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Adaptable waveform design for radar and jammer for multi-target using game theoretic strategies

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Abstract

In the environment of electronic warfare, the countermeasure between radar and jammer has become a hot issue. Finding the solution to optimal waveforms between them is very great significance. Aiming at the problem of multi-target detection in clutter environment, the authors propose the waveform optimization method based on signal-to-interference-plus-noise ratio (SINR) criterion for radar and jammer, respectively. When radar is the dominant player, the maxmin strategy is used to establish the optimization model to optimize the radar transmission waveform. When jammer is the dominant player, the minmax strategy is used to establish the optimization model to optimize the jammer waveform. When dealing with random multi-target, the authors also propose an adaptive weight method to solve the problem of summation of weighted multiple targets. Then, an improved method combined with Jensen's inequality for radar and jammer is proposed, respectively, which could improve the detection performance of the radar system and reduce the computational complexity of maximum SINR-based method. The simulation results show that the proposed maximum SINR-based methods could adaptively allocate the transmission energy to improve the performance of the radar. The energy allocation of radar and jammer is slightly different under maxmin strategy and minmax strategy. The proposed improved methods could output the higher SINR than the maximum SINR-based methods. Both of the proposed methods could improve the performance of radar detecting targets.

Keywords: Electronic warfare, Multi-target, Waveform optimization, SINR

1 Introduction

Cognitive radar (CR) is a new concept for the operation of radar systems. It can adaptively adjust the radar transmission waveform according to the prior information such as environment, which greatly improves the performance of radar system [1–3]. Bell [4] first proposed a water-filling method combining with information theory, which provides a new idea for waveform optimization. Two waveform design problems of deterministic target impulse response and random target impulse response were proposed and solved. Based on the water-filling method, matched signal design in the presence of clutter and noise has been treated in [5, 6], respectively. Moreover, they also summarized the main theoretical content of matched waveform design and derived the optimal

waveforms based on SINR criterion and mutual information (MI) criterion in detail. In [7–9], the optimal waveforms of orthogonal frequency division multiplexing (OFDM) signal were designed according to the MI criterion for random extended target. The solution to waveforms was found by an iterative algorithm, and a good target recognition effect was obtained. The proposed adaptive structure uses the reflected interference to design OFDM signal. Improved Kalman particle filtering algorithms were proposed in [10, 11] for target tracking and target estimation. The proposed Kalman filter-based method could optimize the transmission waveform directly and improve the radar estimation performance. The improved algorithm was proposed on the basis of the target feature expression model and the target probability model. As for the angular uncertainty for range-extended targets, [12] introduced the static multi-model approach and the exponential correlation model and then derived the optimization waveform based on MI, which improve the performance of target recognition.

For the target detection, a joint optimization approach to transmission waveform and receiver filter for cognitive radar was proposed in [13]. The signal processing model in [13] is modeled in Fourier domain rather than in the time domain, which greatly decreases the computing complexity. In addition, Xiong Naixue [14–16] has studied the problems of target location. A diffusion Gauss–Newton (GN) algorithm for range-based target localization was proposed in [14], and it balanced the unbalance noise distribution over the wireless sensor networks. As for the treatment of noise, [15] was embedded with a noise evidence filter in fuzzy probability Bayesian network, and the filter reduces the impact caused by system faults. In automatic identification system (AIS) network [16], an improved density-based spatial clustering of applications with noise algorithm was proposed, and the proposed algorithm has high accuracy and a good clustering performance for trajectory data mining. In addition, as for moving target detection [17, 18], Anthony Martone proposed a spectrum sharing technology to waveform design for target detection. Satyabrata Sen proposed a space-time adaptive processing (STAP) algorithm using an OFDM signal, and thus, the signal increases the frequency diversity of the system and improves the performance of target detection. In [19], the authors presented the algorithm to optimize the radar transmission waveform using the detection probability as the performance parameter. It solved the objective function in combination with Jensen's inequality. Aiming at the problem of hypothesis testing in radar detecting target, Danilo Orlando [20–23] designed several detectors for radar system. The generalized likelihood ratio test (GLRT) detector could evaluate the direction of arrival of target, and the derived finite-sum expression based on GLRT could realize the performance evaluation of radar system. Adaptive GLRT detector solved the problem of anomaly detection in hyperspectral imagery. For the problem of hypothesis testing under multiple alternative hypotheses in radar system, [21] proposed a decision framework relying on the Kullback–Leibler information criterion, which provided a theoretical solution for GLRT dealing with multiple alternative hypotheses. On the basis of [21, 22] analyzed the practical problems of real aperture radar and synthetic aperture radar, such as radar detecting range-spread targets with unknown location information and multiple point-like targets with unknown number and so on. [24–26] combined expectation–maximization algorithm with cyclic estimation procedures to deal with the problem of clutter return clustering. Besides, the expectation–maximization algorithm realized target detection in

heterogeneous Gaussian environments and reduced the energy loss in the detection process, and the cyclic estimation procedures solved the problem of radar system detecting noise-like jammers (NLJ). Different from the previous monostatic radar system, Kay [27] designed the optimal transmission signal for multi-static radar with the derived Neyman–Pearson criterion, and the author proposed the maximum marginal allocation algorithm which is guaranteed to maximize the divergence and is easy to implement. In the condition of uncertainty on the knowledge of the noise or clutter environment, Rossetti et al. [28–30] proposed robust waveform design approaches for multi-static cognitive radars, and the proposed approaches could achieve the desired performance of radar system. In the actual environment, we need to consider the uncertainties of some parameters, because the characteristics of parameters cannot be accurately evaluated by radar system. Based on the consideration above, Akcakaya et al. [31–33] proposed proper methods for target detection in uncertain environments. The proposed data-driven method could adaptively update the detection algorithm with the changed environment and could improve the performance of detection. The proposed robust constant false alarm rate (CFAR) method could be applied to different clutter conditions such as sea clutter. In view of the error in the estimation of the target impulse response in the actual environment, [34] designed the robust transmission waveform and receiving filter banks in the deterministic model and random model.

The researches on waveform optimization above do not take into account the existence of jammer. The influence of jammer is difficult to be eliminated in the actual battlefield environment. Besides, all the above studies are based on the single target assumption; however, the multi-target situation is also common in practice. Based on the above problems, this paper mainly studies the optimal transmission waveform design in radar and jammer game for multiple targets. It takes SINR as the optimization criterion to design transmission waveform of radar and jammer, respectively. Radar and jammer are like a pair of game opponents. When radar is dominant, we adopt maxmin strategy to establish the optimization model to design the radar transmission waveform. When jammer is dominant, the minmax strategy is used to establish the optimization model to design the jammer waveform. The two-order Lagrange multiplier method is adopted to solve optimization models to obtain the maximum SINR-based waveforms and improved maximum SINR-based waveforms.

The main innovations of our work can be described as follows:

(1) The maximum SINR-based waveform design approach in radar and jammer is proposed for multiple targets. Based on the target characteristics obtained, the optimal radar transmission waveform and jamming waveform under the maxmin and minmax strategies are designed.
(2) We also develop an improved approach, which combines maximum SINR-based waveform design approach with Jensen's inequality for multiple targets. We derive a strict lower bound of SINR and optimize the improved waveform by taking the lower bound as objective function. The improved approach could improve the performance of radar and jammer on the basis of maximum SINR-based waveform performance.

(3) We propose an adaptive weight method to solve the problems of summations of weighted multiple targets in the maximum SINR-based waveform design approach and the improved maximum SINR-based waveform design approach.

This paper is organized as follows. We give more work related with radar and jammer in Sect. 2. In Sect. 3, the signal models for multi-target are presented and the output SINR is derived. Maximum SINR-based waveform design method and improved SINR-based waveform design method are also proposed. In Sect. 4, we discuss the simulation results and the final conclusions and the future work are drawn in Sect. 5.

Notations: Throughout this paper, the time domain signals are represented by lowercase letters, the frequency domain signals are represented by uppercase letters, and all signals are represented by no vectors or matrices.

2 The related work

There are some other work which considers the existence of jammer. In view of the existence of interference, Wang Yuxi et al. [35] designed the transmission waveform in the electronic warfare environment and proposed three different countermeasure models: smart radar and dumb target, dumb radar and smart target, and smart radar and smart target. They added the target as an interference into the signal model and optimized the transmission waveform based on the SINR criterion. For the detection problem, Bachmann [36] proposed an interference spectrum design based on a non-cooperative two-person zero-sum (TPZS) game between radar and jammer. The results indicate that the game analysis of radar and jammer can be used to identify the conditions of jammer self-protection and radar detecting targets. Chen Yuzhong [37] proposed a multi-player game theory algorithm by analyzing the competitive and cooperative relationships among sensor nodes. A utility function to achieve efficient intra-cluster data aggregation was also proposed and extensive experiments confirm the accuracy of the proposed algorithm. In [38, 39], Li Kang et al. proposed the egalitarian game strategy and Stackelberg game strategy as technological solutions for the countermeasure between radar and jammer. The work above all investigates the existence condition of Nash equilibrium (NE). A contribution for the presence of interference in OFDM radar waveform is [40], which proposed a radar waveform design method using compressed sensing. The proposed method could achieve the robust signal evaluation in the presence of interference without degradation of target detection performance. Besides, [41] proposed an electronic countermeasure (ECM) system for OFDM-based passive radars and investigated in detail how the system generates and transmits multiple false target echoes for moving targets. In [42], the authors proposed transmit-adaptive jammer nulling waveforms for cognitive radar and introduced knowledge-based noise jammers in the electronic warfare. The simulation results indicate that the proposed transmit-adaptive waveforms have inhibitory effect on knowledge-based noise jammers.

Radar electronic countermeasure (ECM) and electronic counter-countermeasure (ECCM) are a hot topic for radar waveform optimization. The ECM systems aim at preventing the enemy's radar from working correctly, while the ECCM systems aim at protecting the radar from being jammed [38]. In ECM environment, the jammer can generate false targets and false target ranges to confuse the radar [43, 44]. The jamming

signal generated by interrupted-sampling repeater jamming (ISRJ) forms multiple false targets to jam radar, which reduces the detection performance of radar. Besides, the authors proposed smart NLJ, which generates a pulse of noise to hide the true target range. In ECCM environment, [45] proposed a dynamic scheme based on jammer parameter estimation and transmitted signal design. The authors firstly transmitted the linear frequency-modulated (LFM) signal to estimate the main jamming parameters and then designed parameter-adjusted coded signal on the basis of estimated parameters. Besides, [46] designed a novel two-step estimation procedure for the problem of target detection and reduced the sensitivity of jammer. The estimation procedure considers two scenarios where the radar system is under the noise-like interferers and where the coherent jammers are in existence. Andrey Garnaev [47] has taken the uncertainty of the jammer's location into account and solved the game between the joint radar and communication system and jammer with Bayesian game.

3 Materials and methods

3.1 Problem formulation

The main purpose of this paper is to design optimal waveforms for radar and jammer in the environment of electronic warfare. The radar and jammer can adaptively allocate the transmission energy. Without loss of integrality, we introduce the signal models of deterministic multi-target and random multi-target in this subsection.

For the completeness of this paper, we first introduce the signal model for deterministic or random single target, as depicted in Fig. 1. $x(t)$ is the transmission waveform of the radar system and its Fourier transform is $X(f)$. $n(t)$ is a zero-mean additive Gaussian noise process with power spectral density (PSD) $S_{nn}(f)$. $c(t)$ is a zero-mean Gaussian random process with PSD $S_{cc}(f)$. $j(t)$ is the jamming signal that is generated by the jammer, and its PSD is $P_j(f)$. $r(t)$ is the ideal low-pass filter, and its Fourier transform is $R(f)$. $y(t)$ is the received signal, and $Y(f)$ is transformed into the frequency domain. $h_{D1}(t)$ is the deterministic target impulse response of target 1, and its spectral response is $H_{D1}(f)$. $h_{R1}(t)$ is the random target impulse response of target 1, and its spectral response is $H_{R1}(f)$.

For the deterministic single target, its PSD can be used to describe target characteristic because $h_{D1}(t)$ is a Gaussian stationary process. For the random single target, $h_{R1}(t)$ is generated randomly, which can be obtained by $h_{R1}(t) = a(t)g_1(t)$. Among $a(t)$ is a rectangular window function with duration T_h and $g_1(t)$ is a generalized stationary random process; therefore, $h_{R1}(t)$ is a generalized stationary stochastic process with finite duration. Since $h_{R1}(t)$ is an energy limited and not a real Gaussian stationary random

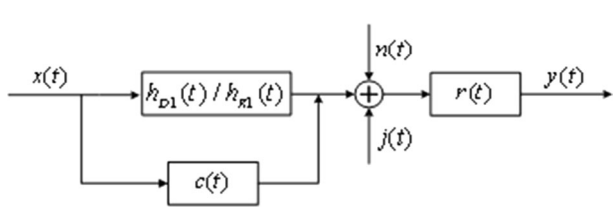

Fig. 1 Signal model for deterministic/random single target

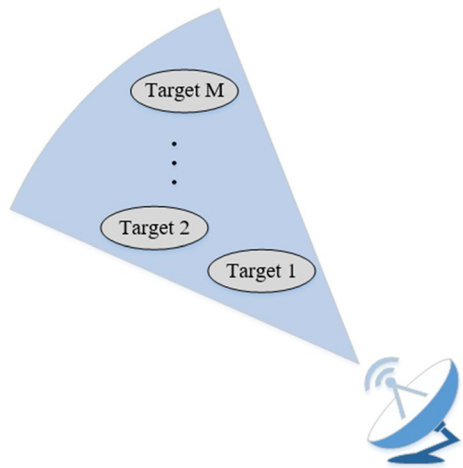

Fig. 2 Multiple targets in the same beam

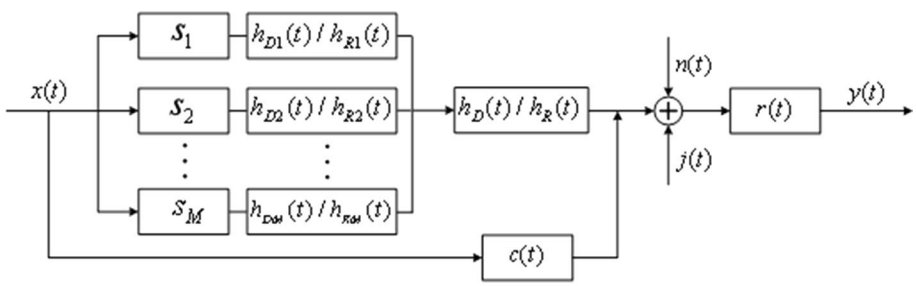

Fig. 3 Signal model for deterministic/random multi-target

process, its PSD cannot be used to describe target characteristic. The target energy spectral density (ESD) can be used to describe target characteristic, and ESD is defined as

$$\xi_{h_1}(f) = E\left[\left|H_{R1}(f)\right|^2\right] \tag{1}$$

where $E[\bullet]$ is the expectation, and $H_{R1}(f)$ is the spectral response of $h_{R1}(t)$. The target energy spectrum variance (ESV) is

$$\sigma^2_{h_1}(f) = E\left[\left|H_{R1}(f) - \mu_{h_1}(f)\right|^2\right] \tag{2}$$

where $\mu_{h_1}(f)$ is the mean of $H_{R1}(f)$. In order to simplify the computation, $\mu_{h_1}(f)$ is assumed to be zero. In this way, the result that ESV is equal to ESD can be obtained. Consequently, the target characteristic can be described by using $\sigma^2_{h_1}(f)$.

On the basis of deterministic or random single target, we assume that all targets are in the same beam, as shown in Fig. 2. We can get that the proposed signal model for deterministic multi-target or random multi-target is depicted in Fig. 3, and the meaning of each part in Fig. 3 is the same as that in the signal model for deterministic or random single target. In addition, M is the number of targets and s_i is the weight value of i_{th} target. $h_D(t)$ is the total target impulse response after weighting the impulse responses of multiple deterministic targets, and its spectral response is $H_D(f)$. $h_R(t)$ is the total target

impulse response after weighting the impulse responses of multiple random targets, and its spectral response is $H_R(f)$.

For deterministic multi-target, the output received signal is

$$y(t) = x(t) * h_D(t) * r(t) + r(t) * (x(t) * c(t) + n(t) + j(t)) \tag{3}$$

where $*$ is convolution symbol. Equation (3) is expressed in the frequency domain as

$$Y(f) = X(f)H_D(f)R(f) + R(f)(X(f)S_{cc}(f) + S_{nn}(f) + P_j(f)) \tag{4}$$

The derived output SINR is

$$\text{SINR} = \int_w \frac{\left|X(f)\right|^2 \left|H_D(f)\right|^2}{S_{cc}(f)\left|X(f)\right|^2 + S_{nn}(f) + P_j(f)} df \tag{5}$$

where w is the bandwidth of the spectrum responses of the transmission signal and the jammer signal.

For random multi-target, $h_R(t)$ is generated randomly, and it represents the summation of weighted multiple random targets. Since $h_R(t)$ is not a real Gaussian stationary random process, we use its ESD to describe target characteristics, and ESD is defined as

$$\xi_H(f) = E\left[\left|H_R(f)\right|^2\right] \tag{6}$$

where $H_R(f)$ is the spectral response of $h_R(t)$. The target ESV is

$$\sigma_H^2(f) = E\left[\left|H_R(f) - \mu_H(f)\right|^2\right] \tag{7}$$

where $\mu_H(f)$ is the mean of $H_R(f)$. In order to simplify the computation, $\mu_H(f)$ is assumed to be zero. In this way, the result that ESV is equal to ESD can be obtained. Consequently, the target characteristics can be described by using $\sigma_H^2(f)$. The output SINR can be approximately as

$$\text{SINR} = \int_w \frac{\left|X(f)\right|^2 \sigma_H^2(f)}{S_{cc}(f)\left|X(f)\right|^2 + S_{nn}(f) + P_j(f)} df \tag{8}$$

Different from the SINR expression of Eq. (5), the target impulse response $h_R(t)$ is a random process with finite duration, and $\left|H_D(f)\right|^2$ is replaced by $\sigma_H^2(f)$. $\sigma_H^2(f)$ is the total spectral response after summation of weighted multiple target spectral responses. We use adaptive weight method to get it. Assuming that there are M targets that are distinguishable in range within the same beam. As for i_{th} target, $\sigma_H^2(f)$ contains the characteristics of all targets in the same frequency band. However, at each frequency point, the characteristic of every target is different. Therefore, each target should be weighted in order to highlight the target with strong characteristic and weaken the target with weak characteristic. The weight value is

$$s_i = \frac{\sigma_{h_i}^2(f)}{\sum_{j=1}^{M} \sigma_{h_j}^2(f)} \tag{9}$$

The weight value s_i represents the proportion of the i_{th} target characteristic in all target characteristics. The total spectral response $\sigma_H^2(f)$ can be expressed as

$$\sigma_H^2(f) = \sum\nolimits_{j=1}^{M} s_j \sigma_{h_j}^2(f) \tag{10}$$

where $j = 1, 2, ..., M$.

3.2 Waveform optimization design methods

Since the optimal waveform design method for random target is similar to that of deterministic target, the waveform optimization method of random target is discussed here. The conclusions obtained are also applicable to deterministic target. In the electronic warfare environment, since the jammer cannot be ignored, the radar transmission waveform and jamming waveform need to be designed at the same time. The waveform design is mainly divided into two cases, the first one is the waveform design with the maxmin strategy when the radar is dominant. It can design the radar transmission waveform according to the jamming signal transmitted by the jammer, so as to reduce the interference of jammer. The other one is the waveform design with the minmax strategy when the jammer is dominant. The jammer can transmit jamming signal according to the radar transmission waveform to degrade the performance of radar detecting targets.

3.2.1 Maximum SINR-based waveform design with maxmin strategy

Radar is the leader, which indicates that the radar designs the transmission waveform according to the target spectrum and the jamming waveform. The waveform optimization strategy selected by radar and jammer is as follows

$$\begin{aligned} &\max_{|X(f)|^2} \min_{P_j(f)} \int_w \frac{|X(f)|^2 \sigma_H^2(f)}{S_{cc}(f)|X(f)|^2 + S_{nn}(f) + P_j(f)} df \\ &\text{s.t.} \int_w P_j(f) df = P, \int_w |X(f)|^2 df = E_x \end{aligned} \tag{11}$$

where P is the transmission power of jammer, and E_x is the transmission energy of radar. $\sigma_H^2(f)$ is the total target spectrum after summation of weighted multiple random targets. The optimal radar transmission waveform and jamming waveform obtained by two-order Lagrange multiplier method are as follows

$$\begin{cases} P_j(f) = \max\left\{ \frac{\sigma_H^2(f)}{|2\lambda_2|} + \frac{\lambda_1 S_{cc}(f)\sigma_H^2(f)}{4\lambda_2^2} - S_{nn}(f), 0 \right\} \\ |X(f)|^2 = \max\left\{ -\frac{\lambda_1 \sigma_H^2(f)}{4\lambda_2^2}, 0 \right\} \end{cases} \tag{12}$$

where λ_1 and λ_2 are Lagrange multipliers. Their values are determined by $\int_w P_j(f) df = P$ and $\int_w |X(f)|^2 df = E_x$, respectively. We use iterative search approach to find the solution of Lagrange multipliers, which mainly includes the following steps:

(1) Obtain the maximum values of Lagrange multipliers according to the constraint conditions, and set the iterative threshold β.
(2) Iteratively search Lagrange multipliers within the range, and substitute the searched values into $\int_w P_j(f) df = P$ and $\int_w |X(f)|^2 df = E_x$.

(3) Stop iteration when the error between $P_j(f)$ and setting value is less than threshold β, or when the error between $\left|X(f)\right|^2$ and setting value is less than threshold β. Get the values of Lagrange multipliers.

The optimal transmission waveform spectrum can be obtained according to Eq. (12).

Proof For the solution to the optimal radar transmission waveform and jammer waveform in Eq. (12), first the jammer waveform is found by solving the constrained optimization problem

$$\begin{aligned} &\min_{P_j(f)} \int_w \frac{|X(f)|^2 \sigma_H^2(f)}{S_{cc}(f)|X(f)|^2 + S_{nn}(f) + P_j(f)} df \\ &\text{s.t.} \int_w P_j(f) df = P \end{aligned} \tag{13}$$

Making use of Lagrangian multiplier, we seek to minimize

$$\Gamma(P_j(f)) = \int_w \frac{\left|X(f)\right|^2 \sigma_H^2(f)}{S_{cc}(f)\left|X(f)\right|^2 + S_{nn}(f) + P_j(f)} df + \lambda_1 (P - \int_w P_j(f) df) \tag{14}$$

We can equivalently minimize

$$\varphi(P_j(f)) = \frac{\left|X(f)\right|^2 \sigma_H^2(f)}{S_{cc}(f)\left|X(f)\right|^2 + S_{nn}(f) + P_j(f)} - \lambda_1 P_j(f) \tag{15}$$

Since the second derivative with respect to $P_j(f)$ in Eq. (15) is positive, $\varphi(P_j(f))$ is a convex function, and there is a minimum point. The first derivative with respect to $P_j(f)$ in Eq. (15) is

$$\varphi'(P_j(f)) = -\frac{\left|X(f)\right|^2 \sigma_H^2(f)}{\left[S_{cc}(f)\left|X(f)\right|^2 + S_{nn}(f) + P_j(f)\right]^2} - \lambda_1 \tag{16}$$

By equating (16) to zero, the jammer waveform is

$$P_j(f) = \sqrt{\frac{\left|X(f)\right|^2 \sigma_H^2(f)}{-\lambda_1}} - S_{cc}(f)\left|X(f)\right|^2 - S_{nn}(f) \tag{17}$$

Next the transmission waveform of the radar is found by solving the constrained optimization problem

$$\begin{aligned} &\max_{|X(f)|^2} \int_w \frac{|X(f)|^2 \sigma_H^2(f)}{S_{cc}(f)|X(f)|^2 + S_{nn}(f) + P_j(f)} df \\ &\text{s.t.} \int_w \left|X(f)\right|^2 df = E_x \end{aligned} \tag{18}$$

Making use of Lagrangian multiplier, we seek to maximize

$$\mathrm{K}(|X(f)|^2) = \int_w \frac{|X(f)|^2 \sigma_H^2(f)}{S_{cc}(f)|X(f)|^2 + S_{nn}(f) + P_j(f)} df + \lambda_2 (E_x - \int_w |X(f)|^2 df) \tag{19}$$

We can equivalently maximize

$$\kappa(|X(f)|^2) = \frac{|X(f)|^2 \sigma_H^2(f)}{S_{cc}(f)|X(f)|^2 + S_{nn}(f) + P_j(f)} - \lambda_2 |X(f)|^2 \tag{20}$$

Substitute Eq. (17) into Eq. (20)

$$\kappa(|X(f)|^2) = \frac{|X(f)|^2 \sigma_H^2(f)}{\sqrt{\frac{|X(f)|^2 \sigma_H^2(f)}{-\lambda_1}}} - \lambda_2 |X(f)|^2 \tag{21}$$

Since the second derivative with respect to $|X(f)|^2$ in Eq. (21) is negative, $\kappa(|X(f)|^2)$ is a concave function, and there is a maximum point. The first derivative with respect to $|X(f)|^2$ in Eq. (21) is

$$\kappa'(|X(f)|^2) = \frac{\frac{1}{2}\sqrt{-\lambda_1 |X(f)|^2 \sigma_H^2(f)}}{|X(f)|^2} - \lambda_2 \tag{22}$$

By equating (22) to zero, the transmission waveform of the radar is

$$|X(f)|^2 = -\frac{\lambda_1}{4\lambda_2^2} \sigma_H^2(f) \tag{23}$$

Substitute Eq. (23) into Eq. (17) to obtain Eq. (12). □

3.2.2 Maximum SINR-based waveform design with minmax strategy

The jammer is the leader, indicating that the jammer can design jamming waveform according to the waveform spectrum of the target and the transmission waveform of the radar. The waveform optimization strategy selected by the radar and jammer is

$$\begin{aligned} &\min_{P_j(f)} \max_{|X(f)|^2} \int_w \frac{|X(f)|^2 \sigma_H^2(f)}{S_{cc}(f)|X(f)|^2 + S_{nn}(f) + P_j(f)} df \\ &\text{s.t.} \int_w |X(f)|^2 df = E_x, \int_w P_j(f) df = P \end{aligned} \tag{24}$$

The optimal radar transmission waveform and jamming waveform obtained by using the two-order Lagrange multiplier method are as follows

$$\begin{cases} |X(f)|^2 = \max\left\{\frac{1}{S_{cc}^2(f)}\left[\frac{\sigma_H^2(f)}{2\lambda_4} - \frac{\lambda_3 \sigma_H^2(f)}{4\lambda_4^2 S_{cc}(f)}\right], 0\right\} \\ P_j(f) = \max\left\{\frac{\lambda_3 \sigma_H^2(f)}{4\lambda_4^2 S_{cc}^2(f)} - S_{nn}(f), 0\right\} \end{cases} \tag{25}$$

where λ_3 and λ_4 are Lagrange multipliers. Their values are determined by $\int_w |X(f)|^2 df = E_x$ and $\int_w P_j(f) df = P$, respectively. We still use iterative search approach to find the values of λ_3 and λ_4, and the specific solution steps are the same as

those for λ_1 and λ_2. When the target spectrum, clutter spectrum, and noise are obtained, the optimal transmission waveforms of radar and jammer can be solved.

Proof For the solution to the optimal radar transmission waveform and jammer waveform in Eq. (25), first the transmission waveform of the radar is found by solving the constrained optimization problem

$$\begin{aligned} &\max_{|X(f)|^2} \int_w \frac{|X(f)|^2 \sigma_H^2(f)}{S_{cc}(f)|X(f)|^2 + S_{nn}(f) + P_j(f)} df \\ &\text{s.t.} \int_w |X(f)|^2 df = E_x \end{aligned} \tag{26}$$

Making use of Lagrangian multiplier, we seek to maximize

$$\Gamma(|X(f)|^2) = \int_w \frac{|X(f)|^2 \sigma_H^2(f)}{S_{cc}(f)|X(f)|^2 + S_{nn}(f) + P_j(f)} df + \lambda_3 (E_x - \int_w |X(f)|^2 df) \tag{27}$$

We can equivalently maximize

$$\varphi(|X(f)|^2) = \frac{|X(f)|^2 \sigma_H^2(f)}{S_{cc}(f)|X(f)|^2 + S_{nn}(f) + P_j(f)} - \lambda_3 |X(f)|^2 \tag{28}$$

Since the second derivative with respect to $|X(f)|^2$ in Eq. (28) is negative, $\varphi(|X(f)|^2)$ is a concave function, and there is a maximum point. The first derivative with respect to $|X(f)|^2$ in Eq. (28) is

$$\varphi'(|X(f)|^2) = \frac{\sigma_H^2(f)\left[S_{nn}(f) + P_j(f)\right]}{\left[S_{cc}(f)|X(f)|^2 + S_{nn}(f) + P_j(f)\right]^2} - \lambda_3 \tag{29}$$

By equating (29) to zero, the transmission waveform of the radar is

$$|X(f)|^2 = \frac{1}{S_{cc}(f)} \left[\sqrt{\frac{\sigma_H^2(f)\left[P_j(f) + S_{nn}(f)\right]}{\lambda_3}} - P_j(f) - S_{nn}(f) \right] \tag{30}$$

Next the jammer waveform is found by solving the constrained optimization problem

$$\begin{aligned} &\min_{P_j(f)} \int_w \frac{|X(f)|^2 \sigma_H^2(f)}{S_{cc}(f)|X(f)|^2 + S_{nn}(f) + P_j(f)} df \\ &\text{s.t.} \int_w P_j(f) df = P \end{aligned} \tag{31}$$

Making use of Lagrangian multiplier, we seek to minimize

$$\mathrm{K}(P_j(f)) = \int_w \frac{|X(f)|^2 \sigma_H^2(f)}{S_{cc}(f)|X(f)|^2 + S_{nn}(f) + P_j(f)} df + \lambda_4 (P - \int_w P_j(f) df) \tag{32}$$

We can equivalently minimize

$$\kappa(P_j(f)) = \frac{|X(f)|^2 \sigma_H^2(f)}{S_{cc}(f)|X(f)|^2 + S_{nn}(f) + P_j(f)} - \lambda_4 P_j(f) \tag{33}$$

Substitute Eq. (30) into Eq. (33)

$$\kappa(P_j(f)) = \frac{\sigma_H^2(f)}{S_{cc}(f)} - \frac{\sqrt{\lambda_3 \sigma_H^2(f)\left[S_{nn}(f) + P_j(f)\right]}}{S_{cc}(f)} - \lambda_4 P_j(f) \tag{34}$$

Since the second derivative with respect to $P_j(f)$ in Eq.(34) is positive, $\kappa(P_j(f))$ is a convex function, and there is a minimum point. The first derivative with respect to $P_j(f)$ in Eq. (34) is

$$\kappa'(P_j(f)) = \frac{-\lambda_3 \sigma_H^2(f)}{2S_{cc}(f)\sqrt{\lambda_3 \sigma_H^2(f)\left[S_{nn}(f) + P_j(f)\right]}} - \lambda_4 \tag{35}$$

By equating (35) to zero, the jammer waveform is

$$P_j(f) = \frac{\lambda_3 \sigma_H^2(f)}{4\lambda_4^2 S_{cc}^2(f)} - S_{nn}(f) \tag{36}$$

Substitute Eq. (36) into Eq. (30) to obtain Eq. (25). □

3.2.3 Improved maximum SINR-based waveform design

In [19], Goodman proposed a waveform design algorithm by deriving a strict lower bound of MI based on ESV and designed the enhanced waveform by maximizing the lower bound. Inspired by this algorithm, we propose an improved waveform design method which derives a strict lower bound of SINR based on ESV. Based on hypothesis test theorem, the relationship between the weight values of targets and the ESV of the multiple random targets is [19]

$$S_R(f) = \sum\nolimits_{i=1}^{M} s_i \sigma_{h_i}^2(f) \tag{37}$$

where M is the number of targets. s_i is the weight value of i_{th} target. $\sigma_{h_i}^2(f)$ is ESV of i_{th} target. The SINR can be obtained as

$$\begin{aligned} \text{SINR} &= \int_w \frac{|X(f)|^2 S_R(f)}{S_{cc}(f)|X(f)|^2 + S_{nn}(f) + P_j(f)} df \\ &= \int_w \frac{|X(f)|^2 \left\{\sum_{i=1}^{M} s_i \sigma_{h_i}^2(f)\right\}}{S_{cc}(f)|X(f)|^2 + S_{nn}(f) + P_j(f)} df \\ &= \int_w \left[\sum\nolimits_{i=1}^{M} s_i \left\{\frac{|X(f)|^2 \left\{\sigma_{h_i}^2(f)\right\}}{S_{cc}(f)|X(f)|^2 + S_{nn}(f) + P_j(f)}\right\}\right] df \end{aligned} \tag{38}$$

According to Jensen's inequality, if $f(x)$ is a concave function on an interval (a, b), there is $f(\sum_{i=1}^{n}\lambda_i x_i) \geq \sum_{i=1}^{n}\lambda_i f(x_i)$ for any $x_1, x_2, x_3, ..., x_n \in (a, b)$, $\lambda_i > 0 (i = 1, 2, 3, ..., n)$, and $\sum_{i=1}^{n}\lambda_i = 1$.

Assume that the total weight value of all targets is 1, that is $\sum_{i=1}^{M} s_i = 1$. We can obtain Eq. (39) by using Jensen's inequality

$$\mathrm{SINR} \geq \sum\nolimits_{i=1}^{M} s_i \int_w \frac{\left|X(f)\right|^2 \left\{\sigma_{h_i}^2(f)\right\}}{S_{cc}(f)\left|X(f)\right|^2 + S_{nn}(f) + P_j(f)} df \tag{39}$$

Therefore, the objective function and constraints of the optimized waveform are

$$\begin{aligned} &\max_{|X(f)|^2} \min_{P_j(f)} \sum\nolimits_{i=1}^{M} s_i \int_w \frac{|X(f)|^2 \sigma_{h_i}^2(f)}{S_{cc}(f)|X(f)|^2 + S_{nn}(f) + P_j(f)} df \\ &\text{s.t.} \int_w P_j(f) df = P, \int_w \left|X(f)\right|^2 df = E_x \end{aligned} \tag{40}$$

Making use of Lagrangian multiplier method, we can obtain

$$\begin{cases} P_j(f) = \max\left\{\frac{\sum_{i=1}^{M} s_i \sigma_{h_i}^2(f)}{|2\lambda'_2|} + \frac{\lambda'_1 S_{cc}(f) \sum_{i=1}^{M} s_i \sigma_{h_i}^2(f)}{4(\lambda_2')^2} - S_{nn}(f), 0\right\} \\ \left|X(f)\right|^2 = \max\left\{-\frac{\lambda'_1}{4(\lambda_2')^2} \sum_{i=1}^{M} s_i \sigma_{h_i}^2(f), 0\right\} \end{cases} \tag{41}$$

The value of λ'_1 is determined by $\int_w P_j(f) df = P$, and the value of λ'_2 is determined by $\int_w \left|X(f)\right|^2 df = E_x$.

The objective function in the case of jammer is the dominant player which can be obtained as

$$\begin{aligned} &\min_{P_j(f)} \max_{|X(f)|^2} \sum\nolimits_{i=1}^{M} s_i \int_w \frac{|X(f)|^2 \sigma_{h_i}^2(f)}{S_{cc}(f)|X(f)|^2 + S_{nn}(f) + P_j(f)} df \\ &\text{s.t.} \int_w \left|X(f)\right|^2 df = E_x, \int_w P_j(f) df = P \end{aligned} \tag{42}$$

The optimal transmission waveform is

$$\begin{cases} \left|X(f)\right|^2 = \max\left\{\frac{1}{S_{cc}^2(f)}\left[\frac{\sum_{i=1}^{M} s_i \sigma_{h_i}^2(f)}{2\lambda'_4} - \frac{\lambda'_3 \sum_{i=1}^{M} s_i \sigma_{h_i}^2(f)}{4(\lambda_4')^2 S_{cc}(f)}\right], 0\right\} \\ P_j(f) = \max\left\{\frac{\lambda'_3 \sum_{i=1}^{M} s_i \sigma_{h_i}^2(f)}{4(\lambda_4')^2 S_{cc}^2(f)} - S_{nn}(f), 0\right\} \end{cases} \tag{43}$$

The value of λ'_3 is determined by $\int_w \left|X(f)\right|^2 df = E_x$, and the value of λ'_4 is determined by $\int_w P_j(f) df = P$.

4 Results and discussion

In order to demonstrate the effectiveness of the proposed optimal radar transmission waveform and jamming waveform, the corresponding simulation results and performance analysis are made in this section.

Under the premise of known clutter and noise PSD, the maxmin strategy and minmax strategy in the case of random multi-target are verified and analyzed. The main simulation parameters are shown in Table 1.

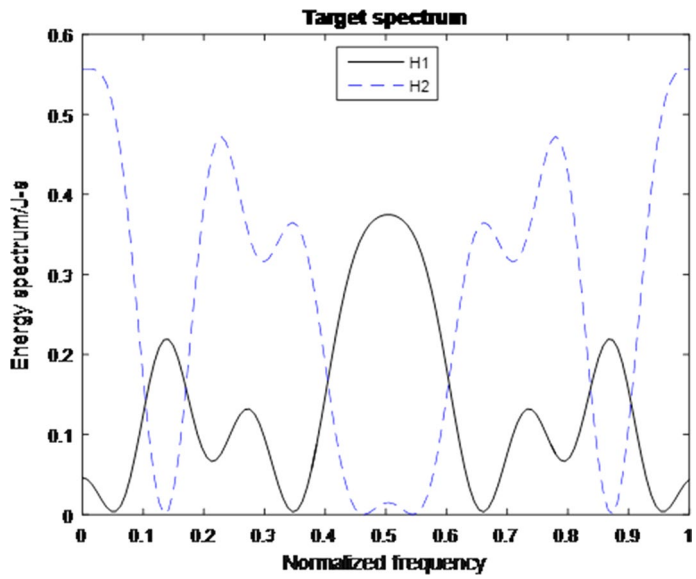

Fig. 4 Energy spectrum allocation of two random targets

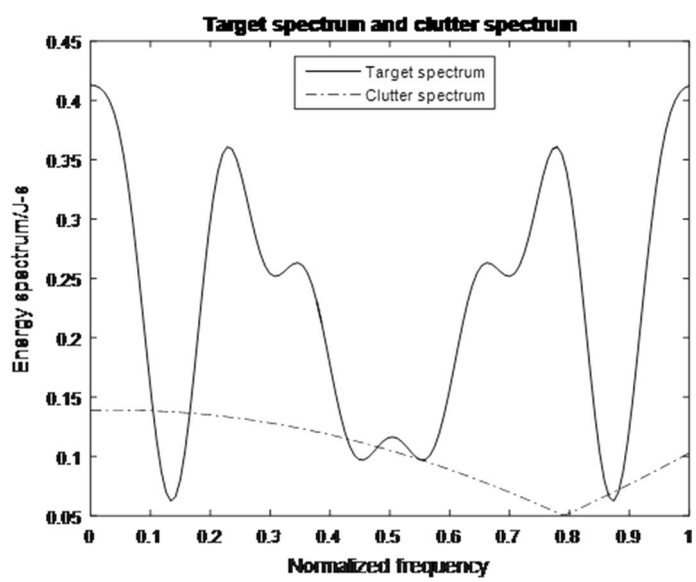

Fig. 5 Allocation of total target spectrum and clutter spectrum

It is assumed that there are two random targets, which are independent of each other and are randomly generated. Figure 4 shows the energy allocation of the two targets. In Fig. 4, each target is independently distributed and has no influence on each other, which has advantages for target recognition. When the weight value of each target is found, the total target spectrum can be obtained by summation of weighted multiple targets. Figure 5 shows the distribution of the total target spectrum and clutter spectrum.

4.1 Maximum SINR-based waveform with maxmin strategy

In this subsection, the energy allocation of radar transmission waveform and jamming waveform is analyzed. The total target spectrum and clutter spectrum are shown in

Table 1 Main simulation parameters

Parameter	Value
Target number	$M = 2$
Target duration	0.5s
Reference threshold	$\beta = 1 * 10^{-}6$
Sampling frequency	2kHz
Sampling points	128
Normalized signal frequency	$f \in (0, 1)$
Noise PSD	$S_{nn}(f) = 0.01$
Clutter PSD	$S_{cc}(f) = \left\| \begin{array}{l} 0.2(\cos(0.3\pi(f+0.45)))+ \\ 0.4(\sin(0.5\pi(f+0.5))) - 0.15 \end{array} \right\| + 0.05$

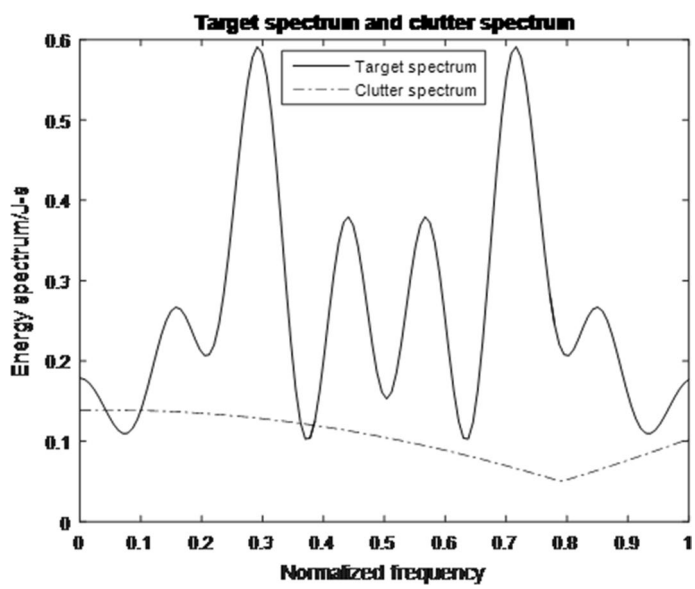

Fig. 6 Total target spectrum and clutter spectrum adopted with maxmin strategy

Fig. 6. The power spectrum of jamming waveform is shown in Fig. 7, it includes the maximum SINR-based waveform and the improved maximum SINR-based waveform, and the SINR-based waveform is a compared waveform [48].

Figure 7 shows that the jammer designs the jamming waveform according to the target spectrum and clutter spectrum with the maxmin strategy. The main characteristics of maximum SINR-based jamming waveform are as follows:

(1) Since the jammer hopes to reduce the SINR of radar receiver in order to degrade the radar performance, jammer allocates the energy according to the target energy spectrum to jam the radar. The jammer allocates more energy to the place with strong target energy spectrum. Comparing between Figs. 6 and 7, the energy spectrum of the target is higher in the frequency bands around 0.3 and 0.45. Therefore, the jammer allocates the main energy in these two frequency bands.

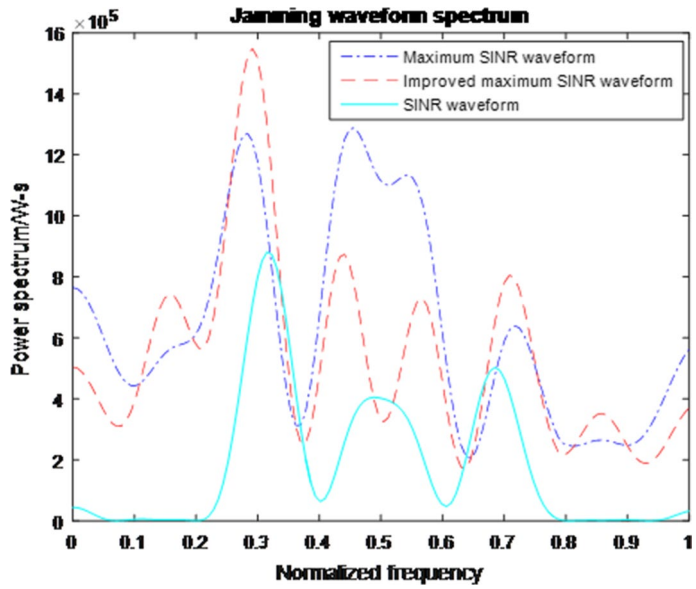

Fig. 7 Jamming waveform spectrum of jammer with maxmin strategy

(2) The jammer allocates more energy in the frequency bands with a larger product of the target spectrum and clutter spectrum according to Eq. (12). For example, jammer allocates more energy in the frequency band around 0.3 than in the frequency band around 0.7. The jammer and clutter could jam the radar jointly according to the target spectrum, which further reduce the radar received SINR.

The main characteristics of improved maximum SINR-based jamming waveform are as follows:

(1) Improved maximum SINR-based jamming waveform has the same characteristics as maximum SINR-based jamming waveform.
(2) In the frequency bands with a large product of the target spectrum and clutter spectrum, the improved maximum SINR-based jamming waveform could allocate more energy than maximum SINR-based jamming waveform, which is determined by the property of Jensen's inequality, which further degrades the radar received SINR.

When jammer transmits the jamming waveform, the radar could adjust the transmission waveform according to the comprehensive analysis of the target spectrum, clutter spectrum, and the jamming waveform. The energy spectrum of radar transmission waveform is shown in Fig. 8, it includes the maximum SINR-based waveform and the improved maximum SINR-based waveform, and the SINR-based waveform is a compared waveform.

The main characteristics of maximum SINR-based radar waveform are as follows: The radar allocates less energy in the frequency bands with strong interference and more energy in the frequency bands with strong target energy spectrum; however, the radar mainly designs the transmission waveform according to the target spectrum when both

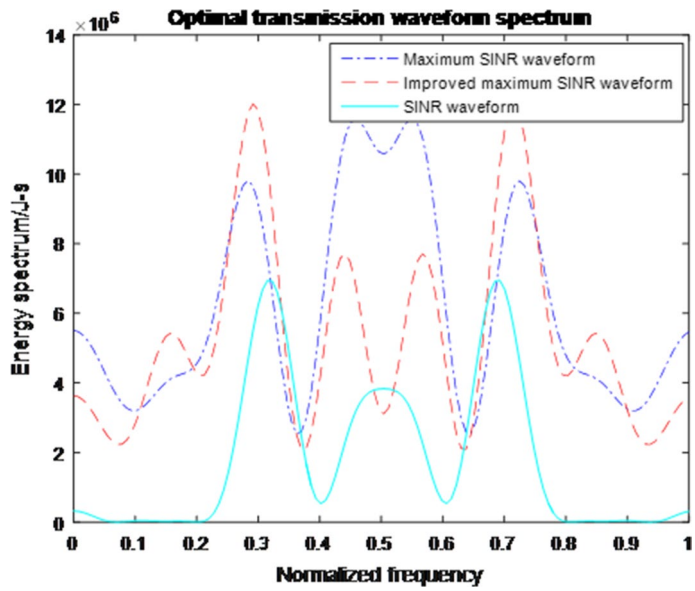

Fig. 8 Transmission waveform spectrum of radar with maxmin strategy

target spectrum and interference are strong. As shown in Fig. 8, the radar allocates much energy in the frequency band around 0.3 because the target spectrum is strong in the frequency band around 0.3 even the interference is strong. The radar reduces the loss of target energy, in this way, the radar can obtain more useful information about the targets and compensate the loss of target information caused by jamming waveform.

Figure 8 also shows that improved maximum SINR-based radar waveform. The improved transmission waveform has a certain improvement in energy allocation compared with the maximum SINR-based radar transmission waveform. The improved waveform allocates more energy in frequency bands where the target has larger spectrum value. The energy allocated in other frequency bands is reduced because the transmission energy is fixed.

4.2 Maximum SINR-based waveform with minmax strategy

According to Eq. (25), when the random multi-target spectrum is shown in Fig. 9, the energy spectrum of radar transmission waveform is shown in Fig. 10, it includes the maximum SINR-based waveform and the improved maximum SINR-based waveform, and the SINR-based waveform is a compared waveform.

In Fig. 10, the radar firstly designs the optimal transmission waveform according to the prior information with the minmax strategy. The radar allocates most of the energy in the frequency bands where the target has larger spectrum value and clutter has smaller spectrum value. As can be seen from Figs. 9 and 10, the radar allocates more transmission energy in the frequency band around 0.8 than in the frequency band around 0.2, because the clutter spectrum is smaller in the frequency band around 0.8 even though the target spectrum is roughly the same in the frequency bands. In this way, the radar could avoid some interference and get more information about targets, while less energy is allocated in where the clutter is strong, so as to avoid the loss of output SINR of radar

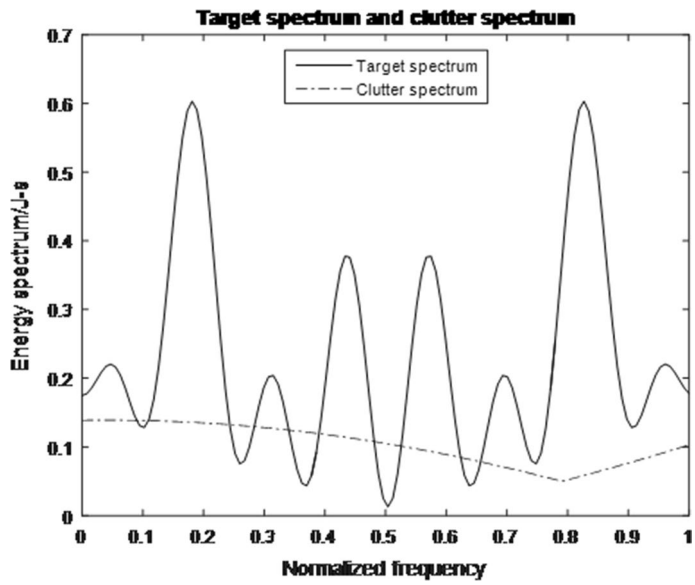

Fig. 9 Total target spectrum and clutter spectrum adopted with minmax strategy

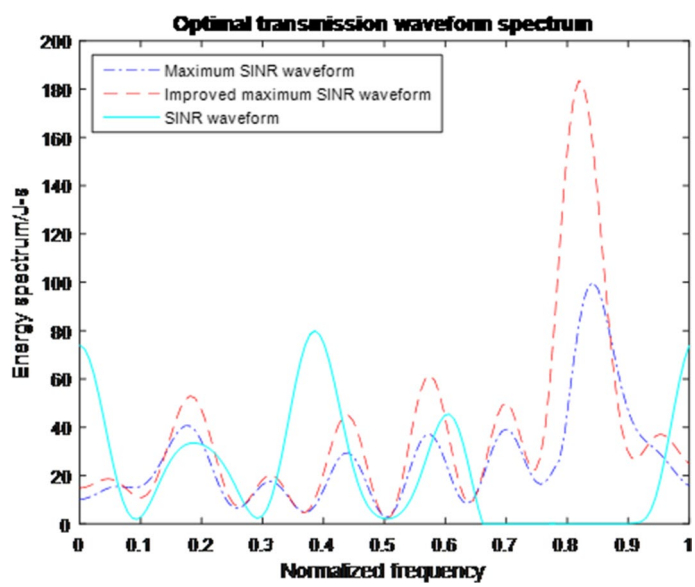

Fig. 10 Transmission waveform spectrum of radar with minmax strategy

and improve the performance of the radar system. In addition, Fig. 10 shows that the improved optimization method allocates more energy in the frequency bands with less clutter. It can further improve the radar received SINR.

The power spectrum of jamming waveform is shown in Fig. 11, it includes the maximum SINR-based waveform and the improved maximum SINR-based waveform, and the SINR-based waveform is a compared waveform. Figure 11 shows the jammer transmits the jamming waveform according to radar transmission waveform and target energy spectrum. The main characteristics of maximum SINR-based jamming waveform are as follows:

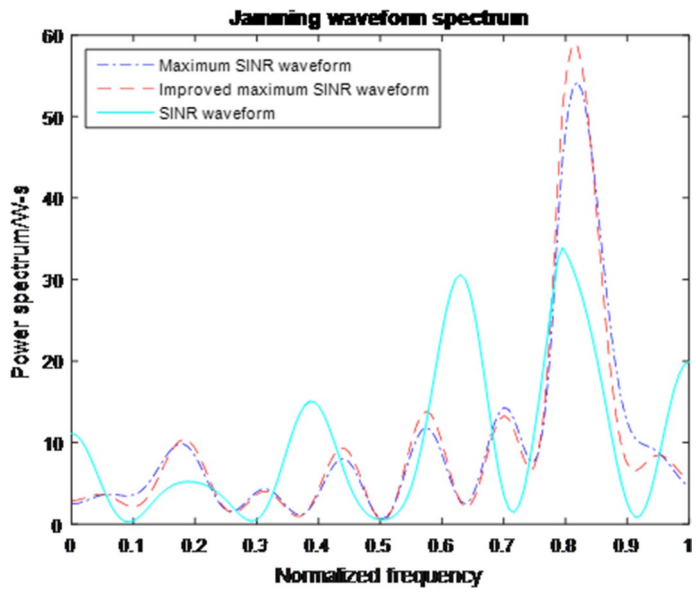

Fig. 11 Jamming waveform spectrum of jammer with minmax strategy

(1) The jammer allocates much energy in the frequency bands where the ratio of target energy spectrum to the clutter spectrum is large, which results in the decrease in the received SINR. For example, the energy spectrum of the target is roughly the same in the frequency bands around 0.2 and 0.8 in Fig. 9. However, the clutter spectrums are different in these frequency bands, and the jammer could allocate more energy in the frequency band around 0.8. In this way, although the radar allocates more energy in the frequency band around 0.8, the jammer also allocates more energy in this frequency band in order to degrade the radar received SINR.

(2) The jammer allocates more energy in the frequency band where the radar has stronger transmission waveform spectrum. As shown in Fig. 11, the jammer allocates more energy in the frequency band around 0.6 than in the frequency band around 0.2, because the radar transmission waveform spectrum is stronger in the frequency band around 0.6 even though the ratio of target spectrum to the clutter spectrum is larger in the frequency band around 0.2. The improved jamming waveform allocates more energy in the frequency bands where the radar transmission waveform spectrum is strong and the ratio of target energy spectrum to the clutter spectrum is large, which has better jamming effect on radar.

4.3 Performance analysis of improved maximum SINR-based waveform optimization method

In this subsection, the improved maximum output SINR with two strategies for multi-target is given. The output SINR curves of proposed maximum SINR-based waveform and improved maximum SINR-based waveform with maxmin strategy are shown in Fig. 12. The output SINR curves of proposed maximum SINR-based waveform and improved maximum SINR-based waveform with minmax strategy are shown in Fig. 13.

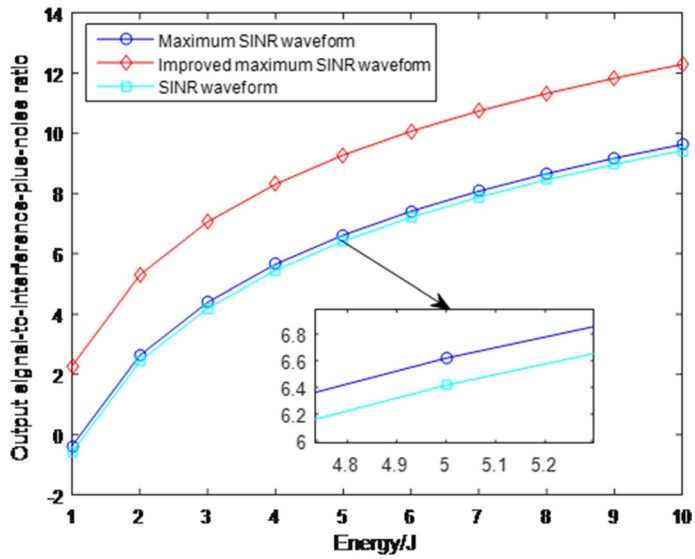

Fig. 12 Output SINR of different waveforms with maxmin strategy

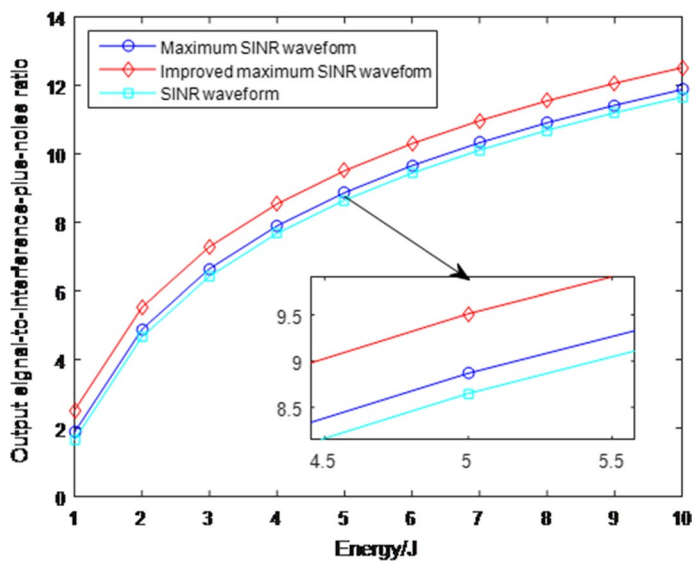

Fig. 13 Output SINR of different waveforms with minmax strategy

In the proposed waveform optimization approaches, the total spectral response is obtained by proposed adaptive weight method. Besides, Figs. 12 and 13 also include output SINR curves of SINR-based waveform [48] for comparison with the proposed waveforms.

(1) Figures 12 and 13 show that the output SINR of improved maximum SINR-based waveform is significantly improved compared with other waveforms. Because the radar could accurately design transmission waveform according to the target char-

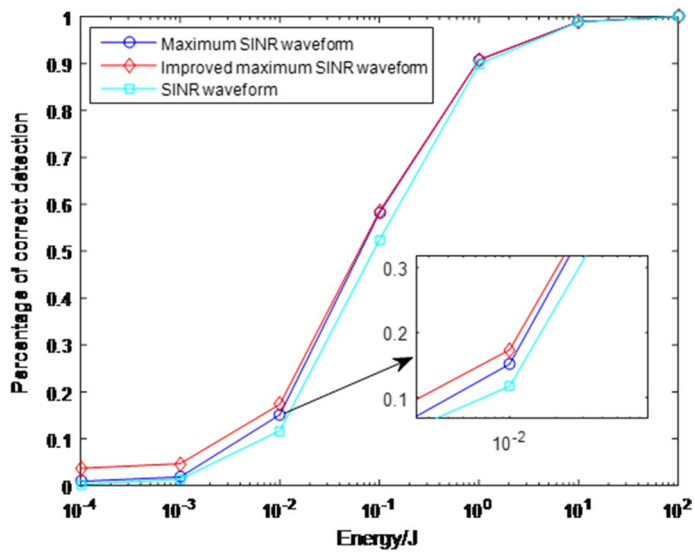

Fig. 14 Percentage of correct detection of different waveforms with maxmin strategy

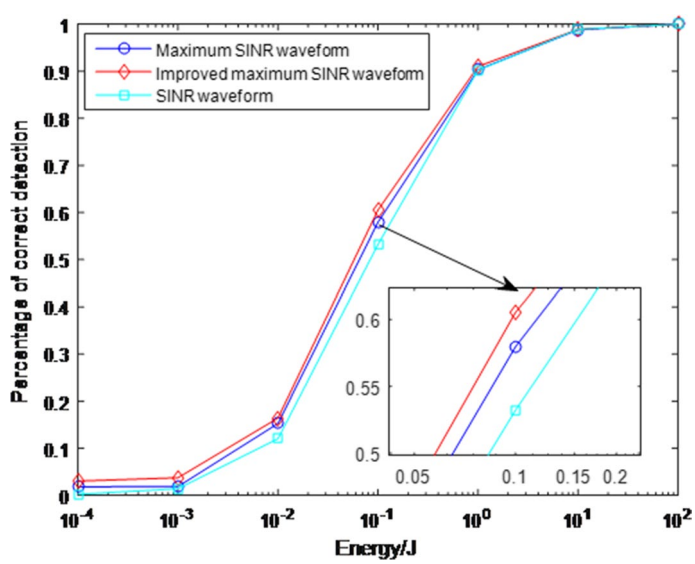

Fig. 15 Percentage of correct detection of different waveforms with minmax strategy

acteristics in the proposed optimization methods, thus the radar could obtain more useful information of the target.

(2) Figures 12 and 13 also show that the proposed maximum SINR-based waveform has a slight difference with SINR-based waveform. The performance of maximum SINR-based waveform is slightly superior to SINR-based waveform. The performance of the proposed improved waveform which has combined with Jensen's inequality is better than the maximum SINR-based waveform.

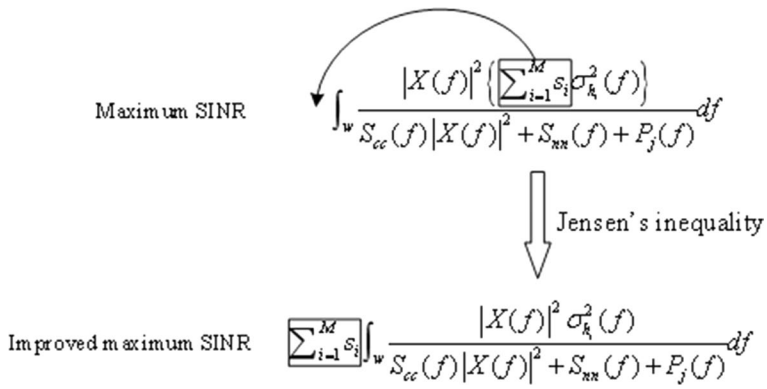

Fig. 16 Change in expressions of maximum SINR and improved maximum SINR

Table 2 The output SINR and average time for solving Lagrange multipliers

Waveform type	SINR(energy=5)	Average time/s
Improved maximum SINR waveform	9.5123	0.3379
Maximum SINR waveform	8.9278	0.3650

The probability of radar correctly detecting targets under maxmin strategy and minmax strategy is shown in Figs. 14 and 15, respectively. Figures 14 and 15 contain proposed maximum SINR waveform and improved maximum SINR waveform, and SINR waveform is a compared waveform.

(1) The performance of proposed optimization waveforms is better than the performance of SINR waveform, and the optimal waveform methods achieve the effect of radar waveform optimization.
(2) The improved waveform has better target detection and recognition performance. The maximum SINR-based optimization waveform and improved maximum SINR-based optimization waveform could improve the detection performance of the radar.

In the maximum SINR expression, the integral calculation will increase the computational complexity of the weighting coefficient. Thus in the improved maximum SINR expression, we combine Jensen's inequality to take the weighting coefficient out of the integral. The specific change is shown in Fig. 16. The improved maximum SINR expression could reduce the computational complexity of the maximum SINR expression and further reduce the algorithm complexity. At the same time, since the weighting within the integral belongs to indirect weighting, and the weighting outside the integral belongs to direct weighting. Therefore, the change will optimize the calculation results of the maximum SINR expression and further optimize the results of algorithm.

The specific results are shown in Table 2. Table 2 contains output SINR value and the average time of the iterative procedure for solving Lagrange multipliers of improved maximum SINR waveform and maximum SINR waveform. As shown in Table 2, the improved maximum SINR waveform could output higher SINR, which confirms that the

change will optimize the calculation results. Besides, the procedure for solving Lagrange multipliers of improved maximum SINR waveform needs less time, which could indirectly confirm that the change could reduce the computational complexity of the maximum SINR waveform.

5 Conclusions and the future work

In this paper, the optimal waveform methods of radar transmission waveform and jamming waveform based on SINR criterion are proposed. The solutions to radar dominated and jammer dominated strategies are carried out for the case of random multi-target. When dealing with random multi-target, we propose an adaptive weight method to solve the problem of summation of weighted multiple targets. The conclusions are also applicable to the case of deterministic multi-target. In addition, an improved maximum SINR-based waveform optimization method is proposed according to the maximum SINR-based waveform optimization method. The optimization waveform with multi-target ESV is analyzed, which leads to a good performance of radar system. The simulation results show that the waveform optimization methods proposed in this paper could effectively improve the overall performance of the radar system. The proposed methods are based on SINR criterion with maxmin strategy and minmax strategy, and they can adaptively allocate the transmission energy. The improved method has higher output SINR than the maximum SINR-based waveform. With improving the radar detection performance and obtaining more target information, the improved method can also reduce the computational complexity of maximum SINR-based waveform and improve the efficiency of computing Lagrange multipliers.

The future work will study the waveform optimization problems for joint radar and communication system when jammer exists. We will design the transmission waveforms for radar and jammer when the communication signal is considered as useful energy for radar. In addition, the performance of radar system will also be investigated in the joint radar and communication system.

Abbreviations

AIS	Automatic identification system
CFAR	Constant false alarm rate
CR	Cognitive radar
ECM	Electronic countermeasure
ECCM	Electronic counter-countermeasure
ESD	Energy spectral density
ESV	Energy spectrum variance
GLRT	Generalized likelihood ratio test
GN	Gauss–Newton
ISRJ	Interrupted-sampling repeater jamming
LFM	Linear frequency-modulated
MI	Mutual information
NE	Nash equilibrium
NLJ	Noise-like jammers
OFDM	Orthogonal frequency division multiplexing
PSD	Power spectral density
SINR	Signal-to-interference-plus-noise ratio
STAP	Space-time adaptive processing
TPZS	Two-person zero-sum

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Author contributions
All authors read and approved the final manuscript.

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Availability of data and materials
Data sharing is not applicable to this article as no datasets were generated or analyzed during the current study.

Declarations

Competing interests
The authors declare that they have no competing interests.

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References

1. S. Haykin, Cognitive radar: a way of the future. IEEE Signal Process. Mag. **23**(1), 30–40 (2006). https://doi.org/10.1109/MSP.2006.1593335
2. K.L. Bell, C.J. Baker, G.E. Smith, J.T. Johnson, M. Rangaswamy, Cognitive radar framework for target detection and tracking. IEEE J. Sel. Top. Signal Process. **9**(8), 1427–1439 (2015). https://doi.org/10.1109/JSTSP.2015.2465304
3. S. Brüggenwirth, M. Warnke, S. Wagner, K. Barth, Cognitive radar for classification. IEEE Aerosp. Electron. Syst. Mag. **34**(12), 30–38 (2019). https://doi.org/10.1109/MAES.2019.2958546
4. M.R. Bell, Information theory and radar waveform design. IEEE Trans. Inf. Theory **39**(5), 1578–1597 (1993). https://doi.org/10.1109/18.259642
5. R.A. Romero, N.A. Goodman, Waveform design in signal-dependent interference and application to target recognition with multiple transmissions. IET Radar Sonar Navig. **3**(4), 328–340 (2009). https://doi.org/10.1049/iet-rsn.2008.0146
6. R.A. Romero, J. Bae, N.A. Goodman, Theory and application of snr and mutual information matched illumination waveforms. IEEE Trans. Aerosp. Electron. Syst. **47**(2), 912–927 (2011). https://doi.org/10.1109/TAES.2011.5751234
7. V. Karimi, R. Mohseni, S. Samadi, OFDM waveform design based on mutual information for cognitive radar applications. J. Supercomput. **75**(5), 2518–2534 (2019). https://doi.org/10.1007/s11227-018-2648-3
8. V. Karimi, R. Mohseni, S. Samadi, Adaptive ofdm waveform design for cognitive radar in signal-dependent clutter. IEEE Syst. J. **14**(3), 3630–3640 (2020). https://doi.org/10.1109/JSYST.2019.2943809
9. Y. Liu, G. Liao, Z. Yang, Robust ofdm integrated radar and communications waveform design based on information theory. Signal Process. **162**, 317–329 (2019). https://doi.org/10.1016/j.sigpro.2019.05.001
10. N. Zhang, C. Wu, Y. Wu, N.N. Xiong, An improved target tracking algorithm and its application in intelligent video surveillance system. Multimed. Tools Appl. **79**(23–24), 15965–15983 (2020). https://doi.org/10.1007/s11042-018-6871-y
11. P. Chen, C. Qi, L. Wu, X. Wang, Waveform design for kalman filter-based target scattering coefficient estimation in adaptive radar system. IEEE Trans. Veh. Technol. **67**(12), 11805–11817 (2018). https://doi.org/10.1109/TVT.2018.2875314
12. Z.-J. Wu, C.-X. Wang, Y.-C. Li, Z.-Q. Zhou, Extended target estimation and recognition based on multimodel approach and waveform diversity for cognitive radar. IEEE Trans. Geosci. Remote Sens. **60**, 1–14 (2022). https://doi.org/10.1109/TGRS.2021.3065335
13. X. Zhang, K. Wang, X. Liu, Joint optimisation of transmit waveform and receive filter for cognitive radar. IET Radar Sonar Navig. **12**(1), 11–20 (2018). https://doi.org/10.1049/iet-rsn.2017.0107
14. M. Wu, N. Xiong, L. Tan, Adaptive range-based target localization using diffusion Gaussnewton method in industrial environments. IEEE Trans. Ind. Inf. **15**(11), 5919–5930 (2019). https://doi.org/10.1109/TII.2019.2909135
15. Q. Zhang, C. Zhou, Y.-C. Tian, N. Xiong, Y. Qin, B. Hu, A fuzzy probability Bayesian network approach for dynamic cybersecurity risk assessment in industrial control systems. IEEE Trans. Ind. Inf. **14**(6), 2497–2506 (2018). https://doi.org/10.1109/TII.2017.2768998
16. H. Li, J. Liu, K. Wu, Z. Yang, R.W. Liu, N. Xiong, Spatio-temporal vessel trajectory clustering based on data mapping and density. IEEE Access **6**, 58939–58954 (2018). https://doi.org/10.1109/ACCESS.2018.2866364
17. A. Martone, K. Gallagher, K. Sherbondy, A. Hedden, C. Dietlein, Adaptable waveform design for enhanced detection of moving targets. IET Radar Sonar Navig. **11**(10), 1567–1573 (2017). https://doi.org/10.1049/iet-rsn.2017.0125
18. S. Sen, Ofdm radar space-time adaptive processing by exploiting spatio-temporal sparsity. IEEE Trans. Signal Process. **61**(1), 118–130 (2013). https://doi.org/10.1109/TSP.2012.2222387
19. H.-S. Kim, N.A. Goodman, C.K. Lee, S.-I. Yang, Improved waveform design for radar target classification. Electron. Lett. **53**(13), 879–880 (2017). https://doi.org/10.1049/el.2017.0536
20. J. Liu, W. Liu, X. Chen, D. Orlando, A. Farina, Performance analysis of the generalized likelihood ratio test in general phased array radar configuration. IEEE Trans. Signal Process. **69**, 4544–4555 (2021). https://doi.org/10.1109/TSP.2021.3097658

21. P. Addabbo, S. Han, F. Biondi, G. Giunta, D. Orlando, Adaptive radar detection in the presence of multiple alternative hypotheses using Kullback-Leibler information criterion-part i: Detector designs. IEEE Trans. Signal Process. **69**, 3730–3741 (2021). https://doi.org/10.1109/TSP.2021.3089440
22. P. Addabbo, S. Han, F. Biondi, G. Giunta, D. Orlando, Adaptive radar detection in the presence of multiple alternative hypotheses using kullback-leibler information criterion-part ii: Applications. IEEE Trans. Signal Process. **69**, 3742–3754 (2021). https://doi.org/10.1109/TSP.2021.3089277
23. J. Liu, Z. Hou, W. Li, R. Tao, D. Orlando, H. Li, Multipixel anomaly detection with unknown patterns for hyperspectral imagery. IEEE Trans. Neural Netw. Learn. Syst. (2021). https://doi.org/10.1109/TNNLS.2021.3071026
24. P. Addabbo, S. Han, D. Orlando, G. Ricci, Learning strategies for radar clutter classification. IEEE Trans. Signal Process. **69**, 1070–1082 (2021). https://doi.org/10.1109/TSP.2021.3050985
25. J. Liu, D. Massaro, D. Orlando, A. Farina, Radar adaptive detection architectures for heterogeneous environments. IEEE Trans. Signal Process. **68**, 4307–4319 (2020). https://doi.org/10.1109/TSP.2020.3009836
26. L. Yan, P. Addabbo, Y. Zhang, C. Hao, J. Liu, J. Li, D. Orlando, A sparse learning approach to the detection of multiple noise-like jammers. IEEE Trans. Aerosp. Electron. Syst. **56**(6), 4367–4383 (2020). https://doi.org/10.1109/TAES.2020.2988960
27. S. Kay, Waveform design for multistatic radar detection. IEEE Trans. Aerosp. Electron. Syst. **45**(3), 1153–1166 (2009). https://doi.org/10.1109/TAES.2009.5259190
28. G. Rossetti, S. Lambotharan, Robust waveform design for multistatic cognitive radars. IEEE Access **6**, 7464–7475 (2018). https://doi.org/10.1109/ACCESS.2017.2782878
29. M. Ben Kilani, Y. Nijsure, G. Gagnon, G. Kaddoum, F. Gagnon, Cognitive waveform and receiver selection mechanism for multistatic radar. IET Radar Sonar Navig. **10**(2), 417–425 (2016). https://doi.org/10.1049/iet-rsn.2015.0319
30. L. Zhong, Y. Li, W. Cheng, W. Zhou, Robust cognitive radar tracking based on adaptive unscented Kalman filter in uncertain environments. IEEE Access **8**, 163405–163418 (2020). https://doi.org/10.1109/ACCESS.2020.3019837
31. M. Akcakaya, S. Sen, A. Nehorai, A novel data-driven learning method for radar target detection in nonstationary environments. IEEE Signal Process. Lett. **23**(5), 762–766 (2016). https://doi.org/10.1109/LSP.2016.2553042
32. R. Elwell, R. Polikar, Incremental learning of concept drift in nonstationary environments. IEEE Trans. Neural Netw. **22**(10), 1517–1531 (2011). https://doi.org/10.1109/TNN.2011.2160459
33. W. Zhou, J. Xie, G. Li, Y. Du, Robust CFAR detector with weighted amplitude iteration in nonhomogeneous sea clutter. IEEE Trans. Aerosp. Electron. Syst. **53**(3), 1520–1535 (2017). https://doi.org/10.1109/TAES.2017.2671798
34. X. Zhang, C. Cui, Robust transmitted waveform and received filter design for cognitive radar in the presence of signal-dependent interference. Circuits Syst. Signal Process. **32**(6), 3013–3029 (2013). https://doi.org/10.1007/s00034-013-9611-6
35. W. Yuxi, H. Guoce, L. Wei, Waveform design for radar and extended target in the environment of electronic warfare. J. Syst. Eng. Electron. **29**(1), 48–57 (2018). https://doi.org/10.21629/JSEE.2018.01.05
36. D.J. Bachmann, R.J. Evans, B. Moran, Game theoretic analysis of adaptive radar jamming. IEEE Trans. Aerosp. Electron. Syst. **47**(2), 1081–1100 (2011). https://doi.org/10.1109/TAES.2011.5751244
37. Y. Chen, S. Weng, W. Guo, N. Xiong, A game theory algorithm for intra-cluster data aggregation in a vehicular ad hoc network. SENSORS (2016). https://doi.org/10.3390/s16020245
38. K. Li, B. Jiu, H. Liu, Game theoretic strategies design for monostatic radar and jammer based on mutual information. IEEE Access **7**, 72257–72266 (2019). https://doi.org/10.1109/ACCESS.2019.2920398
39. K. Li, B. Jiu, H. Liu, S. Liang, Waveform design for cognitive radar in presence of jammer using Stackelberg game. J. Eng. **2019**(21), 7581–7584 (2019). https://doi.org/10.1049/joe.2019.0621
40. C. Knill, B. Schweizer, C. Waldschmidt, Interference-robust processing of OFDM radar signals using compressed sensing. IEEE Sens. Lett. **4**(4), 1–4 (2020). https://doi.org/10.1109/LSENS.2020.2980165
41. E. Giusti, A. Capria, M. Martorella, C. Moscardini, F. Berizzi, Electronic countermeasure for ofdm-based imaging passive radars. IET Radar Sonar Navig. **13**(9, SI), 1458–1467 (2019). https://doi.org/10.1049/iet-rsn.2018.5629.
42. Q.J.O. Tan, R.A. Romero, Jammer-nulling transmit-adaptive radar against knowledge-based jammers in electronic warfare. IEEE Access **7**, 181899–181915 (2019). https://doi.org/10.1109/ACCESS.2019.2960012
43. Q. Wu, F. Zhao, J. Wang, X. Liu, S. Xiao, Improved isrj-based radar target echo cancellation using frequency shifting modulation. Electronics (2019). https://doi.org/10.3390/electronics8010046
44. P. Addabbo, O. Besson, D. Orlando, G. Ricci, Adaptive detection of coherent radar targets in the presence of noise jamming. IEEE Trans. Signal Process. **67**(24), 6498–6510 (2019). https://doi.org/10.1109/TSP.2019.2954499
45. Z. Wei, Z. Liu, B. Peng, R. Shen, Eccm scheme against interrupted sampling repeater jammer based on parameter-adjusted waveform design. SENSORS (2018). https://doi.org/10.3390/s18041141
46. L. Yan, P. Addabbo, C. Hao, D. Orlando, A. Farina, New eccm techniques against noiselike and/or coherent interferers. IEEE Trans. Aerosp. Electron. Syst. **56**(2), 1172–1188 (2020). https://doi.org/10.1109/TAES.2019.2929968
47. A. Garnaev, W. Trappe, A. Petropulu, A prospect theoretic look at a joint radar and communication system, in *Internet of Things, Smart Spaces, and Next Generation Networks and Systems, NEW2AN 2018*. ed. by O. Galinina, S. Andreev, S. Balandin, Y. Koucheryavy (Springer, Cham, 2018), pp.483–495
48. B. Wang, X. Chen, F. Xin, X. Song, Sinr and mi-based maximin robust waveform design. Entropy (2019). https://doi.org/10.3390/e21010033

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