Yu *et al.*
EURASIP Journal on Advances in Signal Processing (2022) 2022:71
https://doi.org/10.1186/s13634-022-00902-2

EURASIP Journal on Advances in Signal Processing

RESEARCH **Open Access**

A signal enhancement method based on the reverberation statistical information

Ge Yu[1,2,3], Jiangjiang Sun[1,2,3] and Xiukun Li[1,2,3*]

*Correspondence:
Lixiukun@hrbeu.edu.cn

[1] Acoustic Science and Technology Laboratory, Harbin Engineering University, Harbin 150001, China
[2] Key Laboratory of Marine Information Acquisition and Security (Harbin Engineering University), Ministry of Industry and Information Technology, Harbin 150001, China
[3] College of Underwater Acoustic Engineering, Harbin Engineering University, Harbin 150001, China

Abstract

This paper proposes a reverberation suppression algorithm utilizing fractional lower-order moments based on statistical properties. As fractional lower-order moments can only be applied on symmetric α-stable random variables, the energy redistribution method is used, so reverberation signals can obtain the characteristic exponents of the symmetric α-stable distribution. To evaluate the proposed algorithm, an experiment involving simulated linear frequency modulation reverberation with the proposed method and comparison methods is performed and discussed. Moreover, an experiment is conducted using reverberation as measured from the lake. The results show that the proposed algorithm can achieve better reverberation suppression and signal enhancement performance compared with other methods.

Keywords: Signal enhancement, Reverberation statistical distribution information, Fractional lower-order moments, Fractional Fourier transform, Energy redistribution

1 Introduction

In shallow waters, reverberation caused by the scattering of transmitted signals during propagation is the main interference in underwater signal processing. Due to the reverberation generation principle, high correlations with target echoes greatly degrade some traditional methods, such as the matched filter. Obviously, it is easy to see that the signal with high signal-to-reverberation ratio (SRR) cannot be obtained by increasing the source level. Usually, reverberation suppression is achieved from two aspects: transmission signal design at the transmitter and signal processing at the receiver [1].

For the former, Hague [2] and Touati [3] designed waveform based on developing CW waveform to detect the target. Dosiy [4], Collin [5] and Guan [6] designed specific type of pulses such as wideband Doppler-sensitive pulses, phase-coded sequence and so on to suppress reverberation. Cox designed geometric comb waveforms and effectively suppressed reverberation in shallow water [7]. Mio improved the performance of low-frequency sonar in handling reverberation by utilizing waveform design and the broadband characteristics of signals [8]. As some designed transmitted pulses can only mitigate the effects of reverberation, signal processing methods should be proposed for suppression. As reverberation is nonstationary colored background noise, autoregression to whiten a signal is a common method for reverberation suppression. Kay proposed a prewhitener which is based on an autoregressive model to make the target more easily detected than current FFT processor [9]. Li gave a novel space time adaptive prewhitener for reverberation based on a two-dimensional autoregressive model to obtain better detection performance [10]. In recent years, nonnegative matrix factorization (NMF) theory has been widely used in reverberation suppression. Jia proposed a method that nonnegative matrix factorization is applied to express the time–frequency matrix as a low-rank matrix [11]. Lee [1] proposed an algorithm for the reverberation suppression of continuous wave signals using non-negative matrix factorization. As NMF-based reverberation suppression algorithms are only applicable to continuous wave reverberation, Kim [12] proposed two pre-processing methods, namely dechirping transformation and modulo operation, to facilitate application of the NMF method to LFM reverberation. Array processing as an effective method to improve the signal-to-noise ratio is also used for reverberation suppression. Zhu proposed an approach of sparse spatial spectral estimation based on singular value decomposition [13]. Xing proposed a direct data domain localized domain joint algorithm that can effectively suppress reverberation by combining the advantages of localized joint adaptive and space–time processing [14]. Li combined image morphology and time–frequency blind separation algorithm to separate the target echo from reverberation and derived the expression of reverberation in the WVD time–frequency domain [15]. High-order cumulates are also widely used to enhance underwater acoustic signals. However, when characteristic index α-value of considering signal is between 0 and 2, the second-order and higher-order statistics of considering signal do not exist anymore. Therefore, the traditional signal processing methods based on second-order statistics (such as power spectrum) will lead to performance degradation or even failure when processing signals are subject to alpha stable distribution.

The classical Gaussian model is generally used to explain the reverberation process when the scatterer distribution approximately satisfies the central limit theorem (CLT) [16]. However, in special marine environments when the scatterer distribution cannot meet the CLT, the reverberation amplitude distribution deviates from Gaussian. As the tail widens in the statistical distribution of seabed reverberation and there are strong non-Gaussian characteristics for the amplitude fluctuations of certain impact characteristics caused by the strong instability of shallow water reverberation, the α-distribution is used to build reverberation in shallow waters. However, considering the nonzero skewness of reverberation statistics that result in its non-compliance with SαS, in this paper, an energy redistribution method based on adaptive fractional Fourier transform (FrFT) is proposed to make signals to satisfy SαS distribution. Consequently, the fractional lower-order moment (FLOM), which is an effective tool to analyze the symmetric α-stable (SαS)-stable distribution signals, is applied to enhance target echoes in reverberation environments.

This paper is organized as follows. Section 2 provides a short description on the reverberation of α-stable model and presents the FLOM theory and the proposed energy redistribution in fractional domain. Section 3 presents the simulation and experimental results of the proposed method in shallow waters. The conclusions follow in Sect. 4.

2 Methods

2.1 Reverberation model which followed α-stable distribution

The reverberation probability density function has the same characteristics of a single peak, bell shape and thick tail as the SαS distribution, which is the most widely used and representative special distribution of the α-stable distribution. Therefore, the SαS distribution can be used to describe the statistical characteristics of underwater acoustic signal noise under a reverberation background [17].

In 1925, Levy proposed α-stable distribution which is also called non-Gaussian stable distribution based on generalized central limit theorem. Except for a few special cases, there is no unified and closed analytical expression for the probability density function of the distribution; characteristic function is generally used to describe its distribution characteristics. The random variable obeys α-distribution law, if and only if the characteristic function is

$$\varphi(t) = \exp\left\{\mathrm{i}\delta t - |\gamma t|^{\alpha}[1 + \mathrm{i}\beta \mathrm{sgn}(t)\omega(t,\alpha)]\right\} \tag{1}$$

$$\omega(t,\alpha) = \begin{cases} \tan\left(\frac{\pi\alpha}{2}\right), & \alpha \neq 1 \\ \frac{2}{\pi}\log|t|, & \alpha = 1 \end{cases} \tag{2}$$

$$\mathrm{sgn}(t) = \begin{cases} 1, & t > 0 \\ 0, & t = 0 \\ -1, & t < 0 \end{cases} \tag{3}$$

where $0 < \alpha \leq 2, -1 \leq \beta \leq 1, \gamma > 0, -\infty < \delta < +\infty$, i is the imaginary unit, and $\mathrm{sgn}(t)$ is a symbolic function.

It is seen that the α-stable distribution is uniquely determined from four characteristic exponents. α is the characteristic index that determines the trailing thickness of the probability density function. Distinct from the Gaussian distribution, the α-stable distribution decays algebraically, and the attenuation velocity is related to α. The β parameter determines the degree of distribution symmetry. When $\beta = 0$, the distribution is also called an SαS. ϒ is the scale parameter that measures the dispersion of the distribution. The distribution is regarded as Gaussian if $S_2(\Upsilon, 1, \delta) = N(\Upsilon, 1, \delta)$, Cauchy if $S_1(\Upsilon, 0, \delta)$, and Levy if $S_{0,5}(\Upsilon, 1, \delta)$.

The time-domain expression of active sonar emission waveform is $s(t)$. Assuming the number of discrete scatterer elements satisfying the i.i.d. condition, reverberation model was built by using the element scattering model based on the generating process of the seafloor reverberation. It divided the scattering element by the scattering coefficient. Therefore, the signal received at the receiver can be expressed as

$$r(t) = a_0 s(t - \tau_0) + \sum_{i=1}^{N} a_i g_i ss(t - \tau_i) + n_w(t) \tag{4}$$

In Eq. (4), the first term on the right of the equation represents the received target signal, where a_0 is the attenuation coefficient of the target signal which is related to the propagation loss of the acoustic signal of the excitation target in the back-and-forth sound path and the absorption loss of the medium to the sound energy. τ_0 is the time delay of target echo signal. In this paper, the Doppler effect caused by target movement is not discussed. The second term represents the reverberation. $ss(t-\tau_i)$ is the waveform expression of sound wave scattered by the ith scatterer element. a_i is the attenuation coefficient of the scattering signal of the ith scatterer element which is related to the propagation loss and absorption loss before and after the scattering of the sound wave; g_i represents the scattering intensity coefficient of the ith scatterer element which is related to the variation coefficient of scattering intensity caused by different scattering coefficient and incident grazing angle when acoustic wave enters the ith scatterer element. The third term represents marine environmental noise.

2.2 Fractional lower-order moments for signal enhancement in reverberation environment

In this part, the basic theory of FLOM is introduced firstly. As it can only be used to process the signal which satisfies SαS distribution [18], an energy redistribution method which is applied to redistribute the received signal energy in fractional domain is proposed in detail. Then, the target echo broadening and compression in fractional domain is analyzed.

2.2.1 Basic knowledge of fractional lower-order moments

Assume random variable $x \sim S\alpha S$, $0<\alpha\leq 2$, the fractional-order moment of the SαS random variable $x \sim S_\alpha(\beta, \Upsilon, 0)$ with zero location parameter ($\delta=0$) is given by

$$\mathrm{E}\left[|X|^p\right]=\frac{2^{p+1}\Gamma\left(\frac{p+1}{2}\right)\Gamma\left(-\frac{p}{\alpha}\right)}{\alpha\sqrt{\pi}\,\Gamma\left(-\frac{p}{2}\right)}\Upsilon^{\alpha/p},\quad p<\alpha \tag{5}$$

where $\Gamma(\bullet)$ is the gamma function,

$$\Gamma(x)=\int_0^\infty t^{x-1}e^{-t}\mathrm{d}t \tag{6}$$

If the random variables X and Y obey SαS distribution, the range of α is from 1 to 2. The fractional lower-order correlation (FLOC) can be expressed as

$$R_{XY}^p=E\left[XY^{<p-1>}\right],\quad 1\leq p<\alpha \tag{7}$$

Based on the Wigner–Ville distribution (WVD) and FLOM theories, if the random variables obey SαS distribution, the fractional lower order of WVD (FLOM-WVD) expression can be written as

$$WVD_{FLOS}(t,f)=\int_{-\infty}^{\infty} x^{\langle P\rangle}(t+\tau/2)x^{-\langle P\rangle}(t-\tau/2)e^{-j2\pi f\tau}d\tau \tag{8}$$

where $x^{\langle P\rangle}=|x|^p\mathrm{sign}(x), p<\alpha/2$.

2.2.2 Received signal redistributed in fractional domain

Considering the size of sonar arrays, as the bandwidth of transmitted acoustic signals increases, the sonar resolution unit decreases, and the number of effective scatterers in the resolution unit cell decreases. Thus, the received signal does not well satisfy an SαS random variable. However, the fractional lower order can only be used on received signals that are SαS random variables. To apply FLOM to suppress reverberations that have any characteristic exponents, an energy redistribution method based on the FrFT is proposed to redistribute the received signal to meet the required characteristic exponent.

The FrFT is a type of linear integral transformation that is performed on LFM signals and used to transform signals from one domain to another. This is known as the FrFT domain and is represented by u. Then, the pth-order FrFT on the function is denoted as $F^p(u)$ and defined as

$$F^p(u) = \int_{-\infty}^{+\infty} \underset{p}{\tilde{K}}(u,t)s(t)\mathrm{d}t \tag{9}$$

where $\underset{p}{\tilde{K}}(u,t) = \sqrt{1 - i\cot\left(\frac{p\pi}{2}\right)}\exp\left[i\pi\left(u^2\cot\left(\frac{p\pi}{2}\right) - 2u\csc\left(\frac{p\pi}{2}\right) + t^2\cot\left(\frac{p\pi}{2}\right)\right)\right]$, $p \neq 2n$, $n \in Z$. With some simplification, the above equation can be written as

$$F^p(u) = \begin{cases} B_p \int_{-\infty}^{+\infty} \exp\left[i\left(\frac{t^2+u^2}{2}\cot\left(\frac{p\pi}{2}\right) - \frac{tu}{\sin(p\pi/2)}\right)\right]s(t)\mathrm{d}t, & p \neq 4n \\ s(t), & p = 4n \\ s(-t), & p \neq 2(2n+1) \end{cases} \tag{10}$$

where $B_p = \sqrt{\frac{1-\cot(p\pi/2)}{2\pi}}$, and $s(t)$ in Eq. (10) is expressed as

$$s(t) = \exp\left[i\pi\left(2f_0t + \mu t^2\right)\right], -\frac{T}{2} < t < \frac{T}{2} \tag{11}$$

where f_0, μ and T are the initial frequency, frequency modulation rate and duration. In particular, when $p = 1$, the FrFT on $s(t)$ can be regarded as Fourier transform.

When we applied (10) to Eq. (4), the received signal in fractional domain with transform p will be

$$F^p(u) = \int_{-\infty}^{+\infty} \underset{p}{\tilde{K}}(u,t)s(t)\mathrm{d}t = \int_{-\infty}^{+\infty} \underset{p}{\tilde{K}}(u,t)\left[a_0s(t-\tau_0) + \sum_{i=1}^{N} a_ig_iss(t-\tau_i) + n_w(t)\right]\mathrm{d}t = S(u_p) + SS(u_p) + N(u_p) \tag{12}$$

where

$$S(u_p) = \int_{-\infty}^{+\infty} \underset{p}{\tilde{K}}(u,t)a_0s(t-\tau_0)\mathrm{d}t \tag{13}$$

$$SS(u_p) = \int_{-\infty}^{+\infty} \underset{p}{\tilde{K}}(u,t)\sum_{i=1}^{N} a_ig_iss(t-\tau_i)\mathrm{d}t \tag{14}$$

$$N(u_p) = \int_{-\infty}^{+\infty} \underset{p}{\tilde{K}}(u,t)\sum_{i=1}^{N} n_w(t)\mathrm{d}t \tag{15}$$

$S(u_p)$, $SS(u_p)$ and $N(u_p)$ are the received target signal, reverberation and marine environmental noise in fractional domain, respectively.

Table 1 Characteristic exponents of the redistributed signal with different transform p values

P value	−0.54	−0.5	−0.02	0.02	0.12	0.22	0.3	0.36	0.4
α	1.503	1.349	1.251	1.163	0.952	0.917	0.893	0.969	1.104
β	0.093	0.10	0.032	0.003	−0.03	−0.132	0.029	0.120	0.056
γ	0.47	0.43	0.112	0.11	0.18	0.22	0.24	0.28	0.34
δ	0.01	0.05	0.01	0	0.05	0.255	0.037	0.741	0.146

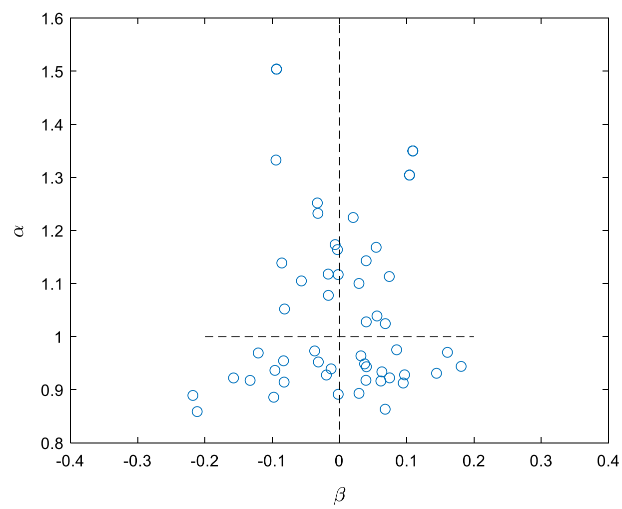


Fig. 1 α and β values of redistributed signal with various transform *p* values

Here, we give a reverberation model example that obeys the α-distribution (1.2176, 0.6, 0.0420, 0.0004). The characteristic exponents of the redistributed signal are shown in Table 1 with different transform *p* values.

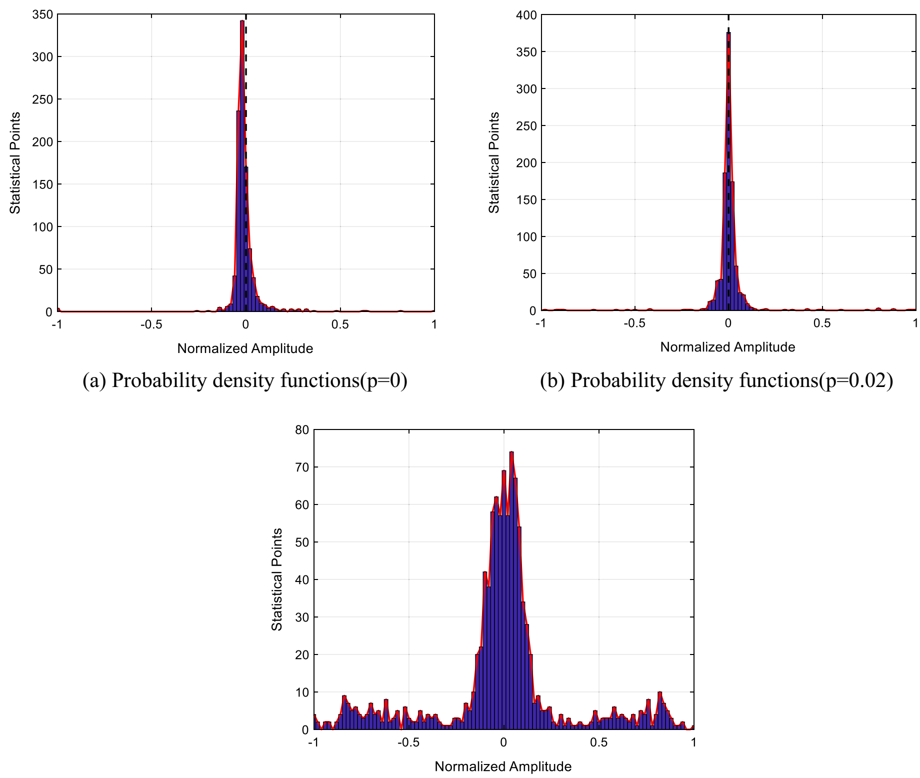
The probability density curves with different transform p values (0, 0.02 and 0.22) are shown in Fig. 1. With the FrFT of different p values, the redistributed signal has different characteristic exponents.

Consider the two cases of $p=0$, $p=0.22$ and $p=0.02$; Fig. 2a–c, respectively, shows the probability density functions. For $p=0.22$, there is a thicker and less smooth tail than for $p=0.02$. Furthermore, $\alpha=1.163$ and $\beta=0.003$ for the redistributed signal ($p=0.02$) are SαS random variables. More specifically, β is closest to 0 for α between 0 and 1 and is the optimal p selection principle, which is completely different from the traditional FrFT. Here, we give the influence of the FrFT on the target echo.

2.2.3 The target echo broadening and compression in fractional domain

In this part, we discuss the influence of the redistributed signal method on the target echo. Figure 3 shows the transformation relationship between the time domain and the fractional domain of the target echo.

According to Fig. 3, there are some changes in the bandwidth of the received signal in the fractional domain due to the FrFT. We assume that the frequency bandwidth of the transmitted signal is B, the duration of the transmitted signal is T and the observation duration



(a) Probability density functions(p=0)

(b) Probability density functions(p=0.02)

(c) Probability density functions(p=0.22)

Fig. 2 Probability density distribution of three cases

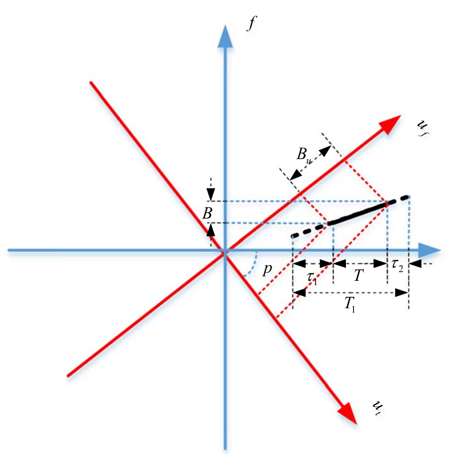


Fig. 3 Corresponding relation between echo signal fractional domain

is T_1. The transform angle of FrFT is p. With the geometric relation, the bandwidth in the fractional domain can be written as

$$B_u = (B - \chi T) \sin (p\pi/2) \tag{16}$$

where $\chi = -\frac{1}{\tan(2p/\pi)}$ is related to the chirp rate of the transmitted signal in the fractional domain. The target echo broadening or compression in the fractional domain is related to the observation signal length T and the transform angle p. In addition, according to Fig. 3, the information and energy of the echo signal are not changed in the fractional domain.

3 Results and discussion

3.1 The performance of the proposed method in shallow water environment

This section presents the simulation results for the proposed method to improve detection and compares it with other methods. The performance of the target echo enhancement in the reverberation environment is investigated using the peak-signal-to-reverberation ratio (PSRR) under different conditions. The PSRR is defined by

$$PSRR = 20\log 10\left(\frac{M_{\text{sig}}}{M_{\text{rbn}}}\right) \tag{17}$$

where M_{sig} is the signal peak value and M_{rbn} is the reverberation peak value.

Figure 4 shows the results by the proposed method and matched filter in reverberation environment (SRR = 10 dB), respectively. In Fig. 4a, it is hard to observe the target component that is located at the true echo time delay (1 s). However, this component is easy to locate the target in Fig. 4b. According to Fig. 4, the proposed algorithm effectively removes reverberation, confirming that the target is clearly detected. It illustrates that the proposed algorithm suppresses the reverberation component effectively in server reverberation environments.

Figure 5 shows the SRR value of the proposed approach along with the matched filter and FLOC. The proposed algorithm (red solid line) achieves a PSRR gain ($\beta = 0$) of 6 to 12 dB under an input SRR from -24 to -9 dB. Compared with the other two methods (blue and black solid lines), the proposed method attains the best performance. Consider the case of $\beta = 0.4$, the performance of the proposed method (red dotted line) is similar to the result of $\beta = 0$ (red solid line), which suggests that the proposed method could force the received signal to obey the SαS distribution when SRR is above -12 dB.

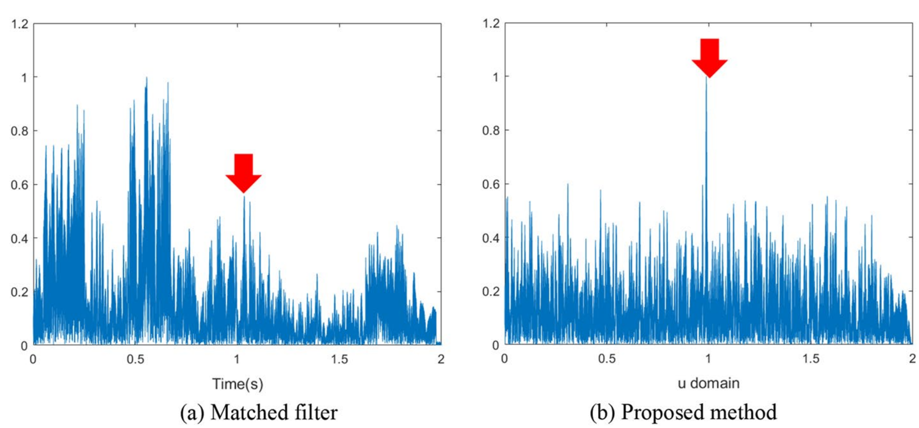


Fig. 4 Processing results by proposed method and comparison methods

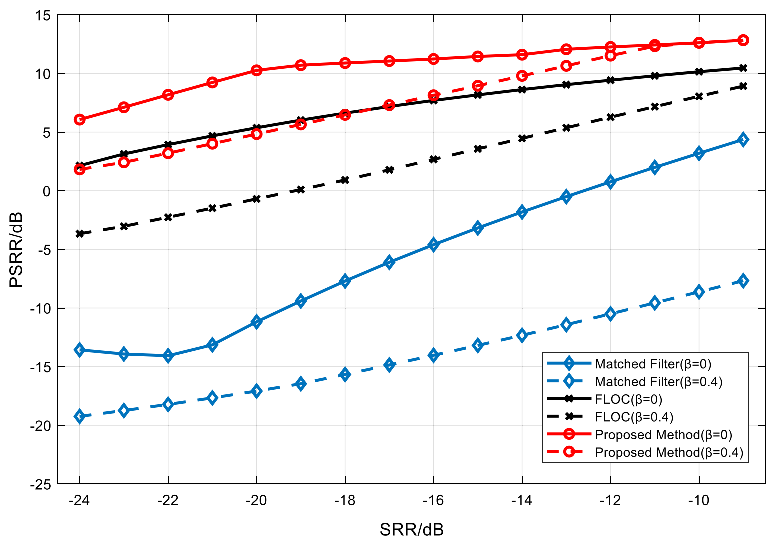


Fig. 5 Enhancement performance of the proposed method and comparison with various approaches

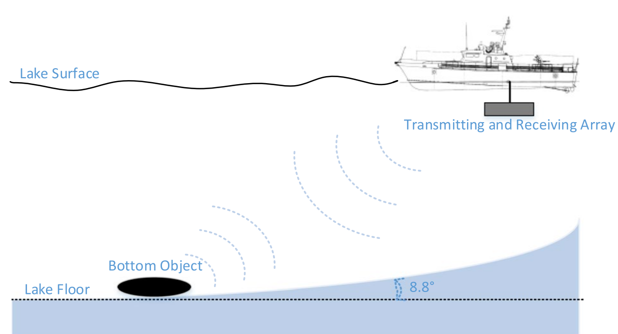


Fig. 6 Schematic diagram of lake experiment

In addition, as the figure shows, the performance of matched filter (blue dashed line) and FLOC (black dashed line) descended more or less when $\beta = 0.4$ (Fig. 5).

3.2 Experimental results and discussion

To further verify the method, lake trial data from a suspended object echo signal are processed. The slope of the lake bottom is approximately 8.8° as Fig. 6 shows. A monostatic sonar is used to transmit a wideband linear frequency modulated (LFM) signal. The frequency range is from 25 to 50 kHz, and the duration is 2 ms. The receiving system is an 18-element uniform linear array placed 1 m from the lake surface.

The probability density characteristics of the instantaneous values are analyzed by intercepting the data over different observation times. We consider the time durations T of 60, 100 and 160 ms. According to [19], the α-stable distribution (α, β, γ, μ) parameter estimation of observation duration could be calculated.

According to Fig. 7a, the probability density curve of the considered time duration ($T = 60$ ms) roughly fits the ideal α-distribution ($\alpha = 1.8093$, $\beta = 0.1010$ and $\gamma = 0.1977$). The fitting performance of Fig. 7b is slightly better than Fig. 7a. Particularly, compared with Fig. 7a, b, there is less fluctuation at the tail of probability density curve in Fig. 7c.

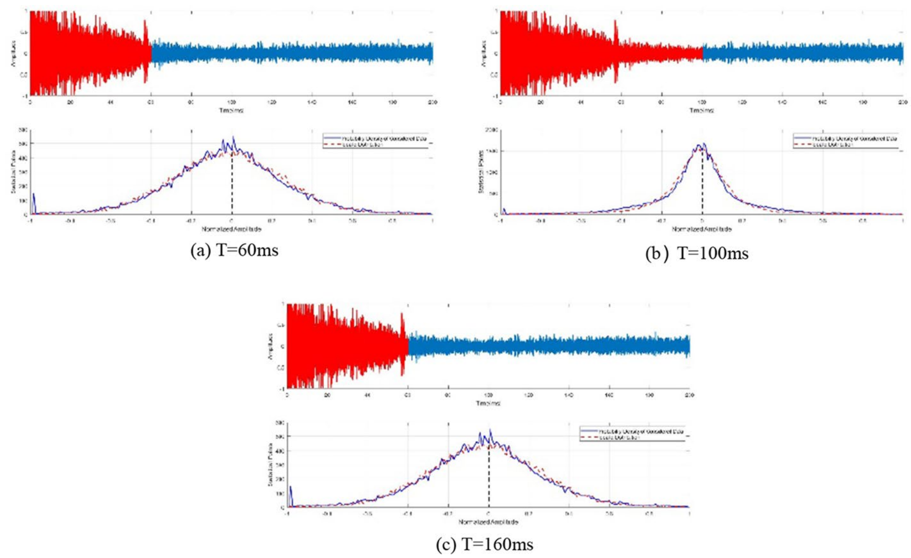


Fig. 7 Time-domain waveform and probability density curve of reverberation in different observation duration

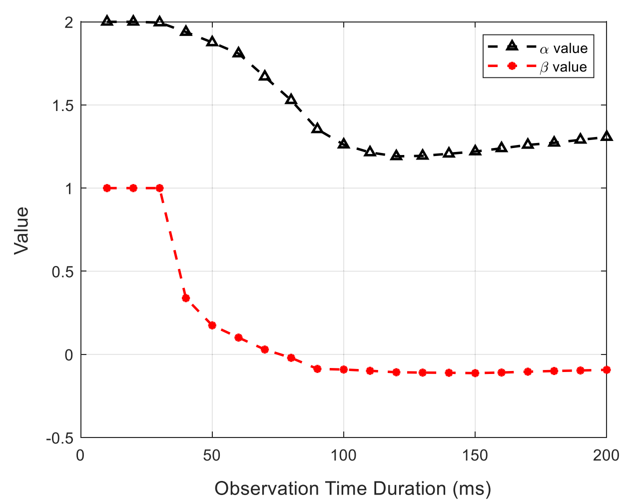


Fig. 8 α and β values with different observation time duration

Therefore, with time duration increasing, probability density curve of reverberation has a better agreement with ideal SαS distribution.

Figure 8 shows the relationship between parameters (α and β) of α-distribution and observation duration. It is easy to observe that α and β tend to be stable with the increase in observation duration. In particular, β gradually closes to zero.

According to Fig. 9b, c, a better fit between the ideal SαS distribution and the experimental data is obtained from the proposed method. As the signal is redistributed in the fractional domain, the probability density curve of the processed signal is much smoother than the received signal.

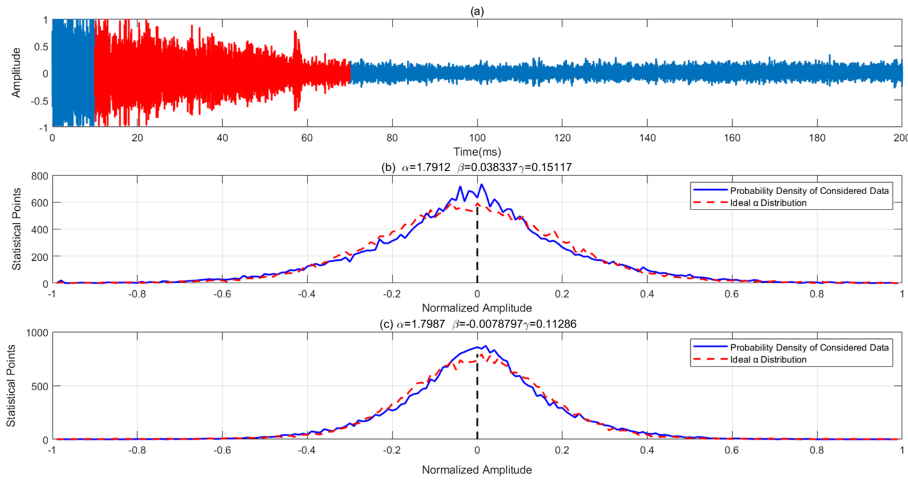


Fig. 9 Probability density curves of time-domain waveform and fractional domain with the optimal transform angle

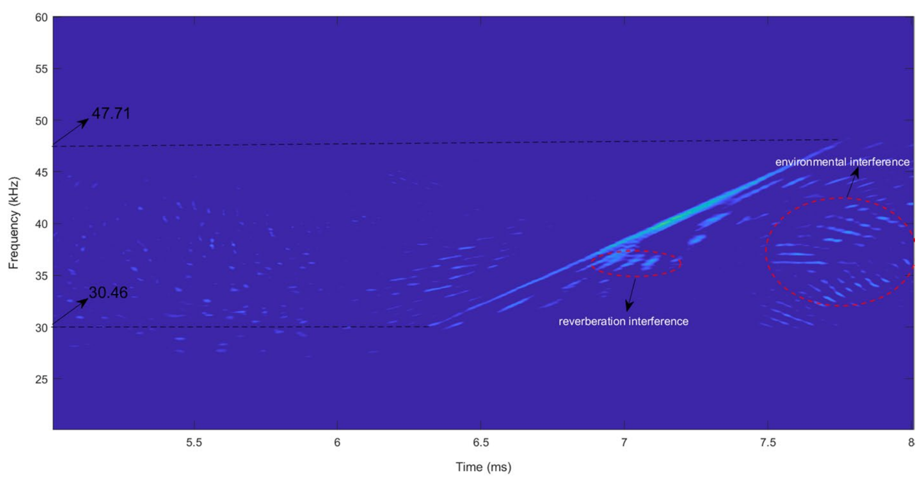


Fig. 10 Time–frequency distribution of observed reverberation signal with traditional WVD

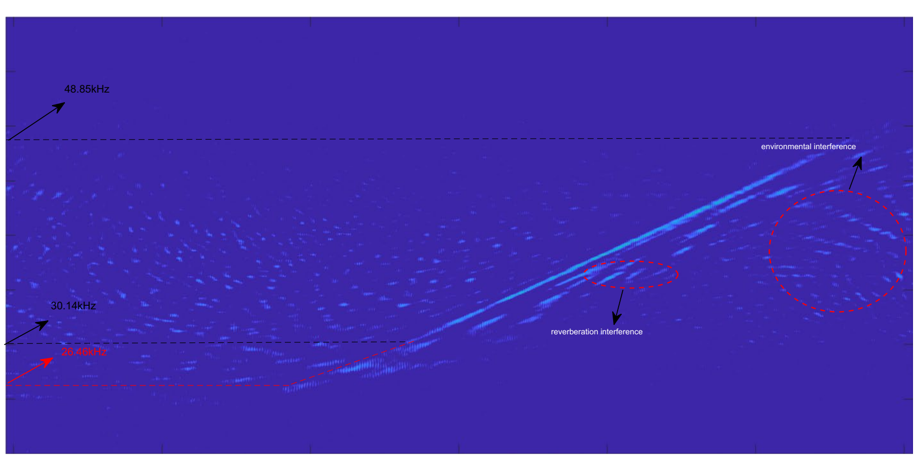


Fig. 11 Time–frequency distribution of observed reverberation signal with proposed method

The observed bandwidth in Fig. 10 is from 30.46 to 47.71 kHz. As the bandwidth of the transmitted signal is 25 kHz, only 69% of the bandwidth is reflected in Fig. 10. The results from the proposed method are shown in Fig. 11, indicating that the reverberation and environment interference suppression performance is much better than the traditional WVD method (marked in the ellipse). In addition, the observed bandwidth is up to 89%, which is a significant improvement. The target echo enhancement performance (marked in the ellipse) is much better than in Fig. 10.

4 Conclusions

Reverberation suppression is an essential function in active sonar systems. A suppression method is proposed using FLOM theory under an α-stable distribution reverberation. Considering the limitations of FLOM theory, the energy redistribution method is applied to force the reverberation signal to obey an SαS distribution. By applying the proposed preprocessing method to the received signal, the FLOM-WVD method achieves a better performance than the traditional WVD method. The simulation and experimental results show that the proposed algorithm can effectively suppress underwater reverberations and improve the SRR of underwater target echoes.

Abbreviations

FrFT	Fractional Fourier transform
FLOM	Fractional lower-order moments
FLOC	Fractional lower-order correlation
WVD	Wigner–Ville distribution
SαS	Symmetric α-stable

Acknowledgements
Not applicable.

Author contributions
GY proposed the framework of the whole algorithm; JJ performed the simulations and analysis of the results. XKL and GY have participated in the design of this research. All authors read and approved the final manuscript.

Funding
This work was supported by the State Key Laboratory of Precision Measuring Technology and Instruments (Tianjin University) (pilab 1903) and the stable supporting fund of the Acoustics Science and Technology Laboratory (JCKYS2021604SSJS006).

Availability of data and materials
Please contact author for data requests.

Declarations

Ethics approval and consent to participate
Not applicable.

Consent for publication
The picture materials quoted in this article have no copyright requirements, and the source has been indicated.

Competing interests
The authors declare that they have no competing interests.

Received: 5 April 2022 Accepted: 15 August 2022
Published online: 26 August 2022

References

1. S. Lee, J.S. Lim, "Reverberation suppression using non-negative matrix factorization to detect low-Doppler target with continuous wave active sonar," EURASIP J. Adv. Sig. Pr. **1** (2019)
2. D.A. Hague, J.R. Buck, An experimental evaluation of the generalized sinusoidal frequency modulated waveform for active sonar systems. J. Acoust. Soc. Am. **145**, 3741–3755 (2019)
3. N. Touati, C. Tatkeu, T. Chonavel, A. Rivenq, Design and performances evaluation of new Costas-based radar waveforms with pulse coding diversity. IET Radar Sonr. Nav. **10**, 877–891 (2016)
4. Y. Doisy, L. Deruaz, S.V. Ijsselmuide, S.P. Beerens, R. Been, Reverberation suppression using wideband doppler-sensitive pulses. IEEE J. Oceanic Eng. **33**, 419–433 (2008)
5. T. Collins, P. Atkins, Doppler-sensitive active sonar pulse designs for reverberation processing. IEE Proc-Radar Son. Nav. **145**, 347–353 (1999)
6. C. Guan, Z. Zhou, X. Zeng, Phase-coded sequence design method for active sonar. Sensors **17**, 4659 (2020)
7. H. Cox, H. Lai, Geometric comb waveforms for reverberation suppression. in Proceedings of 1994 28th Asilomar Conference on Signals, Systems and Computers, 2, IEEE, Pacific Grove, USA, 1994, pp. 1185–1189
8. K. Mio, Y. Chocheyras, Y. Doisy, Space time adaptive processing for low frequency sonar, in OCEANS 2000 MTS/IEEE Conference and Exhibition, 2, IEEE, Providence, USA, 2000, pp. 1315–1319
9. S. Kay, Improved active sonar detection using autoregressive prewhiteners. J. Acoust. Soc. Am. **80**, 1603–1611 (1990)
10. W. Li, X. Ma, Y. Zhu, J. Yang, C. Hou, "Detection in reverberation using space time adaptive prewhiteners," J. Acoust. Soc. Am. **124**(4), EL236 (2008).
11. H. Jia, X. Li, Underwater reverberation suppression based on non-negative matrix factorisation. J. Sound Vib. **506**(1), 116166 (2021)
12. G. Kim, K. Lee, S. Lee, Linear frequency modulated reverberation suppression using non-negative matrix factorization methods, dechirping transformation and modulo operation. IEEE Access **99**, 110720–110737 (2020)
13. Y. Zhu, K. Yang, R. Duan et al., Sparse spatial spectral estimation with heavy sea bottom reverberation in the fractional fourier domain. Appl. Acoust. **160**, 107132 (2020)
14. .X. Xing, Z.M. Cai, Ocean reverberation suppressing by direct data domain based stap, in 2012 IEEE 11th International Conference on Signal Processing, 3, IEEE, Beijing, China, 2012, pp. 2085–2088
15. X. Li, Y. Yang, X.X. Meng, Morphological characteristics separation of underwater target echo and reverberation in time and frequency domain. Acta Acust. **42**(2), 169–177 (2017)
16. R.J. Urick, "Principles of underwater sound," 3rd ed, McGraw-Hill Book Company, (1983).
17. M. Shao, C.L. Nikias, Signal processing with fractional lower order moments: stable processes and their applications. Proc. IEEE **81**, 986–1010 (1993)
18. V.A. Aalo, A. Ackie, C. Mukasa, Performance analysis of spectrum sensing schemes based on fractional lower order moments for cognitive radios in symmetric α-stable noise environments. Signal Process. **154**, 363–374 (2019)
19. I.A. Koutrouvelis, An iterative procedure for the estimation of the parameters of stable laws. Commun. Stat. Simul. Comput. **10**(1), 17–28 (2007)

Publisher's Note

Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.