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Data-precoded algorithm for multiple-relay-assisted systems

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Abstract

A data-precoded relay-assisted (RA) scheme is proposed for a system cooperating with multiple relay nodes (RNs), each equipped with either a single-antenna or a two-antenna array. The classical RA systems using distributed space-time/frequency coding algorithms, because of the half-duplex constraint at the relays, require the use of a higher order constellation than in the case of a continuous link transmission from the base station to the user terminal. This implies a penalty in the power efficiency. The proposed precoding algorithm exploits the relation between QPSK and 4^L-QAM, by alternately transmitting through L relays, achieving full diversity, while significantly reducing power penalty. This algorithm explores the situations where a direct path (DP) is not available or has poor quality, and it is a promising solution to extend coverage or increase system capacity. We present the analytical derivation of the gain obtained with the data-precoded algorithm in comparison with distributed space-frequency block code (SFBC) ones. Furthermore, analysis of the pairwise error probability of the proposed algorithm is derived and confirmed with numerical results. We evaluate the performance of the proposed scheme and compare it relatively to the equivalent distributed SFBC scheme employing 16-QAM and non-cooperative schemes, for several link quality scenarios and scheme configurations, highlighting the advantages of the proposed scheme.

1. Introduction

The use of relays is considered an important technology for future wireless systems, because of its potential to increase capacity, extend coverage, and improve access fairness, as well as to provide additional flexibility in the upgrading of the networks [1]. It can be achieved through cooperation of terminals, either dedicated or user terminals acting as relays, which share their antennas and thereby create a virtual multiple-input multiple-output (MIMO) system [2]. These allow single-antenna devices to benefit from spatial diversity without the need for co-located additional physical antenna arrays.

Several cooperative diversity protocols have been proposed and analyzed to demonstrate the potential benefits of cooperation [3-5]. Some authors research the theoretical diversity-multiplexing trade-off of cooperative systems, such as in [6]. Furthermore, in [7] the Rayleigh performance of a single-relay cooperative scenario with multiple-antenna nodes is investigated, deriving pairwise error probability (PEP) expressions. Research has advanced beyond Rayleigh channels, considering more complex channel models for the cooperative channel links, modeled, for example, by Rician or Nagakami-m models, such as in [8,9].

Other works resulted from the association of two high-performance techniques: the use of relaying channels and multiple antennas at the transmitting and receiving sides. Furthermore, most of the extensive literature on cooperative relaying diversity considers that RNs are equipped with a single-antenna, although some works have explored the benefits of multiple antennas in the cooperating nodes. It is fairly easy to deploy multiple antennas arrays in infrastructure-based fixed relay networks, which increases the interest in MIMO relaying [10].

Despite the advantages mentioned in using the RA schemes, they require the use of constellations with higher cardinality in comparison with the continuous link transmission from the base station(BS) to the user terminal(UT), when this is available. This is due to the half-duplex constraint in RNs [3]. Despite achieving full diversity, these schemes cannot achieve full spectral efficiency, since they use two phases for transmission, thus

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achieving half of the bandwidth efficiency of the equivalent non-cooperative systems. Consequently, the use of constellations with more symbols is considered in these cases as a means to achieve the same transmission rates of the non-cooperative ones, but it leads to a power efficiency penalty. Some examples of these RA schemes use distributed orthogonal algorithms, such as the ones in [11-15]. Capacity for a RA system with one and two RNs with single-antenna terminals was studied in [16]. In such study, it was found that the use of relays to assist a communication with the objective of increasing its capacity is only effective in high path-loss scenarios, because of the half-duplex constraint of RA schemes. It was also concluded that RA schemes that do not have transmission through the DP have lower performances than similar ones having such contribution, when the DP has a good transmission quality. For example, non-orthogonal protocols for cooperative systems with two or more relays were developed with the objective of increasing capacity or diversity order of cooperative systems, such as in [17,18]. These proposals require the existence of the DP; therefore, in situations with poor direct link conditions, their performances are significantly degraded and, in case of outage of one relay, some information can be lost. Motivated by the fact that it is common to have large objects or other hindrances affecting the DP, the authors of [19] proposed a new algorithm for these situations, while bringing RA performances close to the continuous link transmission. This algorithm was derived for a two-relay-assisted scheme, exploiting the relation between QPSK and 16-QAM, by alternately transmitting through the two relays, to achieve full diversity and significantly reduce power penalty. Further along the development of cooperative systems, some relay precoder designs were also proposed, however with different goals, such as providing robustness through the use of relays considering imperfect channel state information (CSI) [20,21].

Concerning the system-oriented application of RA schemes, these have been studied for different cases. For cellular systems, RA techniques have been also applied to multicarrier communications, such as in orthogonal frequency-division multiplexing (OFDM) systems. These are widely used for high-speed data transmission in wireless standard technologies, such as Wimax and LTE, because of the advantages mentioned above, and its ability to eliminate ISI. An OFDM-oriented approach is used in this work, since relay networks combined with OFDM technology can make a strong platform for future wireless communications [11,22].

In this article, we extend the work of [19] on data precoded for two-relay-assisted scheme, to data precoded for a generic multiple L-relay case, where each RN is equipped with either one or two antennas. In this algorithm there is no need to transmit through the direct link, in alternative to the non-orthogonal algorithms proposed previously. This is beneficial for most scenarios, since the direct link is usually strongly affected by path loss or shadowing. A data-precoding of the data symbols prior to transmission is performed, followed by decoding at the UT by using the Viterbi algorithm [23]. The theoretical analysis of the PEP of the proposed algorithm is derived and confirmed with numerical results. Moreover, we show the analytical derivation of the gain obtained with the data-precoded algorithm, in comparison with distributed ones. The performance of the proposed scheme is evaluated and compared relatively to distributed space-frequency block code (SFBC) and non-cooperative schemes, for several channel quality scenarios and scheme configurations.

The remainder of the article is organized as follows: in Section 2, a general description of the system model considered is presented. We then describe the proposed algorithm and derive the main link equations in Section 3. Section 4 follows with the derivation of the theoretical gain obtained with the proposed algorithm against the distributed SFBC algorithms, for a generic system configuration. PEP derivation and diversity analysis are shown for the proposed algorithm in Section 5, including the comparison between theoretical and simulation results. Then, in Section 6, the performance of the data precoded algorithm is assessed and compared with the reference cooperative and non-cooperative systems. Finally, we point out the main conclusions in Section 7.

2. System model

Let us consider a general 4G cooperative communication system, in the downlink transmission. The rates required for downlink transmissions are generally higher than for the uplink, and therefore cooperation will be more beneficial when applied to the downlink, reason why we focus on this case. This RA system includes different configurations with different numbers of nodes and antennas. We further consider that there are L RNs cooperating with a BS and a UT, as shown in Figure 1. When L is zero, the system is considered to be non-cooperative. When at least one RN is cooperating with the point-to-point communication, the system can be referred to as RA system.

We assume that the BS and UT are equipped with N_{B} and N_{U} antennas, respectively. RNs are considered to be dedicated and fixed nodes, equipped with N_{R} antennas. In addition, relays are considered to be half-duplex. Since different cooperative schemes can be considered by changing the number of antennas in each terminal, their designation can be simplified to the form RA L RN-$N_{\mathrm{B}} \times N_{\mathrm{R}} \times N_{\mathrm{U}}$. Similarly, the non-cooperative systems are named non-relay-assisted (NRA) schemes with N_{B}

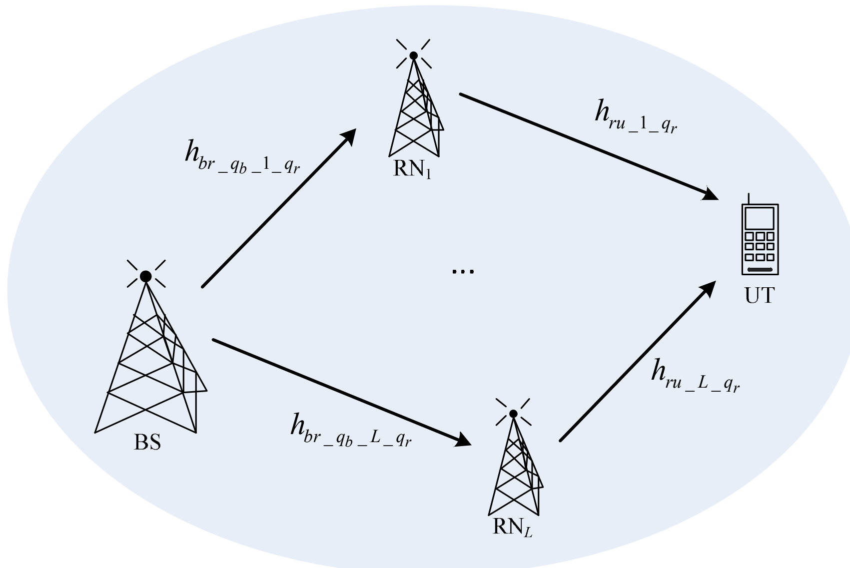

Figure 1 General downlink system with *L* relays cooperating with a BS and a UT.

and N_U antennas at the BS and UT, respectively, which can be generically referred to as NRA $N_B \times N_U$.

In practical systems, the BS is usually equipped with multiple antennas, since the size, cost, and other physical problems are much less stringent than in the UTs. This generally leads to lower bit error rates (BERs) for the links between the BS and the RNs. We consider that the relays are strategically located so that they have a good quality link between the BS and themselves. Furthermore, we can assume to have a selective decode-and-forward relay protocol by considering that each is capable of deciding whether or not it has decoded correctly. If an RN decodes correctly, it will forward the BS data in the second phase, otherwise it remains idle. This can be achieved through the use of cyclic redundancy check codes. This decision can also be approximated by setting a signal-to-noise ratio (SNR) threshold at all the RNs; the RN will only forward the BS data if the received SNR is larger than that threshold [12,24]. Furthermore, we focus our efforts on the special case where the direct link transmission is strongly affected by large-scale losses, such as due to shadowing, and thus no DP is considered for communication.

The expressions modeling the received signals at RNs depend on the space-time-frequency processing at the BS. To simplify, and to allow us to derive theoretical formulas, we assume error-free links between the BS and the RNs, and thus the symbols retransmitted by the RNs are the same as the ones transmitted by the BS. Most of the scenarios consider the BS $\rightarrow$ RN channels as error-free, but we also obtain numerical results assuming non error-free links between those terminals. In this case we assume 2 × 1 space-frequency block coding scheme from BS to each RN. The received signal expressions at the relays were derived in [25].

Since the systems have LN_R independent paths from the relays to the destination, diversity can be achieved. Assuming the half-duplex nature of relays, we consider two algorithms for a RA scheme communication In the first one, distributed SFBC algorithm, we have two phases: in a first phase the BS broadcasts the information to the relays and in the second phase the relays retransmit the received information to the UT, emulating a SFBC in a distributed manner. The flow of signals is described in Figure 2, for the case of single-antenna RNs and an OFDM-based system. The received symbols are represented in blue, while the transmitted ones are in white. The first (second) phase of transmission corresponds to the odd (even) time slots. Concerning the notation used, s_k^p refers to symbols transmitted by the BS at time slot k and subcarrier position p; $z_k^{R_i,p}$ refers to symbols transmitted by the ith RN at time slot k and subcarrier position p; and, y_k^p to the symbols received in the UT. In this approach, spatial diversity can be achieved, but because of the half-duplex constraints of relays, the information has to be transmitted during half of the time that would be needed in the case of a continuous link available from the source to the destination. This means that, assuming that a modulation scheme carrying m bits per symbol could be used in the case when the continuous direct link is available, one would need to switch toward a modulation carrying $2m$ bits per symbol (if the symbol duration was kept identical).

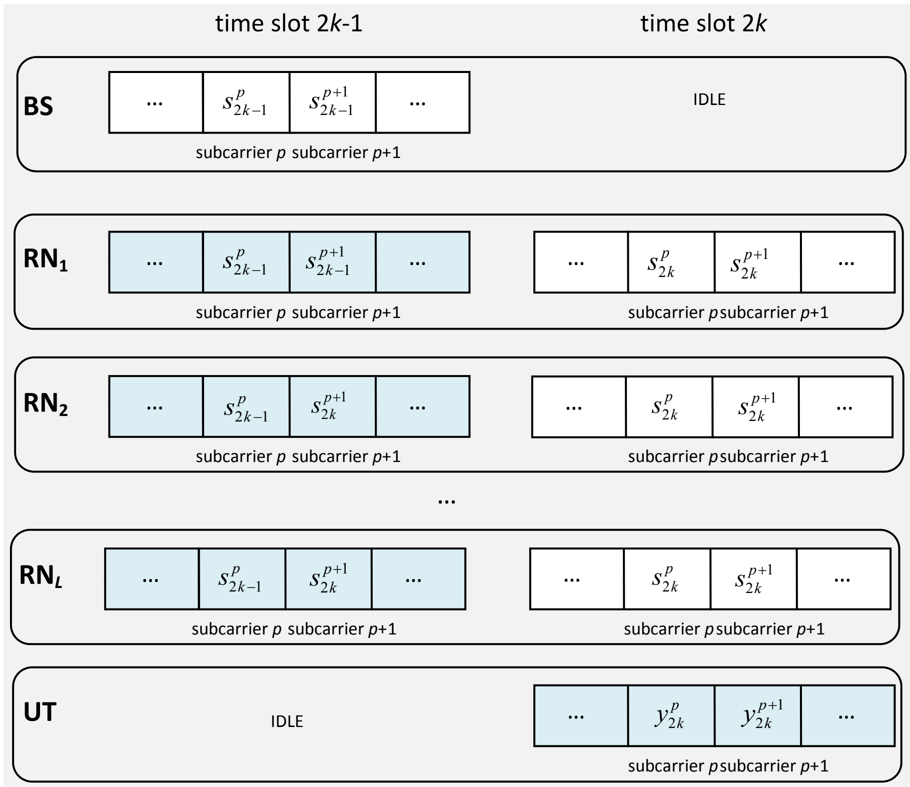

Figure 2 Transmitted (white slots) and received (blue slots) signals for each node for distributed SFBC algorithm.

As a major consequence, the increasing of the modulation order leads to a decrease of power efficiency.

The second algorithm presented, data precoded RA (PRA), aims to solve this spectral efficiency problem. In this proposed algorithm, the relays receive and transmit alternately, while the source is transmitting continuously, maintaining the same spectral efficiency as compared to the non-cooperative scheme. In order to get full diversity the data symbols are precoded at the BS. The flow of data information for this algorithm is shown in Figure 3, considering single-antenna RNs shown. Signal expressions of this scheme are presented in detail in Section 3. The rate of the proposed scheme is $N_l/(N_l+1)$, where N_l is the number of OFDM frames transmitted, which is close to 1 for large values of N_l.

3. Data PRA algorithm

We consider that the number of antennas at the relays can be one or two. Note that a distributed space-frequency code should be implemented in the relays with more than one antenna and that there exist only fully orthogonal codes with unitary rate for a maximum of two antennas [26]. The relays receive and transmit alternately and the source is transmitting continuously, first sending information to RN_1 and then repeating it to RN_2, and then successively until RN_L. Diversity is achieved by using a data-precoding at the BS. There is no need for any extra processing at the relays. At the UT a soft decoding is obtained using MRC, followed by a final decoding based on Viterbi algorithm. This decoding method increases the complexity of the proposed scheme compared to distributed SFBC one, but on the other hand it improves the scheme performance. The complexity of this algorithm requires $O(4N_s)$ arithmetical operations, where 4 comes from the number of QPSK symbols and N_s is the number of states of trellis diagram given by $N_s = 4^{L-1}$. The nodes that are transmitting and receiving in each instant are exemplified in Table 1 where A $\rightarrow$ B represents the transmission from node A to node B. Preliminary derivations and results for two relays, each equipped with one or two antennas, were presented in [19]. In this article, we extend the proposed data-precoded-based algorithm for a generic number of relays. The source produces a sequence of symbols $\{x_k\}$, each one carrying m information bits. The BS transmitter precodes successive groups of symbols $\{x_k, x_{k-1}, ..., x_{k-L}\}$, using a bijective function $F(x_k, x_{k-1}, ..., x_{k-L})$. The precoded symbols, s_k, are alternately transmitted to the relays, allowing each symbol, when all paths are available, to reach the UT through L-independent links.

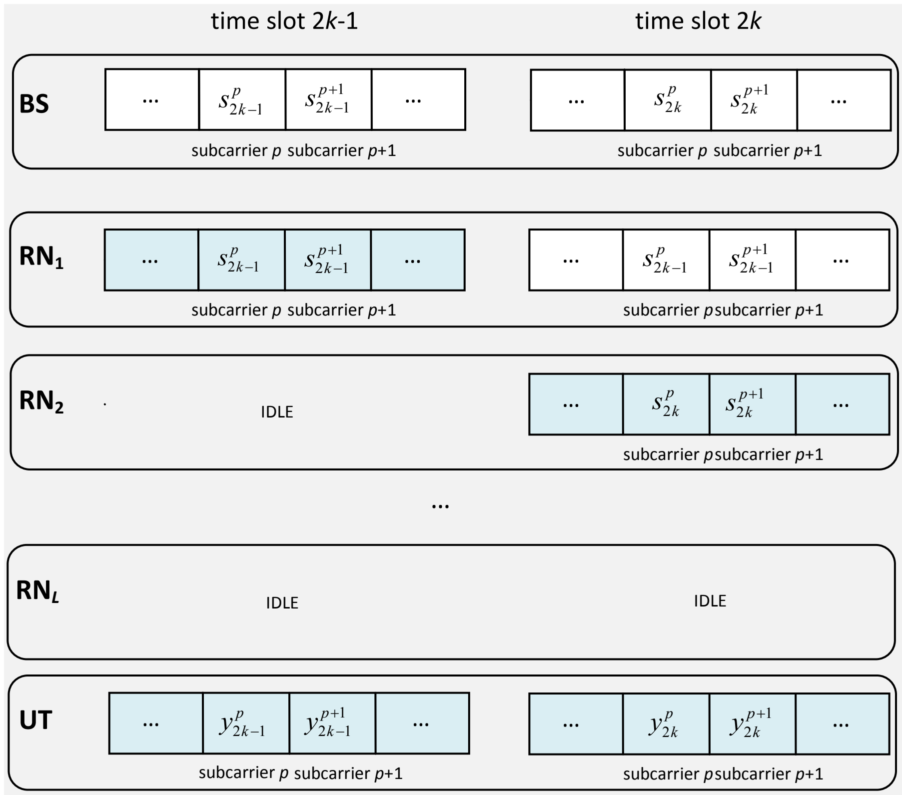

Figure 3 Transmitted (white slots) and received (blue slots) signals for each node for data PRA algorithm.

When one of the links fails, the bijectivity allows for the recovery of the original symbols QPSK. The groups of original symbols that are joined in a single precoded symbol are shown in Figure 4, when considering the particular case of having three RNs.

In the case where original symbols are QPSK, we propose to use a simple precoding operation that relates QPSK and M-QAM. It is easy to verify that a symbol s belonging to a regular 4^L-QAM can be expressed as the superposition of L QPSK symbols, $s = \sum_{n=0}^{L-1} 2^{-n} x_n$, which is easily derived by the definition of M-QAM modulated signals presented in [27]. The precoded symbols, which are transmitted by the BS, are then given by

$$s_k = \mu \sum_{i=0}^{L-1} 2^{-i} x_{k+i}, \tag{1}$$

Table 1 Active links in each time slot for the data-PRA scheme

T_s	L- 1	L	L+ 1	L+ 2
Active links	BS $\rightarrow$ RN_L	BS $\rightarrow$ RN_{L+1}	BS $\rightarrow$ RN_{L+2}	BS $\rightarrow$ RN_{L+3}
	RN_{L-1} $\rightarrow$ UT	RN_L $\rightarrow$ UT	RN_{L+1} $\rightarrow$ UT	RN_{L+2} $\rightarrow$ UT

where x_k is the kth QPSK symbol of the original sequence information, with unitary power; μ is the unitary normalization factor for a generic number of relays, which is independent of the number of antennas in each relay, and was derived by us, according to the presented algorithm:

$$\mu = \sqrt{\frac{4^{L-1}}{\sum_{i=0}^{L-1} 4^i}}. \tag{2}$$

From Equation (1), we easily recognize each symbol s_k as a M-QAM symbol, with M = 4^L. However, the receiver will interpret it as a sum of L QPSK symbols, thus bringing the performance close to the one that would be achieved if the QPSK symbols were transmitted continuously, because of the fact that each QPSK symbol is received through L paths. When L_f (L_f <L) of the links fails, it is possible to recover the original symbols QPSK from the L - L_f available links, although the diversity is reduced to L - L_f.

In this algorithm, while BS continuously transmits data to the RNs, relays transmit and receive alternately, as shown in Table 1. Thus, the received signal at UT, in

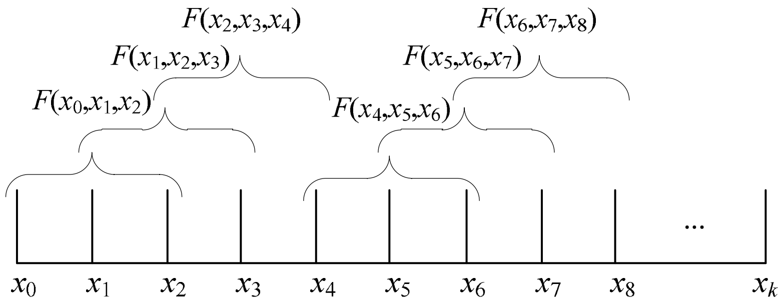

Figure 4 Groups of symbols alternately transmitted to each relay, according to a bijective function $F(x_k,x_{k+1},x_{k+2})$ for the case of $L = 3$.

time slots $Lk + l$, with $l = 1,.., L$ and $k \in \mathbb{N}$, for the case that N_R is equal to one and two, is given by

$$\gamma_{Lk+l} = \begin{cases} \mu^2 h_{ru\_l\_1,Lk+l} \sum_{i=0}^{L-1} 2^{-i} x_{Lk+l-1+i} + n_{Lk+l} & , N_R = 1 \\ \frac{\mu^2}{2}\left(\left|h_{ru\_l\_1,k,p}\right|^2 + \left|h_{ru\_l\_2,k,p}\right|^2\right) \sum_{i=0}^{L-1} 2^{-i} x_{Lk+l-1+i} + n_{Lk+l} & , N_R = 2 \end{cases} \quad (3)$$

where $h_{ru\_l\_q_r,k} = \sqrt{\beta_l} h^R_{ru\_l\_q_r,k}$ represents the cooperative channel for the link between the q_r^{th} antenna of RN_l and the UT; $h^R_{ru\_l\_q_r,k}$ is the complex flat fading Rayleigh channel realization for time slot k, with unit average power; and, β_l represents the long-term channel power.

4. Asymptotic gain of the proposed algorithm over distributed SFBC

The proposed algorithm has a trellis structure for the transmission of the QPSK symbols with four states. The minimum distance of the proposed scheme is obtained assuming a specific symbol is transmitted and calculating the Euclidean distance between the correct decoding path and the minimum erroneous path. For the case of $L = 2$ and assuming that symbol $u^{(1)}$ is transmitted (note that the code is linear), this measurement is obtained through the paths of trellis structure represented in Figure 5, corresponding to the paths that correctly recovers $u^{(1)}$ and that erroneously recover $u^{(i)}$ instead of $u^{(1)}$. Each path in the figure has the corresponding cost-function value. For the general case of having L relays we get similarly the squared minimum distance of the proposed algorithm for single-antenna relays given by

$$d^2_{\min_{\text{PRA}}} = \mu^2 d^2_{\min_{\text{QPSK}}} \min_{j \in J} \left(\sum_{i=0}^{L-1} 4^{-i} \left| h_{ru\_i+L\_1,k} \right|^2 \right), \quad (4)$$

with $J \in \{1, 2, ..., L\}$ and assuming that $h_{ru\_l\_q_r,k} = h_{ru\_l+L\_q_r,k}$.

Let us define ρ_{mi} as the channel power gains of each link comparatively to the maximum channel power gain. Assuming, without loss of generality, that $|h_{ru\_1\_1,k}|^2 \geq |h_{ru\_2\_1,k}|^2 \geq ... \geq |h_{ru\_L\_1,k}|^2$, we obtain the channel power gain for each link for relays with one antenna and for $i = 1,..., L$-1:

$$\rho_{mi} = \frac{\left|h_{ru\_i+1\_1,k}\right|^2}{\left|h_{ru\_1\_1,k}\right|^2}. \quad (5)$$

Thus, using the variables of channel power gains, Equation (4) can be simplified to

$$d^2_{\min_{\text{RA-prec}}} = \mu^2 d^2_{\min_{\text{QPSK}}} \left(\sum_{i=1}^{L-1} 4^i \rho_{mi} + 1 \right) \left| h_{ru\_i\_1,k} \right|^2. \quad (6)$$

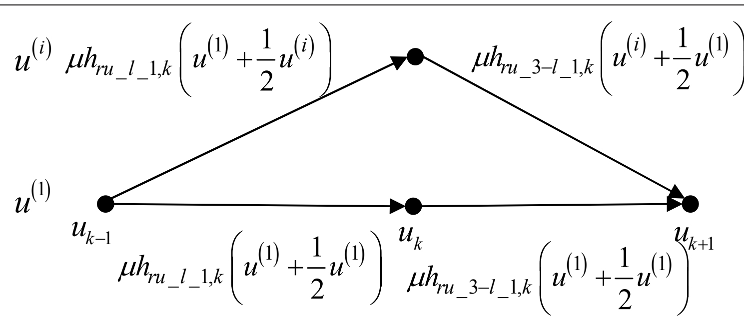

Figure 5 Trellis code for the RA precoded algorithm between symbols $u^{(1)}$ and $u^{(i)}$, for $L = 2$.

Similarly, for the case of relays equipped with two antennas, we can assume that $|h_{ru\_1\_1,k}|^2 + |h_{ru\_1\_2,k}|^2 \geq |h_{ru\_2\_1,k}|^2 + |h_{ru\_2\_2,k}|^2 \geq \ldots \geq |h_{ru\_L\_1,k}|^2 + |h_{ru\_L\_2,k}|^2$, and thus obtain the corresponding channel power gains for $i = 1,\ldots, L$-1 in

$$\rho_{mi} = \frac{\left|h_{ru\_i+1\_1,k}\right|^2 + \left|h_{ru\_i+1\_2,k}\right|^2}{\left|h_{ru\_1\_1,k}\right|^2 + \left|h_{ru\_1\_2,k}\right|^2}, \quad (7)$$

and the squared minimum distance in

$$d^2_{\min_{\text{RA - Prec}}} = \frac{\mu^2}{2} d^2_{\min_{\text{QPSK}}} \left(\sum_{i=1}^{L-1} 4^i \rho_{mi} + 1 \right) \left(\left|h_{ru\_1\_1,k}\right|^2 + \left|h_{ru\_1\_2,k}\right|^2 \right) \quad (8)$$

where the factor 1/2 comes from the normalization of the transmitted power, since in this case we have two antennas in each relay, and thus each antenna transmits one half of the total power.

It is important to evaluate the performance achieved with the precoded algorithm comparatively to the one we get using conventional cooperative algorithms. First we compare the precoded algorithm with one using distributed SFBCs, with unitary multiplexing rate. For the cases of having more than two transmitting antennas, the SFBCs are not fully orthogonal, thus resulting in lower performances. Derivation of the gain and the minimum distance expressions, for the distributed algorithm, is detailed in Appendix 1. We present the theoretical gain obtained with the PRA algorithm considered relatively to the distributed SFBC, for $L \in \mathbb{N}\backslash\{1\}$ and $N_R \in \{1, 2\}$, obtained from Equation (A.5), given by

$$G_L \approx 10 \log \left(\frac{15L \left(\sum_{i=1}^{L-1} 4^i \rho_{mi} + 1 \right)}{2\left(4^L - 1\right) \left(\sum_{i=1}^{L-1} \rho_{mi} + 1 \right)} \right), \quad (9)$$

where the channel gains ρ_{mi} are defined in Equation (5) for the case of a single-antenna relays and in Equation (7) for the two-antenna case. This obtained gain can also be seen as a lower-bound for $N_R > 2$, because of the non-achievement of full orthogonality in those cases. In the asymptotic case of high SNR values and when channels have equal power gains, the gain tends to a constant irrespective of the number of relays, L, given by

$$\lim_{\text{SNR}\to\infty} G_L = 10 \log \left(\frac{5}{2} \right). \quad (10)$$

An alternative scheme for comparison with the proposed one is an equivalent cooperative scheme, also with a unitary rate, though fully orthogonal. This cooperative scheme can use the Alamouti code multiple times according to the number of elements and is referred to as distributed-compound-Alamouti (DCA) algorithm. This algorithm requires more time for transmission, which depends on the number of relays. Thus, for the case of single-antenna relays, it takes twice the time to transmit as compared to the time that the continuous link would take if available for the two-relay case, and thrice the time for three- and four-relay cases and so forth.

The derivation of the gain obtained with the precoded algorithm with the DCA one is also derived in Appendix 1. In the case of high SNR regime and when channels have equal power gain, this asymptotic gain for a generic system with L relays, obtained through Equation (A.8), is given by

$$\lim_{\text{SNR}\to\infty} G_L = \begin{cases} 10 \log \left(\dfrac{4\left(4^{\frac{L}{2}+1} - 1\right)}{3L\,(L+2)} \right), & L \text{ is even} \wedge N_R = 1 \\ 10 \log \left(\dfrac{4\left(4^{\frac{L+3}{2}} - 1\right)}{3L\,(L+3)} \right), & L \text{ is odd} \wedge N_R = 1 \\ 10 \log \left(\dfrac{2\left(4^L - 1\right)}{L^2} \right), & N_R = 2 \end{cases} . \quad (11)$$

Thus, the asymptotic gain that the precoded algorithm achieves relatively to the DCA one, when the channels have equal power gains, depends on the number of RNs and the number of antennas in each one. This gain in dBs in function of the number of RNs is shown in Figure 6. When the number of antennas in each relay is two, the gain increases with the number of relays, since Alamouti code is implemented in each relay. However, in the case of the single-antenna relay the gain decreases when the number of RNs goes from 3 to 4 or from 5 to 6. Note that in those situations we are increasing the cardinality of modulation in a factor 4, because of the lower spectral efficiency of this scheme, since in those cases we need an additional time phase for another Alamouti code implementation.

5. PEP derivation for data-precoded algorithm

5.1. Derivation of error probability

In this section, the bit error probability for a general number of relays, L, and for a general number of antennas equipping each relay, N_R, is derived. For a high SNR regime, the PEP for convolutional codes can be asymptotically approached by [28]:

$$\text{PEP} \simeq \frac{N_{\min}}{2} erfc \left(\frac{d_{\min}}{2\sqrt{N_0}} \right), \quad (12)$$

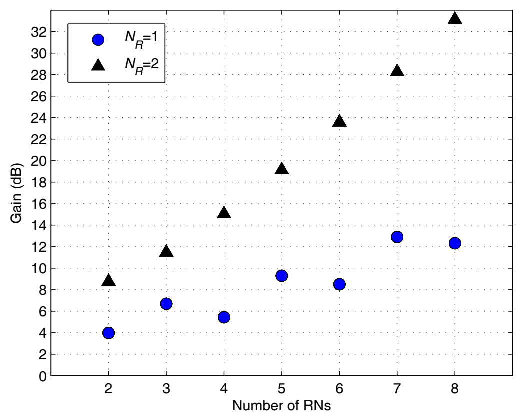

Figure 6 Coding gain obtained with the proposed algorithm in comparison to the DCA one, for a high SNR regime, for equal channel power gains and for N_R = 1.

where $N_{\min}$ is the number of paths with the minimum distance and $erfc(\cdot)$ is the complementary error function. Because of this approximation the error probability derived is not exact, but a lower bound, since for low SNRs error events may correspond to paths that do not have the minimum distance.

Consequently, as there are two minimum paths and since the two minimum paths have a Hamming distance of 1 and have the same weight, the conditioned bit error probability for RA scheme is obtained by replacing the expression of the minimum distance in Equation (12), thus obtaining the following expression

$$P_{b|h_i} = \frac{1}{2} erfc\left(\sqrt{\frac{d_{\min}\left(h_1, ..., h_{LN_R}\right)}{N_0}}\right), \tag{13}$$

We then get the unconditioned probability of error, for the proposed algorithm, as can be seen in the following expression

$$P_b = \underbrace{\int_0^{+\infty} \cdots \int_0^{+\infty}}_{LN_R} Q\left(\sqrt{2\sum_{i=1}^{LN_R} \nu_i}\right) \prod_{i=1}^{LN_R} f_{\nu_i}(\nu_i) d\nu_1 ... d\nu_{LN_R}, \tag{14}$$

where $i = 1, ..., LN_R$; $\nu_i = h_i \bar{\nu}_i$ are i.i.d. variables that follow an exponential distribution with mean $\bar{\nu}_i = \frac{\gamma}{N_R 4^{i-1}}$ and pdf given by

$$f_{\nu_i}(\nu_i) = \begin{cases} \frac{1}{\bar{\nu}_i} e^{-\frac{\nu_i}{\bar{\nu}_i}} & , \nu_i \geq 0 \\ 0 & , \nu_i < 0 \end{cases} \tag{15}$$

$\gamma = \mu^2 \frac{E_b}{N_0}$; and, function Q is defined as [29]

$$Q\left(\sqrt{2\varphi}\right) = \frac{1}{\pi} \int_0^{\frac{\pi}{2}} e^{-\frac{\varphi}{\sin^2 \phi}} d\phi. \tag{16}$$

Replacing Equations (15) and (16) in (14), we present an alternative expression for the bit error probability for the case of L relays and N_R antennas given by

$$P_b = \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \prod_{i=1}^{L} \left(\frac{\sin^2\phi}{\sin^2\phi + \bar{\nu}_i}\right)^{N_R} d\phi. \tag{17}$$

By solving this integral, we obtain the final expression for the bit error probability given by

$$P_b = \frac{1}{2} \sum_{i=1}^{L} \sum_{k=1}^{N_R} A_{k_i} \left[1 - \sqrt{\frac{\bar{\nu}_i}{1 + \bar{\nu}_i}} \sum_{j=0}^{k-1} \binom{2j}{j} \frac{1}{[4(1 + \bar{\nu}_i)]^j}\right], \tag{18}$$

where the auxiliary variable A_{ki} can be obtained based on Equation [27] and is defined as

$$A_{k_i} = \frac{1}{(N_R - k)!(\bar{\nu}_i)^{N_R-k}} \left\{ \frac{d^{N_R-k}}{dx^{N_R-k}} \prod_{\substack{j=1 \\ j \neq i}}^{L} \left(\frac{1}{1 + \bar{\nu}_j x}\right)^{N_R} \right\} \Bigg|_{x=-\frac{1}{\bar{\nu}_i}} \tag{19}$$

and where $\binom{n}{k}$ is the k-combination of a set of n elements.

The generic expression of Equation (18) was derived by assuming fully orthogonal SFBCs for any value of N_R. However, in practice full orthogonal codes with rate one for N_R > 2 do not exist and thus, for these cases, this expression can be seen as a lower bound of the bit error probability.

In order to obtain an approximation to the error probability for high SNR regime, considering a general number of relays and antennas equipping each relay, we try to simplify the bit error expression in Equation (17). For a high SNR, it is reasonable to assume $\nu_i \gg \sin^2 \varphi$. This simplification approximates (18) to an upper bound, based on [30]

$$P_b \approx \frac{(2N_R L - 1)!!}{2^{N_R L+1} \cdot (N_R L)!} \prod_{i=1}^{L} \left(4^{i-1}\right)^{N_R} \cdot \left(\frac{\gamma}{N_R}\right)^{-N_R \cdot L}, \tag{20}$$

where the double factorial operator is defined as

$$n!! = \begin{cases} n(n-2)...5 \cdot 3 \cdot 1 & , n > 0 \text{ and odd} \\ n(n-2)...6 \cdot 4 \cdot 2 & , n > 0 \text{ and even} \\ 1 & , n = -1, 0 \end{cases} \tag{21}$$

A diversity order of LN_R can be achieved, as confirmed by Equation (20), with the proposed algorithm. Note that the general expressions derived in this section are naturally reduced to the most simple case ones presented in [19], by setting $L = 2$, since the scheme presented in that work is a particular case of the one discussed in this manuscript.

5.2. Validation of bit error probability expressions

The analytical derivation of the error probability of the proposed algorithm is corroborated by the BER performance obtained through Monte Carlo simulations. Theoretical and simulated BER curves are shown in Figure 7, including the theoretical upper bound approximation for a high SNR regime derived previously for the particular schemes PRA-2RN $N_{\mathrm{B}} \times 1 \times 1$, PRA-2RN $N_{\mathrm{B}} \times 2 \times 1$ and PRA-3RN $N_{\mathrm{B}} \times 1 \times 1$. The particular expressions of error probability for each scheme are presented in Appendix 2.

The simulation curve for PRA-2RN $N_{\mathrm{B}} \times 1 \times 1$ scheme has approximately the same behavior as the one given by its theoretical approximation shown in Equation (18), only differing for low SNRs. Note that, because of the approximation done in (12), (18) can be seen as a lower bound of the algorithm exact error probability. At low SNRs, error events may correspond to paths that do not have the minimum distance, which results in the differences between the lower bound and the simulated curves. These are nonetheless lower than 1 dB for $E_b/N_0 \geq 12$ dB and thus negligible for high SNR values. We can also observe that the simulated curve has the same linear decay as the asymptotic curve given by Equation (B.2) for high SNRs, confirming the diversity order of 2.

Regarding the simulated performance obtained for the RA scheme with the proposed algorithm for two relays, each one equipped with two antennas, it is compared with the derived theoretical expression. Again, the curves are close to each other, not differing more than 1 dB for any value of SNR. Moreover, the asymptotic curve confirms the order diversity of 4, which is shown in Equation (B.2).

Another scheme simulated, in order to validate the error probability expression derived previously, is the RA scheme with three RNs, all equipped with a single antenna. As in the previous cases, the simulated and theoretical curves approach one another as SNR increases. Again, the small discrepancy is due to the approximation done in the theoretical expressions. These expressions are obtained assuming the recovery of each bit error through one of the minimum distance paths. Furthermore, the slope derived by the approximated expression for high-SNR regime in Equation (B.2) is of order 3, as can be confirmed in Figure 7.

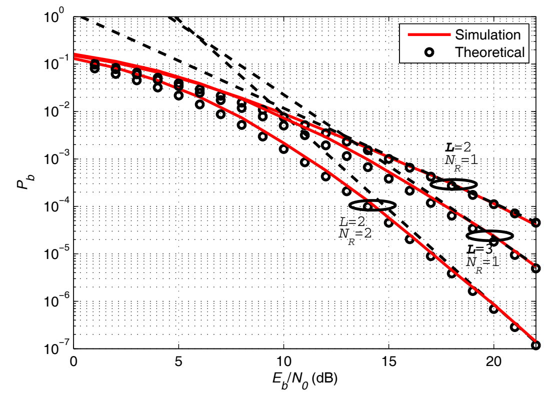

Figure 7 Theoretical and simulated error probability for data-PRA schemes.

6. Numerical results

6.1. Assumptions and conditions

Some assumptions are considered for this work, such as: perfect CSI at the relays and at the UT; normalization of the transmitted power per time slot to one; and distance between antenna elements of each BS and RN far apart to assume uncorrelated antenna propagation channels. The block length used in the simulations, N_l, is of 3600 symbols. In all the considered systems, two information bits are transmitted per symbol interval, and thus all of them have the same spectral efficiency.

In order to characterize propagation aspects as a whole, including the effects of path loss, shadowing, scattering and others, we consider different link quality combinations, quantifying them in terms of SNR, given, for each link, by the ratio between received and noise powers. We define different SNRs for the second-hop cooperative links $\mathrm{RN}_i \rightarrow$ UT for $i = 1,..., L$, referred to as SNR_{ci}, and for the direct link (the link between the BS and the UT of the non-cooperative systems) as SNR_d. For simplicity, as we assume perfect detection in relays, we do not refer to SNR differences in the first cooperative hop.

Three propagation scenarios are accounted for, differing in the link SNRs mentioned above for the schemes with two relays cooperating, as shown in Table 2. In scenario 1, we assume that all the links have the same quality conditions, i.e., $\mathrm{SNR}_d = \mathrm{SNR}_{c1} = \mathrm{SNR}_{c2}$. We also include scenarios where the second-hop cooperative

Table 2 Propagation scenarios considered in Monte Carlo simulations for $L = 2$

	SNR_d	SNR_{c1}	SNR_{c2}
Scenario 1	SNR_d	SNR_d	SNR_d
Scenario 2	SNR_d	SNR_d + 10 dB	SNR_d
Scenario 3	SNR_d	SNR_d + 10 dB	SNR_d + 10 dB

links, i.e., $RN_1 \rightarrow UT$ and $RN_2 \rightarrow UT$, have higher quality than the direct link. The choice of these scenarios derives from the fact that, in most real situations, the cooperative link has better quality conditions of transmission than the direct link. We then define scenario 2, where the link between RN_1 and UT has a SNR 10 dB higher than the other two links, i.e., $SNR_d = SNR_{c2}$ and $SNR_{c1} = SNR_d + 10$ dB. In scenario 3, all the cooperative paths have better transmission quality conditions than the DP, i.e., $SNR_{c1} = SNR_{c2} = SNR_d + 10$ dB.

We consider a typical pedestrian scenario based on LTE specifications, following the system configurations in Table 3[31]. In systems where a space-frequency code is needed for two transmitting antennas, the well-known Alamouti coding is implemented [32]. For the three antennas case, the distributed quasi-orthogonal SFBC (QO-SFBC) implemented is the one proposed by Li, Park and Kim (LPK) [33]. In systems with four antennas transmitting simultaneously, we implement the QO-SFBC proposed by Tirkkonen, Boariu and Hottinen (TBH) in a distributed manner [34].

The schemes considered in our evaluation are presented in the list below. The first two bullets correspond to the proposed schemes and the remaining schemes are used as references:

- RA scheme with the proposed algorithm, using precoded QPSK symbols and Viterbi decoding method, for two relays with one and two antennas (PRA 2RN-$N_B \times 1 \times 1$ and PRA 2RN-$N_B \times 2 \times 1$, respectively);
- RA scheme with the precoded algorithm, for three RNs, each one equipped with one antenna (PRA 3RN-$N_B \times 1 \times 1$);
- Distributed RA (DRA) scheme for two relays, with one and two antennas, using an SFBC and 16-QAM modulation (DRA 2RN-$N_B \times 1 \times 1$ and DRA 2RN-$N_B \times 2 \times 1$, respectively) [34];
- DRA scheme, using a QO-SFBC applied to three relays, with 16-QAM modulation (DRA 3RN-$N_B \times 1 \times 1$) [33];
- Non-cooperative 4 × 1 QPSK with a QO-SFBC with a continuous link available (NRA QO-SFBC 4 × 1) [34];
- Non-cooperative 2 × 1 QPSK Alamouti coding with a continuous link available (NRA 2 × 1).

We also obtain numerical results assuming non error-free links for BS à RNs channels. In this case we assume 2 × 1 space-frequency block coding scheme from BS to each RN.

The numerical results are presented in terms of the average BER as a function of E_b/N_0, where E_b is the received energy per bit at the UT through the direct link (BS à UT) and N_0 is the noise power spectral density.

6.2. Single-antenna two-relay scheme

Cooperative and reference systems performances are shown in Figure 8 for scenario 1, for the case of the two relays are cooperating with the RA schemes, each equipped with a single antenna. In this case, the reference systems presented are the non-cooperative NRA 2 × 1 and DRA $N_B \times 1 \times 1$ ones, both using Alamouti code.

When comparing the PRA scheme against DRA, we observe an improvement of 2.2 dB, for BER = 10^{-3}. This, in turn, derives from the precoding used in the proposed scheme, which mitigates some of the penalty resulting from the half-duplex constraint at the relays, avoiding the use of a higher modulation order.

The proposed cooperative scheme has a penalty of about 1 dB from the best reference, i.e., 2 × 1 QPSK Alamouti coding with a continuous link available, for the same BER conditions. It is, however, worthwhile to point out that in our reference we assume independence between the channels. In practice, using co-located antennas inevitably leads to some correlation between the channels, in fact reducing such 1 dB of penalty, or even outperforming it in the case of high correlation [35].

Table 3 Parameters of simulated scenarios according to LTE standard

LTE general signal definitions	FFT size	1024
	Number of available subcarriers	300
	Sampling frequency	15.36 MHz
	Useful symbol duration	66.6 μs
	Cyclic prefix duration	5.21 μs
	Overall OFDM symbol duration	71.86 μs
	Sub-carrier separation	15 kHz
	Number of OFDM symbols per block	12
Channel model	ITU pedestrian model B	
	Tap delays modified accordingly to the sampling frequency defined for LTE systems	
UT velocity	3 km/h	

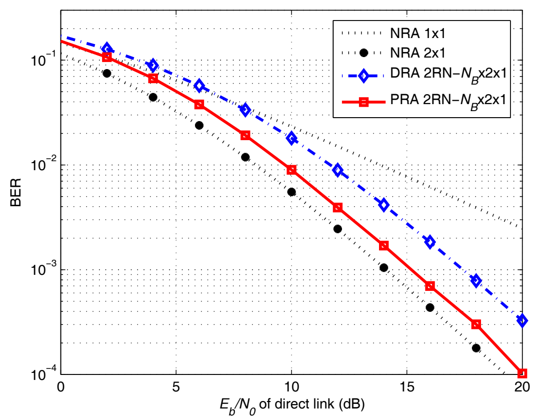

Figure 8 BER of cooperative systems with two single-antenna relays and of reference systems for scenario 1.

In Figure 9, the performance of the same schemes in scenario 2 is shown. Under this scenario conditions, the proposed precoded scheme outperforms the equivalent non-cooperative system. Improvements of 4 dB are obtained in comparison with 2 × 1 Alamouti, for BER = 10^{-3}. However, the RA Alamouti scheme is still worse than the non-cooperative scheme with two antennas in the BS. The coding gain between the precoded scheme and the RA Alamouti is of 6 dB for the same BER conditions, which is higher than in the previous scenario. By this, we extrapolate that when we have quality asymmetry in cooperative links, we have more benefits in using the precoded Viterbi scheme than the other presented schemes.

In Figure 10, both links between relays and UT have SNRs 10 dB higher than the direct link (scenario 3). In this case, the cooperative schemes have the same resulting behavior as in the previous scenarios, although the cooperative schemes achieve better performances, as expected. The difference between non-cooperative 2 × 1 and the PRA schemes is now more than 8 dB, for BER = 10^{-3} (for best visualization purposes, the non-cooperative 2 × 1 curve is not completely shown in the plot). Compared with the DRA using Alamouti, we have an improvement of about 2.2 dB with the proposed code, for BER = 10^{-3}, which is the same difference as in scenario 1.

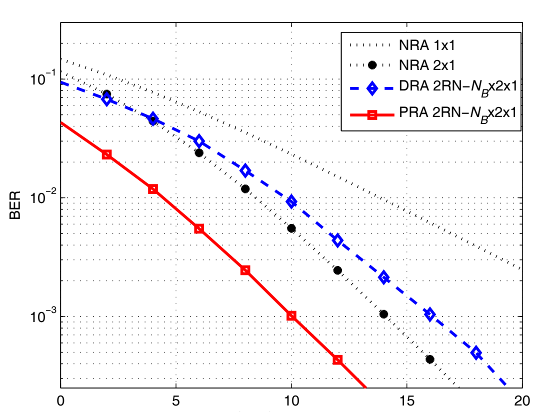

Figure 9 BER of cooperative systems with two single-antenna relays and of reference systems for scenario 2.

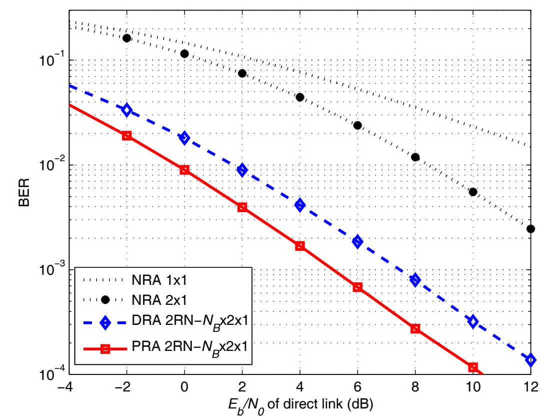

Figure 10 BER of cooperative systems with two single-antenna relays and of reference systems for scenario 3.

In order to observe the improvement obtained with the proposed algorithm in a more realistic situation, where the first cooperative-hops are not considered as ideal or error-free links, other scenarios are considered. It includes correlation between antenna channels [36] and the links between the BS and the RNs have a quality of transmission 10 dB higher than the direct link, since relays are often selected in such way that at least those links have high-quality and due to the possibility of having multiple antennas at the BS. The other links have the same relations defined for each of the scenarios 1, 2 and 3. The resulting performances are presented in Figure 11. We observe that differences between this case and assuming error-free links until the relays are not significant, differing less than 1.5 dB, for BER = 10^{-3}, independently of the considered scenario.

6.3. Two-antenna two-relay scheme

In this sub-section, we assume the two RNs cooperative scheme, with both RNs equipped with two antennas. The considered reference systems are: the non-cooperative 2 × 1 and 4 × 1 systems, using Alamouti and TBH codes respectively; and, the RA scheme with the TBH code applied to the RNs. The results shown in Figure 12, were obtained considering that the all the links have

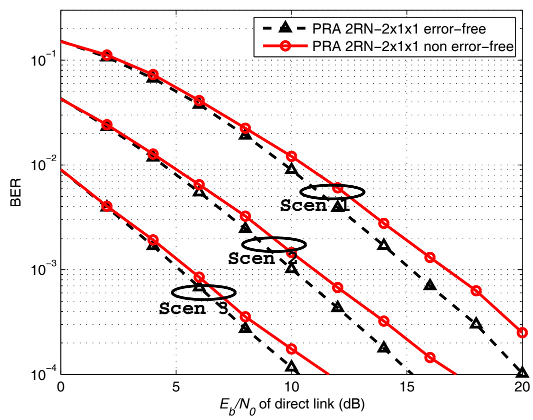

Figure 11 BER of cooperative systems with two single-antenna relays and of reference systems for a scenario with non-ideal first cooperative-hop.

the same transmission conditions. In this scenario, higher coding gains are obtained with the proposed algorithm than in Figure 8, as expected, since we have two antennas in each relay. An enhancement of about 5 dB is achieved with the PRA scheme, compared with the distributed cooperative scheme using TBH code, for BER = 10^{-3}. Comparing with the non-cooperative systems, the proposed scheme outperforms the NRA 2 × 1 system by about 3 dB, for the same BER. The performance of the new algorithm also outperform the non-cooperative system 4 × 1 for high SNRs, specifically for $E_b/N_0 > 9$ dB. This happens because, contrarily to the Alamouti coding, space-time codes for four antennas are not fully orthogonal, thus not achieving full diversity.

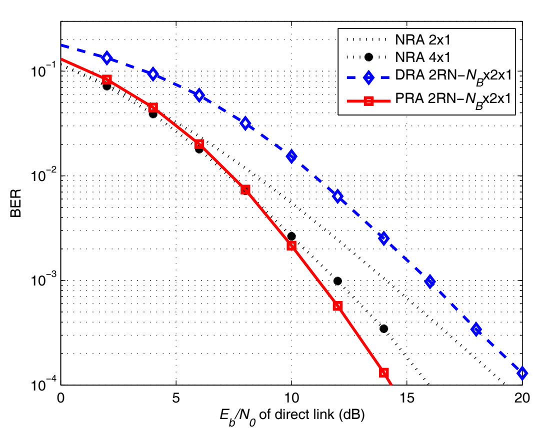

Figure 12 BER of cooperative systems with two relays, each equipped with two antennas per relay, and of reference systems for scenario 1.

6.4. Single-antenna three-relay scheme

The performance of cooperative schemes with three single-antenna RNs is also analyzed for a scenario equivalent to scenario 1, when all the links have the same transmission quality, in Figure 13. The schemes considered are the RA schemes with the precoded algorithm, for two and three RNs, the DRA with the distributed QO-SFBC LPK and the non-cooperative systems 2 × 1 and 3 × 1, using Alamouti and LPK codes, respectively.

When comparing the PRA scheme against DRA, we observe an improvement of 2.5 dB for BER = 10^{-3}. This gain is due to the precoding used in the proposed scheme, which avoids the use of a higher modulation order. Moreover the proposed scheme achieves a diversity order of 3, while the SFBC applied to the three relays does not achieve full diversity, since for three transmitting antennas orthogonality is relaxed in order to maintain a unitary rate.

The proposed cooperative scheme has a penalty of about 1.3 dB from the best reference, i.e., NRA 3 × 1 scheme with a continuous link available, for the same BER = 10^{-3}. It is however worthwhile to point out that we assume independence between the channels. In practice using co-located antennas inevitably leads to some correlation between the channels, in fact reducing such penalty, or even outperforming it in the case of high correlation [35]. The additional relay brings advantage for moderate/high SNR values. The gain increases with SNR, achieving about 2 dB for BER = 10^{-4}.

Comparing with the DRA, we have the same gain in using the proposed algorithm as in the first scenario. We thus infer that improvements are fixed for the cases where all the cooperative links have the same quality of transmission. We can also observe that the difference between having two and three RNs in this scenario, in both cases with single-antenna relays, is of 2.2 dB for BER = 10^{-4}.

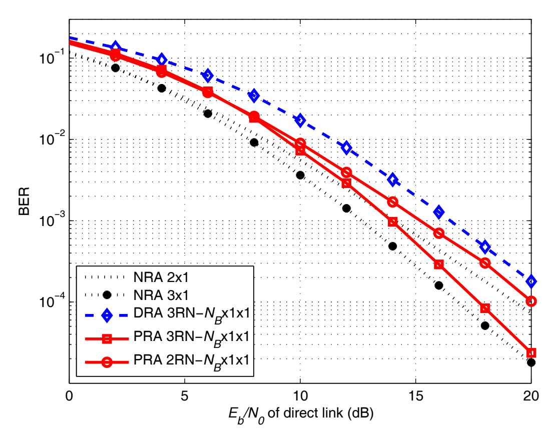

Figure 13 BER of cooperative systems with three single-antenna relays and of reference systems.

7. Discussion and conclusions

We proposed a data-precoded algorithm for multiple-antenna L RA based systems, which ensures spatial diversity, while maximizing spectral efficiency. The algorithm mitigates some of the penalty resulting from the half-duplex constraint at the relays and asymptotically achieves the same performance as the one obtained when a direct continuous link is available. Furthermore, with the precoded algorithm, there is no need to transmit through the direct link, which is beneficial for most scenarios, since the direct link is usually most strongly affected by path loss or shadowing.

We observed that the gain obtained with the precoded algorithm, relatively to the distributed SFBC one, increases with the number of RNs in a nonlinear way. The proposed precoding brings the performance very close to the one achieved when a direct continuous link is available and SFBC coding is used at the BS. Actually, for the case of two antennas in each relay, the precoded scheme outperforms the non-cooperative one for high SNR regime, due to the non-orthogonality of space-frequency codes for four transmitting antennas. Improvements are obtained for scenarios where cooperative links have higher quality than the direct link, being more pronounced as the relative quality of the cooperative links increases.

Moreover, we concluded that independently of the propagation scenario, precoded schemes outperform the equivalent distributed SFBC cooperative schemes, achieving better performance due to the coding gain obtained with data-precoding. Even for the most probable situation of asymmetric quality conditions between cooperative links, results show that the proposed scheme is better than the reference cooperative ones. In these cases, the difference between the two cooperative schemes is higher.

We also observed that the extra antenna in each relay leads to a considerable improvement in the overall system performance. Furthermore, the performance difference between the precoded schemes and the respective equivalent DRA schemes is higher for the case of having two antennas in each relay.

The performance of the PRA scheme was also obtained for three RNs, confirming the previous conclusions for two relays. The performance of this scheme, as expected, outperforms that of the scheme with two relays, both with single antennas equipping the relays, especially for scenarios with high-quality links.

From the presented results, it is clear that the proposed cooperative schemes can be used to extend the coverage mainly in scenarios where the quality of the direct link is poor, as is the case of cluttered urban environments. Through the use of the proposed multiple-relay-assisted scheme we achieve full diversity, with a moderate degradation relatively to the case where a continuous link is available, where the number of RNs and antennas in each one can be selected according to the required quality of service.

Appendix 1

In this section, we derive the gains presented in Section 4 obtained with the data-precoded algorithm comparatively with other cooperative algorithms. First we compare the precoded algorithm with the distributed SFBC one, neglecting the non-achievement of full orthogonality requirement for the cases of more than two antennas in all the relays. In these algorithms a 16-QAM modulation is used to implement such algorithm, due to the two phases of communication: in the first one the source broadcasts its information to the relays and in the second one the relays forward that information to the UT using the QO-SFBC. The difference in having two antennas at the RNs instead of one is in the modulation used, since we need the double of the time. In alternative, we could use a DSFBC designed for $2L$ transmitters, maintaining the same modulation. According to this, the squared minimum distance expression for the precoded algorithm is given by

$$d_{\min_{\text{RA - SFBC}}}{}^2 \approx \frac{d^2_{\min_{\text{16 - QAM}}}}{N_R L} \sum_{i=0}^{L-1} g_{N_R,i,k}, \tag{A.1}$$

where the variable with the equivalent link channel gain is given by

$$g_{j,i,k=} \begin{cases} \left|h_{ru\_i\_1,k}\right|^2 & , j = 1 \\ \left|h_{ru\_i\_1,k}\right|^2 + \left|h_{ru\_i\_2,k}\right|^2 & , j = 2 \end{cases}. \tag{A.2}$$

The ratio between the minimum distance of an M-QAM modulation and of a QPSK modulation is given by

$$\alpha_{M-\text{QAM}} = \frac{d_{\min_{M-\text{QAM}}}}{d_{\min_{\text{QPSK}}}} = \sqrt{\frac{3\log_2 M}{2\,(M-1)}}, \tag{A.3}$$

which we call normalization factor for M-QAM modulation [29]. The corresponding gains expressed in Equation (A.1) are thus transformed into

$$d_{\min_{\text{RA - SFBC}}}{}^2 \approx d^2_{\min_{\text{QPSK}}} \frac{2}{5L} \left(\sum_{i=1}^{L-1} \rho_{mi} + 1 \right) g_{N_R,1,k}. \tag{A.4}$$

The asymptotic lower power gain from the proposed algorithm considered relatively to the SFBC is obtained through the ratio of the minimum distances of both algorithms in Equations (8) and (A.4), and by using (A.3), what results in

$$G_L \approx 10 \log \left(\frac{ \frac{1}{\sum_{i=0}^{L-1} 4^i} \left(\sum_{i=1}^{L-1} 4^i \rho_{mi} + 1 \right) }{ \frac{2}{5LN_R} \left(\sum_{i=1}^{L-1} \rho_{mi} + 1 \right) } \right). \tag{A.5}$$

We can consider $\sum_{i=0}^{L-1} 4^i$ as a sum of the first L terms of a geometric progression of ratio 4 and initial value 1. Thus, the gain for $L \in \mathbb{N}\backslash\{1\}$ and $N_R \in \{1,2\}$ is given by Equation (9), where the channel gains ρ_{mi} are exhibited in Equation (5) for the case of a single-antenna relays and in Equation (7) for the two-antenna case.

An alternative scheme for comparison with the proposed one is the equivalent DCA algorithm. In this algorithm the modulation used is M_A-QAM, where $M_A = 4^{\frac{L}{2}+1}$ if the number of relays is even and $M_A = 4^{\frac{L+3}{2}}$ otherwise. When $N_R = 2$ the modulation used is given by $M_A = 4^L$. The squared minimum distance for the DCA algorithm, obtained by application of Euclidean distance definition, is then given by

$$d_{\min_{\text{RA - DCA}}}{}^2 = \frac{\alpha^2_{M_A-\text{QAM}}}{2} \sum_{i=0}^{L-1} g_{N_R,i,k}. \tag{A.6}$$

The corresponding gain in dBs obtained with the precoded algorithm in comparison with the equivalent DCA algorithm is given by

$$G_L = 10\log\left(\frac{\frac{1}{\sum_{i=0}^{L-1} 4^i}\left(\sum_{i=1}^{L-1} 4^i \rho_{mi} + 1\right)}{\frac{\alpha^2_{M_A-\text{QAM}}}{2}\left(\sum_{i=1}^{L-1} \rho_{mi} + 1\right)}\right). \tag{A.7}$$

By replacing the modulation factor gain and after some simplifications, we get the final expression for the gain obtained when using the precoded algorithm instead of the DCA one, for $L \in \mathbb{N}\backslash\{1\}$, and is given by

$$G_L = \begin{cases} 10\log\left(\dfrac{\frac{1}{4^L-1}\left(\sum_{i=1}^{L-1} 4^i\rho_{mi} + 1\right)}{\frac{(L+2)}{4\left(4^{\frac{L}{2}+1}-1\right)}\left(\sum_{i=1}^{L-1}\rho_{mi} + 1\right)}\right) & , L \text{ is even} \wedge N_R = 1 \\ 10\log\left(\dfrac{\frac{1}{4^L-1}\left(\sum_{i=1}^{L-1} 4^i\rho_{mi} + 1\right)}{\frac{(L+3)}{4\left(4^{\frac{L+3}{2}}-1\right)}\left(\sum_{i=1}^{L-1}\rho_{mi} + 1\right)}\right) & , L \text{ is odd} \wedge N_R = 1 \\ 10\log\left(\dfrac{\frac{1}{4^L-1}\left(\sum_{i=1}^{L-1} 4^i\rho_{mi} + 1\right)}{\frac{3L}{2\left(4^L-1\right)}\left(\sum_{i=1}^{L-1}\rho_{mi} + 1\right)}\right) & , N_R = 2 \end{cases}$$

Appendix 2

Theoretical expressions for BER for the schemes shown in Figure 7 are obtained through the general expression in Equation (18), resulting in the following

$$P_b = \begin{cases} \frac{1}{2}\left(1 - \frac{4}{3}\sqrt{\frac{\gamma}{1+\gamma}} + \frac{1}{3}\sqrt{\frac{\gamma}{4+\gamma}}\right) & , L = 2 \wedge N_R = 1 \\ \frac{1}{2} - \frac{4}{9}\sqrt{\frac{\gamma}{2+\gamma}}\left(\frac{2}{3} + \frac{1}{2+\gamma}\right) - \frac{1}{18}\sqrt{\frac{\gamma}{8+\gamma}}\left(\frac{11}{3} + \frac{4}{8+\gamma}\right) & , L = 2 \wedge N_R = 2 \\ \frac{1}{2}\left(1 - \frac{64}{45}\sqrt{\frac{\gamma}{1+\gamma}} + \frac{4}{9}\sqrt{\frac{\gamma}{4+\gamma}} - \frac{1}{45}\sqrt{\frac{\gamma}{16+\gamma}}\right) & , L = 3 \wedge N_R = 1 \end{cases} \tag{B.1}$$

The error probability expression for high SNR regime, for the same schemes, reached by Equation (20), are represented by

$$P_b = \begin{cases} \frac{3}{4}\gamma^{-2} & , L = 2 \wedge N_R = 1 \\ \frac{35}{16}\gamma^{-4} & , L = 2 \wedge N_R = 2 \\ 10\gamma^{-3} & , L = 3 \wedge N_R = 1 \end{cases}, \tag{B.2}$$

from where it is evident the diversity order achieved by each scheme.

Competing interests

The authors declare that they have no competing interests.

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