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Application of compressed sensing for image compression based on optimized Toeplitz sensing matrices

Yuvraj V. Parkale* and Sanjay L. Nalbalwar

* Correspondence: yuvrajparkale@gmail.com

Department of Electronics and
Telecommunication Engineering, Dr.
Babasaheb Ambedkar Technological
University (Established by
Government of Maharashtra State),
Lonere, Raigad, Maharashtra, India

Abstract

In compressed sensing, the Toeplitz sensing matrices are generated by randomly drawn entries and further optimizes them with suitable optimization methods. However, during an optimization process, state-of-the-art optimization methods tend to lose control over the structure of measurement matrices. In this paper, we proposed the novel approach for optimization of Toeplitz sensing matrices based on evolutionary algorithms such as Genetic Algorithm (GA), Simulated Annealing (SA), and Particle Swarm Optimization (PSO) for compression of an image signal. Furthermore, we investigated the performance of Basis Pursuit (BP) and Orthogonal Matching Pursuit (OMP) algorithms for the reconstruction of the images. The proposed optimized Toeplitz sensing matrices based on evolutionary algorithms such as GA, SA, and PSO exhibit a significant reduction in the mutual coherence (μ) and thus improved the recovery performance of 2D images compared to state-of-the-art non-optimized Toeplitz sensing matrices. The result reveals that the optimized Toeplitz sensing matrices with Basis Pursuit (BP) achieved more accurate results with a robust and uniform reconstruction guarantee compared to the OMP algorithm. However, BP shows the slow reconstruction performance of the image signal. On the other hand, an optimized Toeplitz sensing matrix with OMP shows a fast reconstruction guarantee, but at the cost of a reduction in the PSNR. Furthermore, the proposed approach retains the structure of Toeplitz sensing matrices and improves the image recovery performance of compressed sensing. Finally, the experimental results validate the effectiveness of the proposed method based on evolutionary algorithms for image compression.

Keywords: Compressed sensing, Genetic Algorithm (GA), Simulated Annealing (SA), Particle Swarm Optimization (PSO), Optimization, Basis Pursuit (BP), Orthogonal Matching Pursuit (OMP)

1 Introduction

The conventional Nyquist sampling depends on the highest amount of rate of alteration of a signal. In this sampling scheme, samples of the signal are captured at double the highest frequency in the signal. However, this method restricts the efficient compression of a signal. Since this scheme places an enormous burden on an encoder side

which acquires a vast number of samples of the signal and keeps only a few significant samples that are required to characterize the signal. Furthermore, these methods include complicated multiplications, an exhaustive coefficient search, and sorting procedure along with the arithmetic encoding of the significant coefficients with their locations. Consequently, it results in a vast storage requirement and power consumption.

On the contrary, compressed sensing is emerging as the most recent sampling scheme, which allows compression and signal reconstruction from the minimum number of measurements. In this scheme, the signal acquisition and compression are performed simultaneously at the encoder end. The signal is recovered back with a higher probability of success by using different optimization algorithms. Thus, CS results in a significant reduction in storage requirement and further reduces power consumption.

Compressed sensing has been implemented in diverse fields, including medical imaging, radar imaging, cameras, coding theory, geophysics, and astronomy.

CS-based biomedical imaging has been shown enormous interest and growth in recent times. Recently, researchers Wang, Bresler, and Vasilis [1] had reported an in-depth survey and success on the application of CS in MRI, CT, PET, SPECT, optical imaging, and ultrasound imaging. The researchers Lustig et al. [2] had been successfully used CS to MRI. Thus, CS-based MRI could speed up the data acquisition process by reducing the scan time, and this allows us to examine a higher number of patients.

The dictionary learning-based reconstruction of MR images is one of the recent developments and shows great potential in medical applications. The researchers Ravishankar and Bresler [3] reconstructed MR images based on the dictionary learning approach. Furthermore, they [4] had successfully proposed learning of doubly sparse transform for the images.

Further, the application of CS in radar imaging has been an additional growing field of interest. Yang et al. [5] had successfully designed the segmented recovery scheme for CS-based SAR (Synthetic Aperture Radar) imaging. Bu et al. [6] had developed a CS algorithm for SAR imaging. They had reconstructed the data of good quality with limited observations and thus results in the reduction of storage requirement. Deng et al. [7] had successfully proposed CS-based image coding. They had been achieved a robust performance against the lossy channel compared to conventional coding methods.

Li and Qi [8] proposed a nonlocal Douglas-Rachford (NLDR) algorithm, based on Douglas-Rachford splitting to solve low-rank optimization problems constrained by the CS measurements. Shen et al. [9] sparse Bayesian dictionary learning based compressed sensing-based inpainting of aqua moderate resolution imaging.

Furthermore, some researchers had been implemented and successfully tested the real-time hardware for CS-based applications. For example, the single-pixel camera based on compressed sensing had developed by Duarte et al. [10]. Further, Nagesh and Li [11] had developed color imaging architecture based on the combination of single-pixel CS camera and Bayer color filter. Similarly, single-pixel CS was applied for remote sensing by researcher Ma Jianwei [12], which results in the reduction of storage requirement and the computational cost of imaging.

The author Liquan Zhao et al. [13] has implemented compressed sensing for monitoring the images of the transmission line. These images are compressed and

reconstructed using a compressed sensing technique which, reduces the overall operational cost of a system.

In a nutshell, the Nyquist sampling put an enormous burden on the encoder side due to a massive number of samples of an acquired signal, particularly for audio/speech, ECG, image, and video signals. This fact inspired the study of compressed sensing as a potential solution for sampling, compression, and reconstruction of a signal. Intuitively, sparsity represents a large amount of energy concentration in a few numbers of coefficients. Several real-world signals such as speech and image are sparse or compressible in some transform domain. For example, images are compressible in basis algorithms such as JPEG and JPEG2000. The compressed sensing uses this sparsity property to compress and recover the signal effectively.

Traditionally, the random sensing matrices widely employed for signal compression in CS. The random Gaussian sensing matrices are entirely unstructured. Therefore, these matrices resulted in an enhanced computational complexity and increased memory storage requirement. Hence, the practical implementation of random Gaussian sensing matrices is costly. Further, sensing technologies need structured measurement matrices to accomplish different applications. Thus, the Toeplitz measurement matrix is one of the structured class matrix and widely used in a different field of applications, such as MRI [14], Synthetic Aperture Radar (SAR) [15], and channel estimations [16]. The Toeplitz matrices possess some exceptional features, such as these matrices generated with a smaller number of entries. Moreover, different techniques are available to speed up the matrix multiplication, which further may result in fast signal reconstruction. So far, the work carried out under the statement that the Toeplitz matrices generated using randomly drawn entries. Recently, Dirksen et al. [17] proposed partial Gaussian circulant matrices for 1-bit compressed sensing. Furthermore, Jie et al. [18] proposed compressed sensing matrices using vector spaces for signal processing.

In the literature, so far, different methods are proposed for the optimization of random Gaussian measurement matrices [19, 20]. However, these methods randomly draw entries to generate random Toeplitz matrices and further optimize them with proper optimization methods. Nevertheless, during an optimization process, these methods lose control over the structure of measurement matrices. Also, researchers Abolghasemi, Jarchi, and Sanei [20] proposed the gradient-decent-based method to optimize mutual coherence. In this method, the modified cost function is followed by the gradient-descent minimization method to optimize the sensing matrices iteratively. This method shows the robustness in handling complex values. The researcher Duarte-Carvajalino and Sapiro [21] proposed the non-iterative way to calculate t -averaged mutual coherence. This method intended to make a Gram matrix closer to the identity matrix. However, this method ignores the negative eigenvalues and thus presents the problem of complex values which, causes the algorithm to fail.

It had proved that if sensing matrices satisfy restricted isometry property (RIP) [22], then there has been a high probability of superior quality signal reconstruction. On the contrary, the RIP is impractical to evaluate. Therefore, another way to satisfy the RIP and guarantee the exact reconstruction of a signal is to compute the mutual coherence (μ) between the sensing matrix (Φ) and the sparsifying matrix (Ψ). The mutual coherence (μ) of a dictionary $D_{M \times N} = \Phi_{M \times N} \times \Psi_{N \times N}$ is

defined as the biggest absolute and normalized inner product among different columns of D [19] and given by the equation (1).

$$\text{Minimize } \mu(D) = \sqrt{N} \cdot \max_{1 \leq i, j \leq N, i \neq j} \frac{|d_i^T d_j|}{\|d_i\| \cdot \|d_j\|} \quad (1)$$

where N is the length of the input signal. From linear algebra, $1 \leq \mu(\Phi, \Psi) \leq \sqrt{N}$.

Thus, the minimization of mutual coherence may be one of the effective ways to boost the recovery performance of compressed sensing matrices [23].

This paper proposes the optimization of Toeplitz sensing matrices based on evolutionary algorithms such as Genetic Algorithm (GA), Simulated Annealing (SA), and Particle Swarm Optimization (PSO) algorithm for image compression. The minimization of mutual coherence may be one of the effective ways to boost the recovery performance of compressed sensing matrices. Thus, this paper proposed the minimization of the mutual coherence (μ) between the sensing matrix (Φ) and the sparsifying matrix (Ψ) using evolutionary algorithms. The proposed optimization approach provides the best random Toeplitz vector, which consequently minimizes the mutual coherence (μ) between the sensing matrix (Φ) and the sparsifying matrix (Ψ). Further, the Toeplitz measurement matrix generated using the best random Toeplitz vector and finally applied for image compression. Furthermore, this proposed approach retains the structure of the Toeplitz matrix and improves the image recovery performance of compressed sensing.

Since the novel approach of the proposed optimization method is based on an evolutionary algorithm, and hence, it is entirely different from the state-of-the-art optimization methods. Until now, all the state-of-the-art optimization methods use non-evolutionary approaches for optimization of sensing matrices and thus tend to lose the structure of sensing matrices. Therefore, it is not practicable to compare proposed evolutionary approaches directly with non-evolutionary approaches of state-of-the-art methods. Thus, rather than, we have compared the performance of the proposed optimized Toeplitz sensing matrices based on evolutionary algorithms with non-optimized Toeplitz sensing matrices.

The main contributions of the proposed work are as follows:

1. We proposed a novel approach for the optimization of Toeplitz sensing matrices based on Evolutionary algorithms.
2. We proposed the first approach for the optimization of Toeplitz sensing matrices based on the Genetic Algorithm.
3. We proposed the second approach for the optimization of Toeplitz sensing matrices based on the Simulated Annealing (SA) Algorithm.
4. We proposed the third approach for the optimization of Toeplitz sensing matrices based on the Particle Swarm Optimization (PSO) Algorithm.
5. We investigated the signal reconstruction performance using Basis Pursuit (BP) and Orthogonal Matching Pursuit (OMP) algorithm for GA, SA, and PSO-based optimization approaches.

6. Finally, GA, SA, PSO-based optimization approaches exhibit a significant reduction in the mutual coherence (μ) and thus improved the recovery performance of 2D images compared to non-optimized Toeplitz sensing matrices.

The organization of the paper is as follows: Section 1 presents the formulation of an optimization problem. Section 2 elaborates on the proposed optimization method based on evolutionary algorithms. Section 4 presents results and discussion. Finally, Section 5 presents the conclusions.

2 Method

Let the problem needs to optimize in terms of the minimization problem. Therefore, objective/fitness function is required to minimize. This problem consists of two matrices, namely the measurement/sensing matrix ($\Phi_{M \times N}$, where $M < N$) and the sparsifying transform matrix ($\Psi_{N \times N}$). The Toeplitz matrix is used as a sensing matrix to compress the given signal, whereas the Discrete Cosine Transform (DCT) is used as a sparsifying transform matrix. The problem description is as follows:

The dictionary matrix (D) given as: $D_{M \times N} = \Phi_{M \times N} \times \Psi_{N \times N}$.

Now, the dictionary matrix D must be optimized, such that the mutual coherence (inner product) of matrix D is as small as possible as given by the equation (1).

Thus, the objective function is to optimize (minimize) the mutual coherence (μ) of the dictionary matrix (D) using an evolutionary algorithm such as Genetic Algorithm (GA), Simulated Annealing (SA), and Particle Swarm Optimization.

The minimization of the mutual coherence (μ) of the dictionary matrix (D) will satisfy the RIP condition and further results in improving the recovery performance of a sparse signal.

The statement of the optimization problem is as follows:

The objective function is defined by equation (2) as follows. Here, N is the length of the input signal.

$$\text{Minimize } \mu(D) = \sqrt{N} \cdot \max_{1 \leq i, j \leq N, i \neq j} \frac{|d_i^T d_j|}{\|d_i\| \cdot \|d_j\|} \quad (2)$$

2.1 Proposed optimization of Toeplitz sensing matrices using Genetic Algorithm (GA)

This section presents the optimization of Toeplitz sensing matrices based on the Genetic Algorithm (GA). The initial population was generated by using different random Toeplitz vectors. Then the fitness function $f(x)$ is evaluated for all the random Toeplitz vectors. The new population made using three steps: selection, crossover, and mutation. The Roulette wheel selection technique was used to select the best parents to create new offspring. In this technique, the best parents have chosen to depend on the fitness of the population. Higher fitness indicates a greater chance to get selected. Then the bit sequence of the two parents is swapped to create the new offspring. The diversity in the new population is achieved with a mutation. Next, the old population was replaced by the new population [24]. Then the algorithm is tested for the convergence criterion such as the maximum number of iterations. Finally, when an algorithm is converged, it

will return the best Toeplitz vector, which results in minimum mutual coherence (μ). Thus, this best solution vector is used to generate the Toeplitz matrix.

The algorithmic steps for optimization of Toeplitz sensing matrices using Genetic Algorithm (GA) [25] are given as follows:

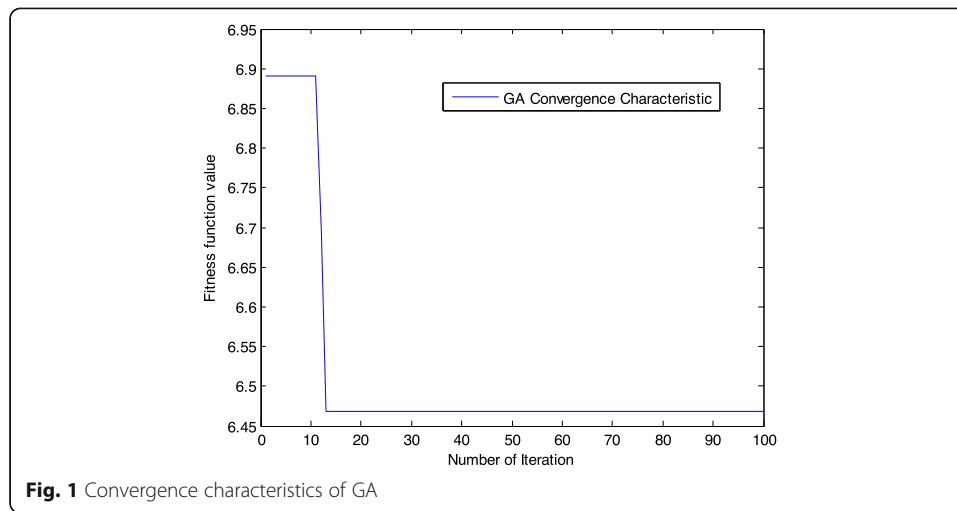
- (i) Generate an initial population of n chromosomes using random Toeplitz vectors: Here, each Toeplitz vector is considered as one of the candidate solutions for the given problem. Each Toeplitz vector corresponding to the Toeplitz matrix is generated.
- (ii) Then evaluate the fitness function $f(x)$ of each chromosome in the population:
- (iii) Further, evaluate the mutual coherence (i.e., fitness function) (μ) between the generated Toeplitz matrix (Φ) and the sparsifying matrix (Ψ). Thus, we had evaluated the fitness function for the generated Toeplitz matrix. Similarly, evaluate the fitness function for all the generated Toeplitz vectors.
 - (a) *Selection*: Select two candidate solutions as a parent chromosome depending on their fitness value. Here, the best parents are selected by using the Roulette wheel selection technique. It works on the principle of the higher the fitness value of a chromosome better is the chance to get selected.
 - (b) *Crossover*: Swap the bit sequence of the chosen parent chromosomes to create a new population. We can select one-point/two-point/three-point crossover for this purpose.
 - (c) *Mutation*: It provides the variety in a new population and thus protects the algorithm to trap at the local optimum solution. However, this results in slow convergence of the algorithm.
- (iv) Now, create a new population using the following steps:
- (v) Replace the old population with a new population.
- (vi) Finally, test for the convergence criterion of the Genetic Algorithm such as the number of iterations, etc.

When an algorithm is converged, it returns the best candidate solution, i.e., it returns the Toeplitz vector, which resulted in minimum mutual coherence (μ). Otherwise, go to step (2) and repeat the procedure till end criterion meets.

The simulation shows excellent results with the following specifications:

- Algorithm: Binary GA Algorithm
- Population Size = 100
- Selection technique: Roulette wheel selection technique
- Selection rate of parents for generating offspring = 0.5 or 50% of the initial population
- Number of encoding bits = 16
- Mutation method: Single point crossover
- Mutation rate = 0.15
- The maximum iterations (stopping criteria) = 100

Figure 1 shows the convergence characteristics of GA. It is observed from Fig. 1 that the value of the fitness function reduces with an increase in the number of iterations.



Moreover, the cost of the fitness function remains constant from the 10th iteration to the 100th iteration. Thus, GA shows good and stable convergence characteristics up to the 100th iteration.

2.2 Proposed optimization of Toeplitz sensing matrices using Simulated Annealing (SA) algorithm

This section presents the optimization of Toeplitz sensing matrices using Simulated Annealing (SA). Primarily, the temperature parameter (T_0) is set to some high value, and then it is gradually reduced using the temperature reduction factor (α). The initial solution vector (s) and the new solution vector (s_0) are randomly generated using a random generator and then evaluate the fitness function for both the solution vectors.

If a new solution has better fitness than the current solution, it is selected as the next solution. On the contrary, if the new solution has a worse fitness compared to the current solution, the algorithm still considers it as the next solution. The acceptance or rejection of the new solution vector as the future solution vector depends on the Metropolis-Hasting criterion [26]. The Metropolis-Hasting principle given as follows:

$$p(r) = e^{\frac{-\delta f}{kT}}$$

where $\delta f = f(s) - f(s_0)$ is the fitness difference between the new solution vector and the old solution vector, T = temperature parameter, $P(r)$ = probability of acceptance or rejection of the new solution vector as a next solution vector and $r \in (0, 1)$, and $k = 1$ (Boltzmann's constant).

A random number (r) is generated such that $r \in (0, 1)$. If a random number (r) $< \exp [(-\delta f)/(T)]$, then a new solution is selected as the next solution ($s_0 = s$); otherwise, a new solution is discarded. Then the temperature parameter is reduced gradually to narrow search the optimum solution. The algorithm repeated until it meets the stopping criterion, such as minimum temperature value or a number of iterations. Finally, when an algorithm is converged, it will return, the best Toeplitz vector which, results in a minimum mutual coherence (μ). Thus, this best solution vector is used to generate the Toeplitz matrix.

The algorithmic steps for the optimization of Toeplitz sensing matrices using Simulated Annealing (SA) [27] are given as follows:

- Solution space: X
 - Objective function: f
 - Neighborhood structure: N
 - Select Initial point: s_0
 - Select Initial temperature: $T_0 > 0$
 - Select temperature reduction function: α
 - Repeat
 - Repeat
 - Randomly select $s \in N(s_0)$
 - $\delta f = f(s) - f(s_0)$
 - If $\delta f < 0$
 - then $s_0 = s$
 - else
 - Generate a random number $r \in (0, 1)$
 - If $r < e^{(-\delta f / T)}$ then
 - $s_0 = s$
 - Until iteration_count = maximum iteration
 - Set $T = \alpha(T)$
 - Until stopping condition = TRUE
- Output: s_0 is the approximation to the global minimum solution

The simulations are conducted with the following specifications:

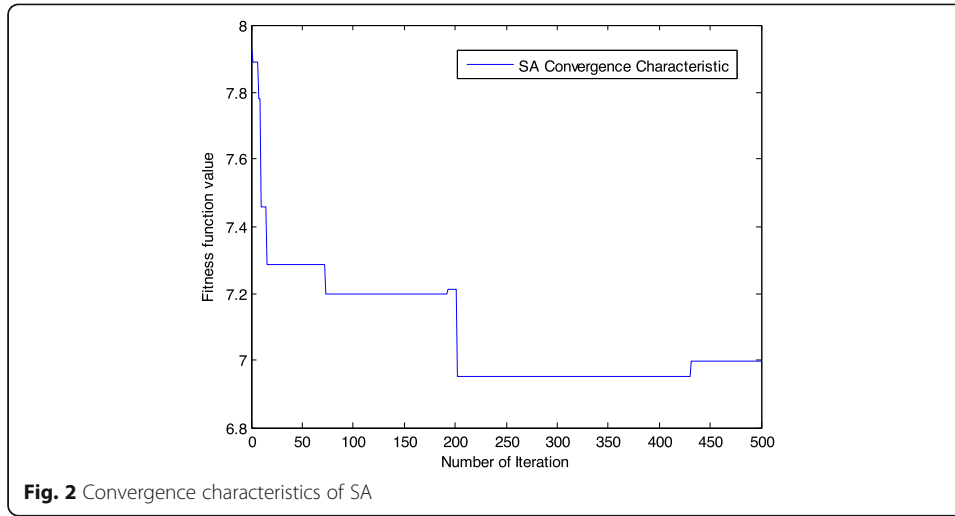
- The maximum numbers of iterations = 500
- Initial Temperature=1
- Temperature reduction factor=0.8
- Stopping Temperature value= 1e-8

Figure 2 shows the convergence characteristics of the SA. It is observed from Fig. 2 that the value of the fitness function reduces with an increase in the number of iterations. Moreover, the value of the fitness function remains relatively constant from the 200th iteration to the 500th iteration. Thus, SA exhibited good and stable convergence characteristics up to the 500th iteration.

2.3 Proposed optimization of Toeplitz sensing matrices using Particle Swarm

Optimization (PSO) algorithm

This section presents the optimization of Toeplitz sensing matrices using Particle Swarm Optimization (PSO). Here, each particle in the population is equivalent to the



candidate solution in the solution space. This solution space is generated using a random number generator. Then the fitness function is evaluated for each solution in the population. If the current particle fitness is higher than the previous local best particle fitness, then we have to update the local best particle position (*PBest*). Similarly, if the current particle fitness is higher than the previous global best particle fitness, then we have to update the global best particle position (*GBest*). Then update the position and velocity of each particle using equation (position update equation) and equation (velocity update equation). Finally, when an algorithm is converged, it will return the best Toeplitz vector, which results in minimum mutual coherence (μ). Thus, this best solution vector is used to generate the Toeplitz matrix.

The algorithmic steps for the optimization of Toeplitz sensing matrices using Particle Swarm Optimization (PSO) [28] are given as follows.

- (i) Randomly generate the initial population.
- (ii) Randomly initialize the positions and velocities of particles in the population.
- (iii) Evaluate the fitness function for each particle in the population.
- (iv) If the current particle fitness is higher than the previous local best particle fitness, then update the local best particle position (*PBest*).
- (v) Similarly, if the current particle fitness is higher than the previous global best particle fitness, then update the global best particle position (*GBest*).
- (vi) Then update the position and velocity of each particle.
 - The velocity update equation is given by the equation number (3), and (4):

$$v_i(k+1) = \text{Inertia} + \text{cognitive} + \text{social}, \quad (3)$$

$$\begin{aligned} \mathbf{v}_i(k+1) = & \omega \times \mathbf{v}_i(k) + c_1 \times \text{random}_1() \times \\ & (\text{PBest}_i - \mathbf{x}_i(k)) + c_2 \times \text{random}_2() \times \\ & (\text{GBest} - \mathbf{x}_i(k)), \end{aligned} \quad (4)$$

where $v_i(k)$ is the initial velocity, and $v_i(k+1)$ is the updated velocity of the i th particle, w : inertia weight, c_1 , c_2 : Positive constant c_1 and c_2 are personal (cognitive) and social

learning factors; PBest: personal past best position of the i th particle; GBest: Global best position of the swarm; random1 (); random2 (): random function in the range [0, 1]; and k denotes the iteration counter.

- The position update equation (5) given as:

$$x_i(k+1) = x_i(k) + v_i(k+1), \quad (5)$$

where $x_i(k)$ is the initial position of the particle, and $x_i(k+1)$ is the updated position of the i th particle.

(vii) Go to step (iii), and repeat until stopping condition meets.

The simulations conducted with the following specifications:

- Population size = 100
- Inertia weight (maximum) = 0.9
- Inertia weight (minimum) = 0.4
- Personal/cognitive factor ($c1$) = 2
- Social learning factor ($c2$) = 2
- The maximum numbers of iterations = 100

Figure 3 shows the convergence characteristics of the PSO. It is observed from Fig. 3 that the value of the fitness function reduces with an increase in the number of iterations. Moreover, the value of the fitness function remains constant from the 50th iteration to the 100th iteration. Thus, PSO showed good and stable convergence characteristics up to the 100th iteration.

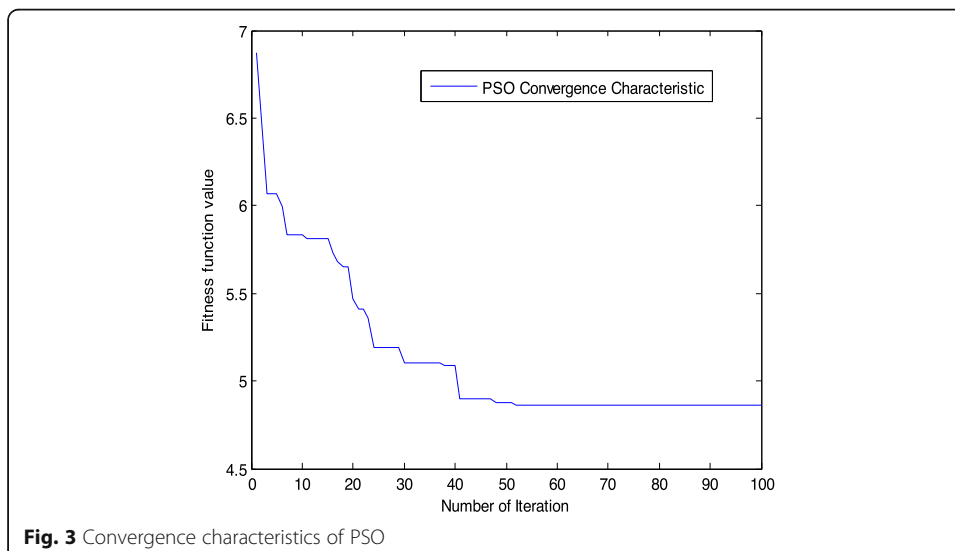


Fig. 3 Convergence characteristics of PSO

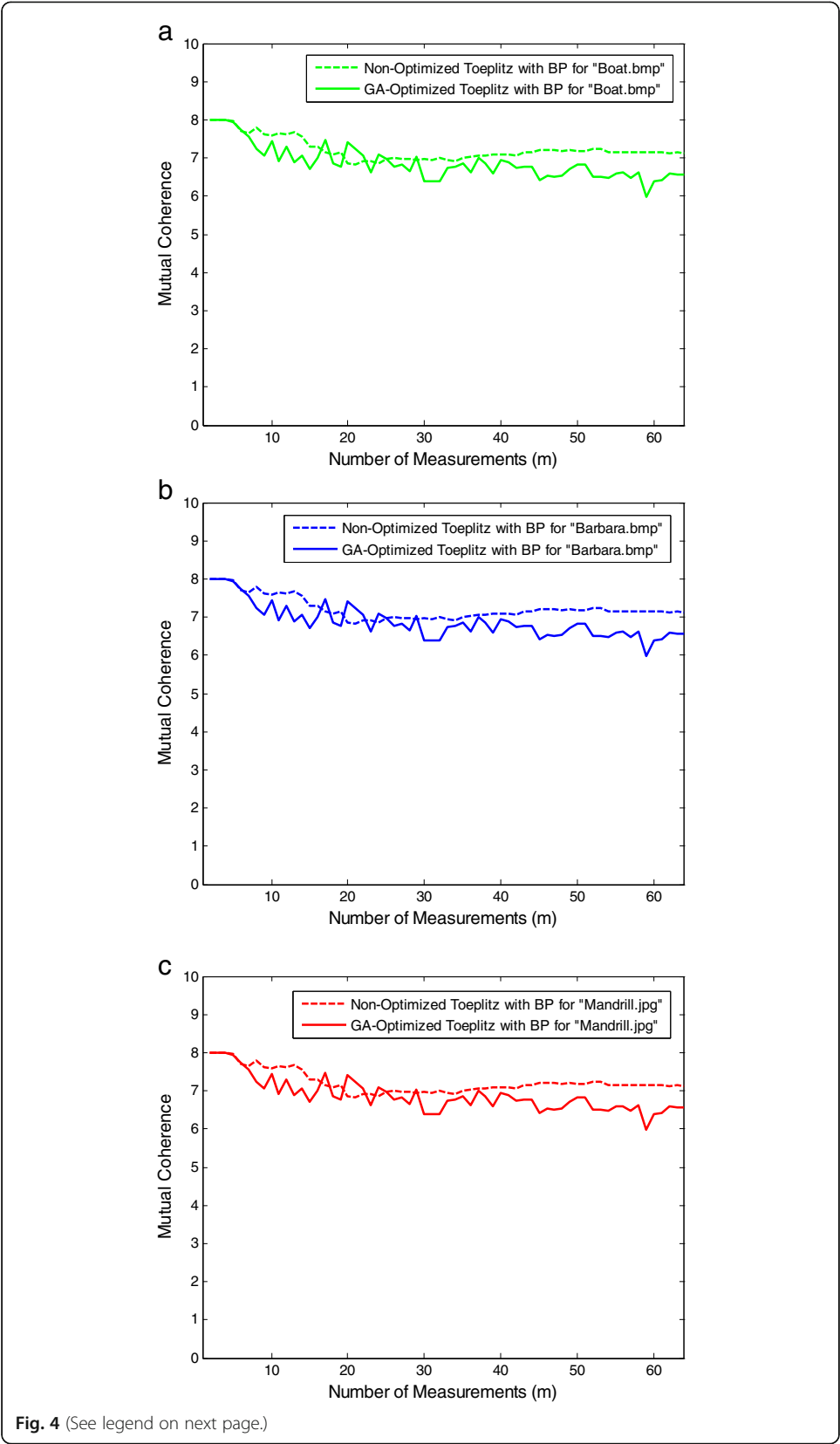


Fig. 4 (See legend on next page.)

(See figure on previous page.)

Fig. 4 a The mutual coherence (μ) of the proposed GA-optimized Toeplitz sensing matrix for different values of measurements (m), for “Boat.bmp” image. **b** The mutual coherence (μ) of proposed GA-optimized Toeplitz sensing matrix for different values of measurements (m), for “Barbara.bmp” image. **c** The mutual coherence (μ) of the proposed GA-optimized Toeplitz sensing matrix for different values of measurements (m), for “Mandrill.jpg” image

3 Results and discussion

The proposed work is evaluated on 256×256 test images namely: “Boat.bmp,” “Barbara.bmp,” and “Mandrill.jpg” [29]. The 8×8 DCT block image processing is used on 256×256 test images. The optimized random Toeplitz sensing matrices are used to compress the image signal.

The Discrete Cosine Transform (DCT) is used as the sparsifying basis for an image signal because of its higher sparseness. The test images recovered using Basis Pursuit (BP) [30] and Orthogonal Matching Pursuit (OMP) algorithm [31]. The experimental work performed using MATLAB 9.0 software with Intel (R) Core (TM) i3-4130 CPU @ 3.40 GHz, 8 GB RAM system specifications. The reconstruction performance of test images evaluated using metrics such as mutual coherence (μ), number of measurements, mean square error (MSE), peak signal-to-noise ratio (PSNR), signal reconstruction time, and a sensing matrix construction time.

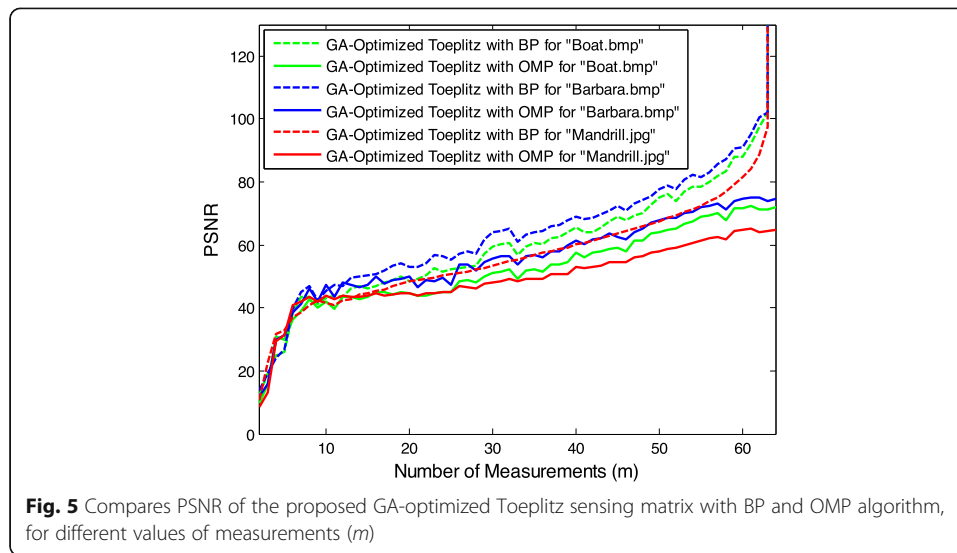
3.1 Performance analysis of GA-optimized Toeplitz sensing matrices with BP and OMP

This section presents the performance analysis of the proposed genetic algorithm-based optimized Toeplitz sensing matrices with Basis Pursuit (BP) and Orthogonal Basis Pursuit (OMP) as a reconstruction algorithm. The reconstruction performance of test images evaluated using error metrics such as mutual coherence (μ), number of measurements (m), mean square (MSE), peak signal-to-noise ratio (PSNR), signal reconstruction time, and sensing matrix construction time.

Figure 4a–c compared the mutual coherence (μ) of proposed GA-optimized Toeplitz sensing matrices with non-optimized Toeplitz sensing matrices for different values of measurements (m). The result shows that the proposed GA-optimized Toeplitz sensing matrices exhibit a significant reduction in the mutual coherence (μ) compared to non-optimized Toeplitz sensing matrices for “Boat.bmp,” “Barbara.bmp,” and “Mandrill.jpg” images. The reduction in mutual coherence indicates the improvement in the reconstruction performance of the test images and vice versa.

Figure 5 shows that the proposed GA-Optimized Toeplitz sensing matrices with BP achieve excellent results with higher PSNR compared to OMP-based GA-optimized Toeplitz sensing matrices.

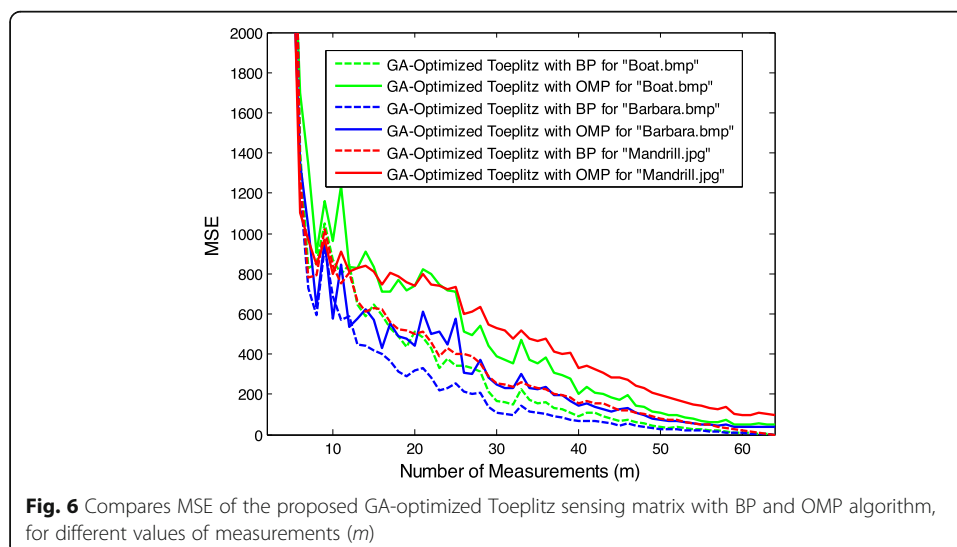
Here, the higher values of PSNR indicate the better quality of the reconstructed images. Figure 6 shows the comparison of mean square error (MSE) between the proposed GA-Optimized Toeplitz sensing matrices with BP and OMP algorithms. The proposed GA-optimized Toeplitz sensing matrices with BP achieve more



accurate results (i.e., reduction in MSE) compared to the GA-optimized Toeplitz sensing matrices with OMP, as shown in Fig. 6. Here, smaller values of MSE indicate a more accurate result and thus gives better quality of the reconstructed images.

Furthermore, Figure 7a–c compared the image reconstruction time required for the GA-optimized Toeplitz sensing matrices with Basis Pursuit (BP) and OMP algorithms for “Boat.bmp,” “Barbara.bmp,” and “Mandrill.jpg” images. The result shows that the GA-optimized Toeplitz sensing matrices with the OMP algorithm achieve fast image reconstruction compared to the Basis Pursuit (BP)-based image reconstruction.

Table 1 shows the performance comparison of the proposed GA-optimized sensing matrices with Basis Pursuit (BP) and OMP algorithms for compression ratio $CR (N/m) = 0.5$.



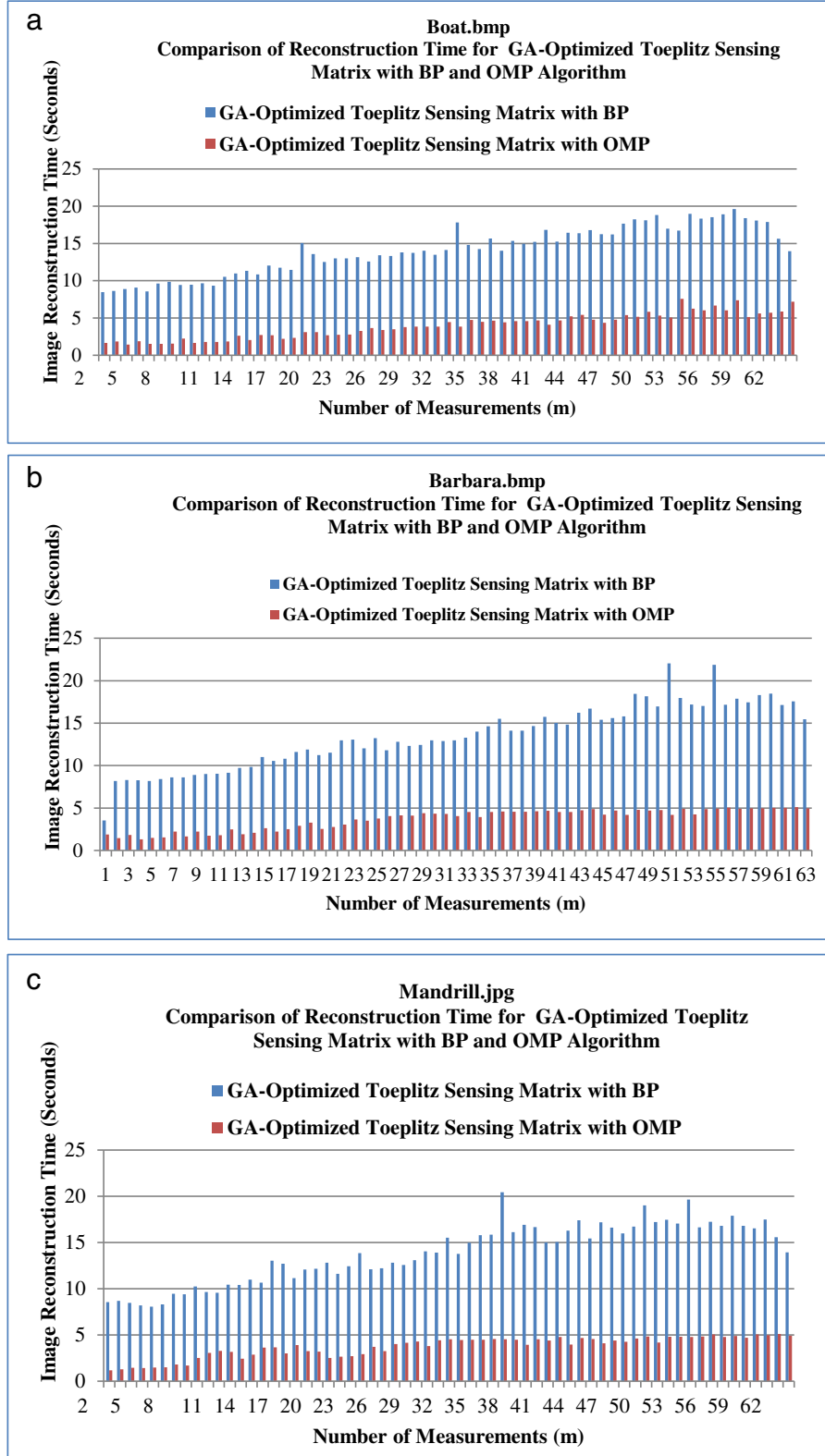
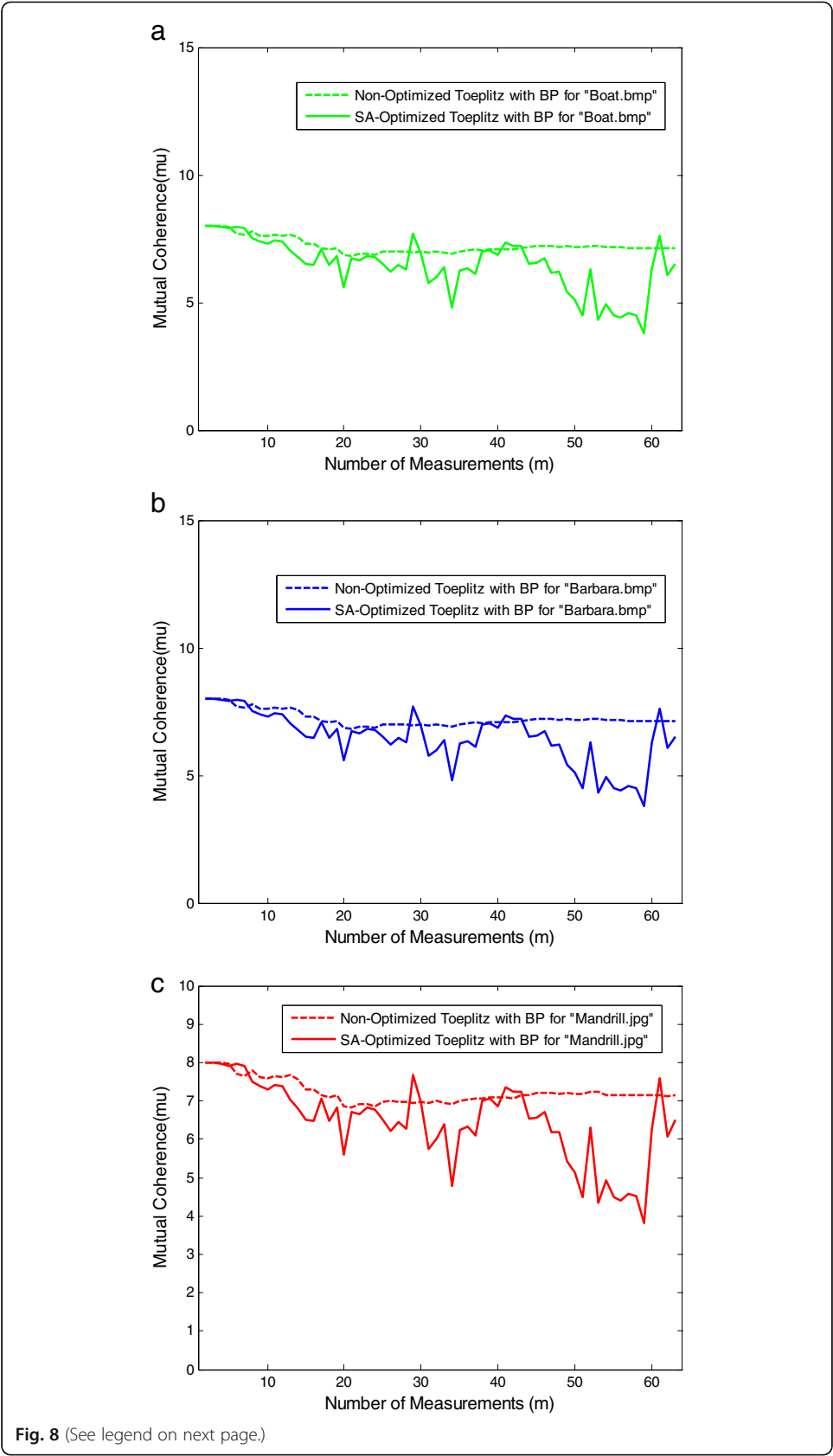


Fig. 7 **a** The image reconstruction performance of the proposed GA-optimized Toeplitz sensing matrix with BP and OMP algorithm, for “Boat.bmp” image. **b** The image reconstruction performance of the proposed GA-optimized Toeplitz sensing matrix with BP and OMP algorithm, for “Barbara.bmp” image. **c** The image reconstruction performance of the proposed GA-optimized Toeplitz sensing matrix with BP and OMP algorithm, for “Mandrill.jpg” image

Table 1 Performance comparison of the proposed GA-optimized sensing matrix with Basis Pursuit (BP) and OMP algorithms, for compression ratio CR (N/M) = 0.5

Method used	Proposed GA-optimized sensing matrices with BP and OMP									
	Boat.bmp			Barbara.bmp			Mandrill.jpg			
Sensing methods and performance measures	Non-optimized Toeplitz with BP	GA-optimized Toeplitz with BP	GA-optimized Toeplitz with OMP	Non-optimized Toeplitz with BP	GA-optimized Toeplitz with BP	GA-optimized Toeplitz with OMP	Non-optimized Toeplitz with BP	GA-optimized Toeplitz with BP	GA-optimized Toeplitz with BP	GA-optimized Toeplitz with OMP
Mutual Coherence	6.9924	6.3988	6.3988	6.9924	6.3988	6.3988	6.9924	6.3988	6.3988	6.3988
PSNR (dB)	59.1519	60.6576	52.0795	62.9252	65.0589	56.3420	55.0343	56.2108	56.2108	49.1046
MSE	175.4469	150.9232	355.8757	120.3017	97.1869	232.3700	264.8315	235.4382	235.4382	479.1770
Reconstruction Time (seconds)	27.4589	13.4688	3.8421	15.3074	12.9837	4.2958	13.3598	13.9166	13.9166	4.4473
Sensing matrix construction time	0.0045	0.0015	0.0012	0.0047	0.0019	0.0014	0.0044	0.0016	0.0016	0.0014



(See figure on previous page.)

Fig. 8 a The mutual coherence (μ) of proposed SA-optimized Toeplitz sensing matrix for different values of measurements (m), for “Boat.bmp” image. **b** The mutual coherence (μ) of the proposed SA-optimized Toeplitz sensing matrix for different values of measurements (m), for “Barbara.bmp” image. **c** The mutual coherence (μ) of proposed SA-optimized Toeplitz sensing matrix for different values of measurements (m), for “Mandrill.jpg” image

From Table 1, it is observed that the proposed GA-optimized sensing matrices with BP achieved higher PSNR for “Boat.bmp,” “Barbara.bmp,” and “Mandrill.jpg” test images. Similarly, it is observed that the GA-optimized sensing matrices with BP showed lower MSE compared to GA-optimized sensing matrices with OMP. On the contrary, GA-optimized sensing matrices with OMP achieved considerably fast image reconstruction compared to GA-optimized sensing matrices with BP. Additionally, GA-optimized sensing matrices with OMP show the rapid construction of sensing matrices. (Please refer to Figure 16 from Appendix 1 for reconstruction quality of test images using proposed GA-Optimized sensing matrices with BP and OMP algorithm.)

3.2 Performance analysis of SA-optimized Toeplitz sensing matrices with BP and OMP

This section presents the comparative performance analysis between the SA-BP and SA-OMP based optimized Toeplitz sensing matrices.

It is seen from Fig. 8a–c that the proposed SA-optimized Toeplitz sensing matrices exhibited a significant reduction in the mutual coherence (μ) compared to non-optimized Toeplitz sensing matrices, for “Boat.bmp” image, “Barbara.bmp,” and “Mandrill.jpg” images.

It is noted from Fig. 9 that the proposed SA-optimized Toeplitz sensing matrices with BP achieved excellent results with higher PSNR compared to OMP based image reconstruction for “Boat.bmp,” “Barbara.bmp,” and “Mandrill.jpg” images.

It is observed from Fig. 10 that the proposed SA-optimized Toeplitz sensing matrices with BP achieve more accurate results (i.e., reduction in MSE) compared to the SA-optimized Toeplitz sensing matrices with OMP.

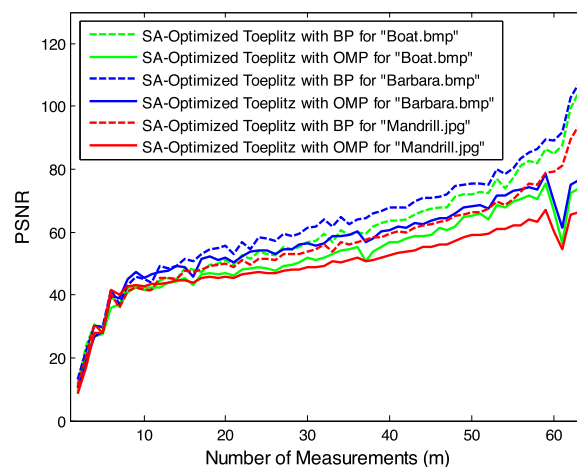
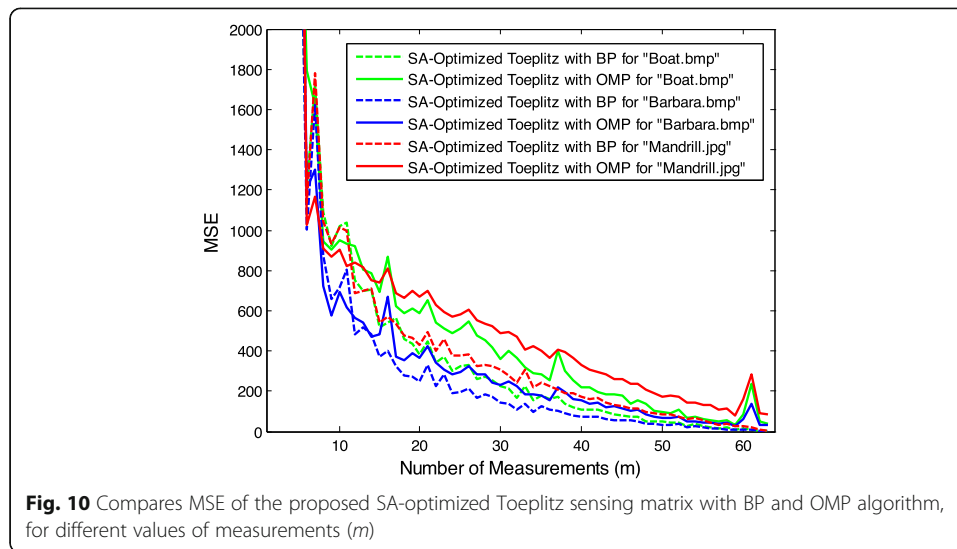


Fig. 9 Compares PSNR of the proposed SA-optimized Toeplitz sensing matrix with BP and OMP algorithm, for different values of measurements (m)



It is noted from Fig. 11a–c that the SA-optimized Toeplitz sensing matrices with the OMP algorithm achieved fast image reconstruction compared to the Basis Pursuit (BP) based reconstruction of the test images.

It is noted from Table 2, that the proposed SA-optimized sensing matrices with BP achieved higher PSNR for “Boat.bmp,” “Barbara.bmp,” and “Mandrill.jpg” test images. Similarly, SA-optimized sensing matrices with BP show lower MSE compared to SA-optimized sensing matrices with OMP. On the contrary, SA-optimized sensing matrices with OMP achieve considerably fast image reconstruction compared to SA-optimized sensing matrices with BP. Additionally, it also shows the rapid construction of sensing matrices. (Please refer to Figure 17 from Appendix 2 for reconstruction quality of test images using proposed SA-Optimized sensing matrix with BP and OMP algorithm.)

3.3 Performance analysis of PSO-optimized Toeplitz sensing matrices with BP and OMP

This section presents the comparative performance analysis between the PSO-BP and PSO-OMP based optimized Toeplitz sensing matrices.

It is seen from Fig. 12a–c that the proposed PSO-optimized Toeplitz sensing matrices achieve a significant reduction in the mutual coherence (μ) compared to non-optimized Toeplitz sensing matrices, for “Boat.bmp” image, “Barbara.bmp,” and “Mandrill.jpg” images.

It is noted from Fig. 13 that the proposed PSO-optimized Toeplitz sensing matrices with OMP attain marginally lower PSNR compared to non-optimized Toeplitz sensing matrices with BP.

Similarly, it is observed from Fig. 14 that the proposed PSO-optimized Toeplitz sensing matrices with OMP exhibit marginally higher MSE compared to the non-optimized Toeplitz sensing matrices with BP. However, this is due to the fact that the reconstruction guarantees of OMP are weak and show non-uniform behavior.

It is observed from Fig. 15a–c that the PSO-optimized Toeplitz sensing matrices with OMP exhibit significantly faster image reconstruction performance as compared to the non-optimized Toeplitz sensing matrices for “Boat.bmp,” “Barbara.bmp,” and “Mandrill.jpg,” respectively.

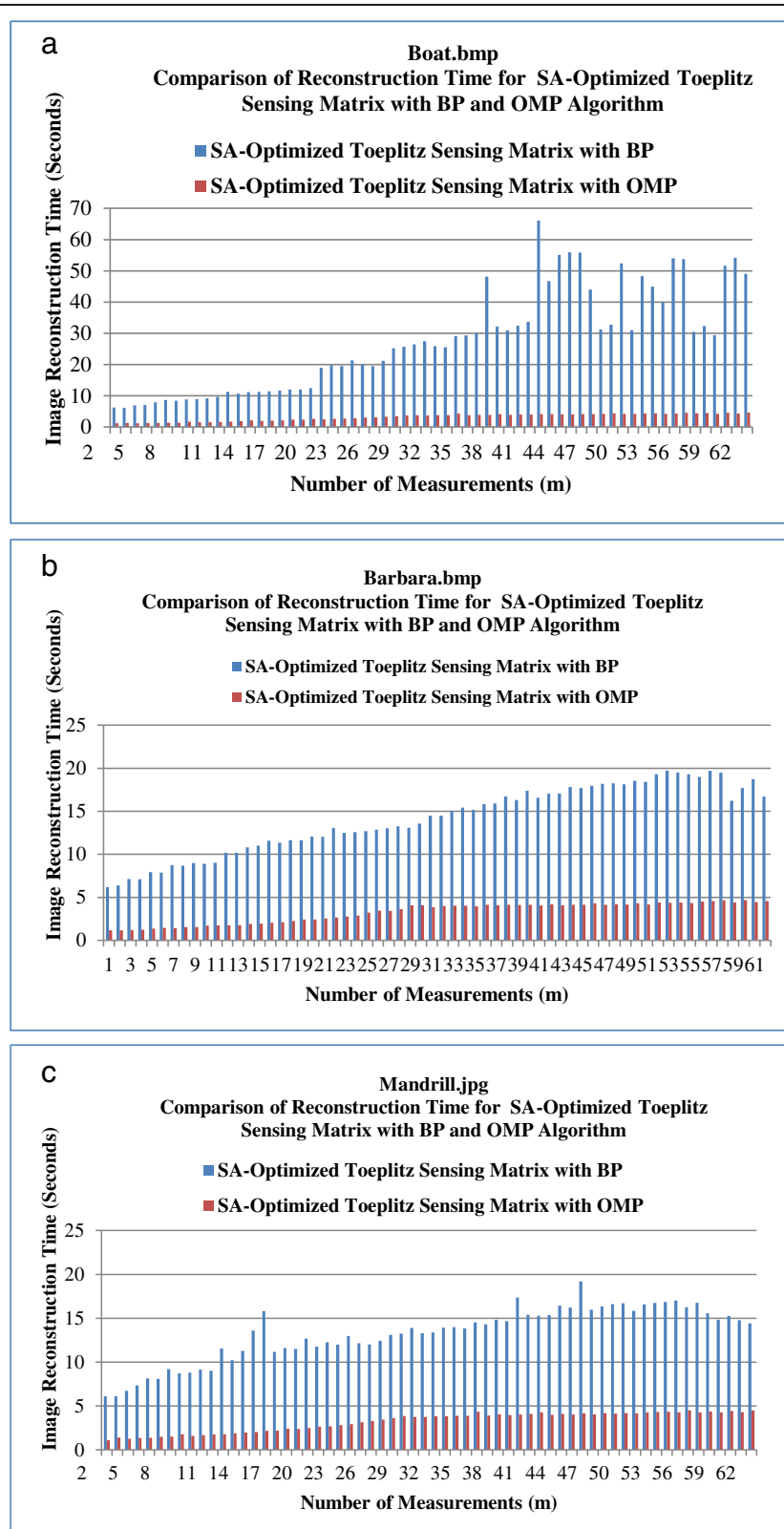


Fig. 11 **a** The image reconstruction performance of proposed SA-optimized Toeplitz sensing matrix with BP and OMP algorithm, for "Boat.bmp." **b** The image reconstruction performance of the SA-optimized Toeplitz sensing matrix with BP and OMP algorithm, for "Barbara.bmp." **c** The image reconstruction performance of SA-optimized Toeplitz sensing matrix with BP and OMP algorithm, for "Mandrill.jpg"

Table 2 Performance comparison of the proposed SA-optimized sensing matrix with Basis Pursuit (BP) and OMP algorithms, for compression RATIO CR (N/M) = 0.5

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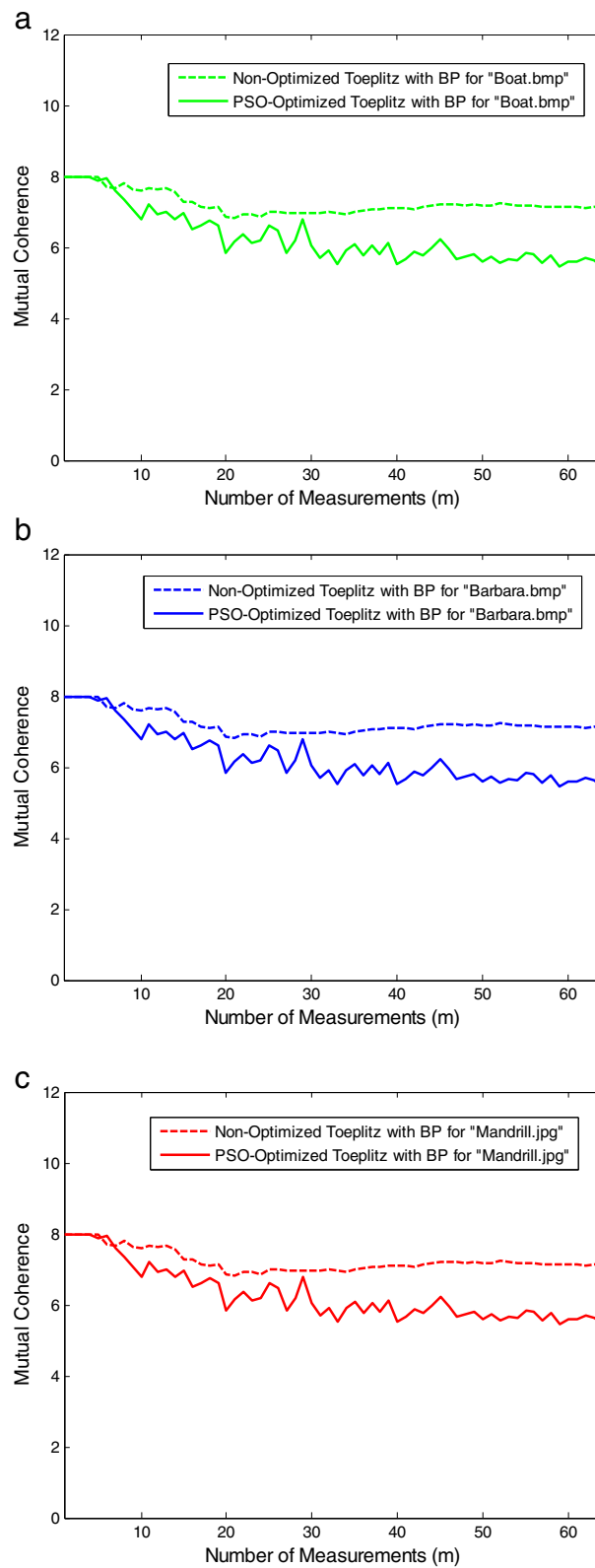
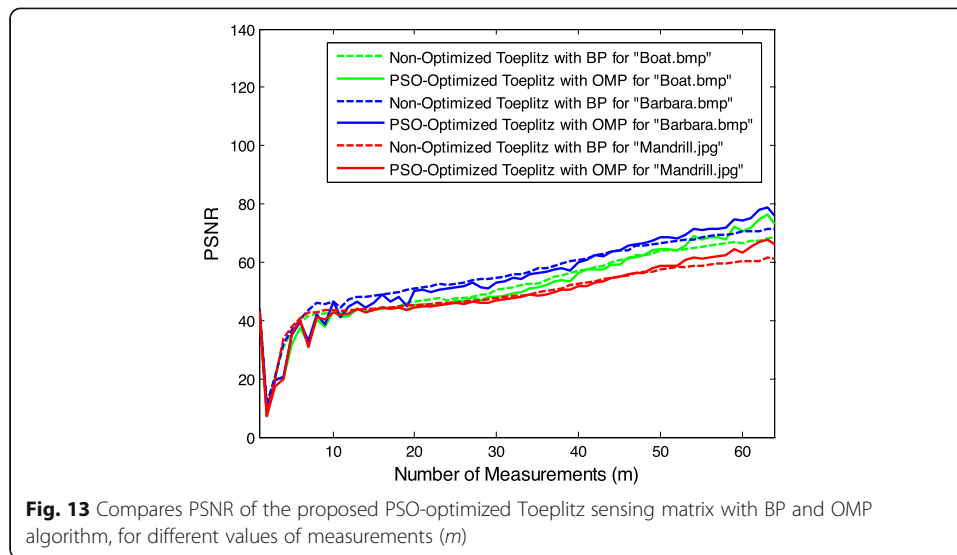


Fig. 12 **a** The mutual coherence (μ) of proposed PSO-optimized Toeplitz sensing matrix for different values of measurements (m), for "Barbara.bmp" image. **b** The mutual coherence (μ) of proposed PSO-optimized Toeplitz sensing matrix for different values of measurements (m), for "Barbara.bmp" image

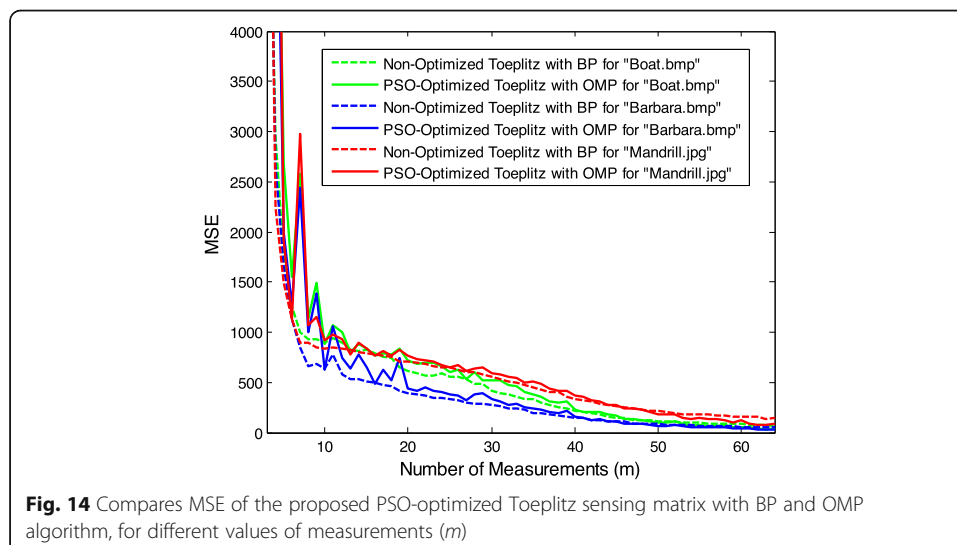


It is noted from Table 3 that the proposed PSO-optimized sensing matrices with BP achieve higher PSNR for “Boat.bmp,” “Barbara.bmp,” and “Mandrill.jpg” test images. Similarly, PSO-optimized sensing matrices with BP attain lower MSE compared to OMP-Optimized sensing matrices with OMP.

On the contrary, PSO-optimized sensing matrices with OMP achieve considerably fast image reconstruction compared to PSO-optimized sensing matrices with BP. Additionally, it also shows the rapid construction of sensing matrices. (Please refer to Figure 18 from Appendix 3 for reconstruction quality of test images using proposed PSO-Optimized sensing matrix with BP and OMP algorithm.)

4 Conclusion

In this paper, a novel approach to optimize Toeplitz sensing matrices using evolutionary algorithms such as Genetic Algorithm (GA), Simulated Annealing (SA), and Particle Swarm Optimization (PSO) algorithms for compressed sensing is discussed. Furthermore, the performance of Basis Pursuit (BP) and Orthogonal Matching



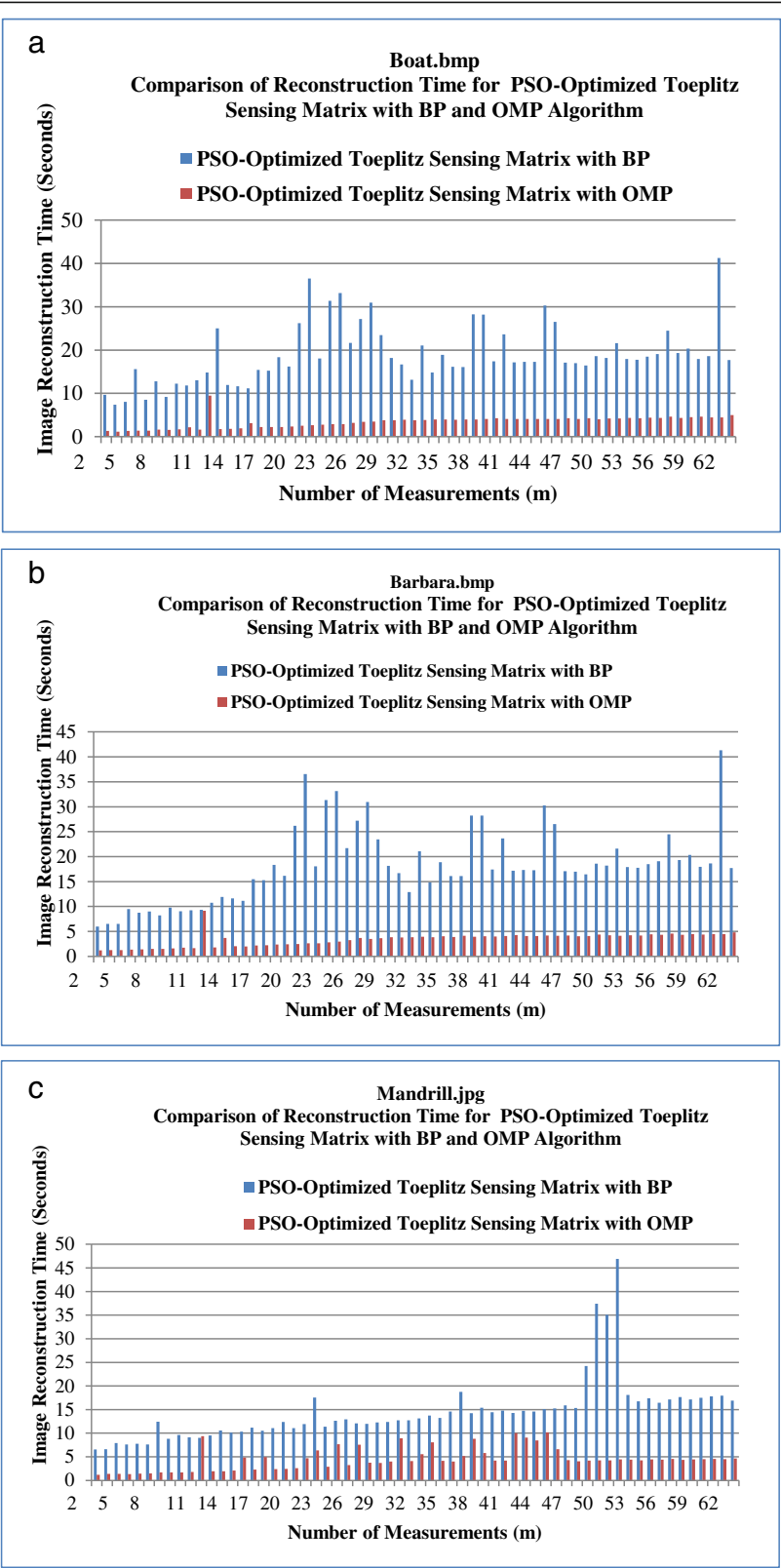


Fig. 15 (See legend on next page.)

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Fig. 15 **a** The image reconstruction performance of the proposed PSO-optimized Toeplitz sensing matrix with BP and OMP algorithm, for "Boat.bmp." **b** The image reconstruction performance of PSO-optimized Toeplitz sensing matrix with BP and OMP algorithm, for "Barbara.bmp." **c** The image reconstruction performance of PSO-optimized Toeplitz sensing matrix with BP and OMP algorithm, for "Mandrill.jpg"

Pursuit (OMP) algorithms are investigated for the reconstruction of the 2D image signal.

The minimization of mutual coherence (μ) between the sensing matrix (Φ) and the sparsifying matrix (Ψ) is one of the effective ways to boost the recovery performance of compressed sensing matrices.

The following significant conclusions are drawn based on the investigations:

The proposed optimized Toeplitz sensing matrices based on evolutionary algorithms exhibit a significant reduction in the mutual coherence (μ). Thus the proposed method clearly outperforms the non-optimized Toeplitz sensing matrices for image compression application.

The proposed optimized Toeplitz sensing matrices based on Genetic Algorithm (GA), Simulated Annealing (SA), and Particle Swarm Optimization (PSO) algorithms with Basis Pursuit (BP) as a reconstruction algorithm achieve more accurate results (i.e., reduction in MSE) with higher PSNR compared to the non-optimized Toeplitz sensing matrices with Basis Pursuit (BP). Furthermore, these matrices achieve significantly faster image reconstruction as well as faster construction of sensing matrices.

Further, it is noted that the proposed optimized Toeplitz sensing matrices based on Genetic Algorithm (GA), Simulated Annealing (SA), and Particle Swarm Optimization (PSO) algorithms with Orthogonal Matching Pursuit (OMP) as a reconstruction algorithm exhibits the lower values of PSNR and higher values of MSE compared to the non-optimized Toeplitz sensing matrices with BP. Generally, this behavior present due to the greedy approach of the OMP algorithm. Moreover, this is due to the fact that the reconstruction guarantees of OMP are weak and show non-uniform behavior.

On the contrary, the optimized Toeplitz sensing matrices with OMP achieved significantly faster image reconstruction as well as rapid sensing matrix construction performance as compared to the non-optimized Toeplitz sensing matrices.

Thus, in general, the optimized Toeplitz sensing matrices with Basis Pursuit (BP) achieve more accurate results with a robust and uniform reconstruction guarantee. However, they showed a slow signal reconstruction performance. On the other hand, the optimized Toeplitz sensing matrices with OMP show a fast reconstruction guarantee, but at the cost of the reduction in the PSNR.

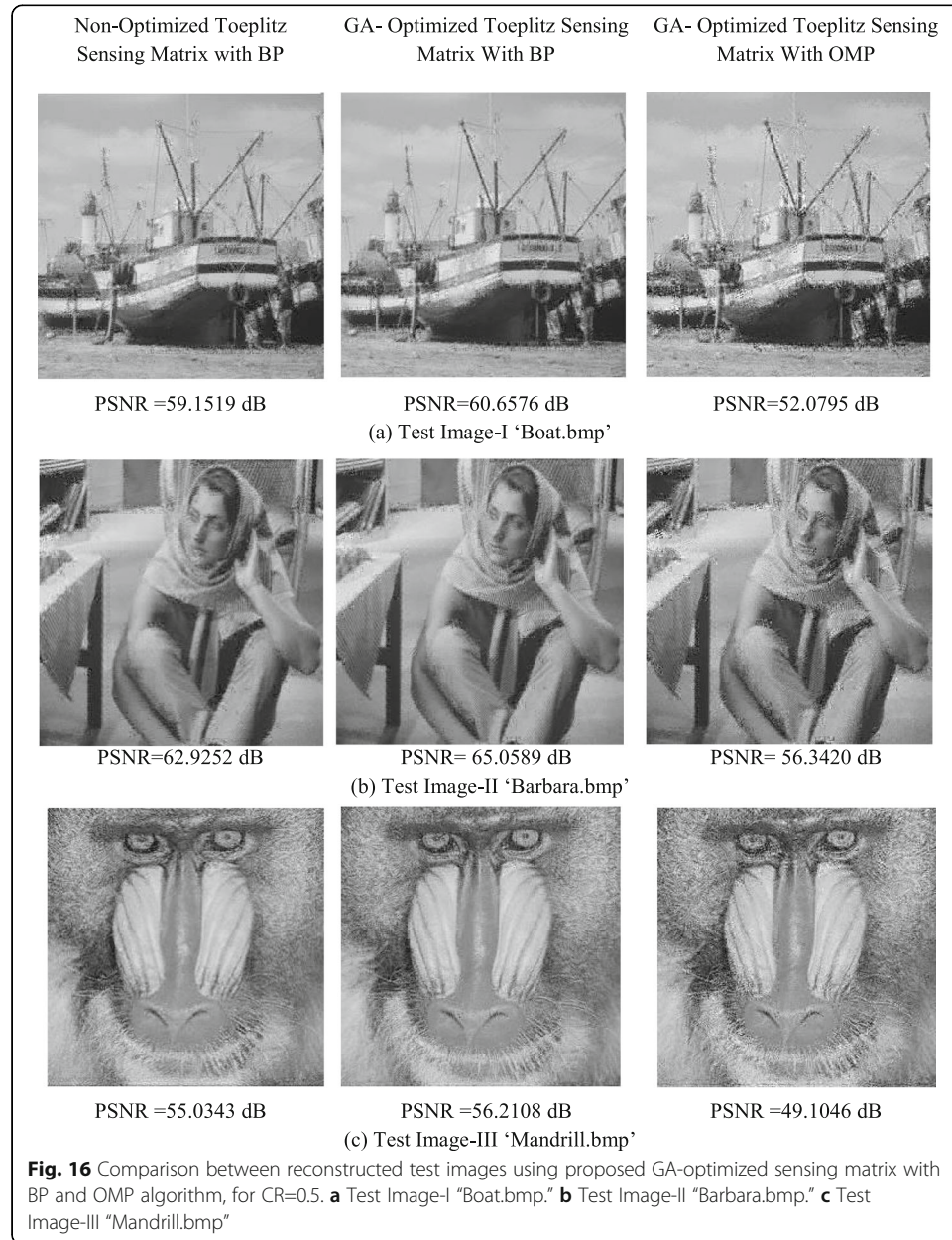
Finally, the result shows the successful implementation of the proposed optimized Toeplitz sensing matrices using evolutionary algorithms such as Genetic Algorithm (GA), Simulated Annealing (SA), and Particle Swarm Optimization (PSO) algorithms for compressed sensing. Furthermore, Toeplitz sensing matrices are easy for hardware realization because of its lower computational complexity compared to random Gaussian sensing matrices.

Table 3 Performance comparison of the proposed PSO-optimized sensing matrix with Basis Pursuit (BP) and OMP algorithms, for compression ratio CR (N/M) = 0.5

Method used	Proposed PSO-optimized sensing matrices with BP and OMP					
	Boat.bmp		Barbara.bmp		Mandrill.jpg	
Sensing methods and performance measures	Non-optimized Toeplitz with BP	PSO-optimized Toeplitz with OMP	Non-optimized Toeplitz with BP	PSO-optimized Toeplitz with BP	Non-optimized Toeplitz with BP	PSO-optimized Toeplitz with OMP
Mutual coherence	6.9924	6.6991	6.9924	5.7139	6.9924	6.6991
PSNR (dB)	59.1519	49.2790	62.9252	64.1856	55.0343	47.7248
MSE	175.4469	470.8930	120.3017	106.0561	264.8315	550.0716
Reconstruction time (seconds)	27.4589	3.8159	15.3074	23.8537	13.3598	4.1206
Sensing matrix construction time	0.0045	0.0012	0.0047	0.0015	0.0044	0.0014

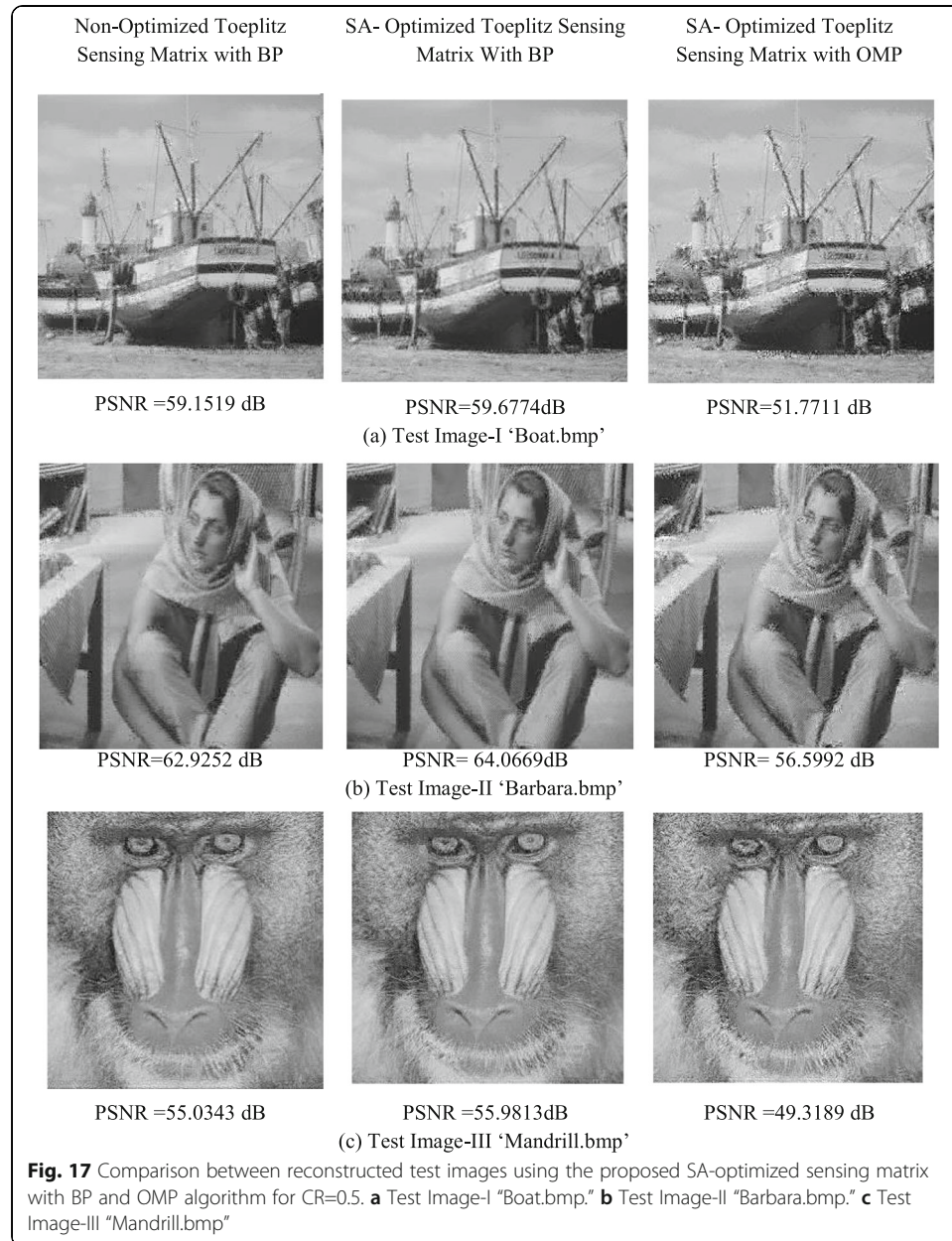
5 Appendix 1

5.1 Reconstructed test images using proposed GA-Optimized sensing matrix with BP and OMP algorithm



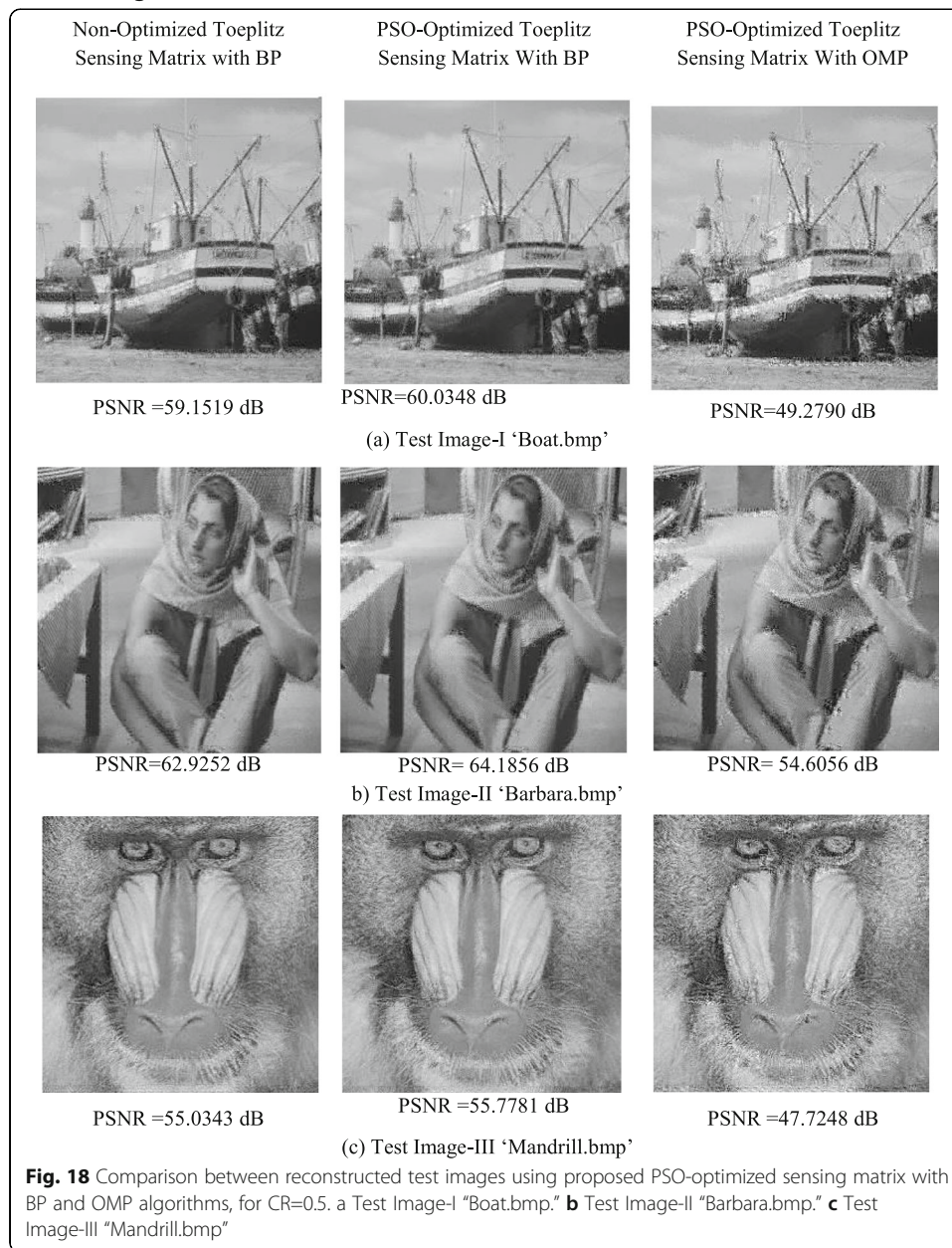
6 Appendix 2

6.1 Reconstructed test images using proposed SA-optimized sensing matrix with BP and OMP algorithm



7 Appendix 3

7.1 Reconstructed test images using proposed PSO-optimized sensing matrix with BP and OMP algorithm



Supplementary Information

The online version contains supplementary material available at <https://doi.org/10.1186/s13634-021-00743-5>.

Additional file 1.

Abbreviations

CS: Compressed sensing; GA: Genetic Algorithm; SA: Simulated Annealing; PSO: Particle Swarm Optimization; BP: Basis Pursuit; OMP: Orthogonal Matching Pursuit; RIP: Restricted isometry property; MRI: Magnetic resonance imaging; CT: Computerized tomography; PET: Positron emission tomography; SPECT: Single-photon emission computed tomography; SAR: Synthetic Aperture Radar; DCT: Discrete Cosine Transform; MSE: Mean square error; PSNR: Peak signal-to-noise ratio

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Authors' contributions

All authors contributed to the study conception and design. Material preparation, data collection, and analysis were performed by YVP and SLN. YVP wrote the software code, performed the experiments and data analysis, and wrote the first draft of the manuscript. SLN substantially revised the manuscript and contributed in additional revisions of the draft. All authors read and approved the final manuscript.

Authors' information

Yuvraj V. Parkale (M'15) received a B.E. degree in Electronics and Telecommunication Engineering from the University of Pune, Maharashtra, India in 2004 and M.Tech. Degree in Electronics and Telecommunication Engineering from Government College of Engineering, Pune (COEP), Maharashtra, India, in 2010. He is recently received a Ph.D. degree in Electronics and Telecommunication Engineering from Government College Dr. Babasaheb Ambedkar Technological University (DBATU), State Technical University, Lonere, Raigad, Maharashtra, India in 2019.

He is the author of more than 40 articles published in International/National Journals and conferences. He has delivered expert lectures on Compressed Sensing and several other topics in signal processing and embedded systems. Also, he has worked as a project consultant. He is also working as a reviewer for Springer, IEEE Access, and some other reputed journals and conferences. He has also worked as an Editorial board member for International and national journals and conferences.

His current research interests include compressed sensing (CS), optimization techniques and algorithms, signal processing, embedded systems, internet of things, and machine learning.

Dr. Yuvraj V. Parkale is a life member of the Indian Society for Technical Education (ISTE).

Sanjay L. Nalbalwar (M'2005-SM'2011) received a B.E. degree in computer science engineering (CSE) in 1990 and M.E. (Electronics) in 1996 from SGGS College of Engineering and Technology, Nanded, Maharashtra, India. He had received a Ph.D. from IIT Delhi in August 2008.

He has more than 30 years of teaching experience. He is working as a Professor and Head of the Electronics and Telecommunication Engineering Department at Dr. Babasaheb Ambedkar Technological University, State Technical University, Maharashtra State, India. He has additional administrative responsibilities such as Director, Innovation, Incubation & Entrepreneurship Cell, Chairman, Board of Information and Communication Technology, and TEQIP-III Co-ordinator of Dr. Babasaheb Ambedkar Technological University, State Technical University, Maharashtra State, India.

He has published over 300 papers in International/National Journals and Conferences. He is the author of 110 international journal articles, 80 national journals, 60 international conferences, and 50 national conferences. He has written four books namely, Basic Electronics, Signals & Systems, Electronics Devices & Circuits, and Digital Signal Processing.

He has guided 10 Research students and presently guiding 8 Research students. He has guided more than 100 PG projects and 300 UG projects in the field of signal processing. His area of interest includes Multirate Signal Processing and Wavelet, Stochastic Process Modeling. He is also working on various research projects in the area of biomedical signal processing, signal representation, signal matched wavelets, smart grid, solar passenger car, wireless sensor network. He has delivered several expert lectures in the field of signal processing and worked as a reviewer for different international journals and conferences.

Dr. Sanjay L. Nalbalwar has received the Best Teacher award for the year 2006–2007 of Dr. Babasaheb Ambedkar Technological University, Lonere. He is a Life Member of IETE, CSI, ISTE, IE, ISCEE, and IEEE society.

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Availability of data and materials

Authors can confirm that all relevant data are included in the article and/or its supplementary information files. Furthermore, please contact the corresponding author for simulation results and code.

Declarations

Ethics approval and consent to participate

The authors declare that they have no human participants, their data, or biological material used in this work.

Consent for publication

Informed consent was obtained from all authors included in the study.

Competing interests

The authors declare that they have no competing interests.

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