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Automatic and online setting of similarity thresholds in content-based visual information retrieval problems

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Abstract

Several information recovery systems use functions to determine similarity among objects in a collection. Such functions require a similarity threshold, from which it becomes possible to decide on the similarity between two given objects. Thus, depending on its value, the results returned by systems in a search may be satisfactory or not. However, the definition of similarity thresholds is difficult because it depends on several factors. Typically, specialists fix a threshold value for a given system, which is used in all searches. However, an expert-defined value is quite costly and not always possible. Therefore, this study proposes an approach for automatic and online estimation of the similarity threshold value, to be specifically used by content-based visual information retrieval system (image and video) search engines. The experimental results obtained with the proposed approach prove rather promising. For example, for one of the case studies, the performance of the proposed approach achieved 99.5 % efficiency in comparison with that obtained by a specialist using an empirical similarity threshold. Moreover, such automated approach becomes more scalable and less costly.

Keywords: Information retrieval, Content-based retrieval systems, Similarity thresholds

1 Introduction

Nowadays, multimedia databases are applied in various fields, storing large amounts of data. Thereby, the development of image and video content-based retrieval systems (capable of efficiently managing such data) has increased as well the academic interest in the area [1, 2]. For the development of such systems, some inherent problems must be solved: the selection of appropriate descriptors to represent images [1, 3]; the selection of appropriate similarity function to measure the similarity of the images being compared [1, 4]; and the definition of a suitable similarity threshold to be used by the systems [5–9].

The scope of possible techniques to solve the problems involved in the retrieval of content-based visual information makes necessary to use metrics that can assess the quality of the results obtained by different techniques. Such systems recover not only equal but also images similar to the searched image, making it necessary to evaluate responses returned to users. In this regard, some evaluation metrics are commonly used to describe the results obtained on an information retrieval system, namely precision, recall and F1 [10], which are classical metrics commonly used in information retrieval tasks.

After pre-processing input images (consultation), information retrieval systems compare them to the collection system and categorize them as either similar or dissimilar. Such verification is done through a similarity function f, which measures similarity between images. After image comparison, the result of the similarity function is tested against a δ similarity threshold to determine image similarity (the similarity threshold's minimum acceptable score [6–9]). Images compared with the input image whose similarity values are greater or equal to δ are considered similar, and the remaining are considered dissimilar. Typically, the values calculated by the similarity function f and defined for the value of δ similarity threshold are in a [0,1] range [5]; therefore, the closer the value returned by f is to 1, the more similar such images are.

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Often, the solutions used to define δ are based on specialist-developed templates for specific searches, aimed at representing the entire collection. For each search, templates compute precision, recall and F1 values, considering different δ values. In this case, δ values maximizing F1 measurements for the highest number of searches are selected and fed to the information retrieval system. Setting the δ value in this fashion is laborious, making it unfeasible in most cases (mainly because of the need to know the entire collection).

Considering, for example, Fig. 1a, b, c, d displays the results of analysis of four separate (content-based retrieval) searches per image with the SAPTE system (a content-based multimedia information retrieval system [11, 12] (FLA Conceição, FLC Pádua, ACM Pereira, GT Assis, GD Silva, AAB Andrade: Semiodiscursive Analysis of TV Newscasts based on Data Mining and Image Pre-processing, to appear). For each image analysis result (Fig. 1a, b, c, d), corresponding F1 values are determined for different similarity threshold values. Here, the searched images are part of templates specifically designed to evaluate the system's performance; such images consist of key frames extracted from Rede Minas TV videos, present in SAPTE.

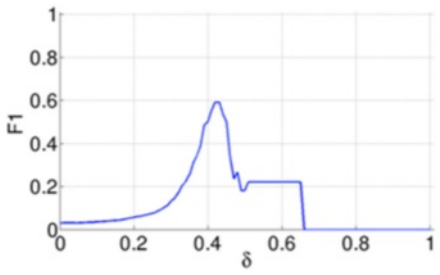
Figure 1a, b, c, d shows that to obtain the largest F1 value for a given search, a distinct δ value should also be considered for producing the best results. Therefore, automatic online setting of a δ value increases search effectiveness.

To automatically estimate δ in online applications, this study advocates the use of a metric based on internal criteria, possible to calculate without human intervention, as opposed to F1 calculation tied to external criteria and thus impossible to automate. Such metric is the silhouette coefficient [13] used to evaluate image clusters returning values within a $[-1 \ldots 1]$ interval (better clusters presenting values closer to 1).

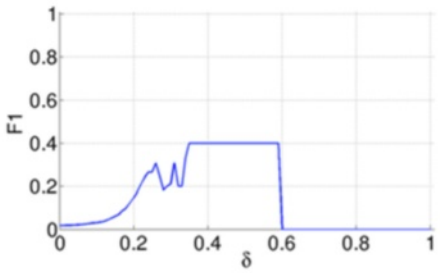
Our approach with automatic and online estimation of the objective similarity threshold through a dynamic similarity threshold value associated to individual searches enhances the efficiency of content-based visual information retrieval. More efficient automatic information retrieval (without human intervention) contributes to generate specific web page repositories, along with improved information retrieval system feedback to users.

An example of improved search feedback to users is provided by SAPTE systems, as shown in Fig. 2a, b. These respectively represent the response of SAPTE to a given search using $\delta = 0.83$ (expert defined) and $\delta = 0.86$ (estimated by this study's approach). In Fig. 2a, the definition of a fixed threshold reached 0.53 in terms of F1 for a specific search, whereas this study's proposal F1 reached a more efficient 0.89 (Fig. 2b).

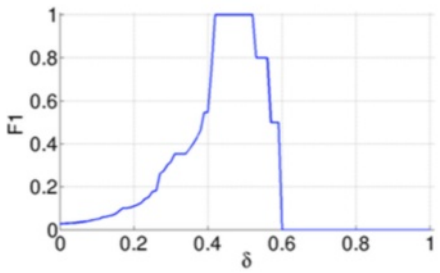
The main contribution of the present work consists in to present and validate a new effective and efficient



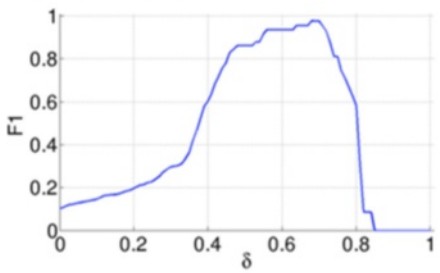
(a) Template 6 from CAPTE.



(b) Template 8 from CAPTE.



(c) Template 55 from CAPTE.



(d) Template 33 from CAPTE.

Fig. 1 F1 values obtained in searches extracted from a CAPTE collection template considering different δ values

(a) Results obtained with expert-defined fixed δ.

(b) Results obtained with a dynamic δ estimated automatically.

Fig. 2 Results returned by a search held with a fixed δ (expert-defined) and a dynamic δ (estimated automatically)

approach, named automatic setting similarity threshold (ASTS), to automatically estimate similarity thresholds in content-based visual information retrieval (CBVIR) problems. As far as we know, this is the first work in the literature to propose a simple and successful solution for such a problem, which is specially challenging in online applications. We claim that ASTS represents a promising alternative to the current solutions, which are commonly based on specialist-developed templates. Unfortunately, the threshold setting processes of those solutions are prone to human error and demand on significant time and financial costs. Moreover, even though the aforementioned templates are computed by considering only subsets of specific searches, their application is extended to searches involving the entire dataset, what frequently produces unsatisfactory results to end-users. Unlike those solutions, ASTS is capable to automatically and online estimate similarity thresholds associated to individual searches, enhancing the effectiveness and the efficiency of CBVIR processes.

This paper is organized as follows: Section 2 presents related work. Section 3 presents and discusses our proposed approach. Section 4 presents experimental results. Section 5 shows final considerations and future work.

2 Related work

Among studies aimed at improving content-based visual information retrieval systems, this one particularly addresses effectiveness improvement (i.e. system's quality improvements through search, setting the automatic and online similarity threshold to be adopted).

Previous works outlined (aimed at improving such systems) address the development of techniques solving several related problems, namely the description mode of images contained in the system [2, 14–16], selection of the best similarity function [4, 17–19], creation of user profiles [20], machine learning, task clustering and indexing, stochastic algorithms [3, 20–26] and similarity threshold estimation [5–7, 9, 27, 28], the latter being the problem addressed in this paper.

For instance, in [3], the authors present a method that combines a k-means clustering algorithm with a B + tree structure to improve system results obtained through search, returning only images of the closest clusters. Another study [22] aims at improved search results. Here, we combine an evolutionary stochastic algorithm (particle swarm optimization) with feedback relevance to understand, by iterative learning, the most relevant features to users and then properly consider the image feature descriptors according to what was learned during the interaction with the user. In a previous study [21], a two-stepped recovery of multimedia information was developed, with content-based visual information retrieval being performed only in the second stage. The first step consists of searching the collection for a top-K cluster, and only after this, the cluster formation was compared by content-based visual information retrieval. Content-based visual information retrieval occurs on a reduced cluster of the collection, thus enhancing recovery efficiency.

Based on collection samples, a semi-automatic approach for the estimation of recall and precision values for various similarity thresholds minimizes efforts involved by static similarity threshold definitions [28, 29]. It requires expert input only where the number of distinct objects contained in each sample is concerned and uses

two techniques to reduce human interaction, namely (i) sample use and (ii) similarity cluster process. Hence, the formed groups are used for automatic calculation of recall and precision then that resultant groups contain only objects that represent singular real objects. These clusters identify relevant objects so that when a particular object cluster is used for search, all its remaining objects are labeled relevant. Consequently, this approach generates a table of estimated recall and precision values from which it becomes possible to determine the appropriate similarity threshold for the application. Despite significantly reducing heavy expert reliance (usual in classical approaches), the proposed approach depends on specialist intervention to indicate the number of distinct objects contained in each sample. This limits the size of generated samples and reduces the number of distinct objects, making it possible for specialists to quantify them.

A new approach [5] combines two strategies to eliminate human intervention [28, 29], during the recall and precision values estimation process. They are (i) use of agglomerative hierarchical clustering algorithms and (ii) use of the silhouette coefficient for cluster evaluation. The first form clusters from different similarity thresholds without notifying the number of groups to be generated. Those are evaluated by the silhouette coefficient selecting the cluster with the highest silhouette coefficient value. Expert dependency is thus eliminated from the estimation process of the best similarity threshold and similarity function. This process is based on the premise that selected clusters (according to the validation of the clustering process) properly partition examined objects, meaning that each group represents only one real object. This premise, for example, was very important for the high linear correlation between the silhouette coefficient and F1 formally recorded in [5]. In expert-prepared templates used for recall and precision calculation, subjective external criteria may come into play. The absence of such information during the silhouette coefficient calculation potentially affects the linear correlation between this and F1 values.

In another study [5], the authors obtained relevant results by eliminating human intervention during the estimation of static similarity thresholds used by the system. However, the resultant information was used only as metadata for future similar applications, given the high computational cost of calculating silhouette coefficients, various different clusters and similarity functions for the entire collection (which excludes it from the dynamic definition of similarity thresholds). Despite being defined automatically, similarity threshold values remain used as a static similarity threshold, set and fixed for a given application. However, given that different searches may require different similarity threshold values, the definition of a single value for the latter ultimately compromises the efficiency of searches. Another relevant question regarding the use of a single similarity threshold value is that it may lose quality with the addition of new objects to the collection.

Unlike the previous methods, this work proposes an automatic and online approach, named automatic setting similarity threshold (ASTS), capable of estimating the most suitable similarity threshold value in accordance with the silhouette coefficient for individual searches, without any previous knowledge of the collection. The silhouette coefficient, originally proposed in [13] for interpretation and validation of consistency within general clusters of data, is used as a simple quality measure of the clustering step of ASTS, allowing the automatic estimation of similarity thresholds associated to individual searches. Note that the silhouette coefficient is an internal evaluation measure [30], which does not require an evaluated dataset, i.e. it does not require matching data instances to be known. As a result, our approach does not require human intervention.

3 Proposed approach

When using information retrieval systems, users expect to obtain a set of images somewhat connected with the searched object. Thus, provided answers should match the best image cluster present in the collection (collection images similar to the query). A previous study [13] indicates that a silhouette coefficient is a good cluster quality indicator, suggesting that this metric leads to a good F1 value if a good answer is obtained for any given search.

Figure 3 shows the proposed approach for automatic online setting of similarity thresholds. It can be described as follows: during searches, an input image supplied to the search system goes through a preprocessing (module 1) responsible for extracting the image's signature. A search and comparison (module 2) then analyzes the image signatures stored in the repository (database) and compares them with the desired image using a similarity function.

Importantly, the visual signatures of images are based on color, shape and texture information and are estimated by using the method proposed in [12]. In that work, the authors address the development of a unified approach to content-based indexing and retrieval of digital videos from television archives and estimate visual signatures to represent key frames of video recordings. More specifically, by using the method described in [12], we compute a visual signature for each image involved in our problem, containing 79 components (54 refer to color, 18 refer to texture and 7 refer to shape positions).

To ensure good recall, a low similarity threshold value ($\delta \approx 0$) is initially considered. When obtaining the result of the search and comparison module (possibly with high recall), the cluster of returned images is refined to increase the response's accuracy. Such refinement regroups the

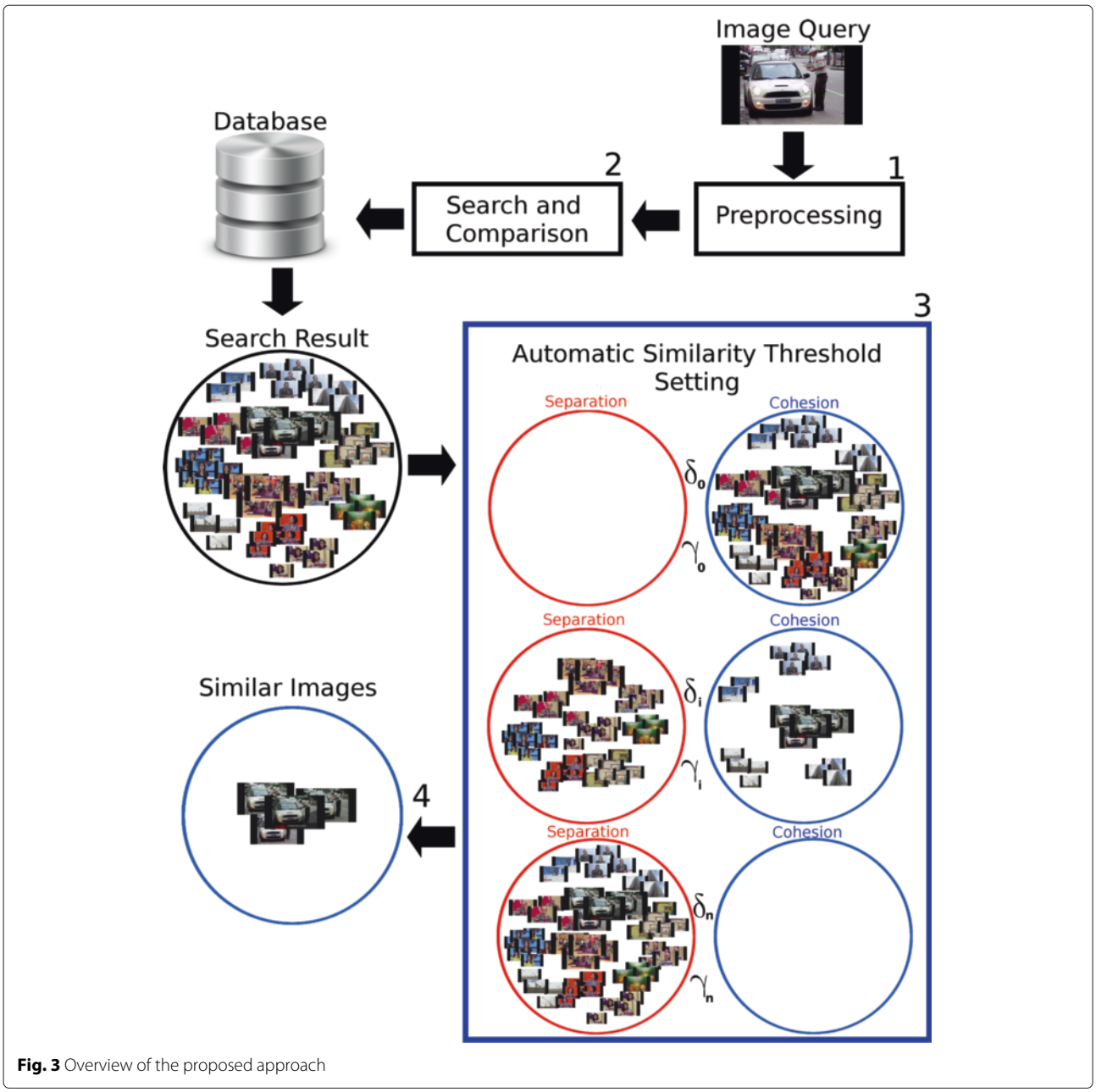


Fig. 3 Overview of the proposed approach

cluster of images initially returned by the search and comparison module into two clusters: (i) similar and (ii) dissimilar images to the query image. This step is executed by the similarity threshold automatic setting (module 3). The grouping is based on different similarity threshold values $(\delta_0, \ldots, \delta_{i-1}, \delta_i, \delta_{i+1}, \ldots, \delta_n)$, where the group of similar images contains objects with a degree of similarity greater than the δ_i similarity threshold value and the dissimilar images contain the remaining ones. Each δ_i is thus associated with a cluster evaluated by a silhouette coefficient (γ). The latter is associated with its corresponding δ_i. Each δ_i associated with a cluster is hereby also associated with a γ_i $(\gamma_0, \ldots, \gamma_{i-1}, \gamma_i, \gamma_{i+1}, \ldots, \gamma_n)$. Finally, users receive a group of similar images (module 4) tied to the largest γ_i value and to a more appropriate similarity threshold. In other words, it is a cluster of similar images matching the group that obtained the best silhouette coefficient evaluation.

Algorithm 1 generically describes the proposed approach steps to such an extent that it encompasses implementations ranging from deterministic algorithms such as ASTS proposed in this study and presented in Section 3.2. The proposed approach does not previously establish a stop condition, in a way that such a condition is set according to the implementation and what is expected of the application being used.

Algorithm 1: Proposed approach

Input: query image i_q, silimilarity threshold $\delta \approx 0$
Output: Sub-cluster of similar images (C_m) associated with the best δ

1 Search of i_q with a δ collection similarity threshold;
2 Response set I from the system;
repeat
3 Generate new δ value;
4 Regroup I according to δ in a sub-cluster of similar images (C_m) and dissimilar images sub-cluster (C_t);
5 Evaluate C_m and C_t generated from δ, to obtain the γ related to this evaluation;
until *a stop criterion is achieved*;
6 Return a sub-cluster of similar images (C_m) associated with the hightest γ value;

This section is arranged as follows. In Section 3.1, we present the complexity analysis of the proposed approach. In Section 3.2, we present the ASTS algorithm, which is used to define the similarity threshold.

3.1 Complexity analysis

Once the silhouette coefficient is calculated for different pairs (δ, $[C_m, C_t]$), its computational cost is determined considering that the proposal tries to solve the problem online. The computational cost of the similarity threshold setting strategy uses a silhouette coefficient function to select the best answer. Algorithm 1 acts as the basis for this strategy cost calculation since it presents the proposed approach.

The operations related to distance calculation between vectors are used here as a metric to determine the computational cost of silhouette coefficients, adopting the formula used to calculate computational costs, as defined in [13]. The cohesion's computational cost is given by $O(n_m - 1)$, where n_m is the amount of images in a C_m cluster, and the computational cost of separation is $O\left(\sum_{j=1}^{K} n_t^j\right)$, where K is the number of groups of dissimilar images (C_t) and n_t is the number of images in each group C_t^j. The amount of clusters for the proposed approach is always two (one of similar images and other of dissimilar images to the query image). The result ($K = 1$) thus reduces the separation computational cost to $O(n_t)$. The silhouette coefficient calculation cost for a single object is $O(n_m - 1 + n_t)$; the silhouette coefficient calculation cost for cluster equals $O\left(n_m^2 - n_m + n_t\right)$. The total computational cost is $O\left(n_m^2\right)$ for each cluster silhouette coefficient calculation. The computational silhouette coefficient is therefore dictated by the quantity of images in C_m: the more images in a group, the higher is the computational cost of its silhouette coefficient calculation.

The computational cost of the proposed approach is defined by the number of times the silhouette coefficient calculation is performed. If N equals the number of times a silhouette coefficient is calculated to define the similarity threshold, the proposed approach cost is $O\left(N \times n_m^2\right)$. Thus, our proposed algorithm presents constant running time (apart from silhouette coefficient computation, which presents squared complexity).

Once the computational cost of the silhouette coefficient and the used similarity threshold setting strategy is known, adjustments can be made to achieve a better balance between runtime and effectiveness.

3.2 ASTS algorithm

The ASTS algorithm is a deterministic algorithm designed for automatic setting of similarity thresholds on the basis of a greedy paradigm. It consists of the heuristic used to estimate the best similarity threshold used in image searches (module 3 in Fig. 3). The ASTS is based on two steps: first, different similarity thresholds are explored to evaluate the quality of each one; second, the thresholds close to the best similarity threshold found by step 1 are exploited.

ASTS implementation requires establishing the following variables: l' and l'' – are the lower and upper limits for δ, respectively; α is the increment value of l' to l'', which generates δ values having the initial exploration function $(\delta_1, \ldots, \delta_{i-1}, \delta_i, \delta_{i+1}, \ldots, \delta_n)$, and $\alpha < (l'' - l')$; β evaluates the answers closer to the best solution found until then (with $\beta < \alpha$). The operation performed with the aid of β values is made through increases and decreases of the δ value associated with the best solution. Algorithm 2 details the ASTS operation.

According to Algorithm 2, once input variables (l', l'', α, β) are defined and a set of answers (I) is obtained via an information retrieval system, Eq. 1 generates δ values contained within an l' to l'' range (line 1). A cluster is made from each δ value $(\delta_1, \ldots, \delta_{i-1}, \delta_i, \delta_{i+1}, \ldots, \delta_n)$ generated by Eq. 1 so that for each δ_i, there is a cluster $(C_m^{\delta_i}, C_t^{\delta_i})$ (line 3). For each cluster $(C_m^{\delta_i}, C_t^{\delta_i})$ generated by δ_i, a silhouette coefficient (γ_{δ_i}) associated with the cluster is calculated reflecting its quality (line 4). This cluster generator method and respective silhouette coefficient evaluation is repeated for all δ values generated in Eq. 1 (line 2).

Once all solutions generated for the initial operation are evaluated, the solution with the highest γ_{δ_i} value is selected to ensure that it is the best one. The solution is then stored in two clusters with two variables $\left(\left(\delta^+_{\text{better}}, \gamma^+_{\text{better}}\right), \left(\delta^-_{\text{better}}, \gamma^-_{\text{better}}\right)\right)$ upon which improvement is refined by increases and decreases (line 5). Thus, the refining process of $\left(\delta^+_{\text{better}}, \gamma^+_{\text{better}}\right)$ is accomplished by increases and decreases in β values to the same β value in $\left(\delta^-_{\text{better}}, \gamma^-_{\text{better}}\right)$.

Algorithm 2: ASTS

Input: I, l', l'', α, β

Output: R

1 generation of the initial δ values for exploration of the search space:

$$\delta_i = \begin{cases} l' & \text{se } i = 1, \\ \delta_{i-1} + \alpha & \text{se } 1 < i < n,\ n = \dfrac{l'' - l'}{\alpha} + 1 \\ l'' & \text{se } i = n \end{cases} \quad (1)$$

/* Creation of clusters for each threshold generated by Eq. 1, being evaluated by the silhouette coefficient */

2 **for** *todo* δ_i **do**

3 reassembles the answer set I, acording to δ_i, into two subgroups $C_m^{\delta_i}$ and $C_t^{\delta_i}$;

4 compute the silhouette coefficient γ_{δ_i} to get $C_m^{\delta_i}$ e $C_t^{\delta_i}$;

end

5 selects δ_i associated with the highest γ_{δ_i}, assigning it to δ^+_{better} and γ^+_{better}; the same values are assigned to δ^-_{better} and γ^-_{better};

/* Refinement of the best similarity threshold found earlier and defined in δ^+_{better} and γ^+_{better}through β increases */

6 **repeat**

7 $\delta_x = \delta^+_{melhor} + \beta$;

8 reassembles the answer set I according to δ_x into two subgroups $C_m^{\delta_x}$ and $C_t^{\delta_x}$;

9 compute silhouette coefficient γ_{δ_x} to obtain $C_m^{\delta_x}$ and $C_t^{\delta_x}$;

10 the γ^+_{better} value is replaced by γ_{δ_x} if $\gamma_{\delta_x} > \gamma^+_{better}$;

until *($\gamma^+_{better} \leq \gamma_{\delta_x}$) or ($\delta^+_{better} - \alpha \leq \delta_x \leq \delta^+_{better} + \alpha$) or ($l' \leq \delta_x \leq l''$)*;

/* Refinement of the previous best similarity threshold and defined by δ^-_{better} and γ^-_{better} by β decreases */

11 **repeat**

12 $\delta_x = \delta^-_{better} - \beta$;

13 reassembles the answer set I according to δ_x into two subgroups $C_m^{\delta_x}$ and $C_t^{\delta_x}$;

14 compute silhouette coefficient γ_{δ_x} for $C_m^{\delta_x}$ and $C_t^{\delta_x}$;

15 γ^-_{better} is replaced by γ_{δ_x} if $\gamma_{\delta_x} > \gamma^-_{better}$;

until *($\gamma^-_{better} \leq \gamma_{\delta_x}$) or ($\delta^-_{better} - \alpha \leq \delta_x \leq \delta^-_{better} + \alpha$) or ($l' \leq \delta_x \leq l''$)*;

16 Assigns R to the C_m cluster associated with the greatest value between γ^+_{better} and γ^-_{better};

With the best current solution selected, δ values are increased and decreased through β. These produce new δ_x similarity thresholds, which in turn generate new clusters that are evaluated and related to γ_{δ_x} values (lines 7, 8, 9, 12, 13, and 14). This procedure is repeated until a certain δ_x similarity threshold produces a γ_{δ_x} worse than that of the current best solution, i.e. $\gamma^+_{\text{better}} > \gamma_x$ e $\gamma^-_{\text{better}} > \gamma_x$ or until a limit value is reached: $\left(\delta^+_{\text{better}} - \alpha \leq \delta_x \leq \delta^+_{\text{better}} + \alpha\right)$ e $\left(\delta^-_{\text{better}} - \alpha \leq \delta_x \leq \delta^-_{\text{better}} + \alpha\right)$ or $(l' \leq \delta_x \leq l'')$ (lines 6 and 11). When a better solution is found, it replaces the current best (rows 10 and 15). Identical in their logic, both second and third operations repeat Algorithm 2 blocks (lines 6 and 11, respectively); while the first explores solutions close to the best current solution by similarity threshold increases, the second does so with decreases in the same value.

After finishing exploring solutions close to the initially selected better similarity threshold, Algorithm 2 returns to users a cluster of similar images associated with the highest value of γ – in range γ^+_{better} and γ^-_{better}.

4 Experimental results

To better present experimental results, this section is divided into four subsections. Section 4.1 describes experimental planning. Section 4.3 details the baselines used for performance comparison. Section 4.4 analyzes the linear correlation between F1 and silhouette coefficient metrics. Finally, Section 4.5 presents experimental results obtained by the proposed approach, comparing them against those of the defined baseline.

4.1 Experiment planning

The first experimental step was to select and organize the templates of three collections: "CAPTE" [11, 12], "Corel" [31] and "The INRIA Holidays dataset" [32].

The "CAPTE" collection consists of a set of 575 key frames extracted from 90 video blocks found on 11 TV shows aired on Rede Minas television channel [12]. The CAPTE collection is mainly composed of face images from the mentioned TV shows. The "Corel" collection consists of approximately 10,000 general purpose images, which are then reduced to 202 and distributed among 32 similar images classes manually labeled by researchers [31] (the template that was used in this study). "The INRIA Holidays dataset" collection [32] consists of 1491 general images separated into 316 semantically similar image classes.

With the collections and respective templates defined and ready to be used, its images were processed, generating a database of image signatures for each one of them. A previously described method [12] was used to estimate images signatures (module 3 in Fig. 3).

To calculate image similarity, five functions were analyzed to assess the similarity function with best F1

results, namely cosine of the θ angle, Manhattan distance, Euclidean distance, Pearson correlation coefficient and histogram intersection. These similarity functions were chosen because for their low computational cost and workability of the image signatures used.

4.2 Evaluation metric

We use a standard evaluation metric in information retrieval literature: f-measure (F1). This metric is defined in terms of precision (P) and recall (R). Precision is the ratio of the number of correctly returned images to the total number of returned images. Recall is the ratio of the number of correctly returned images to the total number of images that should be returned according to ground truth information. Finally, F1 measure is defined as the harmonic mean of precision and recall, as given by

$$F1 = \frac{2 \times P \times R}{P + R}$$

4.3 Baselines

Following, we detail each baseline used for performance comparison.

4.3.1 Threshold impact on performance

With the collections, the image description and the similarity functions established for each pair [template of a collection/similarity function], we calculated the F1 values associated with different values of similarity threshold. This threshold was expanded from zero to one, by 0.01 increments.

Hence, it was possible to determine which similarity threshold leads to higher F1 (optimal δ_{best}) for a given search using a specific similarity function. The optimal δ can thus be formally defined in Eq. 2:

$$\delta_{\text{best}} = \text{argmax}\left(F1_{\delta_i}^{g_k f}\right), \tag{2}$$

where g_k represents the template related to a particular collection search k, f is the similarity function used and δ_i, for $i = 1, \ldots, n$ represents the similarity threshold considered. The optimal delta represents the value of similarity threshold associated with the highest F1 in a given search k for a specific collection when using similarity functions f. It is expected that optimal δ values close to the automatically set similarity threshold proposed (δ_γ) produce more effective results.

Figure 4a, b, c presents the cumulative F1 sum reached by each pair [collection/similarity function] for all tested δ values, representing "CAPTE", "The INRIA Holidays dataset" and "Corel" collections, respectively.

Figure 4a, b, c also shows that a unique similarity threshold value does not guarantee optimum performance

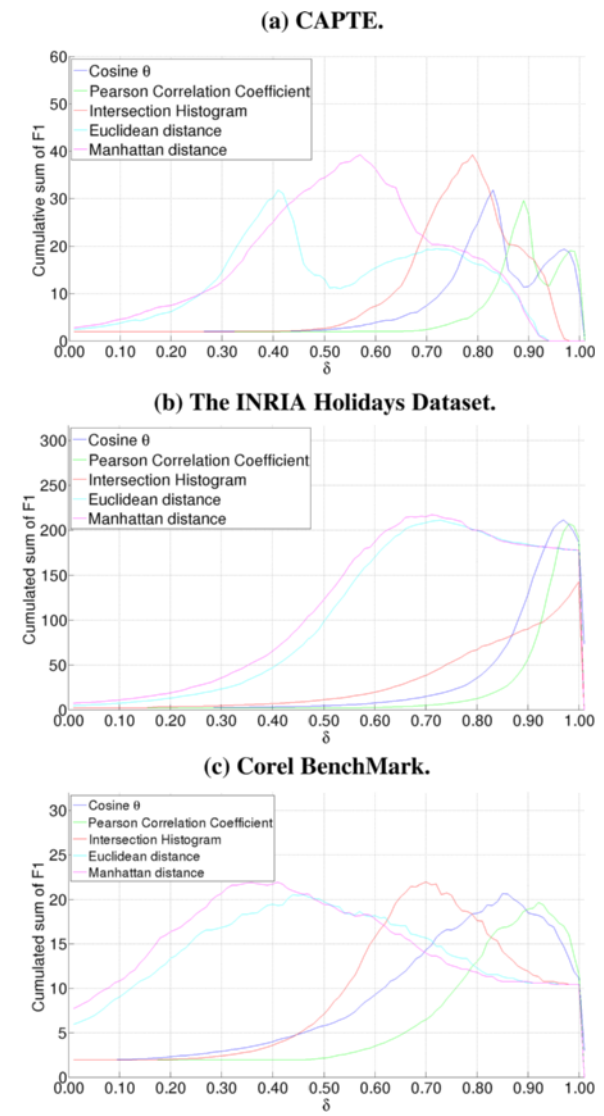


Fig. 4 Curve of the cumulative F1 sum for each pair [f, δ]: similarity function f; similarity threshold considered δ

in a content-based visual information retrieval system. However, it can produce satisfactory results if well defined. Through these figures, it is possible to determine similarity thresholds for each similarity function, leading to the system's greatest cumulative F1 sum. These similarity thresholds (fixed δ) form the baseline, as presented in Table 1. Cumulative F1 sum values for each fixed δ are used to compare and validate the results achieved with the approach proposed hereby (Table 2).

From Table 2, we observe that the best F1 values for all collections is given by the similarity function based on the Manhattan's distance, excepting "Corel benchmark" collection.

Table 1 Fixed δ values defined for each pair [collection/similarity function]

f	CAPTE	INRIA	Corel
Cosine of the θ angle	0.82	0.96	0.85
Pearson correlation coefficient	0.88	0.97	0.91
Histogram intersection	0.78	0.98	0.69
Euclidean distance	0.40	0.72	0.45
Manhattan distance	0.56	0.70	0.40

4.3.2 k-means analysis

Given that our proposed approach is based on clustering similar images, we investigate the performance of *k*-means algorithm [33] retrieving relevant images. The *k*-means algorithm is a classical method for clustering. Here, we represent images by their descriptors and vary the number of clusters (i.e. the *k* parameter) to evaluate the performance of *k*-means when compared to our proposed approach.

Specifically, we start by finding an initial set of images by conducting a query over the whole dataset. Hence, this set is used as input to *k*-means and a given number of expected clusters (i.e. *k* parameter). Finally, given the returned clusters found by *k*-means, we return to the user the cluster of images that have the highest average similarity value among the images within the cluster and the input image (i.e. the one used as query).

In Table 3, we present the results of cumulative F1 measure with distinct *k* values. The best results obtained by *k*-means refer to "Corel" datasets, which is also the best results for our approach (i.e. ASTS) when considering *k* equals to 4 and 8. For all tested *k* values, the proposed approach presents better results than the strong baseline, i.e. the *k*-means algorithm.

Moreover, the experiments show that as we increase the values of *k*, the performance of the *k*-means algorithm also increases. We believe that this happens because as we increase the number of clusters, the precision also increases. Another possibility is that precision increases but recall decreases, which is a well known trade-off in information retrieval literature. We avoid this by limiting the number of generated clusters to avoid empty empty groups of images.

Table 2 Cumulative F1 sum for each pair [collection/similarity function] for fixed δ values set (Table 1)

f	CAPTE	INRIA	Corel
Cosine of the θ angle	*31.73*	*211.96*	*20.68*
Pearson correlation coefficient	29.55	*207.22*	19.65
Histogram intersection	39.18	134.87	21.97
Euclidean distance	*31.73*	*211.10*	*20.57*
Manhattan distance	39.18	216.90	21.92

Table 3 Cumulative F1 sum for *k*-means algorithm with distinct *k* values

k	CAPTE	INRIA	Corel
$k = 2$	3.9492	6.0073	9.2071
$k = 4$	5.6523	9.6352	14.1657
$k = 8$	-	13.6408	18.3246

We also compare the performance of *k*-means algorithm and our proposed approach. For this comparison, we used the cumulative F1 measure and the Euclidean distance as similarity metric. The results are summarized in Table 4. We vary the *k* values for each query for all datasets and present the mean, standard deviation, absolute F1 values and the relative gains of our approach over the *k*-means algorithm.

The results presented in Table 4 show that the performance of our ASTS algorithm is mostly higher than the performance of the *k*-means baseline. A different behavior is observed for the "Corel" dataset when considering $k = 4$ and $k = 8$. Note that the performance of the *k*-means algorithm is worst than the performance of our approach when using fixed δ. We observe that our approach presents a better performance than the tested baseline. Besides the lower performance of *k*-means, it has a disadvantage that the number of clusters needs to be specified beforehand. This can be difficult for large datasets and, as shown in experiments, has considerable impact on the method's performance. Contrarily, the proposed ASTS algorithm avoids this problem, which represents an advantage for researchers and practitioners. Finally, we observe that even when keeping fixed values for δ the proposed ASTS algorithm has a higher performance than *k*-means algorithm for datasets "CAPTE" and "The INRIA Holidays dataset".

4.4 Linear correlation

A high linear correlation between F1 and silhouette coefficients ensures quality in this approach's estimations. Therefore, for each collection searched, linear correlation between these metrics was calculated. For all collections, observations revealed both high and low/almost non-linear correlation between the metrics. Only the "Corel" collection obtained results where the linear correlation was less than zero.

However, a good linear correlation between these metrics is not always guaranteed because F1 calculations consider external criteria absent during the silhouette coefficient calculation (which works with internal criteria). To calculate F1, specialists may include subjective criteria for deciding on similarity or dissimilarity of search-returned images: criteria missing on the characteristics described by the images signatures that directly

Table 4 Cumulative F1 sum reached by ASTS and *k*-means for each *k* value

CAPTE				
k value	F1 mean	Standard deviation	Cumulative F1 sum by *k*-means	Cumulative F1 sum *k*-means relative to δ_γ
2	0.0658	0.0669	3.9492	13.18 %
4	0.0942	0.1014	5.6523	18.86 %
The INRIA Holidays dataset				
k value	F1 mean	Standard deviation	Cumulative F1 sum by *k*-means	Cumulative F1 sum *k*-means relative to δ_γ
2	0.0190	0.0429	6.0073	3.3 %
4	0.0304	0.0728	9.6352	5.3 %
8	0.0431	0.1125	13.5408	7.5 %
Corel benchmark				
k value	F1 mean	Standard deviation	Cumulative F1 sum by *k*-means	Cumulative F1 sum *k*-means relative to δ_γ
2	0.2877	0.2474	9.2071	86.92 %
4	0.4426	0.3378	14.1657	133.73 %
8	0.5726	0.3562	18.3246	172.99 %

affect cluster evaluation through silhouette coefficient calculation. Therefore, a higher or lower linear correlation between F1 and silhouette coefficient may be obtained for different templates of the same collection.

Figure 5a, b, c[1] maps the linear correlation between such metrics for each search of the "CAPTE", "Corel" and "The INRIA Holidays dataset" collections. The "CAPTE" collection template reveals that 63 % of the searches achieved a linear correlation above 70 %, with an average linear correlation of 54 %. In total, 80 % of the "The INRIA Holidays dataset" templates queries reached a linear correlation above 70 %, with an average linear correlation of 78 %. The "Corel" template results were not as good as the first two: only 12 % of the searches attained linear correlation results above 70 %, with an average linear correlation of only 5 %.

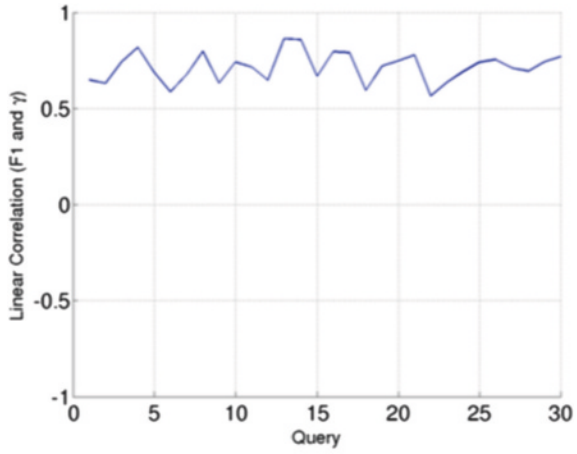
As previously stated, a satisfactory linear correlation between F1 and silhouette coefficients is not always possible. However, the linear correlation between these metrics may improve: the closer the decision criteria on the similarity between images is to that used by experts and by the system, the better the linear correlation between F1 and the silhouette coefficient is. Thus, according to the results obtained in this subsection, a lower efficacy is expected for the "Corel" collection when using the proposed approach.

4.5 Proposed approach evaluation

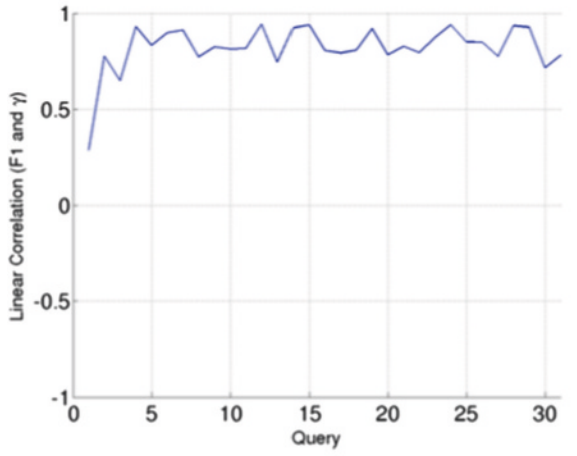
To evaluate the proposed approach, the ASTS algorithm was implemented and executed for the three collections and for five defined similarity functions. The evaluation of the retrieval process is the same for all datasets. The input query image is randomly chosen from a given cluster and the ground truth data are the other images that compose the given cluster. The aim is having as high as possible values for precision and recall, which leads to high F1 values.

In [5], the authors conducted a set of experiments to evaluate the correlation between F1 and silhouette coefficient metrics, using four similarity functions on six datasets. These experiments demonstrated that, when the silhouette coefficient is highly correlated with the F1, the similarity threshold value that maximizes the F1, on a pair dataset/similarity function, also maximizes the silhouette coefficient. Based on this result, we can say that a high linear correlation between F1 and silhouette coefficient metrics ensures efficiency in our proposed approach. Therefore, for each collection searched, linear correlation between these metrics was calculated. For all collections, observations revealed both high and low/almost non-linear correlation between the metrics. Only the "Corel" collection obtained results where the linear correlation was less than zero.

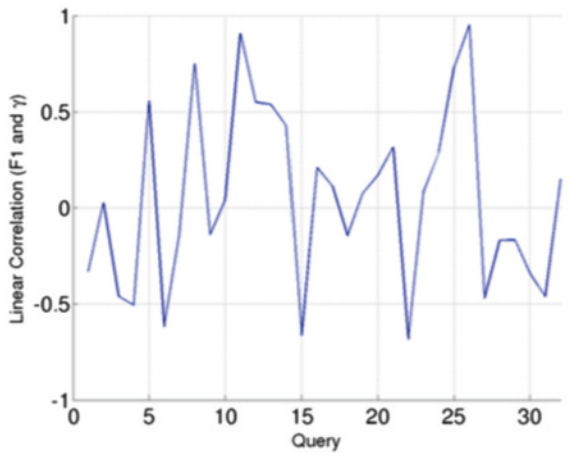
ASTS automatically estimated similarity thresholds for each search with a different similarity threshold (δ_γ) set for each of those. Figure 6a, b, c shows this value and an optimal δ. We therefore used the Manhattan distance similarity function *f*, which performed better with fixed δ (Table 2).



(a) CAPTE.



(b) The INRIA Holidays Dataset.



(c) Corel Bench Mark.

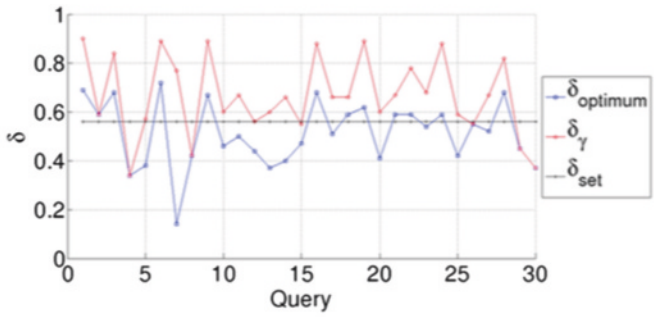
Fig. 5 Linear correlation between F1 and the silhouette coefficient for each template

Figure 6a, b shows that ASTS could estimate similarity thresholds (δ_γ) close to δ optimal values. Such behavior often leads to high F1 values. Figure 6c shows that most of ASTS searches estimated δ_γ values distant from optimal δ (given the linear correlation observed in this collection; see Section 4.4).

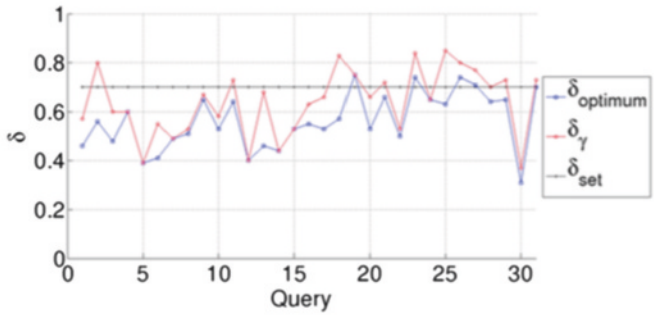
To assess result quality of the proposed approach, we compared our cumulative F1 sums with those attained by expert-set fixed similarity thresholds (baseline). Table 5 shows the cumulative F1 sum reached by ASTS, by similarity function used, considering all searches of the same collection. Table 2 presents the ASTS performance achieved by the baseline, in which we present F1 mean, F1 standard deviation, cumulative F1 for *k*-means and cumulative F1 for *k*-means relative to a fixed delta.

Table 5 shows the collections where ASTS achieved the best results ("CAPTE" and "The INRIA Holidays" dataset). In the "CAPTE" collection, all ASTS-evaluated similarity functions achieved performances greater than 70 % when compared with a cumulative F1 sum for a fixed similarity threshold. In addition, in two other cases, the sums resemble those obtained with a fixed similarity threshold (99.5 and 94.4 %). "The INRIA Holidays dataset" collection also obtained good results, except for the similarity histogram intersection function, which achieved only 55.2 % of what was achieved with this same function when using a fixed similarity threshold. However, the remaining collection reveals performances superior to 60 %: two cases stood out with 86.0 % (for Euclidean distance) and 83.3 % (for Manhattan distance). Only one case of the "Corel" collection achieved a performance superior to 70 %.

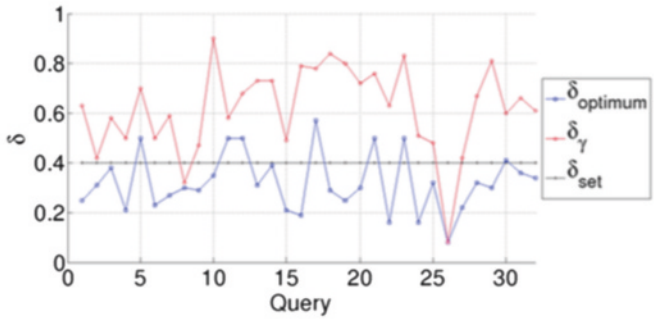
The greatest contribution of this study is the achievement of automatic results, without specialist input, matching the best results attained by expert-defined similarity thresholds (baseline), for a given collection. Other advantages of the proposed approach are as follows: (i) scalability, (ii) resistance to changes in image content, (iii) change in the similarity function, (iv) insertion and removal of new images in the collection, and (v) no previous knowledge of the collection. Furthermore, the approach proposed hereby has the potential to achieve results superior to those obtained via fixed similarity thresholds (because the dynamic setting of a suitable value for this threshold increases result effectiveness). Improvement of the linear correlation between F1 and silhouette coefficients is required though. As described above, expert F1 calculation presupposes a template of desired queries, separating similar from dissimilar images. The criteria considered when determining image similarity or dissimilarity depends on experts. Silhouette coefficient calculation groups images according to certain thresholds and similarity functions with the similarity calculation performed on the image



(a) Valores de δ_γ estimados para a coleção CAPTE.



(b) Valores de δ_γ estimados para a coleção The INRIA Holidays Dataset.



(c) Valores de δ_γ estimados para a coleção Corel Bench Mark.

Fig. 6 δ_γ, δ fixed and δ optimal values with the Manhattan distance similarity function

signature. Therefore, the closer the criterion used by specialists is to that used for similarity calculation by similarity functions f, the greater is the linear correlation between these measures and the better results become.

We also investigate the computational costs per query. The experiments were implemented in MATLAB and run on a Intel Core i7 2.1 GHz with 32 GB of memory. In Fig. 7, we show the running time for each query as well as the average for the CAPTE dataset[2]. As we can see, there is variation on running time for each query.

4.6 Parameter sensitivity

In this section, we investigate the impacts of different parameter settings on the overall performance of our method. First, we discuss the sensitivity of the α parameter, which refers to the number of initial solutions to be searched in the first step of our algorithm. Hence, we discuss the sensitivity of the β parameter, which refers to the number of answers closer to the best solution found at a given interaction.

Effect of α with fixed β. In Fig. 8, we present the resulting performance costs of our ASTS algorithm when varying α

Table 5 Cumulative F1 sum reached by ASTS for each similarity function

CAPTE				
f	Mean	Standard deviation	Cumulative F1 sum	δ_γ compared to δ_{fixo}
Cosine of the θ angle	0.5263	0.3578	31.58	99.5 %
Pearson correlation coefficient	0.3591	0.3840	21.55	72.9 %
Histogram intersection	0.5244	0.3196	31.47	80.3 %
Euclidean distance	0.4995	0.3319	29.97	94.4 %
Manhattan distance	0.4913	0.3047	29.48	75.2 %
The INRIA Holidays dataset				
f	Mean	Standard deviation	Cumulative F1 sum	δ_γ compared to δ_{fixo}
Cosine of the θ angle	0.5218	0.2330	164.90	77.8 %
Pearson correlation coefficient	0.4134	0.2923	130.64	63.0 %
Histogram intersection	0.2358	0.2262	74.54	55.2 %
Euclidean distance	0.5742	0.1498	181.47	86.0 %
Manhattan distance	0.5718	0.1467	180.70	83.3 %
Corel benchmark				
f	Mean	Standard deviation	Cumulative F1 sum	δ_γ compared to δ_{fixo}
Cosine of the θ angle	0.3751	0.2078	12.00	58.0 %
Pearson correlation coeficcient	0.4520	0.2293	14.46	73.6 %
Histogram intersection	0.3318	0.1423	10.62	48.3 %
Euclidean distance	0.3310	0.1406	10.59	51.5 %
Manhattan distance	0.3310	0.1406	10.59	48.3 %

parameter. As we increase the α values from 0.1, the computational costs also increase, because we are considering more candidate solutions.

Effect of α and β combinations. In Fig. 9, we investigate how different combinations of α and β parameters affect computational performance of our ASTS algorithm. When decreasing values of β leads to more computational costs because we are considering a larger solution space. The opposite is also valid, i.e. increasing β values leads to less computational costs because we are considering more restricted regions in solution space.

5 Conclusions

The main challenge of this study is the dynamic and automatic setting of an appropriate similarity threshold value to be used in searches of content-based visual information retrieval systems. To accomplish this, we proposed an approach using a metric silhouette coefficient in which basic principles were implemented through an ASTS algorithm (also adopted here).

Performed tests revealed promising ASTS results. For example, the "CAPTE" and "The INRIA Holidays dataset" collections (with the greatest number of images) showed performance close to 100 %, obtained automatically (with cumulative F1 sums of the proposed approach compared with the baseline cumulative F1 sum). Notably, however, the templates of each collection contained semantically similar images (simple image signature techniques combined with similarity functions appropriate to this type of signature), thus producing good results. The techniques used on the "Corel" collection were not sufficient for the image descriptors to properly represent the characteristics that determine image resemblance, thus being irrelevant to the proposed approach.

In general, our approach achieved good results, often reaching high performances very close to those achieved by fixed specialist set similarity thresholds. Such results encourage the continuation of this work, to solve a question that significantly enhances the quality of content-based visual information retrieval systems.

We believe that pairing more robust image signatures with appropriate similarity functions to compare the first will lead to better results and a more efficient approach. The computational cost of signature calculation and comparison must be accounted for in proposals seeking to automatically determine similarity thresholds

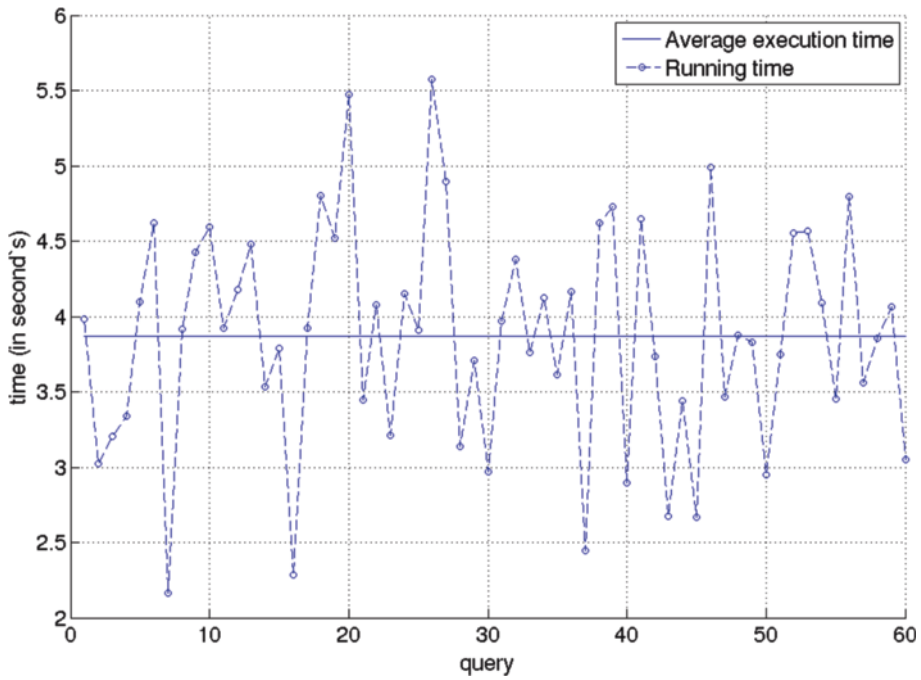


Fig. 7 Computational costs per query for CAPTE dataset

online. Image signatures used hereby are classic models. They work with global characteristic descriptors, describing images with a single vector (with low computational cost for both signature calculation and for its comparison through similarity functions). However, such signatures may not be so effective to describe image signatures working with local features. On the other hand, signature calculation on the basis of local characteristics and the calculation of its similarity (comparison of a set of vectors rather than a single vector) have high computational

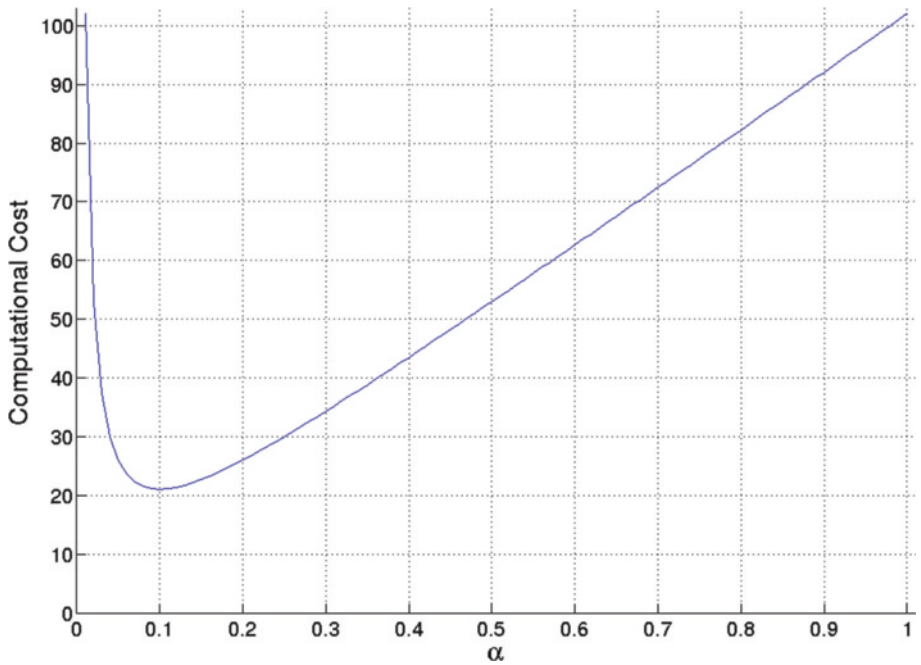


Fig. 8 Computational costs of ASTS algorithm varying α parameter with β parameter equals to 0.01

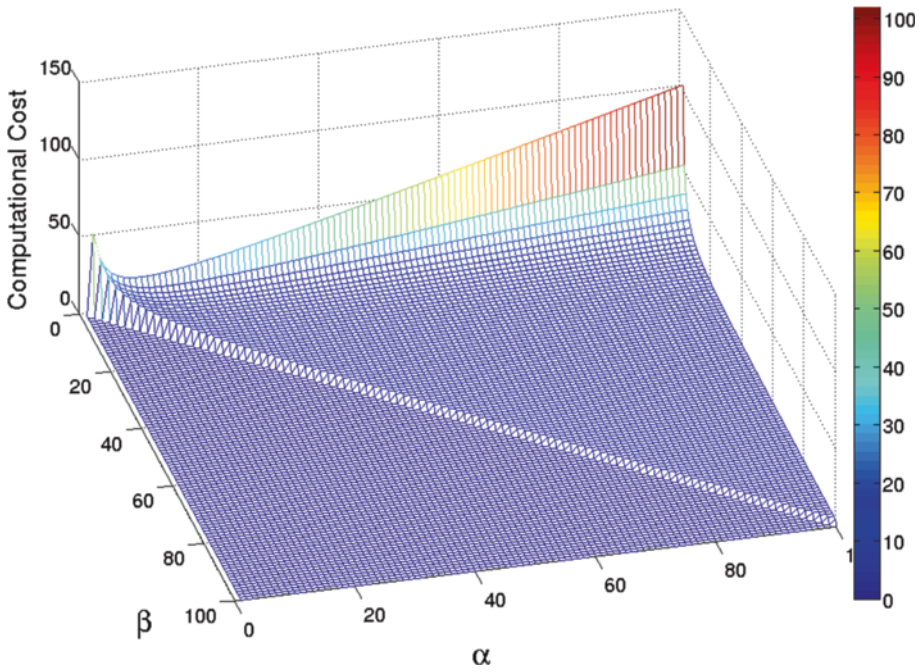


Fig. 9 Computational costs of ASTS algorithm varying α and β parameters

cost, particularly when compared to techniques working with global characteristics. It is therefore necessary to prevent silhouette coefficient computational cost increases, rendering the proposed approach unfeasible.

As future work, we plan to investigate better image representation as well as appropriate similarity functions to compare their signatures. Such way of representation and comparison directly impacts this proposal's effectiveness and efficiency. A balance between these two characteristics (or the predominance of one over the other) depends on what is expected of the system to which the proposed approach is applied. The improvement of such techniques aims to produce high linear correlation between silhouette coefficients and F1s, which will help to generate good results in the information retrieval process.

Endnotes

[1]Figures concerning the linear correlation for "CAPTE" and "The INRIA Holidays dataset" collection searches present only a data sample.

[2]Since results for Corel and INRIA datasets are similar, we omit these results.

Competing interests
The authors declare that they have no competing interests.

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