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Distributed piecewise $\mathscr{H}_\infty$ filtering design for large-scale networked nonlinear systems

Zhenhua Shao* and Tianxiang Chen

Abstract

This paper investigates the problem of distributed piecewise $\mathscr{H}_\infty$ filtering for discrete-time large-scale nonlinear systems. The considered large-scale system is composed of a number of nonlinear subsystems and exchanges its information through communication network. Each nonlinear subsystem is described by a Takagi-Sugeno (T-S) model, and data-packet dropouts happen intermittently in communication network, and its stochastic variables are assumed to satisfy the Bernoulli random-binary distribution. Our objective is to design a distributed piecewise filter such that the filtering error system is stochastically stable with an $\mathscr{H}_\infty$ performance. Based on a piecewise Lyapunov function and some convexifying techniques, less conservative results are developed for the distributed piecewise $\mathscr{H}_\infty$ filtering design of the considered system in the form of linear matrix inequalities (LMIs). The effectiveness of the proposed method is validated by two examples.

Keywords: Large-scale fuzzy systems, Distributed $\mathscr{H}_\infty$ filter, Piecewise Lyapunov function

1 Introduction

In practical application, some complex systems, such as transportation systems, power systems, communication networks, and industrial processes, are referred to as large-scale systems [1, 2]. Due to strong interconnection and high dimensionality, large-scale systems lead to severe difficulties for their analysis and control synthesis. To date, three main control approaches, centralized, decentralized, and distributed control, have been proposed for large-scale systems with interconnection. Since the centralized control suffers from the excessive information processing and heavy computational burdens, there has been recently an increasing interest in the use of decentralized control for large-scale systems [3]. The decentralized control is firstly to partition the overall control problem of a large-scale system into several independent or almost independent subproblems. Then, instead of a single controller, a set of independent controllers can be designed to achieve the overall control of large-scale system [4]. However, the decentralized control strategy appears weaker stability margins and performance, especially when the interconnections among subsystems are strong [5]. In the distributed control, the supplemental feedbacks with the interconnected information are provided for the local controllers to enhance the requirements of stability and performance. As a result, the distributed control avoids those shortages appearing in both centralized and decentralized controls [6, 7].

On the other hand, an important issue is to consider the control problems of nonlinear systems because most control plants are nonlinear. Recently, Takagi-Sugeno (T-S) model has been proved to be a powerful solution to represent any smooth nonlinear functions at any preciseness [8, 9]. The T-S model employs a group of IF-THEN fuzzy rules to describe the global behavior of the nonlinear system in which a number of linear models are connected smoothly by fuzzy membership functions. T-S fuzzy approach combining the merits of both fuzzy logic theory and linear system theory is successfully implemented in embedded microprocessors and is widely applied in a variety of engineering fields [10–13]. During the past few years, a great number of results on function approximation, systematic stability analysis, controller and filtering design for T-S fuzzy systems have been reported in the open literature [14–19].

With the rapid development of digital technology, in the feedback loops, communication networks are often used instead of point-to-point connections due

*Correspondence: zhixiongzhong2012@126.com
High-voltage Key Laboratory of Fujian Province, Xiamen University of Technology, Xiamen 361024, People's Republic of China

to their great advantages, such as simple maintenance and installation, and low cost [20–24]. Unfortunately, the network-induced imperfections, such as quantization errors, packet dropouts, and time delays, can degrade significantly the performance of control systems and may even lead to instability [25–29]. Recently, based on fuzzy/piecewise Lyapunov functions, some results on stability analysis and controller synthesis of fuzzy systems have been presented. It has been demonstrated that the inherently conservatism in common Lyapunov function can be relaxed by using piecewise/fuzzy Lyapunov functions. More recently, T-S fuzzy control has been developed to investigate large-scale nonlinear systems [30–35]. To mention a few, some results on analysis and synthesis methods for decentralized control of large-scale systems have been presented in [30–32]. In [33, 34], the decentralized $\mathscr{H}_\infty$ filtering problem was studied for the discrete-time large-scale system with time-varying delay. To the best knowledge of the authors, few results on the distributed $\mathscr{H}_\infty$ filtering design have been given for large-scale networked T-S fuzzy systems by using piecewise Lyapunov function, which motivates us for the research presented in this paper.

This paper will deal with the distributed $\mathscr{H}_\infty$ filtering problem for discrete-time large-scale nonlinear systems. The large-scale system is composed of several nonlinear subsystems and exchanges its information through communication network. Each nonlinear subsystem is described by a T-S model, and data-packet dropouts occur intermittently in communication network, and its stochastic variables satisfy the Bernoulli random-binary distribution. Based on a piecewise Lyapunov functional (PLF) and some convexifying techniques, the distributed $\mathscr{H}_\infty$ filtering design result will be proposed. It will be shown that the filtering error system is stochastically stable with an $\mathscr{H}_\infty$ performance, and the filtering gains can be given by the form of LMIs. Two simulation examples will be presented to demonstrate the advantage of the proposed methods.

Notations. $\Re^{n\times m}$ is the n-dimensional Euclidean space and $\Re^{n\times m}$ denotes the set of $n \times m$ matrices. $P > 0\,(\geq 0)$ means that matrix P is positive definite (positive semidefinite). Sym$\{A\}$ denotes $A + A^T$. $\mathbf{I}_n$ and $0_{m\times n}$ are the $n \times n$ identity matrix and $m \times n$ zero matrix, respectively. The subscripts n and $m \times n$ are omitted when the size is irrelevant or can be determined from the context. For matrices $A \in \Re^{n\times n}$, A^{-1} and A^T denote the inverse and transpose of the matrix A, respectively. $l_2[0,\infty)$ is the space of square-summable infinite vector sequences over $[0,\infty)$. diag$\{\cdot\ \cdot\}$ is a block-diagonal matrix. The notation $\|\cdot\|$ denotes the Euclidean vector norm, and $\|\cdot\|_2$ is the usual $l_2[0,\infty)$ norm. $\mathbb{E}\{\cdot\}$ denotes the mathematical expectation. The notation $\star$ denotes the symmetric terms. □

2 Model description and problem formulation

This paper considers a class of discrete-time large-scale systems, which consist of N nonlinear subsystems, the ith nonlinear subsystem is described by the T-S model as below.

Plant rule $\mathscr{R}_i^l$: **IF** $\zeta_{i1}(t)$ is $\mathscr{F}_{i1}^l$ and $\zeta_{i2}(t)$ is $\mathscr{F}_{i2}^l$ and $\cdots$ and $\zeta_{ig}(t)$ is $\mathscr{F}_{ig}^l$, **THEN**

$$\begin{cases} x_i(t+1) = A_{il}x_i(t) + \sum\limits_{\substack{k=1\\k\neq i}}^{N} \bar{A}_{ikl}x_k(t) + B_{il}w_i(t) \\ y_i(t) = C_{il}x_i(t) + D_{il}w_i(t) \\ z_i(t) = L_{il}x_i(t), l \in \mathscr{L}_i := \{1,2,\ldots,r_i\} \end{cases} \tag{1}$$

where $i \in \mathcal{N} := \{1,2,\ldots,N\}$; $\mathscr{R}_i^l$ denotes the lth fuzzy inference rule; r_i is the number of fuzzy inference rules; $\mathscr{F}_{i\phi}^l\ (\phi = 1,2,\ldots,g)$ are fuzzy sets; $x_i(t) \in \Re^{n_{xi}}$, $y_i(t) \in \Re^{n_{yi}}$, and $z_i(t) \in \Re^{n_{zi}}$ are the system state, the measured output, and the estimated signal, respectively; $w_i(t) \in \Re^{n_{wi}}$ is the disturbance input, which belongs to $l_2[0,\infty)$; $\zeta_i(t) := [\zeta_{i1}(t), \zeta_{i2}(t), \ldots, \zeta_{ig}(t)]$ are some measurable variables of the ith subsystem; $(A_{il}, B_{il}, C_{il}, D_{il}, L_{il})$ denotes the lth local model for the ith subsystem; $\bar{A}_{ikl}$ is the interconnection matrix between the ith and kth subsystems.

Let us define $\mu_{il}[\zeta_i(t)]$ as the normalized membership function of the inferred fuzzy set $\mathscr{F}_i^l := \prod_{\phi=1}^{g}\mathscr{F}_{i\phi}^l$; it yields

$$\mu_{il}[\zeta_i(t)] := \frac{\prod_{\phi=1}^{g}\mu_{il\phi}[\zeta_{i\phi}(t)]}{\sum_{\varsigma=1}^{r_i}\prod_{\phi=1}^{g}\mu_{i\varsigma\phi}[\zeta_{i\phi}(t)]} \geq 0, \sum_{l=1}^{r_i}\mu_{il}[\zeta_i(t)] = 1 \tag{2}$$

where $\mu_{il\phi}[\zeta_{i\phi}(t)]$ denotes the grade of membership of $\zeta_{i\phi}(t)$ in $\mathscr{F}_{i\phi}^l$. For convenience, in the sequel, the argument of $\mu_{il}[\zeta_i(t)]$ will be dropped for the situations without ambiguity, i.e., we denote $\mu_{il} := \mu_{il}[\zeta_i(t)]$.

By using a standard fuzzy inference, we obtain the ith global T-S fuzzy subsystem as below:

$$\begin{cases} x_i(t+1) = A_i(\mu_i)x_i(t) + \sum\limits_{\substack{k=1\\k\neq i}}^{N} \bar{A}_{ik}(\mu_i)x_k(t) + B_i(\mu_i)w_i(t) \\ y_i(t) = C_i(\mu_i)x_i(t) + D_i(\mu_i)w_i(t) \\ z_i(t) = L_i(\mu_i)x_i(t), i \in \mathcal{N} \end{cases} \tag{3}$$

where

$$\begin{cases} A_i(\mu_i) := \sum_{l=1}^{r_i} \mu_{il} A_{il},\ \bar{A}_{ik}(\mu_i) := \sum_{l=1}^{r_i} \mu_{il} A_{ikl},\ B_i(\mu_i) := \sum_{l=1}^{r_i} \mu_{il} B_{il} \\ C_i(\mu_i) := \sum_{l=1}^{r_i} \mu_{il} C_{il},\quad D_i(\mu_i) := \sum_{l=1}^{r_i} \mu_{il} D_{il} \quad L_i(\mu_i) := \sum_{l=1}^{r_i} \mu_{il} L_{il}. \end{cases} \tag{4}$$

In this paper, we will address the distributed $\mathscr{H}_\infty$ filtering design problem of the discrete-time large-scale fuzzy system in (3) based on a piecewise Lyapunov functional (PLF). For each nonlinear subsystem $i \in \mathcal{N}$, we follow the idea proposed in [36, 37], where the premise variable space is partitioned into two different kinds of regions: fuzzy regions and crisp regions. The region with $0 < \mu_{il}[\zeta_i(t)] < 1$ is defined as the fuzzy region, where the system dynamics are governed by a convex combination of several local models dropped into that region. In addition, the crisp region is the region where $\mu_{il}[\zeta_i(t)] = 1$ for some rules l, and the rest of membership functions equal to zero. The system dynamics in crisp region are governed by the l-th local model within that region.

Let $\{\mathcal{S}_{ij}\}_{j\in\mathscr{J}_i}$ be the premise variable space partition for the ith subsystem, and $\mathscr{J}_i$ be the set of region indices. Based on the partition policy, the global T-S fuzzy system in (3) can be rewritten as

$$\begin{cases} x_i(t+1) = \mathcal{A}_{ij} x_i(t) + \sum_{k=1, k\neq i}^{N} \bar{\mathcal{A}}_{ikj} x_k(t) + \mathcal{B}_{ij} w_i(t) \\ y_i(t) = \mathcal{C}_{ij} x_i(t) + \mathcal{D}_{ij} w_i(t) \\ z_i(t) = \mathcal{L}_{ij} x_i(t), \zeta_i(t) \in \mathcal{S}_{ij}, j \in \mathscr{J}_i, i \in \mathcal{N} \end{cases} \tag{5}$$

where

$$\begin{cases} \mathcal{A}_{ij} := \sum_{m\in\mathcal{I}_i(j)} \mu_{im} A_{im},\ \mathcal{B}_{ij} := \sum_{m\in\mathcal{I}_i(j)} \mu_{im} B_{im}\ \ \bar{\mathcal{A}}_{ikj} := \sum_{m\in\mathcal{I}_i(j)} \mu_{im} A_{ikm} \\ \mathcal{C}_{ij} := \sum_{m\in\mathcal{I}_i(j)} \mu_{im} C_{im},\ \mathcal{D}_{ij} := \sum_{m\in\mathcal{I}_i(j)} \mu_{im} D_{im}\ \ \mathcal{L}_{ij} := \sum_{m\in\mathcal{I}_i(j)} \mu_{im} L_{im} \end{cases} \tag{6}$$

with $0 < \mu_{im}[\zeta_i(t)] < 1, \sum_{m\in\mathcal{I}_i(j)} \mu_{im}[\zeta_i(t)] = 1$. For each local region $\mathcal{S}_{ij}, \mathcal{I}_i(j)$ contains the matrix indices used in that region. For a crisp region, $\mathcal{I}_i(j)$ contains a single index.

For convenience, we denote a new set Ω_i, which represents all possible region transitions for the ith subsystem:

$$\Omega_i := \left\{(j,s) \mid \zeta_i(t) \in \mathcal{S}_{ij}, \zeta_i(t+1) \in \mathcal{S}_{is}, (j,s) \in \mathscr{J}_i\right\} \tag{7}$$

where $j \neq s$ when $\zeta_i(t)$ transits from the region $\mathcal{S}_{ij}$ to $\mathcal{S}_{is}$, and $j = s$ when $\zeta_i(t)$ stays in the same region $\mathcal{S}_{ij}$.

Given the large-scale fuzzy system (5) with the premise variable space partition, we propose a distributed piecewise filter of the following form:

$$\begin{cases} x_{Fi}(t+1) = A_{Fij} x_{Fi}(t) + B_{Fij} y_{Fi}(t) + \sum_{k=1, k\neq i}^{N} B_{Fikj} y_{Fk}(t) \\ z_{Fi}(t) = C_{Fij} x_{Fi}(t), j \in \mathscr{J}_i, i \in \mathcal{N} \end{cases} \tag{8}$$

where $x_{Fi}(t) \in \Re^{n_{fi}}$ is the filter state, $z_{Fi}(t) \in \Re^{n_{zi}}$ is an estimation of $z_i(t), y_{Fi}(t)$ is the measured output applied to filter, and $(A_{Fij}, B_{Fij}, B_{Fikj}, C_{Fij})$ are filter gains to be designed, where $n_{fi} = n_{xi}$ for the full-order filter, and $1 \leq n_{fi} < n_{xi}$ for the reduced-order one.

Here, we assume that the data loss happens in the communication links between the filter and physical plain, the measured output $y_i(t)$ is no longer equivalent to $y_{Fi}(t)$. The ith filtering subsystem with unreliable communication network is shown in Fig. 1, where the ith filter takes all measured outputs via unreliable communication links. This paper models the data-loss condition based on a stochastic approach, thus, it yields [38]:

$$y_{Fi}(t) = \alpha_i(t)\, y_i(t), i \in \mathcal{N} \tag{9}$$

where $\alpha_i(t)$ is the independent Bernoulli processes, which represent the unreliable condition of the links from the sensor to the filter. Specifically, $\alpha_i(t) \equiv 0$ when the link fails, i.e., data are lost, and $\alpha_i(t) \equiv 1$ represents successful transmission, and $\alpha_i(t)$ is supposed to be given by

$$\text{Prob}\{\alpha_i(t) = 1\} = \mathbb{E}\{\alpha_i(t)\} = \bar{\alpha}_i,\ \text{Prob}\{\alpha_i(t) = 0\} = 1 - \bar{\alpha}_i. \tag{10}$$

In addition, let us define

$$\tilde{\alpha}_i(t) = \alpha_i(t) - \bar{\alpha}_i. \tag{11}$$

It is easy to see from (11) that

$$\mathbb{E}\{\tilde{\alpha}_i(t)\} = 0, \mathbb{E}\{\tilde{\alpha}_i(t)\,\tilde{\alpha}_i(t)\} = \bar{\alpha}_i(1 - \bar{\alpha}_i). \tag{12}$$

By defining $\bar{x}_i(t) = \left[x_i^T(t)\ x_{Fi}^T(t)\right]^T, \bar{z}_i(t) = z_i(t) - z_{Fi}(t)$, and based on the large-scale fuzzy system in (5) and distributed piecewise filter in (8), the filtering error system is given by

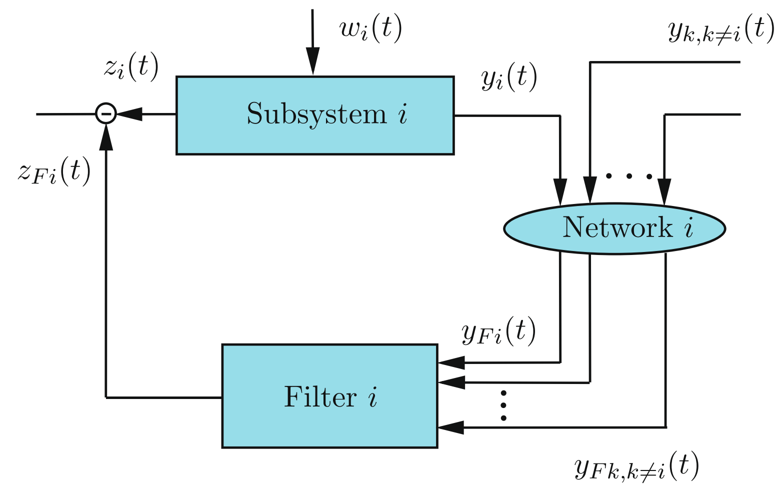

Fig. 1 The *i*th subsystem with unreliable communication network

$$\begin{cases} \bar{x}_i(t+1) = \mathbb{A}_{ij}\bar{x}_i(t) + \sum_{\substack{k=1\\k\neq i}}^{N} \bar{\mathbb{A}}_{ikj}x_k(t) + \mathbb{B}_{ij}w_i(t) + \sum_{\substack{k=1\\k\neq i}}^{N} \bar{\mathbb{B}}_{ikj}w_k(t) \\ \bar{z}_i(t) = \mathbb{C}_{ij}\bar{x}_i(t), \zeta_i(t) \in \mathcal{S}_{ij}, j \in \mathscr{I}_i, i \in \mathcal{N} \end{cases} \tag{13}$$

where

$$\begin{cases} \mathbb{A}_{ij} = \mathscr{A}_{ij}^{(1)} + \tilde{\alpha}_i(t)\mathscr{A}_{ij}^{(2)}, \bar{\mathbb{A}}_{ikj} = \bar{\mathscr{A}}_{ikj}^{(1)} + \tilde{\alpha}_i(t)\bar{\mathscr{A}}_{ikj}^{(2)}, \\ \mathbb{B}_{ij} = \mathscr{B}_{ij}^{(1)} + \tilde{\alpha}_i(t)\mathscr{B}_{ij}^{(2)}, \bar{\mathbb{B}}_{ikj} = E\left(\bar{\alpha}_i B_{Fikj}\mathcal{D}_{kj} + \tilde{\alpha}_i(t) B_{Fikj}\mathcal{D}_{kj}\right), \\ \mathscr{A}_{ij}^{(1)} = \begin{bmatrix} \mathcal{A}_{ij} & 0 \\ \bar{\alpha}_i B_{Fij}\mathcal{C}_{ij} & A_{Fij} \end{bmatrix}, \mathscr{A}_{ij}^{(2)} = \begin{bmatrix} 0 & 0 \\ B_{Fij}\mathcal{C}_{ij} & 0 \end{bmatrix}, \\ \bar{\mathscr{A}}_{ikj}^{(1)} = \begin{bmatrix} \bar{\mathcal{A}}_{ikj} \\ \bar{\alpha}_i B_{Fikj}\mathcal{C}_{kj} \end{bmatrix}, \bar{\mathscr{A}}_{ikj}^{(2)} = \begin{bmatrix} 0 \\ B_{Fikj}\mathcal{C}_{kj} \end{bmatrix}, E = \begin{bmatrix} 0 \\ \mathbf{I} \end{bmatrix}, \\ \mathscr{B}_{ij}^{(1)} = \begin{bmatrix} \mathcal{B}_{ij} \\ \bar{\alpha}_i B_{Fij}\mathcal{D}_{ij} \end{bmatrix}, \mathscr{B}_{ij}^{(2)} = \begin{bmatrix} \mathcal{B}_{ij} \\ B_{Fij}\mathcal{D}_{ij} \end{bmatrix}, \mathbb{C}_{ij} = \begin{bmatrix} \mathcal{L}_{ij} & -C_{Fij} \end{bmatrix}. \end{cases} \tag{14}$$

Definition 1. *Let* $\tilde{x}(t) = \left[\tilde{x}_1^T(t)\ \tilde{x}_2^T(t)\ \cdots\ \tilde{x}_N^T(t)\right]^T$ *and* $\tilde{w}(t) = \left[w_1^T(t)\ w_2^T(t)\ \cdots\ w_N^T(t)\right]^T$. *Then, the filtering error system in (13) is stochastically stable in the mean square if there exists matrix* $W > 0$ *such that*

$$\mathbb{E}\left\{\sum_{t=0}^{\infty} \left|\tilde{x}(t)\right|^2 \middle| \tilde{x}(0)\right\} < \tilde{x}^T(0)\, W \tilde{x}(0) \tag{15}$$

for any initial condition $\tilde{x}(0)$ *when* $\tilde{w}(t) \equiv 0$.

Now, we formulate the distributed piecewise $\mathscr{H}_\infty$ filtering problem as below.

Given the fuzzy filtering system shown in Fig. 1, assume that the communication link parameter $\bar{\alpha}_i$ is available. Given a prescribed scalar $\gamma > 0$, design a distributed piecewise filter in the form of (8) such that the following two conditions are satisfied simultaneously.

1) The filtering error system in (13) is stochastically stable in the sense of Definition 1;
2) Under zero-initial conditions, the estimated error $\tilde{z}(t)$ satisfies

$$\|\tilde{z}\|_{\mathbb{E}} \le \gamma \|\tilde{w}\|_2, \tag{16}$$

where $\|\tilde{z}\|_{\mathbb{E}} := \mathbb{E}\left\{\sqrt{\sum_{t=0}^{\infty} \left|\tilde{z}(t)\right|^2}\right\}$.

If both the conditions are satisfied, then the filtering error system in (13) is stochastically stable with an $\mathscr{H}_\infty$ performance γ.

3 Main results

In this section, based on a piecewise Lyapunov functional (PLF), the performance analysis and design of the distributed piecewise $\mathscr{H}_\infty$ filter will be developed for the considered system in (3). The filter gains will be given for both full-order and reduced-order filters by solving a number of LMIs.

3.1 Distributed $\mathscr{H}_\infty$ filtering performance analysis

Here, we will present a distributed $\mathscr{H}_\infty$ filtering performance analysis, and the result can be summarized in the following lemma.

Lemma 1. *Given the fuzzy system in (3) and distributed piecewise filter in (8), then the filtering error system in (13) is stochastically stable with an $\mathscr{H}_\infty$ performance γ, if there exist matrices $0 < P_{ij} = P_{ij}^T \in \Re^{(n_{xi}+n_{fi})\times(n_{xi}+n_{fi})}, j \in \mathscr{J}_i, i \in \mathcal{N}$, and matrix multiplier $\mathcal{G}_i \in \Re^{(2n_{xi}+2n_{fi}+n_{wi})\times(n_{xi}+n_{fi})}$, $i \in \mathcal{N}$, and matrices $0 < H_{ikj} < H_{ik0} \in \Re^{n_{xi}\times n_{xi}}, 0 < M_{ikj} < M_{ik0} \in \Re^{n_{wi}\times n_{wi}}, j \in \mathscr{J}_i, k \neq i, (i,k) \in \mathcal{N}$, such that for all $(j,s) \in \Omega_i, j \in \mathscr{J}_i, (i,k) \in \mathcal{N}$, the following matrix inequalities hold:*

$$\begin{bmatrix} \Theta_{ijs} + Sym\left\{\Pi_{ij}\right\} & \Psi_{(1)} & f_i\Psi_{(2)} & \Psi_{(3)} & f_i\Psi_{(4)} & \Psi_{(5)} \\ \star & -\mathbb{H}_{ij} & 0 & 0 & 0 & 0 \\ \star & \star & -\mathbb{H}_{ij} & 0 & 0 & 0 \\ \star & \star & \star & -\mathbb{M}_{ij} & 0 & 0 \\ \star & \star & \star & \star & -\mathbb{M}_{ij} & 0 \\ \star & \star & \star & \star & \star & -\mathbf{I} \end{bmatrix} < 0, \tag{17}$$

where

$$\begin{cases} \Theta_{ijs} = diag\left\{ P_{is} \sum\limits_{\substack{k=1\\k\neq i}}^{N} \bar{E} H_{ki0} \bar{E}^T - P_{ij} \sum\limits_{\substack{k=1\\k\neq i}}^{N} M_{ki0} - \gamma^2 \mathbf{I} \right\}, \\ \Pi_{ij} = \left[-\mathcal{G}_i \ \mathcal{G}_i \mathscr{A}_{ij}^{(1)} \ \mathcal{G}_i \mathscr{B}_{ij}^{(1)} \right], \\ \Psi_{(1)} = \underbrace{\left[\Psi_{i1j(1)} \cdots \Psi_{ikj(1),k\neq i} \cdots \Psi_{iNj(1)}\right]}_{N-1}, \Psi_{ikj(1)} = \mathcal{G}_i \bar{\mathscr{A}}_{ikj}^{(1)}, \\ \Psi_{(2)} = \underbrace{\left[\Psi_{i1j(2)} \cdots \Psi_{ikj(2),k\neq i} \cdots \Psi_{iNj(2)}\right]}_{N-1}, \Psi_{ikj(2)} = \mathcal{G}_i \bar{\mathscr{A}}_{ikj}^{(2)}, \\ \Psi_{(3)} = \underbrace{\left[\Psi_{i1j(3)} \cdots \Psi_{ikj(3),k\neq i} \cdots \Psi_{iNj(3)}\right]}_{N-1}, \Psi_{ikj(3)} = \bar{\alpha}_i \mathcal{G}_i E B_{Fikj} \mathcal{D}_{kj}, \\ \Psi_{(4)} = \underbrace{\left[\Psi_{i1j(4)} \cdots \Psi_{ikj(4),k\neq i} \cdots \Psi_{iNj(4)}\right]}_{N-1}, \Psi_{ikj(4)} = \mathcal{G}_i E B_{Fikj} \mathcal{D}_{kj}, \\ \Psi_{(5)} = \left[0 \ \mathbb{C}_{ij} \ 0\right]^T, f_i = \sqrt{\bar{\alpha}_i (1-\bar{\alpha}_i)}, E = \begin{bmatrix} 0 \\ \mathbf{I} \end{bmatrix}, \bar{E} = \begin{bmatrix} \mathbf{I} \\ 0 \end{bmatrix}, \\ \mathbb{H}_{ij} = diag\underbrace{\left\{H_{i1j} \cdots H_{ikj,k\neq i} \cdots H_{iNj}\right\}}_{N-1}, \mathbb{M}_{ij} = diag\underbrace{\left\{M_{i1j} \cdots M_{ikj,k\neq i} \cdots M_{iNj}\right\}}_{N-1}. \end{cases} \tag{18}$$

Proof. Choose the following piecewise Lyapunov functional (PLF):

$$\begin{aligned} V(t) &= \sum_{i=1}^{N} V_i(t) \\ &= \bar{x}_i^T(t) P_{ij} \bar{x}_i(t), \zeta_i(t) \in \mathcal{S}_{ij}, j \in \mathscr{J}_i \end{aligned} \tag{19}$$

where $P_{ij} \in \Re^{(n_{xi}+n_{fi})\times(n_{xi}+n_{fi})}, j \in \mathscr{J}_i, i \in \mathcal{N}$, are positive definite symmetric Lyapunov matrices. □

Define $\Delta V_i(t) = V_i(t+1) - V_i(t)$, one has

$$\begin{aligned} \Delta V_i(t) = &\bar{x}_i^T(t+1) P_{is} \bar{x}_i(t+1) \\ &- \bar{x}_i^T(t) P_{ij} \bar{x}_i(t), (j,s) \in \Omega_i. \end{aligned} \tag{20}$$

It follows from the error system in (13) that

$$\begin{aligned} \left[-\mathbf{I} \ \mathbb{A}_{ij} \ \mathbb{B}_{ij} \right] \chi_i(t) &+ \sum_{\substack{k=1\\k\neq i}}^{N} \bar{\mathbb{A}}_{ikj} x_k(t) \\ &+ \sum_{\substack{k=1\\k\neq i}}^{N} \bar{\mathbb{B}}_{ikj} w_k(t) = 0, \end{aligned} \tag{21}$$

where $\chi_i(t) = \left[\bar{x}_i^T(t+1) \ \bar{x}_i^T(t) \ w_i^T(t)\right]^T, \zeta_i(t) \in \mathcal{S}_{ij}, j \in \mathscr{J}_i, i \in \mathcal{N}$.

Note that

$$2\bar{x}^T \bar{y} \leq \bar{x}^T M^{-1} \bar{x} + \bar{y}^T M \bar{y}, \tag{22}$$

where $\bar{x}, \bar{y} \in \Re^n$ and matrix $M = M^T > 0$.

Define the matrix multipliers $\mathcal{G}_i \in \Re^{(2n_{xi}+2n_{fi}+n_{wi})} \times (n_{xi}+n_{fi})$, $i \in \mathcal{N}$, and the matrices $0 < H_{ikj} < H_{ik0} \in \Re^{n_{xi}\times n_{xi}}$, and it follows from (21), (22) and Lemma A1 in the "Appendix" section that

$$\begin{aligned} &2\sum_{i=1}^{N} \chi_i^T(t) \mathcal{G}_i \sum_{\substack{k=1\\k\neq i}}^{N} \bar{\mathbb{A}}_{ikj} x_k(t) \\ &\leq \sum_{i=1}^{N} \sum_{\substack{k=1\\k\neq i}}^{N} \chi_i^T(t) \mathcal{G}_i \bar{\mathbb{A}}_{ikj} H_{ikj}^{-1} (\star) \chi_i(t) + \sum_{i=1}^{N} \sum_{\substack{k=1\\k\neq i}}^{N} x_k^T(t) H_{ikj} x_k(t) \\ &\leq \sum_{i=1}^{N} \sum_{\substack{k=1\\k\neq i}}^{N} \chi_i^T(t) \mathcal{G}_i \bar{\mathbb{A}}_{ikj} H_{ikj}^{-1} (\star) \chi_i(t) + \sum_{i=1}^{N} \sum_{\substack{k=1\\k\neq i}}^{N} x_k^T(t) H_{ik0} x_k(t) \\ &= \sum_{i=1}^{N} \sum_{\substack{k=1\\k\neq i}}^{N} \chi_i^T(t) \mathcal{G}_i \bar{\mathbb{A}}_{ikj} H_{ikj}^{-1} (\star) \chi_i(t) + \sum_{i=1}^{N} \sum_{\substack{k=1\\k\neq i}}^{N} x_i^T(t) H_{ki0} x_i(t). \end{aligned} \tag{23}$$

Similarly, by introducing $0 < M_{ikj} < M_{ik0} \in \Re^{n_{wi}\times n_{wi}}$, it yields

$$
\begin{aligned}
&2\sum_{i=1}^{N}\chi_i^T(t)\,\mathcal{G}_i\sum_{\substack{k=1\\k\neq i}}^{N}\bar{\mathbb{B}}_{ikj}w_k(t)\\
&\le \sum_{i=1}^{N}\sum_{\substack{k=1\\k\neq i}}^{N}\chi_i^T(t)\,\mathcal{G}_i\bar{\mathbb{B}}_{ikj}M_{ikj}^{-1}(\star)\,\chi_i(t)+\sum_{i=1}^{N}\sum_{\substack{k=1\\k\neq i}}^{N}w_k^T(t)\,M_{ikj}w_k(t)\\
&\le \sum_{i=1}^{N}\sum_{\substack{k=1\\k\neq i}}^{N}\chi_i^T(t)\,\mathcal{G}_i\bar{\mathbb{B}}_{ikj}M_{ikj}^{-1}(\star)\,\chi_i(t)+\sum_{i=1}^{N}\sum_{\substack{k=1\\k\neq i}}^{N}w_k^T(t)\,M_{ik0}w_k(t)\\
&= \sum_{i=1}^{N}\sum_{\substack{k=1\\k\neq i}}^{N}\chi_i^T(t)\,\mathcal{G}_i\bar{\mathbb{B}}_{ikj}M_{ikj}^{-1}(\star)\,\chi_i(t)+\sum_{i=1}^{N}\sum_{\substack{k=1\\k\neq i}}^{N}w_i^T(t)\,M_{ki0}w_i(t)\,.
\end{aligned}
\tag{24}
$$

Given the following index

$$
\begin{aligned}
J(t)&=\mathbb{E}\left\{\sum_{i=1}^{N}J_i(t)\right\}\\
&=\mathbb{E}\left\{\sum_{i=1}^{N}\sum_{t=0}^{\infty}\left[\bar{z}_i^T(t)\bar{z}_i(t)-\gamma^2w_i^T(t)\,w_i(t)\right]\right\}.
\end{aligned}
\tag{25}
$$

It follows from (19)–(21) and (23)–(25) that

$$
\begin{aligned}
J(t)&\le\mathbb{E}\{J(t)+V(\infty)-V(0)\}\\
&=\mathbb{E}\left\{\sum_{i=1}^{N}\sum_{t=0}^{\infty}\left[\Delta V_i(t)+\bar{z}_i^T(t)\bar{z}_i(t)-\gamma^2w_i^T(t)\,w_i(t)\right]\right\}\\
&\le\mathbb{E}\left\{\sum_{i=1}^{N}\sum_{t=0}^{\infty}\chi_i^T(t)\left[\Theta_{ijs}+\mathrm{Sym}\left\{\Pi_{ij}\right\}+\sum_{\substack{k=1\\k\neq i}}^{N}\mathcal{G}_i\bar{\mathbb{A}}_{ikj}H_{ikj}^{-1}(\star)\right.\right.\\
&\qquad\left.\left.+\sum_{\substack{k=1\\k\neq i}}^{N}\mathcal{G}_i\bar{\mathbb{B}}_{ikj}M_{ikj}^{-1}(\star)\right]\chi_i(t)\right\}\\
&=\sum_{i=1}^{N}\sum_{t=0}^{\infty}\chi_i^T(t)\left\{\Theta_{ijs}+\mathrm{Sym}\left\{\Pi_{ij}\right\}+\sum_{\substack{k=1\\k\neq i}}^{N}\Psi_{ikj(1)}H_{ikj}^{-1}\Psi_{ikj(1)}^T\right.\\
&\qquad+f_i^2\sum_{\substack{k=1\\k\neq i}}^{N}\Psi_{ikj(2)}H_{ikj}^{-1}\Psi_{ikj(2)}^T+\sum_{\substack{k=1\\k\neq i}}^{N}\Psi_{ikj(3)}M_{ikj}^{-1}\Psi_{ikj(3)}^T\\
&\qquad\left.+f_i^2\sum_{\substack{k=1\\k\neq i}}^{N}\Psi_{ikj(4)}M_{ikj}^{-1}\Psi_{ikj(4)}^T+\Psi_{(5)}\Psi_{(5)}^T\right\}\chi_i(t),
\end{aligned}
\tag{26}
$$

where $f_i=\sqrt{\bar{\alpha}_i(1-\bar{\alpha}_i)}$, $\left\{\Theta_{ijs},\Pi_{ij},\Psi_{ikj(1)},\Psi_{ikj(2)},\Psi_{ikj(3)},\Psi_{ikj(4)}\right\}$ are given by (18).

By using Schur complement lemma on (17), it is easy to see from (26) that the inequality (17) implies $\mathbb{E}\{\Delta V(t)\}<0$ when $w_i(t)\equiv 0$. Thus, it yields

$$
\mathbb{E}\{V(t+1)\}<V(0)\le\left(\lambda_{\max}\left(P_{ij}\right)\right)x^T(0)x(0),j\in\mathscr{J}_i,i\in\mathcal{N}
\tag{27}
$$

and

$$
\mathbb{E}\{V(t+1)\}\ge\left(\lambda_{\min}\left(P_{ij}\right)\right)x^T(t+1)x(t+1),j\in\mathscr{J}_i,i\in\mathcal{N}.
\tag{28}
$$

According to Definition 1, it is easy to see that the filtering error system is stochastically stable in the mean square. Then, considering $\mathbb{E}\{V(t)\}>0$ for all $t\ge 0$, under zero-initial conditions it yields

$$
\mathbb{E}\left\{\sum_{i=1}^{N}\sum_{t=0}^{\infty}\bar{z}_i^T(t)\bar{z}_i(t)\right\}-\sum_{i=1}^{N}\sum_{t=0}^{\infty}\gamma^2w_i^T(t)\,w_i(t)<0,
\tag{29}
$$

which means $\left\|\tilde{z}\right\|_{\mathbb{E}}<\gamma\left\|\tilde{w}\right\|_2$, thus completing this proof.

3.2 Distributed $\mathscr{H}_\infty$ filtering design

In this subsection, we will consider the distributed $\mathscr{H}_\infty$ filtering design for the system in (5). Based on Lemma 1, and by specifying the multiplier $\mathcal{G}_i$, the nonlinear matrix inequalities are formulated into the linear ones, the corresponding result is summarized as below.

Theorem 1. *Given the fuzzy system in (3) and a distributed filter in (8), the filtering error system in (13) is stochastically stable with an $\mathscr{H}_\infty$ performance γ, if there exist matrices $0<P_{ij}=P_{ij}^T\in\Re^{(n_{xi}+n_{fi})\times(n_{xi}+n_{fi})},j\in\mathscr{J}_i,i\in\mathcal{N}$, and matrices $\bar{A}_{Fij}\in\Re^{n_{fi}\times n_{fi}}$, $\left\{\bar{B}_{Fij},\bar{B}_{Fij}\right\}\in\Re^{n_{fi}\times n_{yi}}$, $\bar{C}_{Fij}\in\Re^{n_{zi}\times n_{fi}},G_{ij(1)}\in\Re^{n_{xi}\times n_{xi}},G_{ij(2)}\in\Re^{n_{fi}\times n_{fi}},G_{ij(3)}\in\Re^{n_{fi}\times n_{xi}},G_{ij(5)}\in\Re^{(n_{xi}+n_{fi}+n_{wi})\times n_{xi}},j\in\mathscr{J}_i,i\in\mathcal{N}$, and matrices $0<H_{ikj}<H_{ik0}\in\Re^{n_{xi}\times n_{xi}},0<M_{ikj}<M_{ik0}\in\Re^{n_{wi}\times n_{wi}},j\in\mathscr{J}_i,(i,k)\in\mathcal{N}$, such that for all $(j,s)\in\Omega_i,j\in\mathscr{J}_i,m\in\mathcal{I}_i\left(j\right),(i,k)\in\mathcal{N}$ the following LMIs hold:*

$$
\begin{bmatrix}
\Theta_{ijs}+Sym\left\{\bar{\Pi}_{im}\right\} & \bar{\Psi}_{(1)} & f_i\bar{\Psi}_{(2)} & \bar{\Psi}_{(3)} & f_i\bar{\Psi}_{(4)} & \Psi_{(5)}\\
\star & -\mathbb{H}_{ij} & 0 & 0 & 0 & 0\\
\star & \star & -\mathbb{H}_{ij} & 0 & 0 & 0\\
\star & \star & \star & -\mathbb{M}_{ij} & 0 & 0\\
\star & \star & \star & \star & -\mathbb{M}_{ij} & 0\\
\star & \star & \star & \star & \star & -\mathbf{I}
\end{bmatrix}<0,
\tag{30}
$$

where

$$\begin{cases} \Theta_{i1} = diag\left\{ P_{is} \sum_{\substack{k=1 \\ k\neq i}}^{N} \bar{E} H_{ki0} \bar{E}^T - P_{ij} \sum_{\substack{k=1 \\ k\neq i}}^{N} M_{ki0} - \gamma^2 \mathbf{I} \right\}, \\ \bar{\Pi}_{im} = \begin{bmatrix} -G_{ij(1)} & -KG_{ij(2)} & \Theta_{i2}^{(14)} & K\bar{A}_{Fij} & G_{ij(1)}B_{im} + \bar{\alpha}_i K \bar{B}_{Fij} D_{im} \\ -G_{ij(3)} & -G_{ij(2)} & \Theta_{i2}^{(24)} & \bar{A}_{Fij} & G_{ij(3)}B_{im} + \bar{\alpha}_i \bar{B}_{Fij} D_{im} \\ -G_{ij(5)} & 0 & G_{ij(5)}A_{im} & 0 & G_{ij(5)}B_{im} \end{bmatrix}, \\ \Theta_{i2}^{(14)} = G_{ij(1)}A_{im} + \bar{\alpha}_i K \bar{B}_{Fij} C_{im}, \Theta_{i2}^{(24)} = G_{ij(3)}A_{im} + \bar{\alpha}_i \bar{B}_{Fij} C_{im}, \\ \bar{\Psi}_{(1)} = \underbrace{\left[\bar{\Psi}_{i1j(1)} \cdots \bar{\Psi}_{ikj(1),k\neq i} \cdots \bar{\Psi}_{iNj(1)} \right]}_{N-1}, \bar{\Psi}_{ikj(1)} = \begin{bmatrix} G_{ij(1)}\bar{A}_{ikm} + \bar{\alpha}_i K \bar{B}_{Fikj} C_{km} \\ G_{ij(3)}\bar{A}_{ikm} + \bar{\alpha}_i \bar{B}_{Fikj} C_{km} \\ G_{ij(5)}\bar{A}_{ikm} \end{bmatrix}, \\ \bar{\Psi}_{(2)} = \underbrace{\left[\bar{\Psi}_{i1j(2)} \cdots \bar{\Psi}_{ikj(2),k\neq i} \cdots \bar{\Psi}_{iNj(2)} \right]}_{N-1}, \bar{\Psi}_{ikj(2)} = \begin{bmatrix} K\bar{B}_{Fikj} C_{km} \\ \bar{B}_{Fikj} C_{km} \\ 0 \end{bmatrix}, \\ \bar{\Psi}_{(3)} = \underbrace{\left[\bar{\Psi}_{i1j(3)} \cdots \bar{\Psi}_{ikj(3),k\neq i} \cdots \bar{\Psi}_{iNj(3)} \right]}_{N-1}, \bar{\Psi}_{ikj(3)} = \begin{bmatrix} \bar{\alpha}_i K\bar{B}_{Fikj} D_{km} \\ \bar{\alpha}_i \bar{B}_{Fikj} D_{km} \\ 0 \end{bmatrix}, \\ \bar{\Psi}_{(4)} = \underbrace{\left[\bar{\Psi}_{i1j(4)} \cdots \bar{\Psi}_{ikj(4),k\neq i} \cdots \bar{\Psi}_{iNj(4)} \right]}_{N-1}, \bar{\Psi}_{ikj(4)} = \begin{bmatrix} K\bar{B}_{Fikj} D_{km} \\ \bar{B}_{Fikj} D_{km} \\ 0 \end{bmatrix}, \\ \Psi_{(5)} = \begin{bmatrix} 0 & \begin{bmatrix} L_{im} & -\bar{C}_{Fij} \end{bmatrix} & 0 \end{bmatrix}^T, f_i = \sqrt{\bar{\alpha}_i (1-\bar{\alpha}_i)}, \\ \bar{E} = \begin{bmatrix} \mathbf{I} \\ 0 \end{bmatrix}, K = \begin{bmatrix} \mathbf{I}_{n_{fi}} & 0_{n_{fi}\times(n_{xi}-n_{fi})} \end{bmatrix}^T, \\ \mathbb{H}_{ij} = diag\underbrace{\left\{ H_{i1j} \cdots H_{ikj,k\neq i} \cdots H_{iNj} \right\}}_{N-1}, \mathbb{M}_{ij} = diag\underbrace{\left\{ M_{i1j} \cdots M_{ikj,k\neq i} \cdots M_{iNj} \right\}}_{N-1}. \end{cases} \tag{31}$$

Furthermore, a distributed piecewise filter in the form of (8) is given by

$$\begin{aligned} A_{Fij} &= \left(G_{ij(2)}\right)^{-1} \bar{A}_{Fij}, B_{Fij} = \left(G_{ij(2)}\right)^{-1} \bar{B}_{Fij}, \\ B_{Fikj} &= \left(G_{ij(2)}\right)^{-1} \bar{B}_{Fikj}, C_{Fij} = \bar{C}_{Fij}, j \in \mathscr{I}_i, i \in \mathcal{N}. \end{aligned} \tag{32}$$

Proof. For matrix inequality linearization purpose, the multipliers $\mathcal{G}_i, i \in \mathcal{N}$ are firstly specified by

$$\mathcal{G}_i = \begin{bmatrix} \begin{bmatrix} G_{ij(1)} & KG_{ij(2)} \\ G_{ij(3)} & G_{ij(4)} \end{bmatrix} \\ 0_{(n_{xi}+n_{fi}+n_{wi})\times(n_{xi}+n_{fi})} \end{bmatrix}, j \in \mathscr{I}_i, i \in \mathcal{N} \tag{33}$$

where $K = \begin{bmatrix} \mathbf{I}_{n_{fi}} & 0_{n_{fi}\times(n_{xi}-n_{fi})} \end{bmatrix}^T$, $G_{ij(1)} \in \Re^{n_{xi}\times n_{xi}}$, $G_{ij(2)} \in \Re^{n_{fi}\times n_{fi}}$, $G_{ij(3)} \in \Re^{n_{fi}\times n_{xi}}$, $G_{ij(4)} \in \Re^{n_{fi}\times n_{fi}}$. □

Then, similar to [39], defining $\Gamma := \text{diag}\left\{\mathbf{I}_{n_{xi}}, G_{ij(2)}G_{ij(4)}^{-1}\right\}$, and performing a congruence transformation to

$$\begin{bmatrix} G_{ij(1)} + G_{ij(1)}^T & KG_{ij(2)} + G_{ij(3)}^T \\ \star & G_{ij(4)} + G_{ij(4)}^T \end{bmatrix}, j \in \mathscr{I}_i, i \in \mathcal{N} \tag{34}$$

by Γ, it yields

$$\begin{aligned} &\begin{bmatrix} \mathbf{I}_{n_{xi}} & 0 \\ 0 & G_{ij(2)}G_{ij(4)}^{-1} \end{bmatrix} \begin{bmatrix} G_{ij(1)} + G_{ij(1)}^T & KG_{ij(2)} + G_{ij(3)}^T \\ \star & G_{ij(4)} + G_{ij(4)}^T \end{bmatrix} \begin{bmatrix} \mathbf{I}_{n_{xi}} & 0 \\ 0 & G_{ij(4)}^{-T} G_{ij(2)}^T \end{bmatrix} \\ &= \begin{bmatrix} G_{ij(1)} + G_{ij(1)}^T & KG_{ij(2)}G_{ij(4)}^{-T}G_{ij(2)}^T + G_{ij(3)}^T G_{ij(4)}^{-T} G_{ij(2)}^T \\ \star & G_{ij(2)}G_{ij(4)}^{-T}G_{ij(2)}^T + G_{ij(2)}G_{ij(4)}^{-1}G_{ij(2)}^T \end{bmatrix} \\ &:= \begin{bmatrix} G_{ij(1)} + G_{ij(1)}^T & K\bar{G}_{ij(2)} + \bar{G}_{ij(3)}^T \\ \star & \bar{G}_{ij(2)} + \bar{G}_{ij(2)}^T \end{bmatrix}, i \in \mathcal{N}. \end{aligned} \tag{35}$$

Without loss of generalit, we can specify $G_{ij(4)} = G_{ij(2)}$. Thus, we can directly specify the multipliers $\mathcal{G}_i$ as

$$\mathcal{G}_i = \begin{bmatrix} \begin{bmatrix} G_{ij(1)} & KG_{ij(2)} \\ G_{ij(3)} & G_{ij(2)} \end{bmatrix} \\ 0_{(n_{xi}+n_{fi}+n_{wi})\times(n_{xi}+n_{fi})} \end{bmatrix}, j \in \mathscr{I}_i, i \in \mathcal{N}. \tag{36}$$

It is easy to see that the matrix variable $G_{ij(2)}, j \in \mathscr{I}_i$ is absorbed by the filter gain variables $A_{Fij}, B_{Fij}, B_{Fikj}, j \in \mathscr{I}_i$ when introducing

$$\bar{A}_{Fij} = G_{ij(2)}A_{Fij}, \bar{B}_{Fij} = G_{ij(2)}B_{Fij}, \bar{B}_{Fikj} = G_{ij(2)}B_{Fikj} \tag{37}$$

with $j \in \mathscr{I}_i, k \neq i, (i,k) \in \mathcal{N}$.

It is noted that the first block-row of $\left\{\mathbb{A}_{ij}, \bar{\mathbb{A}}_{ikj}, \mathbb{B}_{ij}, \bar{\mathbb{B}}_{ikj}\right\}$ does not involve in the filter gain variables. Thus, the multipliers $\mathcal{G}_i, i \in \mathcal{N}$ is finally specified as below,

$$\mathcal{G}_i = \begin{bmatrix} G_{ij(1)} & KG_{ij(2)} \\ G_{ij(3)} & G_{ij(2)} \\ G_{ij(5)} & 0 \end{bmatrix}, j \in \mathscr{I}_i, i \in \mathcal{N} \tag{38}$$

where $G_{ij(5)} \in \Re^{(n_{xi}+n_{fi}+n_{wi}) \times n_{xi}}$.

Then, substituting (38) into (17), and by extracting the fuzzy basis functions, the inequality (30) can be obtained.

In addition, the inequality in (17) imply that

$$P_{is} + \mathrm{Sym}\left\{\begin{bmatrix} -G_{ij(1)} & -KG_{ij(2)} \\ -G_{ij(3)} & -G_{ij(2)} \end{bmatrix}\right\} < 0 \tag{39}$$

with $(j, s), \in \Omega_i, j \in \mathscr{I}_i, i \in \mathcal{N}$.

Due to the fact that $P_{is} > 0$, we have $G_{ij(2)} + G_{ij(2)}^T > 0$, which means that the matrix variable $G_{ij(2)}$ are nonsingular. Thus, the filtering gains $\left\{A_{Fij}, B_{Fij}, B_{Fikj}, C_{Fij}\right\}$ can be obtained by (32) and completing this proof.

4 Simulation examples

In the following, let us consider two examples to illustrate the result proposed in this paper.

Example 1. Consider a discrete-time large-scale T-S model with three fuzzy rules as below:

Plant rule $\mathscr{R}_i^l$: IF $x_{i1}(t)$ is $\mathscr{F}_i^l$, THEN

$$\begin{cases} x_i(t+1) = A_{il}x_i(t) + \sum\limits_{\substack{k=1 \\ k \neq i}}^{2} \bar{A}_{ikl}x_k(t) + B_{il}w_i(t) \\ y_i(t) = C_{il}x_i(t) + D_{il}w_i(t) \\ z_i(t) = L_{il}x_i(t), l = \{1, 2, 3\}, i = \{1, 2\} \end{cases}$$

where

$$[A_{11} \quad A_{12} \quad A_{13}] = \left[\begin{array}{cc|cc|cc} 0.98 & 0.01 & 0.87 & 0.02 & 0.95 & 0.01 \\ -0.18 & 0.74 & -0.12 & 0.83 & -0.16 & 0.86 \end{array}\right],$$

$$[B_{11}\ B_{12}\ B_{13}] = \left[\begin{array}{c|c|c} 0 & 0 & 0 \\ 0.28 & 0.25 & 0.21 \end{array}\right],$$

$$C_{11} = [1.03 \quad 0], C_{12} = [0.95 \quad 0], C_{13} = [0.89 \quad 0],$$

$$D_{11} = 0.74, D_{12} = 0.69, D_{13} = 0.58, L_{11} = 0.85, L_{12} = 0.97, L_{13} = 1.04$$

for the first subsystem, and

$$[A_{21} \quad A_{22} \quad A_{23}] = \left[\begin{array}{cc|cc|cc} 0.75 & 0.02 & 0.83 & 0.01 & 0.92 & 0.02 \\ -0.20 & 0.82 & -0.15 & 0.88 & -0.17 & 0.75 \end{array}\right],$$

$$\begin{bmatrix} B_{21} & B_{22} & B_{23} \end{bmatrix} = \left[\begin{array}{c|c|c} 0 & 0 & 0 \\ 0.21 & 0.28 & 0.23 \end{array}\right],$$

$$C_{21} = \begin{bmatrix} 0.94 & 0 \end{bmatrix}, C_{22} = \begin{bmatrix} 0.97 & 0 \end{bmatrix}, C_{23} = \begin{bmatrix} 0.86 & 0 \end{bmatrix},$$

$$D_{21} = 0.49, D_{22} = 0.65, D_{23} = 0.58, L_{21} = 1.05, L_{22} = 0.95, L_{23} = 0.92$$

for the second subsystem.

Figure 2 shows the membership functions. According to the premise variable space partition, there are three subspaces for each subsystem:

$$\begin{aligned} \mathcal{S}_{i1} &= \{x_{i1}(t) | r_{i1} \le |x_{i1}(t)| \le r_{i2}\} \\ \mathcal{S}_{i2} &= \{x_{i1}(t) | r_{i2} < |x_{i1}(t)| \le r_{i3}\} \\ \mathcal{S}_{i3} &= \{x_{i1}(t) | r_{i3} < |x_{i1}(t)| \le r_{i4}\}. \end{aligned}$$

It can be observed that $\mathcal{S}_{i1}$ is a crisp region; both $\mathcal{S}_{i2}$ and $\mathcal{S}_{i3}$ are fuzzy regions. The region index set is $\mathscr{I}_i = \{1, 2, 3\}$.

Here, we consider the case of a full-order filter with $a_i = 0.6$. It is noted that the common Lyapunov function proposed in [40] is not applicable to the distributed $\mathscr{H}_\infty$ filtering design for this case in this example. However, by applying Theorem 1, the feasible solutions of $\gamma_{\min} = 3.9555$ for the full-order filter and $\gamma_{\min} = 6.5599$ for the reduced-order filter are obtained, and the corresponding filter gains are

$$\begin{bmatrix} A_{F11} & A_{F12} & A_{F13} \\ C_{F11} & C_{F12} & C_{F13} \\ A_{F21} & A_{F22} & A_{F23} \\ C_{F21} & C_{F22} & C_{F23} \end{bmatrix} = \left[\begin{array}{cc|cc|cc} 0.7667 & -0.0097 & 0.8152 & 0.0008 & 0.7634 & 0.0059 \\ -0.6453 & 0.5704 & -1.1507 & 0.7841 & -0.5374 & 0.8278 \\ \hline -0.3352 & -0.6075 & -0.0669 & -0.9218 & -0.0618 & -1.0613 \\ \hline 0.1465 & 0.0021 & 0.2362 & -0.0231 & 0.3978 & -0.0336 \\ -4.8483 & 0.6215 & -5.8832 & 0.5941 & -5.6262 & 0.4767 \\ \hline 3.7046 & -0.7545 & 2.3695 & -0.8122 & 0.0207 & -0.9256 \end{array}\right],$$

$$\begin{bmatrix} B_{F11} & B_{F12} & B_{F13} \\ B_{F121} & B_{F122} & B_{F123} \end{bmatrix} = \left[\begin{array}{c|c|c} -0.2872 & -0.2136 & -0.2099 \\ -1.4420 & -1.6868 & -1.9660 \\ \hline -0.0003 & -0.0002 & -0.0002 \\ -0.0029 & -0.0025 & -0.0030 \end{array}\right],$$

$$\begin{bmatrix} B_{F21} & B_{F22} & B_{F23} \\ B_{F211} & B_{F212} & B_{F213} \end{bmatrix} = \left[\begin{array}{c|c|c} -0.6604 & -0.9939 & -0.9347 \\ -7.1466 & -10.5250 & -9.6167 \\ \hline -0.0255 & -0.0721 & -0.0875 \\ -0.4124 & -0.9239 & -1.0579 \end{array}\right]$$

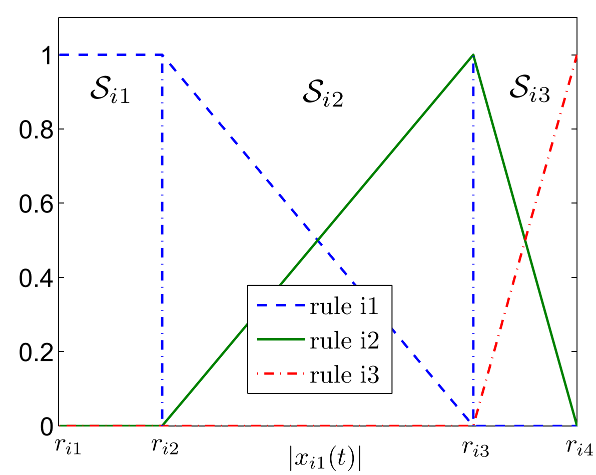

Fig. 2 Membership functions in example 1

for full-order filter and

$$\begin{bmatrix} A_{F11} & A_{F12} & A_{F13} \\ C_{F11} & C_{F12} & C_{F13} \\ A_{F21} & A_{F22} & A_{F23} \\ C_{F21} & C_{F22} & C_{F23} \\ B_{F11} & B_{F12} & B_{F13} \\ B_{F121} & B_{F122} & B_{F123} \\ B_{F21} & B_{F22} & B_{F23} \\ B_{F211} & B_{F212} & B_{F213} \end{bmatrix} = \begin{bmatrix} 0.6646 & 0.7019 & 0.6069 \\ -1.1355 & -1.3025 & -7.8639 \\ 0.3419 & 0.2325 & 0.6842 \\ -2.8731 & -6.5894 & -3.2918 \\ -0.4212 & -0.3742 & -0.5079 \\ -0.0007 & -0.0009 & -0.0014 \\ -0.4754 & -0.6358 & -0.5075 \\ -0.0001 & -0.0003 & -0.0004 \end{bmatrix}$$

for the reduced-order case.

Example 2. Consider a modified double-inverted pendulum, which is connected by a spring [34]. The motional equations of the interconnected pendulum are given by

$$\begin{cases} \dot{x}_{i1} = x_{i2} \\ \dot{x}_{i2} = \frac{1}{100J_i} u_i - \frac{kr^2}{4J_i} x_{i1} + \left[\frac{m_i gr}{J_i} - \frac{kr^2}{4J_i} x_{i2} \right] \sin(x_{i1}) \\ \qquad + \frac{1}{J_i} x_{i2} + \sum\limits_{\substack{k=1 \\ k \neq i}}^{2} \frac{3kr^2}{J_k} x_{k1}, i = \{1, 2\} \end{cases}$$

where x_{i1} and x_{i2} denote the angle from the vertical and the angular velocity, respectively.

In this simulation, the moments of inertia are $J_1 = 2$ kg and $J_2 = 2.5$ kg; the masses of two pendulums are chosen as $m_1 = 2$ kg and $m_2 = 2.5$ kg; the constant of the connecting torsional spring is $k = 8$ N·m/rad; the length of the pendulum is $r = 1$ m; the gravity constant is $g = 9.8$ m/s^2. Here, the interconnected pendulum is linearized around the origin, $x_{i1} = (\pm 80°, 0)$ and $x_{i1} = (\pm 88°, 0)$; each pendulum is described by the T-S model with three fuzzy rules. Given $u_1 = -18x_{11} - 16x_{12}$ and $u_2 = -20x_{21} - 14x_{22}$, it can be known that these two closed-loop T-S fuzzy subsystems are stable. Then, by discretizing the T-S fuzzy systems with sampling period $T = 0.01$ s, the discrete-time interconnected T-S fuzzy system can be obtained as below:

Plant rule $\mathscr{R}_i^l$: IF $|x_{i1}(t)|$ is $\mathscr{F}_i^l$, THEN

$$\begin{cases} x_i(t+1) = A_{il} x_i(t) + \sum\limits_{\substack{k=1 \\ k \neq i}}^{2} \bar{A}_{ik} x_k(t) + B_{il} w_i(t) \\ y_i(t) = C_{il} x_i(t) + D_{il} w_i(t) \\ z_i(t) = L_{il} x_i(t), l = \{1, 2, 3\}, i = \{1, 2\} \end{cases}$$

where

$$[A_{11} \quad A_{12} \quad A_{13}] = \left[\begin{array}{cc|cc|cc} 1 & 0.0120 & 1 & 0.0120 & 1 & 0.0120 \\ -1.3200 & -0.1540 & -1.1818 & -0.1658 & -1.3760 & -0.0352 \end{array} \right],$$

$$\bar{A}_{12} = \begin{bmatrix} 0 & 0 \\ 0.12 & 0 \end{bmatrix}, B_{1l} = \begin{bmatrix} 0 \\ 0.5 \end{bmatrix}, C_{1l} = \begin{bmatrix} 1 & 0 \end{bmatrix}, D_{1l} = 1, L_{1l} = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

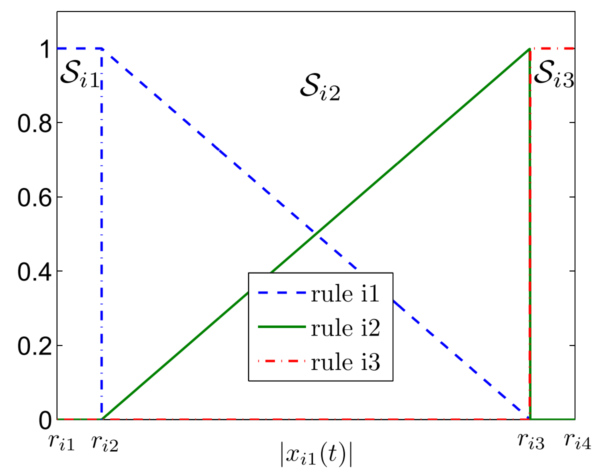

Fig. 3 Membership functions in example 2

for the first subsystem and

$$[A_{21} \quad A_{22} \quad A_{23}] = \left[\begin{array}{cc|cc|cc} 1 & 0.0120 & 1 & 0.0120 & 1 & 0.0120 \\ -1.3760 & -0.0352 & -1.2378 & -0.0447 & -1.2485 & -0.0448 \end{array} \right],$$

$$\bar{A}_{21} = \begin{bmatrix} 0 & 0 \\ 0.096 & 0 \end{bmatrix}, B_{2l} = \begin{bmatrix} 0 \\ 0.4 \end{bmatrix}, C_{2l} = \begin{bmatrix} 1 & 0 \end{bmatrix}, D_{2l} = 1, L_{2l} = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

for the second subsystem.

Figure 3 shows the membership functions. Based on the premise variable space partition, it can be known that there exist three subspaces for each subsystem:

$$\mathcal{S}_{i1} = \{x_{i1}(t) | r_{i1} < |x_{i1}(t)| \leq r_{i2}\}$$
$$\mathcal{S}_{i2} = \{x_{i1}(t) | r_{i2} < |x_{i1}(t)| \leq r_{i3}\}$$
$$\mathcal{S}_{i3} = \{x_{i1}(t) | r_{i3} < |x_{i1}(t)| < r_{i4}\}$$

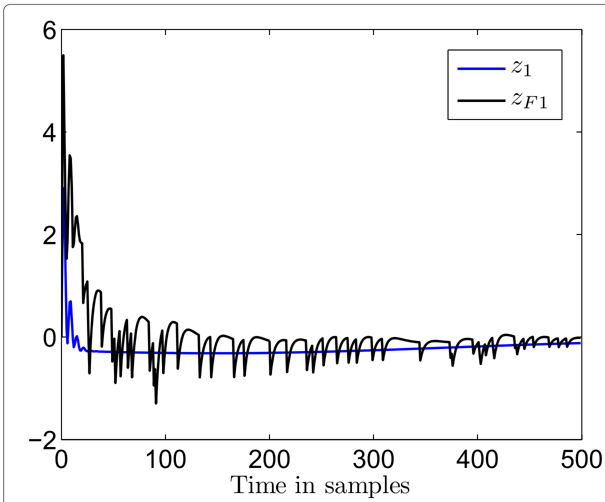

Fig. 4 Responses of $z_1(t)$ and $z_{F1}(t)$

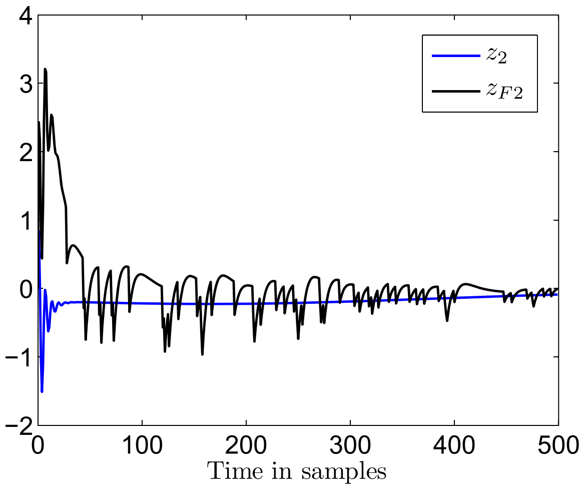

Fig. 5 Responses of $z_2(t)$ and $z_{F2}(t)$

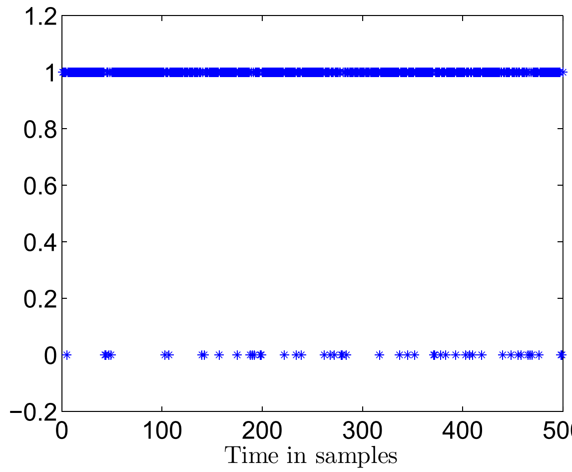

Fig. 7 Data-packet dropout for subsystem 2

where $r_{i1} = 0, r_{i2} = 8°, r_{i3} = 80°$, and $r_{i4} = 88°$. It is shown that both $\mathcal{S}_{i1}$ and $\mathcal{S}_{i3}$ are crisp regions and $\mathcal{S}_{i2}$ is a fuzzy region. The region index set is $\mathscr{I}_i = \{1, 2, 3\}$.

Now, considering the case of full-order filter with $a_i = 0.9$, the common Lyapunov function proposed in [40] is not applicable to the distributed $\mathscr{H}_\infty$ filtering design for this case in this example. However, by applying Theorem 1, the $\mathscr{H}_\infty$ filtering performance $\gamma_{\min} = 3.3755$ is obtained, and the obtained filter gains are

$$[A_{F11} \quad A_{F12} \quad A_{F13}] = \left[\begin{array}{cc|cc|cc} 0.9530 & 0.0064 & 0.9645 & 0.0061 & 0.9645 & 0.0060 \\ -0.8230 & 0.8049 & -0.7860 & 0.7924 & -0.7696 & 0.7620 \end{array}\right],$$

$$[B_{F11} \quad B_{F12} \quad B_{F13}] = \left[\begin{array}{c|c|c} -0.0462 & -0.0357 & -0.0356 \\ -0.6725 & -0.7136 & -0.7026 \end{array}\right],$$

$$[B_{F121} \quad B_{F122} \quad B_{F123}] = \left[\begin{array}{c|c|c} -0.0009 & -0.0009 & -0.0009 \\ -0.1312 & -0.1346 & -0.1314 \end{array}\right],$$

$$[C_{F11} \quad C_{F12} \quad C_{F13}] = \left[\begin{array}{cc|cc|cc} 0.0324 & -0.9570 & 0 & -1.0475 & -0.0006 & -1.0285 \end{array}\right],$$

and

$$[A_{F21} \quad A_{F22} \quad A_{F23}] = \left[\begin{array}{cc|cc|cc} 0.9626 & 0.0062 & 0.9634 & 0.0064 & 0.9634 & 0.0063 \\ -0.7417 & 0.8157 & -0.6866 & 0.8275 & -0.6732 & 0.8048 \end{array}\right],$$

$$[B_{F21} \quad B_{F22} \quad B_{F23}] = \left[\begin{array}{c|c|c} -0.0360 & -0.0361 & -0.0361 \\ -0.5731 & -0.5792 & -0.5732 \end{array}\right],$$

$$[B_{F211} \quad B_{F212} \quad B_{F213}] = \left[\begin{array}{c|c|c} -0.0009 & -0.0010 & -0.0009 \\ -0.1094 & -0.1103 & -0.1085 \end{array}\right],$$

$$[C_{F21} \quad C_{F22} \quad C_{F23}] = \left[\begin{array}{cc|cc|cc} 0.0132 & -1.0074 & 0 & -1.0111 & 0.0004 & -1.0007 \end{array}\right].$$

Given the initial conditions $x_1(0) = [1.3, 0]^T, x_2(0) = [1.1, 0]^T$, and assume that the external disturbances satisfy $w_1(t) = 5e^{-0.02t}\sin(t)$ and $w_2(t) = 5e^{-0.02t}\cos(t)$, it is easy to see from Figs. 4 and 5 that the responses of he time responses of $z_i(t)$ and $z_{Fi}(t), i = \{1, 2\}$ converge to zero. The data-packet dropouts are shown in Figs. 6 and 7, respectively. Then, it can also be shown in Fig. 8 that the

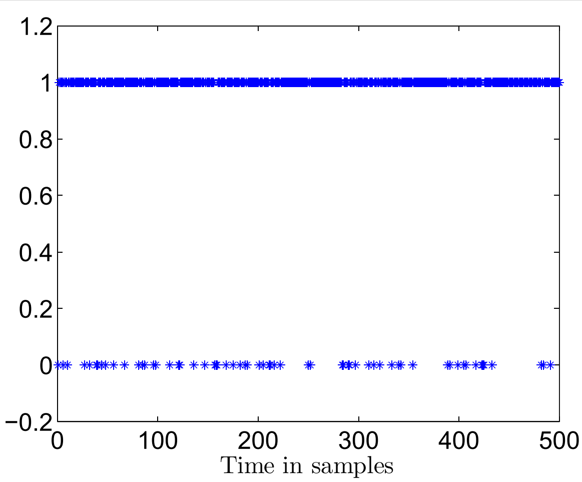

Fig. 6 Data-packet dropout for subsystem 1

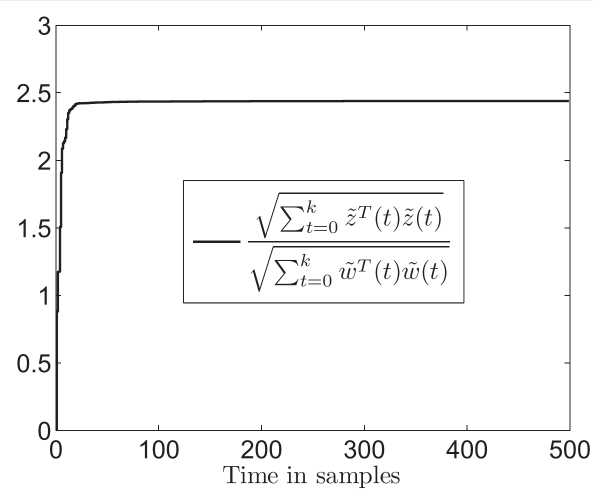

Fig. 8 Response of $\mathscr{H}_\infty$ performance

$\mathscr{H}_\infty$ performance is satisfactory under zero-initial conditions thus verifying the effectiveness of the distributed $\mathscr{H}_\infty$ filtering design.

5 Conclusions

This paper has investigated the distributed $\mathscr{H}_\infty$ filtering design for discrete-time large-scale T-S fuzzy systems, which exchange their information through unreliable communication network. Based on a piecewise Lyapunov function and some convexifying techniques, less conservative results on the distributed piecewise $\mathscr{H}_\infty$ filtering design were derived for the considered system in terms of LMIs. The effectiveness of the method proposed in this paper was validated by using two examples.

Appendix

Lemma A1. [41] *Given matrix* $0 < W = W^T \in \Re^{n\times n}$, *two positive integers* d_2 *and* d_1 *satisfy* $d_2 \geq d_1 \geq 1$. *Then, it yields*

$$\left(\sum_{n=d_1}^{d_2} x^T(n)\right)^T W \left(\sum_{n=d_1}^{d_2} x(n)\right) \leq \bar{d} \sum_{n=d_1}^{d_2} x^T(n)\, Wx(n)$$

where $\bar{d} = d_2 - d_1 + 1$.

Competing interests
The authors declare that they have no competing interests.

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