{d}(k))\,d\mathbf{d}(k) \tag{25}$$

with

$$p(\mathbf{d}(k+1)\,|\,\mathbf{d}(k)) = p(\mathbf{G}\mathbf{x}(k+1)). \tag{26}$$

The *a priori* density function of the plant noise $\mathbf{G}\mathbf{x}(k+1)$ is also supposed to be approximated by a weighted sum of Gaussian density functions. $\mathbf{x}(k+1)$ has $\{\mathbf{x}_q\}_{1\le q\le 2^K}$ values associated with the probabilities $\{p_q\}_{1\le q\le 2^K}$. Then the density function of $\mathbf{x}(k+1)$ is

$$p(\mathbf{x}(k+1)) = \begin{cases} p_q & \text{if } \mathbf{G}\mathbf{x}(k+1) = \mathbf{x}_q,\ 1\le q\le 2^K, \\ 0 & \text{otherwise.} \end{cases} \tag{27}$$

This density function is approximated by a WSG density function centered on the discrete values $\{\mathbf{G}\mathbf{x}_l\}_{1\le l\le 2^K}$.

This assumption yields to

$$p(\mathbf{Gx}(k+1)) = \sum_{q=1}^{2^K} p_q \mathcal{N}(\mathbf{Gx}(n+1) - \mathbf{Gx}_q, \boldsymbol{\Delta}_q) \quad (28)$$

with $p_q = 1/2^K$ and $\boldsymbol{\Delta}_q = \epsilon_0 \mathbf{G}\mathbf{G}^T$ ($\epsilon_0 \ll 1$),[2] ϵ_0 is chosen small enough so that each Gaussian density function is located on a neighborhood of $\mathbf{Gx}_q$ with a probability mass equal to p_q.

Now, We denote

$$\begin{aligned} \mathbf{d}_{i,j,q}(k+1|k) &= \mathbf{F}\mathbf{d}_{i,j}(k|k) + \mathbf{Gx}_q, \\ \mathbf{P}_{i,j,q}(k+1|k) &= \mathbf{F}\mathbf{P}_{i,j}(k|k)\mathbf{F}^T + \boldsymbol{\Delta}_q. \end{aligned} \quad (29)$$

We can show that $p(\mathbf{d}(k+1)|\mathbf{R}^k)$ can be obtained as

$$\begin{aligned} p(\mathbf{d}(k+1)\,|\,\mathbf{R}^k) &= \sum_{j=1}^{2} \sum_{i=1}^{\xi'(k)} \sum_{q=1}^{2^K} \alpha'_{i,j,q}(k+1) \\ &\quad \times \mathcal{N}(\mathbf{d}(k+1) - \mathbf{d}_{i,j,q}(k+1|k), \\ &\quad \mathbf{P}_{i,j,q}(k+1|k)), \end{aligned} \quad (30)$$

where $\alpha'_{i,j,q}(k+1) = p_q \alpha_{i,j}(k)$.

Finally, we can resume the algorithm as follows, by supposing that we have, at iteration $(k-1)$, $\xi(k-1)$ Gaussian terms in the expression given by (16).

(i) *Prediction*:

$$\begin{aligned} \xi'(k) &= \xi(k-1) \times 2^K, \\ \alpha'_{i,j,q}(k) &= p_q \alpha_{i,j}(k-1), \\ \mathbf{d}_{i,j,q}(k|k-1) &= \mathbf{F}\mathbf{d}_{i,j}(k-1|k-1) + \mathbf{Gx}_q, \\ \mathbf{P}_{i,j,q}(k|k-1) &= \mathbf{F}\mathbf{P}_{i,j}(k-1|k-1)\mathbf{F}^T + \boldsymbol{\Delta}_q. \end{aligned} \quad (31)$$

(ii) *Estimation*:

$$\begin{aligned} \xi(k) &= 2\xi'(k), \\ \mathbf{d}_{i,j,q}(k|k) &= \mathbf{d}_{i,j,q}(k|k-1) \\ &\quad + \mathbf{K}_{i,j,q}(k)[\mathbf{r}(k) - \mathbf{A}\mathbf{d}_{i,j,q}(k|k-1)], \\ \mathbf{P}_{i,j,q}(k|k) &= \mathbf{P}_{i,j,q}(k|k-1) \\ &\quad - \mathbf{K}_{i,j,q}(k)\mathbf{A}\mathbf{P}_{i,j,q}(k|k-1), \\ \mathbf{K}_{i,j,q}(k) &= \frac{\mathbf{P}_{i,j,q}(k|k-1)\mathbf{A}^T}{\sigma_j^2 \mathbf{I}_{L\times L} + \mathbf{A}\mathbf{P}_{i,j,q}(k|k-1)\mathbf{A}^T}, \\ \alpha_{i,j,q}(k) &= \frac{\lambda_j \alpha'_{i,j,q}(k)\beta_{i,j,q}(k)}{\sum_{j=1}^{2}\sum_{i=1}^{\xi(k-1)}\sum_{q=1}^{2^K} \lambda_j \alpha'_{i,j,q}(k)\beta_{i,j,q}(k)}, \\ \beta_{i,j,q}(k) &= \mathcal{N}(\mathbf{r}(k) - \mathbf{A}\mathbf{d}_{i,j,q}(k|k-1), \\ &\quad \sigma_j^2 \mathbf{I}_{L\times L} + \mathbf{A}\mathbf{P}_{i,j,q}(k|k-1)\mathbf{A}^T). \end{aligned} \quad (32)$$

The estimated state vector $\hat{\mathbf{d}}(k|k)$ of the state vector $\mathbf{d}(k)$ solution of the minimum mean-square error estimation problem is given by the conditional expectation $E(\mathbf{d}(k)|\mathbf{R}^k)$. The MMSE solution is the convex combination of $\xi(k)$ Kalman filters operating in parallel:

$$\begin{aligned} \hat{\mathbf{d}}_{\text{MMSE}}(k|k) &= \sum_{j=1}^{2} \sum_{i=1}^{\xi(k-1)} \sum_{q=1}^{2^K} \alpha_{i,j,q}(k)\mathbf{d}_{i,j,q}(k|k) \\ &= \sum_{i,j,q}^{\xi(k)} \alpha_{i,j,q}(k)\mathbf{d}_{i,j,q}(k|k). \end{aligned} \quad (33)$$

Each predicted state $\mathbf{d}_{i,j,q}(k|k)$ at the output of the Kalman filter indexed by (i, j, q) is weighted by a coefficient $\alpha_{i,j,q}$ that depends on the probability of appearance of the impulsive noises ($1-\epsilon$ or ϵ) and the density $\beta_{i,j,q}$. The algorithm contains an implicit localization mechanism of impulses in the received signal via the $\beta_{i,j,q}$ terms. When an impulse occurs in the observation window $\mathbf{r}(k)$, the $\beta_{i,1,q}$ terms tend to zero, otherwise, the $\beta_{i,2,q}$ terms tend to zero. Therefore, the algorithm tries to extract the information symbol even when the impulses are present by exploiting the memory in the received signal introduced by the ISI channel.

The associated error covariance matrix $\overline{\mathbf{P}}(k)$ is defined as

$$E\Big[(\mathbf{d}(k) - \hat{\mathbf{d}}_{\text{MMSE}}(k|k))(\mathbf{d}(k) - \hat{\mathbf{d}}_{\text{MMSE}}(k|k))^T\Big]. \quad (34)$$

It yields the following equation:

$$\begin{aligned} \overline{\mathbf{P}}(k) &= \sum_{i,j,q}^{\xi(k)} \alpha_{i,j,q}\Big(\mathbf{P}_{i,j,q}(k|k) + (\mathbf{d}_{i,j,q}(k|k) - \hat{\mathbf{d}}_{\text{MMSE}}(k|k)) \\ &\quad \times (\mathbf{d}_{i,j,q}(k|k) - \hat{\mathbf{d}}_{\text{MMSE}}(k|k))^T\Big). \end{aligned} \quad (35)$$

The complexity of the algorithm, evaluated by $\xi(k)$, grows exponentially through iterations. To make it of practical use for on-line processing, the sum giving $p(\mathbf{d}(k)|\mathbf{R}^k)$ in (23) will be constrained to contain only one term after the filtering step, let $\xi(k) = 1$. This is done by setting $\mathbf{d}_i(k|k)$, to the value of the estimated state $\hat{\mathbf{d}}_{\text{MMSE}}(k)$, $\mathbf{P}_i(k|k)$, to $\overline{\mathbf{P}}(k)$ and $\alpha_{i,j,q}(k)$ to 1 for the next prediction step. This approximation is rational because the *a posteriori* state pdf is assumed to be localized around the MMSE estimation.

The resulting algorithm, called the *extended NKF detector* and shown in Figure 3, can be viewed as two NKF detectors working in parallel, where each of them is optimized for the observation Gaussian noise case. The two NKF detectors are coupled via the contamination constant. The first NKF detector considers σ_w^2 as the background noise whereas the second considers $\kappa\sigma_w^2$ as the background noise. The commutation between the two NKF detectors is governed by the $(\beta_{i,j,q})$ terms.

The study of performance of the proposed structure is presented in Section 6. We notice that the *extended* NKF-MMSE structure jointly cancels the MAI and the ISI.

[2]In case of unit symbol variance $\boldsymbol{\Delta}_q = \epsilon_0 \mathbf{I}_{\tilde{k}K}$ ($\epsilon_0 \ll 1$).

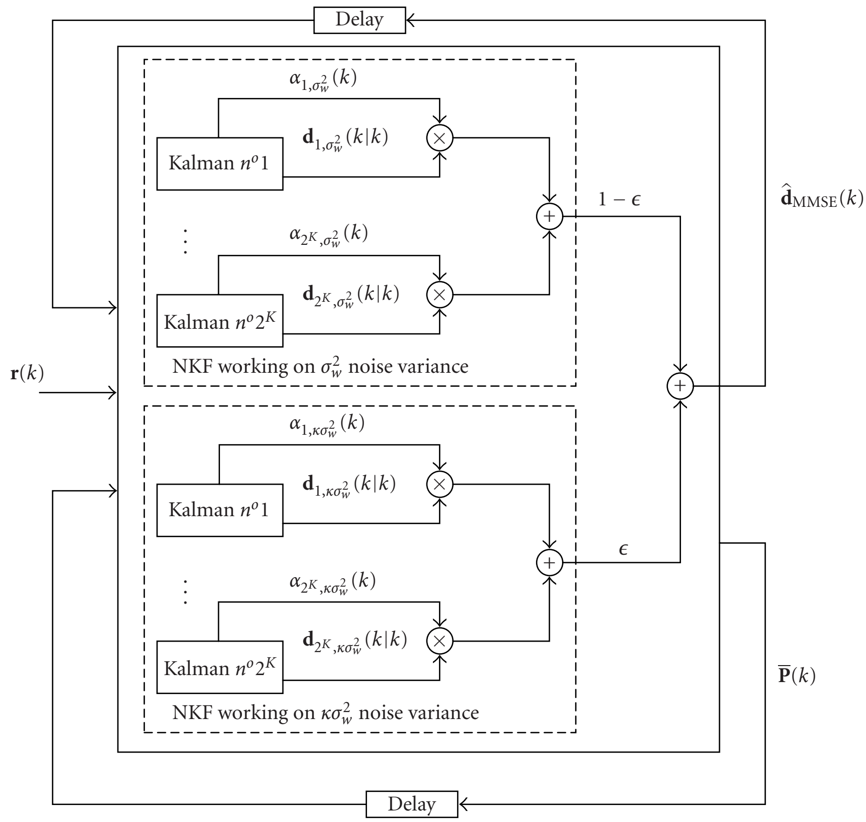

FIGURE 3: The proposed *extended* NKF-MMSE detector structure.

In case of $\epsilon = 0$, that is, the observation noise is Gaussian, the proposed structure reduces to one NKF.

In the second part of this paper, we propose a modified version of the *extended* NKF structure by incorporating a feedback based on a likelihood ratio test. In the next section, we first present the localization procedure of the impulses in the received signal based on a classical hypothesis test.

4. IMPULSE LOCALIZATION BASED ON A LIKELIHOOD RATIO TEST

The detection of impulses in the received signal can be cast as a binary hypothesis testing problem as follows:

(i) H_1: presence of impulsive noise,
(ii) H_0: absence of impulsive noise.

Denote by p_0 and p_1 the *a priori* probability associated with H_0 and with H_1 (i.e., $p_0 + p_1 = 1$). Thus, each time the experiment is conducted, one of these four alternatives can happen: (i) choose H_0 when H_0 is true, (ii) choose H_1 when H_1 is true, (iii) choose H_0 when H_1 is true, (iv) choose H_1 when H_0 is true.

The first and the second alternatives correspond to the correct choices. The third and the fourth alternatives correspond to errors. The purpose of a decision criterion is to attach some relative importance to the four alternatives and reduce the risk of an incorrect decision. Since we have assumed that the decision rule must say either H_0 or H_1, we can view it as a rule for dividing the total observation space denoted by Σ into two parts; Σ_0 and Σ_1. In order to highlight the importance relative to each alternative, we introduce the cost's coefficient. We denote the cost of the four courses of action by C_{00}, C_{11}, C_{10}, and C_{01}, respectively. The first subscript indicates the hypothesis chosen and the second the hypothesis that was true. We assume that the cost of a wrong decision is higher than the cost of a correct decision: $C_{01} > C_{11}$ and $C_{10} > C_{00}$.

Usually, we assume that $C_{00} = C_{11} = 0$. Denoting by $\bar{C}$ the risk, we then have

$$\bar{C} = \sum_{i=0}^{1}\sum_{j=0}^{1} C_{ij}\,p(H_i, H_j) = \sum_{i=0}^{1}\sum_{j=0}^{1} C_{ij}\,p(H_i \mid H_j)\,p_j. \tag{36}$$

We suppose that we know the costs C_{ij} and the *a priori* probabilities $p_0 = 1-\epsilon$ and $p_1 = \epsilon$ where ϵ is the probability of impulse occurrence.

To establish the Bayes test, we must choose the decision regions, Σ_0 and Σ_1, in such a manner that the risk $\bar{C}$ will be minimized. By rewriting the risk $\bar{C}$ with the *a priori*

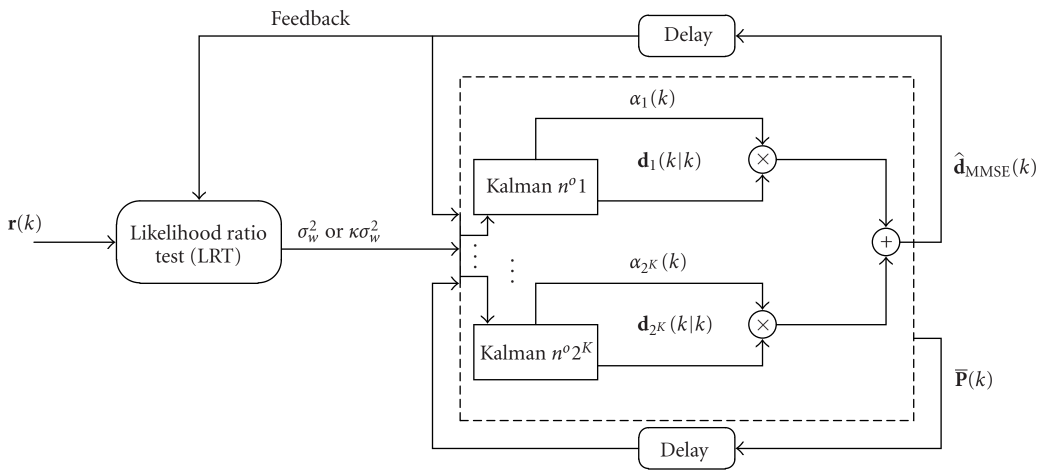

Figure 4: The NKF based on LRT detector structure.

probability and the likelihood, we have

$$\Upsilon(\mathbf{r}) = \frac{P_{R|H_1}(\mathbf{r}\,|\,H_1)}{P_{R|H_0}(\mathbf{r}\,|\,H_0)} \underset{H_0}{\overset{H_1}{\gtrless}} \frac{p_0}{p_1}\frac{C_{10}}{C_{01}}, \tag{37}$$

where $\Upsilon(\mathbf{r})$ is called the likelihood ratio.

The decision rule (37) relies on the comparison of the likelihood ratio to a threshold, which is determined by a cost function and the contamination impulsive noise parameter. If the *a priori* probabilities are unknown, we can use the minmax or Neyman-Pearson criterion [42].

The implementation of the decision rule (37) requires the expression of $P_{R|H_0}(\mathbf{r}|H_0)$ and $P_{R|H_1}(\mathbf{r}|H_1)$ which are based on the knowledge of the unknown state vector $\mathbf{d}(k)$. Therefore, in order to overcome this problem, we exploit the prediction equation $\mathbf{d}_i(k|k-1)$ of each Kalman filter in the NKF structure: $\mathbf{d}_i(k|k-1) = \mathbf{F}\mathbf{d}_i(k-1|k-1)+\mathbf{G}\mathbf{x}_i(k)$. Therefore, we introduce $\tilde{\mathbf{r}}(k)$ defined as follows:

$$\tilde{\mathbf{r}}(k) = \big(\mathbf{r}(k), \mathbf{x}_i(k)\big) = \mathbf{A}\mathbf{d}_i(k|k-1) + \mathbf{b}(k). \tag{38}$$

Each Kalman filter, in the NKF structure, works on the hypothesis that $\mathbf{x}(k) = \mathbf{x}_i(k)$ is transmitted. Therefore, we can determine the expression of $P(\tilde{\mathbf{r}}|H_0, \mathbf{x}(k))$ and $P(\tilde{\mathbf{r}}|H_1, \mathbf{x}(k))$ as follows:

$$\begin{aligned} P\big(\tilde{\mathbf{r}}\,|\,H_0, \mathbf{x}(k)\big) &\propto \mathcal{N}\big(\mathbf{A}\mathbf{d}_i(k|k-1), \mathbf{A}\mathbf{P}_i(k|k-1)\mathbf{A}^T + \sigma_w^2\mathbf{I}_L\big),\\ P\big(\tilde{\mathbf{r}}\,|\,H_1, \mathbf{x}(k)\big) &\propto \mathcal{N}\big(\mathbf{A}\mathbf{d}_i(k|k-1), \mathbf{A}\mathbf{P}_i(k|k-1)\mathbf{A}^T + \kappa\sigma_w^2\mathbf{I}_L\big). \end{aligned} \tag{39}$$

By supposing that the symbols are i.i.d and by taking the expectation over $\mathbf{x}(k)$, the likelihood ratio $\Upsilon(\mathbf{r})$ can be computed as follows:

$$\Upsilon(\mathbf{r}) = \frac{\sum_{i=1}^{2^K}\mathcal{N}\big(\mathbf{A}\mathbf{d}_i(k|k-1), \mathbf{A}\mathbf{P}_i(k|k-1)\mathbf{A}^T + \kappa\sigma_w^2\mathbf{I}_L\big)}{\sum_{i=1}^{2^K}\mathcal{N}\big(\mathbf{A}\mathbf{d}_i(k|k-1), \mathbf{A}\mathbf{P}_i(k|k-1)\mathbf{A}^T + \sigma_w^2\mathbf{I}_L\big)}. \tag{40}$$

The established Bayes test detects the presence of impulses in the received signal.

5. NETWORK OF KALMAN FILTERS BASED ON THE LIKELIHOOD RATIO TEST: DETECTION ALGORITHM

For the optimization of the proposed receiver, we propose to incorporate a decision feedback assuming the knowledge of the adjacent state vectors. We propose here to reject the impulses rather than to clip them as is done in many previous works [29, 30, 31, 32, 41]. In this case, the transmitted symbol estimation at this iteration is taken from the adjacent decided state vector via the proposed feedback. The proposed structure, called the NKF based on likelihood ratio test (LRT), is given in Figure 4.

Compared to the proposed *extended* NKF, we now have only an NKF stage operating on the variance of the observation noise decided by the LRT. In the case of detection of an impulse in the received signal, the following structure rejects the sample $\mathbf{r}(k)$ and exploits the feedback which supposes the knowledge of an adjacent state vector. For clarification, we present in Figure 5 the NKF-based LRT algorithm. We suppose, without loss of generality, that we do not have two successive corrupted received samples.

Suppose that, at iteration $(k-1)$, we detect the presence of an impulse. Therefore, the received sample $\mathbf{r}(k-1)$ is rejected. To estimate the subvector $\mathbf{x}(k-\tilde{k})$, we exploit the same component in the last estimated vector, that is, $\mathbf{d}(k-2|k-2)$ by supposing that this former is consistent.

By assuming that the next received sample $\mathbf{r}(k)$, at iteration k, is impulsive noise free (since we assume that we do not have two successive corrupted received samples), the estimated state vector $\mathbf{d}(k|k)$ is obtained by exploiting the last estimate at time $(k-2)$ and a generalized transition equation of two steps (from $k-2$ to k) (see (41)).

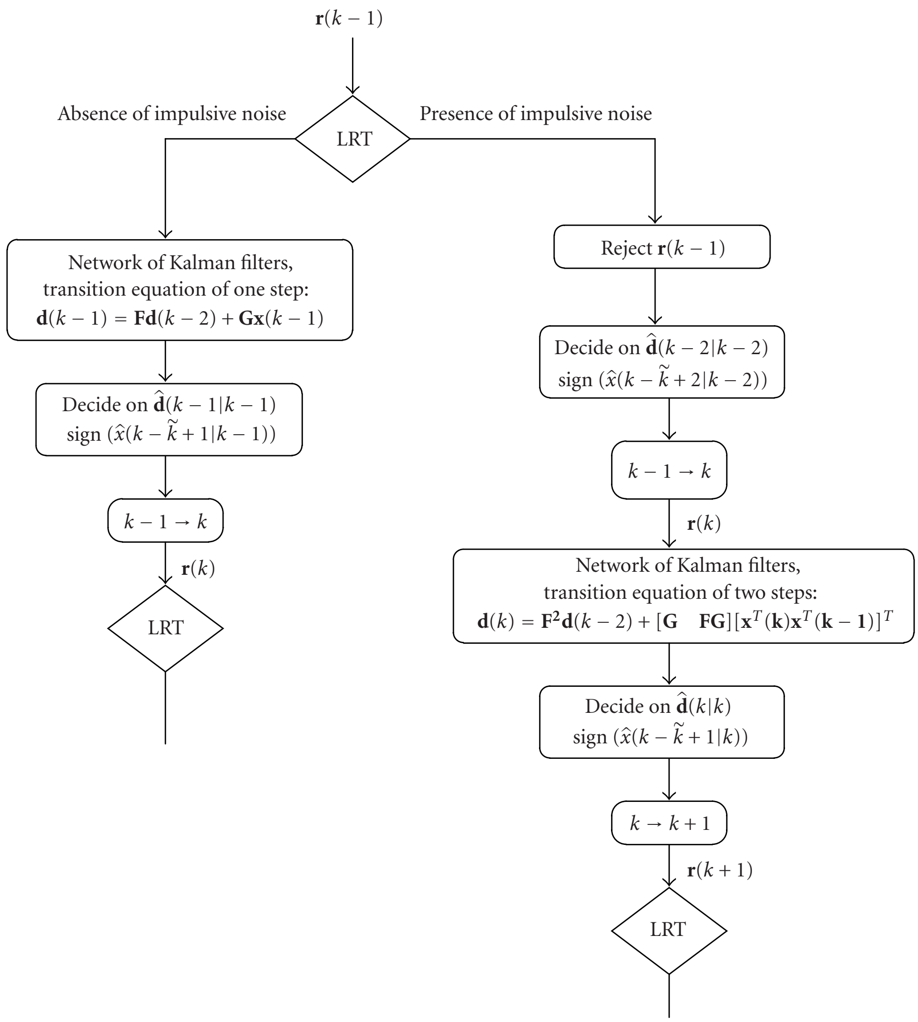

Figure 5: The NKF-based LRT algorithm.

This equation is the generalization of the transition equation of one step (see (7)):

$$\mathbf{d}(k)=\mathbf{F}^2\mathbf{d}(k-2)+[\mathbf{G}\;\;\mathbf{FG}]_{\tilde{k}K\times 2K}\left[\mathbf{x}^T(k)\mathbf{x}^T(k-1)\right]^T_{2K\times 1}. \quad (41)$$

The new plant noise $[\mathbf{x}^T(k)\mathbf{x}^T(k-1)]^T$ is composed of $2K$ components. So, the NKF detector is working on two more hypotheses. An estimate is obtained by combining convexly the output of the Kalman filters in the MMSE sense. After that, the NKF algorithm takes its initial representation based on a prediction equation of one step. The algorithm proposed here can be generalized to many successive impulses, a rare case, by introducing a transition equation of 3, 4, ... steps.

We notice that the proposed algorithm exploits the Kalman structure, especially the prediction equation, and the diversity introduced by the ISI. This is not surprising, since an ISI channel introduces memory to the received signal and the channel essentially serves as a trellis code. When a symbol is hit by a large noise impulse, if the channel is ISI free, then this symbol cannot be recovered; in an ISI channel, however, it is possible to recover this symbol from adjacent received signals.

6. SIMULATION RESULTS

In this section, we assess the performance of the algorithms proposed in the previous sections, namely, the *extended* NKF and NKF based on LRT detectors, via computer simulations. For comparison purposes, and in order to compare our proposed algorithms with the classical approach based on the M estimator of Huber [43], we propose to simulate the NKF-MMSE detector obtained by taking $\epsilon = 0$ in the equation of the *extended* NKF coupled with a nonlinear front end in an attempt to minimize the effect of large noise peaks by eliminating, or at least de-emphasizing, them. For this reason, we consider the following nonlinear clipping functions:

$$\Psi\left(\left[\mathbf{e}(k)\right]_j\right)=\begin{cases}-\tau & \text{if } \left[\mathbf{e}(k)\right]_j<-\tau,\\ \left[\mathbf{e}(k)\right]_j & \text{if } -\tau<\left[\mathbf{e}(k)\right]_j<\tau,\\ \tau & \text{if } \left[\mathbf{e}(k)\right]_j>\tau,\end{cases} \quad (42)$$
$$\text{for } j=1,\ldots,L,$$

where $\mathbf{e}(k) = \mathbf{r}(k) - \mathbf{A}\mathbf{d}_i(k|k-1)$ and $[\mathbf{e}(k)]_j$ represents the jth component of the vector $\mathbf{e}(k)$.

In the NKF-MMSE equations, only the first recursive equation is substituted by the following equation:

$$\mathbf{d}_{i,j}(k|k) = \mathbf{d}_i(k|k-1) + \mathbf{K}_{i,j}(k)\Psi\big([\mathbf{r}(k) - \mathbf{A}\mathbf{d}_i(k|k-1)]\big). \tag{43}$$

The function $\Psi(\cdot)$ is known as Huber's function.

In this paper, we choose a threshold which reflects the effect of impulsive noise:

$$\tau = \sqrt{\sigma_T^2}, \tag{44}$$

where σ_T^2 represents the total noise variance

$$\sigma_T^2 = (1-\epsilon)\sigma_w^2 + \epsilon\kappa\sigma_w^2. \tag{45}$$

Therefore, we propose here to compare

(1) the *extended* NKF-MMSE algorithm,
(2) the NKF + LRT algorithm: exact knowledge of the impulses,
(3) the NKF + LRT algorithm: Bayes test,
(4) the NKF + clipping algorithm,
(5) the NKF-MMSE without any correction of impulsive noise,
(6) the NKF-MMSE in a Gaussian case without impulsive noise, $\epsilon = 0$.

Theses cases are able to give us an idea about the behavior of the proposed algorithms compared to the classical approach (based on the M estimator of Huber) and to study the importance of taking into account the structure of the noise in the problem of the state estimation via the Kalman approach.

Throughout the following example, we consider a synchronous CDMA network employing gold codes of length $L = 7$ as the spreading sequences, $K = 3$ users, and a BPSK modulation. The bit error rate (BER) of the first user is investigated. The channel is a 5-path symmetric Rayleigh channel with the energy profile [0.227; 0.4600; 0.6880; 0.4600; 0.227]. So we have 2 interfering symbols. The estimation delay is taken equal to 1 symbol. The parameter κ is chosen equal to 200, 2000, and 5000. The probability of appearance of the impulsive noise is 10^{-2}. C_{10}/C_{01} is chosen equal to 10.

From Figures 6, 7, and 8, we remark that the performances obtained in the case when we know exactly the appearance instants of the impulses are close to those obtained in the case of Gaussian noise only. However, when we employ the LRT for impulse localization, we remark a loss in the BER at high SNR and, especially, when κ is weak. In fact, the probability of nondetection of impulses increases at high SNR because the Bayes test cannot distinguish the Gaussian noise from the impulsive one if their power are similar.

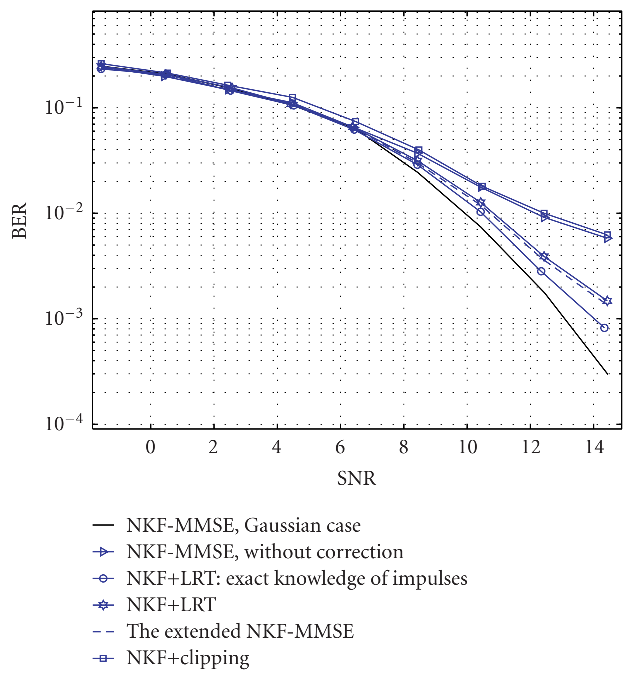

Figure 6: Performance of the proposed algorithms: the *extended* NKF and NKF + LRT in an impulsive environment with $\kappa = 200$ and $\epsilon = 10^{-2}$.

We also remark that the proposed algorithm, NKF based on the LRT, is not very sensitive to the power of the impulsive component in the received signal since the curves corresponding to different simulated κ values are equivalent. On the other hand, the curves related to the NKF + clipping perform like our proposed *extended* NKF detector. But when the value of κ is around 200, our proposed *extended* NKF algorithm is better. The major problem in the clipping is essentially related to the threshold which is determined generally by simulation. It should be noted that for a large threshold level τ, the detection performance should degrade only slightly when the channel noise is actually Gaussian. On the other hand, a smaller value of τ offers an increased robustness against impulses at the cost of deteriorated performance under Gaussian noise. Clearly, the threshold τ controls the trade-off between the degree of robustness and the performance degradation under Gaussian noise.

The simulations presented in Figures 6, 7, and 8 show that the proposed schemes significantly improve the efficiency and filtering performance of the classical Kalman algorithm.

7. CONCLUSION

Two new algorithms have been developed in this paper: the *extended* NKF and NKF based on the LRT detectors for the DS-CDMA system modelized as a discrete-time linear system that have non-Gaussian state and measurement noises. By approximating their non-Gaussian densities by a weighted sum of Gaussian terms and under the common MMSE estimation criterion, the *extended* NKF is derived.

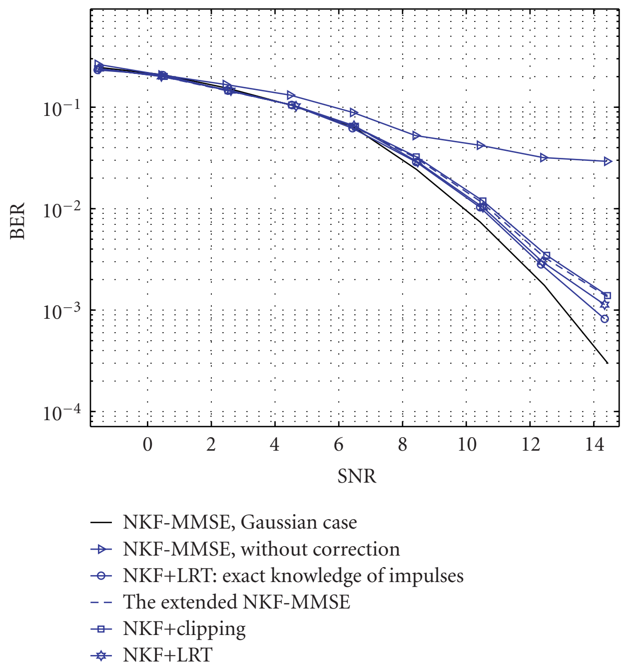

Figure 7: Performance of the proposed algorithms: the *extended* NKF and NKF + LRT in an impulsive environment with $\kappa = 2000$ and $\epsilon = 10^{-2}$.

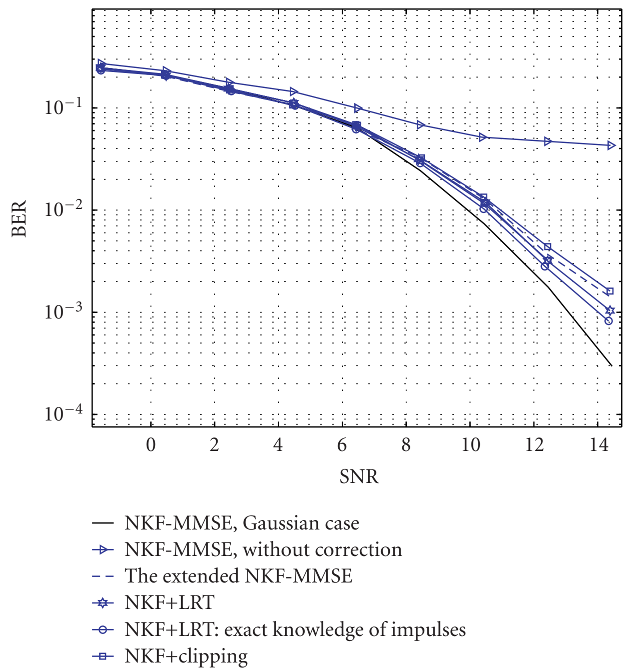

Figure 8: Performance of the proposed algorithms: the *extended* NKF and NKF + LRT in an impulsive environment with $\kappa = 5000$ and $\epsilon = 10^{-2}$.

The resulting multiuser detector cancels jointly the MAI and ISI in the presence of impulsive noise.

We also propose a second version of the NKF detector in which we reduce the complexity of the proposed *extended* NKF detector and improve its performance by incorporating a feedback based on a likelihood ratio test. The NKF based on an LRT scheme exploits the Kalman structure, especially the prediction equation, and the diversity introduced by the ISI. Monte-Carlo simulations have shown that the *extended* NKF detector and the NKF based on an LRT detector significantly improve the efficiency and filtering performance of the classical Kalman algorithm. Some further works are envisaged on this most promising structure, such as the extension to the nonlinear channels.

APPENDIX

GAUSSIAN SUM APPROXIMATION

The Gaussian sum representation p_A [25] of a density function associated with a random m-dimensional vector $\mathbf{y}$ is defined as

$$p_A(y) = \sum_{i=1}^{l} \alpha_i \mathcal{N}(\mathbf{y} - \mathbf{a}_i, \mathbf{G}_i), \tag{A.1}$$

where $\mathcal{N}(\mathbf{a}, \mathbf{G}) = \exp\{-(1/2)\mathbf{a}^T\mathbf{G}^{-1}\mathbf{a}\}/(2\pi)^{n/2}|\mathbf{G}|^{1/2}$ with $\sum_{i=1}^{l} \alpha_i = 1$, $\alpha_i \geq 0$ for all i.

It can be shown in [25] that p_A converges uniformly to any density function of practical concern as the number of terms l increases and the covariance $\mathbf{G}_i$ approaches the zero matrix. In fact, the parameters α_i, $\mathbf{a}_i$, and $\mathbf{G}_i$ can be selected in various ways. For example, the mean values $\mathbf{a}_i$ establish a grid in the region of the state space that contains the probability mass. The α_i are chosen as the values $p(\mathbf{a}_i)$ of the density function $p(\cdot)$ that is to be approximated. The covariance matrices $\mathbf{G}_i$ are determined in order that the approximation error $p - p_A$ is minimized according to a desired criterion.

REFERENCES

[1] R. De Gaudenzi, F. Giannetti, and M. Luise, "Advances in satellite CDMA transmission for mobile and personal communications," *Proc. IEEE*, vol. 84, no. 1, pp. 18–39, 1996.
[2] R. Prasad and T. Ojanpera, "A survey on CDMA: evolution towards wideband CDMA," in *Proc. IEEE 5th International Symposium on Spread Spectrum Techniques and Applications (ISSSTA '98)*, vol. 1, pp. 323–331, Sun City, South Africa, February 1998.
[3] S. Glisic and B. Vucetic, *Spread Spectrum CDMA Systems for Wireless Communications*, Mobile Communications Series. Artech House Publishers, London, UK, 1997.
[4] P. Jung, P. W. Baier, and A. Steil, "Advantages of CDMA and spread spectrum techniques over FDMA and TDMA in cellular mobile radio applications," *IEEE Trans. Veh. Technol.*, vol. 42, no. 3, pp. 357–364, 1993.
[5] S. Verdu, *Multiuser Detection*, Cambridge University Press, New York, NY, USA, 1998.

[6] T. J. Lim and Y. Ma, "The Kalman filter as the optimal linear minimum mean-squared error multiuser CDMA detector," *IEEE Trans. Inform. Theory*, vol. 46, no. 7, pp. 2561–2566, 2000.

[7] T. J. Lim, L. K. Rasmussen, and H. Sugimoto, "An asynchronous multiuser CDMA detector based on the Kalman filter," *IEEE J. Select. Areas Commun.*, vol. 16, no. 9, pp. 1711–1722, 1998.

[8] X. D. Zhang and W. Wei, "Blind adaptive multiuser detection based on Kalman filtering," *IEEE Trans. Signal Processing*, vol. 50, no. 1, pp. 87–95, 2002.

[9] Z. Xu and T. Wang, "Blind detection of CDMA signals based on Kalman filter," in *Proc. 35th Asilomar Conference on Signals, Systems and Computers*, vol. 2, pp. 1545–1549, Pacific Grove, Calif, USA, November 2001.

[10] D. Middleton, "Statistical-physical models of electromagnetic interference," *IEEE Trans. Electromagn. Compat.*, vol. 19, no. 3, pp. 106–127, 1977.

[11] D. Middleton, "Man-made noise in urban environments and transportation systems: models and measurements," *IEEE Trans. Communications*, vol. 1, no. 11, pp. 1232–1241, 1973.

[12] K. L. Blackard, T. S. Rappaport, and C. W. Bostian, "Measurements and models of radio frequency impulsive noise for indoor wireless communications," *IEEE J. Select. Areas Commun.*, vol. 11, no. 7, pp. 991–1001, 1993.

[13] T. K. Blankenship, D. M. Kriztman, and T. S. Rappaport, "Measurements and simulation of radio frequency impulsive noise in hospitals and clinics," in *Proc. IEEE Vehicular Technology Conference (VTC '97)*, vol. 3, pp. 1942–1946, Phoenix, Ariz, USA, May 1997.

[14] D. Middleton, "Non-Gaussian noise models in signal processing for telecommunications: new methods and results for class A and class B noise models," *IEEE Trans. Inform. Theory*, vol. 45, no. 4, pp. 1129–1149, 1999.

[15] S. M. Zabin and H. V. Poor, "Recursive algorithms for identification of impulse noise channels," *IEEE Trans. Inform. Theory*, vol. 36, no. 3, pp. 559–578, 1990.

[16] C. L. Nikias and M. Shao, *Signal Processing with Alpha-Stable Distribution and Applications*, John Wiley & Sons, New York, NY, USA, 1995.

[17] A. Spaulding and D. Middleton, "Optimum reception in an impulsive interference environment—part I: coherent detection," *IEEE Trans. Communications*, vol. 25, no. 9, pp. 910–923, 1977.

[18] A. Spaulding and D. Middleton, "Optimum reception in an impulsive interference environment—part II: incoherent reception," *IEEE Trans. Communications*, vol. 25, no. 9, pp. 924–934, 1977.

[19] B. Picinbono, "Spherically invariant and compound Gaussian stochastic processes (Corresp.)," *IEEE Trans. Inform. Theory*, vol. 16, no. 1, pp. 77–79, 1970.

[20] B. Aazhang and H. V. Poor, "Performance of DS/SSMA communications in impulsive channels—part I: linear correlation receivers," *IEEE Trans. Communications*, vol. 35, no. 11, pp. 1179–1188, 1987.

[21] B. Aazhang and H. V. Poor, "Performance of DS/SSMA communications in impulsive channels—part II: hard-limiting correlation receivers," *IEEE Trans. Communications*, vol. 36, no. 1, pp. 88–97, 1988.

[22] X. Wang and H. V. Poor, "Robust multiuser detection in non-Gaussian channels," *IEEE Trans. Signal Processing*, vol. 47, no. 2, pp. 289–305, 1999.

[23] R. E. Kalman, "A new approach to linear filtering and prediction problems," *Transactions of the ASME Journal of Basic Engineering*, vol. 82, pp. 35–45, 1960.

[24] C. J. Masreliez, "Approximate non-Gaussian filtering with linear state and observation relations," *IEEE Trans. Automat. Contr.*, vol. 20, no. 1, pp. 107–110, 1975.

[25] D. L. Alspach and H. W. Sorenson, "Nonlinear Bayesian estimation using Gaussian sum approximation," *IEEE Trans. Automat. Contr.*, vol. 17, no. 4, pp. 439–448, 1972.

[26] X. Wang and H. V. Poor, "Joint channel estimation and symbol detection in Rayleigh flat-fading channels with impulsive noise," *IEEE Communications Letters*, vol. 1, no. 1, pp. 19–21, 1997.

[27] S. Marcos, "A network of adaptive Kalman filters for data channel equalization," *IEEE Trans. Signal Processing*, vol. 48, no. 9, pp. 2620–2627, 2000.

[28] H. V. Poor and M. Tanda, "Multiuser detection in flat fading non-Gaussian channels," *IEEE Trans. Communications*, vol. 50, no. 11, pp. 1769–1777, 2002.

[29] H.-S. Kim, J.-S. Lim, S. Baek, and K.-M. Sung, "Robust Kalman filtering with variable forgetting factor against impulsive noise," *IEICE Transactions on Fundamentals*, vol. E84-A, no. 1, pp. 363–366, 2001.

[30] P. Spasojevic and X. Wang, "Improved robust multiuser detection in non-Gaussian channels," *IEEE Signal Processing Lett.*, vol. 8, no. 3, pp. 83–86, 2001.

[31] A. M. Zoubir and A. T. Lane-Glover, "Multiuser detection in impulsive noise," in *Proc. 11th IEEE Signal Processing Workshop on Statistical Signal Processing*, pp. 102–105, Singapore, August 2001.

[32] T. C. Chuah, B. S. Sharif, and O. R. Hinton, "Nonlinear decorrelator for multiuser detection in non-Gaussian impulsive environments," *Electronics Letters*, vol. 36, no. 10, pp. 920–922, 2000.

[33] E. W. Kalman and J. K. Su, *Introduction to Optimal Estimation*, Springer-Verlag, New York, NY, USA, 1999.

[34] J. Thielecke, "A state-space multiuser detector for CDMA systems," in *Proc. IEEE International Conference on Communications (ICC '93)*, vol. 3, pp. 1762–1767, Geneva, Switzerland, March 1993.

[35] G. Chen, *Approximate Kalman Filtering*, World Scientific, Singapore, 1993.

[36] B. Sayadi and S. Marcos, "A network of Kalman filters based detector for joint MAI and ISI suppression in asynchronous DS-CDMA systems," in *Proc. IEEE International Symposium on Spread Spectrum Techniques and Applications (ISSSTA '02)*, vol. 2, pp. 511–515, Prague, Czech Republic, September 2002.

[37] C. Tidestav, "Designing equalizers based on explicit channel models of direct-sequence code-division multiple access systems," Tech. Rep. UPTEC 96025R, Uppsala university, Sweden, 1996.

[38] E. Baccarelli, A. Fasano, and A. Zucchi, "A reduced-state soft-statistics-based MAP/DF equalizer for data transmission over long ISI channels," *IEEE Trans. Communications*, vol. 48, no. 9, pp. 1441–1446, 2000.

[39] M. Ghosh, "Analysis of the effect of impulse noise on multicarrier and single carrier QAM systems," *IEEE Trans. Communications*, vol. 44, no. 2, pp. 145–147, 1996.

[40] H. V. Poor and M. Tanda, "An analysis of some multiuser detectors in impulsive noise," in *Proc 16th GRETSI Symposium on Signal Images Processing*, Grenoble, France, September 1997.

[41] R. Pupeikis, "On the optimization of an adaptive state estimation of dynamic systems in the presence of time-varying outliers," *Informatica*, vol. 13, no. 1, pp. 89–104, 2002.

[42] H. L. Van Trees, *Detection, Estimation, and Modulation Theory*, John Wiley & Sons, New York, NY, USA, 1968.

[43] P. J. Huber, *Robust Statistics*, vol. 43, John Wiley & Sons, New York, NY, USA, 1981.

Bessem Sayadi received the B.S.E.E. degree in signal processing from the École Supérieure des Télécommunications de Tunis (Sup'Com Tunis), Tunisia, in 1999, and both the M.Phil. (2000) and the Ph.D. (2003) degrees from the Signals and Systems Laboratory (LSS) at Supélec, Gif-sur-Yvette, the Paris XI University, Orsay, France. In 1999, he joined France Telecom, a research department in France, where he was engaged in research on echo cancellation and adaptive filtering. He has also served as a Teaching Assistant in several courses on communications, signal processing, and electronics in the Department of Electronic and Electrical Engineering, Supélec, ENSEA, and University Paris IX, since September 2000. From 2003 until now, he is an Associate Researcher in the Image and Signal Processing Team (ETIS), ENSEA, Cergy-Pontoise. His current research interests include adaptive filtering, linear and nonlinear channel equalization, Bayesian method, multiuser detection for CDMA systems, wireless packet, and cross-layer design.

Sylvie Marcos received the Engineer degree from the École Centrale de Paris (1984) and both the Doctorate (1987) and the Habilitation (1995) degrees from the Paris XI University, Orsay, France. She is Directeur de Recherche at the National Center for Scientific Research (CNRS) and works in the Signals and Systems Laboratory (LSS) at Supélec, Gif-sur-Yvette, France. Her main research interests are in presently adaptive filtering, linear and nonlinear channel equalization, multiuser detection for CDMA systems, array processing, and spatio-temporal signal processing (STAP) for radar and radiocommunications.