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RESEARCH

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An efficient computation of discrete orthogonal moments for bio-signals reconstruction

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Abstract

Bio-signals are extensively used in diagnosing many diseases in wearable devices. In signal processing, signal reconstruction is one of the essential applications. Discrete orthogonal moments (DOMs) are effective analysis tools for signals that can extract digital information without redundancy. The propagation of numerical errors is a significant challenge for the computation of DOMs at high orders. This problem damages the orthogonality property of these moments, which restricts the ability to recover the signal's distinct and unique components with no redundant information. This paper proposes a stable computation of DOMs based on QR decomposition methods: the Gram–Schmidt, Householder, and Given Rotations methods. It also presents a comparative study on the performance of the types of moments: Tchebichef, Krawtchouk, Charlier, Hahn, and Meixner moments. The proposed algorithm's evaluation is done using the MIT-BIH arrhythmia dataset in terms of mean square error and peak signal to noise ratio. The results demonstrate the superiority of the proposed method in computing DOMs, especially at high moment orders. Moreover, the results indicate that the Householder method outperforms Gram–Schmidt and Given Rotations methods in execution time and reconstruction quality. The comparative results show that Tchebichef, Krawtchouk, and Charlier moments have superior reconstruction quality than Hahn and Meixner moments, and Tchebichef generally has the highest performance in signal reconstruction.

Keywords: Discrete orthogonal moments, Bio-signal reconstruction, Orthogonalization methods, Propagation error, Discrete orthogonal polynomials

1 Introduction

With the explosive growth in computer technology and signal analysis tools, computer-aided analysis of bio-signals has become a common part of clinical. Bio-signal reconstruction is one of the important applications in bio-signal processing. The study of the literature on image analysis techniques indicates that the method of orthogonal moments plays a significant role in each of its important fields. These fields include image reconstruction [1, 2], face recognition [3], image classification [4, 5], image watermarking [6], image encryption [7], image compression [8, 9], color stereo image analysis [10]. Orthogonal moments are classified as continuous or discrete depending on whether the kernel functions are orthogonal in the continuous or discrete domain. Continuous orthogonal can be utilized to characterize an image with minimal redundant information. But even so, computing these moments needs a coordinate transformation and an estimate of the continuous moment's integrals. This adds computational complexity and introduces approximation errors [11]. To this end, many researchers have started to use discrete orthogonal moments [12, 13]. Zhu et al. [14, 15] demonstrated that discrete orthogonal moments are more effective than continuous orthogonal moments at representing images. The types of discrete orthogonal moments (DOMs) according to their corresponding discrete orthogonal polynomials include Tchebichef [16, 17], Krawtchouk[18–21], Charlier [22–24], Hahn [25, 26] and Meixner [27, 28] moments. At the present time, Discrete Orthogonal Moments (DOMs) are gaining popularity in analyzing one-dimensional signals due to their effectiveness in capturing digital information without redundancy. In order to compute DOMs, we have to compute Kernel discrete orthogonal polynomials (DOPs).

The computation of high-order DOPs faces a major problem which is the propagation of numerical errors. This problem destroys the orthogonality property of these polynomials, which affects the ability to extract the signal's distinct and unique components with no information redundancy. To address this problem, we propose using QR decomposition methods to maintain the orthogonality property by re-orthonormalization DOPs. There are many ways for QR decomposition, like the Gram–Schmidt method, the Householder method, and the Given Rotations method [29]. These methods maintain the high-order DOPs orthogonality property effectively. Thus, using the DOMs to analyze large-size signals will become highly efficient Due to significant improvements in the computation of DOPs. Our paper presents several contributions that can be summarized as follows:

- Testing Discrete Orthogonal Moments (DOMs) in bio-signals analysis and reconstruction.
- Proposing a new modified version of DOPs by the QR decomposition methods like the Gram–Schmidt, Householder, and Given Rotations methods. In addition to comparing methods of QR decomposition to estimate the best methods.
- Presenting comparative study between the different types of moments to estimate the best moment in analyzing and reconstructing bio-signals.

The rest of the paper is structured into five sections: Sect. 2 outlines the Recursive relation of Discrete Orthogonal Polynomials (DOPs). Discrete Orthogonal Moments (DOMs) will be discussed in Sect. 3. Section 4 shows the proposed procedure for ensuring the orthogonality property of discrete polynomials. Experimental results and discussion are presented in Sect. 5. Discussions are shown in Sect. 6. In Sect. 7, we conclude our work.

2 Recursive relation of discrete orthogonal polynomials (DOPs)

The discrete orthogonal polynomials are the polynomial solutions of the given difference equation

Table 1 Values of A, B, C, D, and E for each of the polynomial set Tchebichef $t_n(x;N)$, Krawtchouk $k_n(x;P,N)$, Charlier $c_n(x)$, Hahn $h_n^{(\alpha,\beta)}(x)$ and Meixner $m_n^{(a,b)}(x)$

	$t_n(x;N)$	$k_n(x;P,N)$	$c_n(x)$	$h_n^{(\alpha,\beta)}(x)$	$m_n^{(a,b)}(x)$
A	$\frac{n}{2(2n-1)}$	n	$-a_1$	$\frac{n}{(\alpha+\beta+2n-1)} \times \frac{(\alpha+\beta+n)}{(\alpha+\beta+2n)}$	$\frac{b}{b-1}$
B	$x-\frac{N-1}{2}$	$x-n+1-p(N-2n+2)$	$x-n+1-a_1$	$x-\frac{\alpha-\beta+2N-2}{4} - \frac{(\beta^2-\alpha^2)(\alpha+\beta+2N)}{4(\alpha+\beta+2n-2)(\alpha+\beta+2n)}$	$\frac{x-xb-n+1-bn+b-ab}{1-b}$
C	$-\frac{(n-1)\left[N^2-(n-1)^2\right]}{2(2n-1)}$	$-p(1-p)(N-n+2)$	$n-1$	$-\frac{(\alpha+n-1)(\beta+n-1)}{(\alpha+\beta+2n-2)} \times \frac{(\alpha+\beta+N+n-1)(N-n+1)}{(\alpha+\beta+2n-1)}$	$\frac{(n-1)(n-2+a)}{1-b}$
D	$\sqrt{\frac{(2n+1)}{(N^2-n^2)(2n-1)}}$	$\sqrt{\frac{n}{p(1-p)(N-n+1)}}$	$\sqrt{\frac{a_1}{n}}$	$\sqrt{\frac{n(\alpha+\beta+n)(\alpha+\beta+2n+1)}{(\alpha+n)(\beta+n)(\alpha+\beta+n+N)(N-n)(\alpha+\beta+2n-1)}}$	$\sqrt{\frac{b}{n(a+n-1)}}$
E	$\sqrt{\frac{2n+1}{(N^2-n^2)\left[N^2-(n-1)^2\right](2n-3)}}$	$\sqrt{\frac{n(n-1)}{(p(1-p))^2(N-n+2)(N-n+1)}}$	$\sqrt{\frac{a_1^2}{n(n-1)}}$	$\sqrt{\frac{n(n-1)(\alpha+\beta+n)}{(\alpha+n)(\alpha+n-1)(\beta+n)(\beta+n-1)(N-n+1)(N-n)}} \times \sqrt{\frac{(\alpha+\beta+n-1)(\alpha+\beta+2n+1)}{(\alpha+\beta+2n-3)(\alpha+\beta+n+N)(\alpha+\beta+n+N-1)}}$	$\sqrt{\frac{b^2}{n(n-1)(a+n-2)(a+n-1)}}$

$$\sigma(x)\Delta\nabla p_n(x) + \tau(x)\Delta p_n(x) + \lambda_n p_n(x) = 0 \tag{1.a}$$

where $\Delta p_n(x) = p_n(x+1) - p_n(x) and \nabla p_n(x) = p_n(x) - p_n(x-1)$ indicates backward finite-difference operator and forward finite difference operator, respectively. $\sigma(x)$ and $\tau(x)$ denote first and second degree functions. λ_n indicate a suitable constant.

The polynomials $p_n(x)$ satisfy an orthogonality relation of the form

$$\sum_{x=0}^{s} p_n(x)p_m(x)w(x) = d_n^2 \cdot \delta_{mn}, \quad 0 \le m, n \le s \tag{1.b}$$

where $w(x)$ is the weight function, d_n^2 denotes the square of the norm of the corresponding orthogonal polynomials and δ_{mn} denotes the Dirac function. The normalized orthogonal polynomials can be obtained by utilizing the square norm and weighted function

$$\widetilde{p}_n(x) = p_n(x)\sqrt{\frac{w(x)}{d_n^2}}, \quad n = 0, 1, \ldots, s \tag{1.c}$$

Therefore, the orthogonal property of normalized orthogonal polynomials in (1b) can be rewritten as

$$\sum_{x=0}^{s} \widetilde{p}_m(x)\widetilde{p}_n(x) = \delta_{mn}, \quad 0 \le m, n \le s \tag{1.d}$$

A general formula for getting the normalized discrete orthogonal polynomials $\widetilde{p}_n(x)$ of order n is defined three-term recursive relation as follows [11]:

$$\widetilde{p}_n(x) = \left(\frac{B * D}{A}\right)\widetilde{p}_{n-1}(x) + \left(\frac{C * E}{A}\right)\widetilde{p}_{n-2}(x) \tag{1.e}$$

where A, B, C, D, and E are coefficients independent of each polynomial set shown in Table 1. These discrete orthogonal polynomials include Tchebichef$t_n(x; N)$, Krawtchouk $k_n(x; P, N)$, Charlier $c_n(x)$, Hahn $h_n^{(\alpha,\beta)}(x)$ and Meixner $m_n^{(a,b)}(x)$ Polynomials.$\widetilde{p}_{n-1}(x)$ and $\widetilde{p}_{n-2}(x)$ are the zero-order and first-order polynomials, respectively.

2.1 Tchebichef polynomials

The nth Tchebichef polynomials $t_n(x)$ are defined by hypergeometric function as the follows

$$t_n(x) = (1-N)_n {}_3F_2(-n, -x, 1+n; 1, 1-N; 1), \; n, x = 0, 1, 2, \ldots, N-1. \tag{2}$$

From Eq. (1.e) and Table 1, we obtain the recursive relation of discrete orthogonal Tchebichef polynomials as follows:

$$t_n(x) = (\beta_1 x + \beta_2)t_{n-1}(x) + \beta_3 t_{n-2}(x) \tag{3}$$

with

$$
\begin{aligned}
t_0(x) &= \frac{1}{\sqrt{N}}, \\
t_1(x) &= (2x + 1 - N)\sqrt{\frac{3}{N\left(N^2 - 1\right)}}.
\end{aligned} \tag{4}
$$

where

$$
\begin{aligned}
\beta_1 &= \frac{2}{n}\sqrt{\frac{4n^2 - 1}{N^2 - n^2}} \\
\beta_2 &= \frac{1 - N}{n}\sqrt{\frac{4n^{2-1}}{N^2 - n^2}}, \\
\beta_3 &= \frac{1 - n}{n}\sqrt{\frac{2n + 1}{2n - 3}}\sqrt{\frac{N^2 - (n - 1)^2}{N^2 - n^2}}.
\end{aligned} \tag{5}
$$

2.2 Krawtchouk polynomials

Krawtchouk polynomials $k_n(x, p)$ of order n are defined by hypergeometric function as the follows

$$
k_n(x, p) =_2 F_1\left(-n, -x; -N; \frac{1}{p}\right), \quad n, x = 0, 1, 2, \ldots, N - 1. \tag{6}
$$

The recursive relation of discrete orthogonal Krawtchouk polynomials can be calculated using Eq. (1.e) and Table 1 as:

$$
k_n(x, p) = (x + \beta_1)\beta_2 k_{n-1}(x, p) - \beta_3 k_{n-2}(x, p) \tag{7}
$$

with

$$
\begin{aligned}
k_0(x, p) &= \sqrt{\frac{N! p^x (1 - p)^{N - x}}{x!(N - x)!}}, \\
k_1(x, p) &= (-p(N - x) + x(1 - p)) \times \sqrt{\frac{(N - 1)! p^{x - 1} (1 - p)^{N - x - 1}}{x!(N - x)!}}
\end{aligned} \tag{8}
$$

where

$$
\begin{aligned}
\beta_1 &= (1 - n - p\,(N - 2n + 2)), \\
\beta_2 &= \sqrt{\frac{1}{p(1 - p)(N - n + 1)n}} \\
\beta_3 &= \sqrt{\frac{(N - n + 2)(n - 1)}{(N - n + 1)n}}, \quad 0 < p < 1.
\end{aligned} \tag{9}
$$

2.3 Charlier polynomials

Charlier polynomials $C_n^{a_1}(x)$ of order n are defined by hypergeometric function as the follows

$$c_n^{a_1}(x) = {}_2F_0(-n, -x; ; -1/a_1), \quad n, x = 0, 1, 2, \ldots, N-1 \text{ and } a_1 > 0 \tag{10}$$

By substituting coefficients of A, B, C, D, and E from Table 1 in Eq. (1.e), we conclude the recursive relation of discrete orthogonal Charlier polynomials as follows:

$$C_n^{a_1}(x) = (\beta_1 - x)\beta_2 C_{n-1}^{a_1}(x) - \beta_3 C_{n-2}^{a_1}(x) \tag{11}$$

with

$$\begin{aligned} c_0^{a_1}(x) &= \sqrt{\frac{e^{-a_1} a_1^x}{x!}}, \\ C_1^{a_1}(x) &= \frac{a_1 - x}{a_1}\sqrt{\frac{e^{-a_1} a_1^{x+1}}{x!}} \end{aligned} \tag{12}$$

where

$$\begin{aligned} \beta_1 &= (a_1 + n - 1), \\ \beta_2 &= \sqrt{\frac{1}{n a_1}}, \\ \beta_3 &= \sqrt{\frac{n-1}{n}}. \end{aligned} \tag{13}$$

2.4 Hahn polynomials

The nth Hahn polynomials $h_n^{(\alpha,\beta)}(x)$ are defined by hypergeometric function as the follows.

$$\begin{aligned} h_n^{(\alpha,\beta)}(x) = &\frac{(-1)^n(\beta+1)_n(N-n)_n}{n!} \times {}_3F_2(-n, -x, n+1+\alpha+\beta; \beta+1, 1-N; 1), \\ &n, x = 0, 1, 2, \ldots, N-1. \end{aligned} \tag{14}$$

By substituting coefficients of A, B, C, D, and E from Table 1 in Eq. (1.e), we conclude the recursive relation of discrete orthogonal Charlier polynomials as follows:

$$h_n^{(\alpha,\beta)}(x) = (x - \beta_1)\beta_2 h_{n-1}^{(\alpha,\beta)}(x) - \beta_3 h_{n-2}^{(\alpha,\beta)}(x) \tag{15}$$

with

$$\begin{aligned} h_0^{(\alpha,\beta)}(x) &= \sqrt{\frac{(\alpha+1)_\beta(\alpha+\beta+1)}{(N-\alpha)_{\beta+1}}} \\ h_1^{(\alpha,\beta)}(x) &= (\alpha+\beta+2)x - (\beta+1)(N-1) \times \sqrt{\frac{\alpha+\beta+3}{(\alpha+1)(\beta+1)(n-1)(N+1+\beta+1)}}. \end{aligned} \tag{16}$$

where

$$\begin{aligned}
\beta_1 &= \frac{\alpha-\beta+2N-2}{4} + \frac{\left(\beta^2-\alpha^2\right)(\alpha+\beta+2N)}{4(\alpha+\beta+2n-2)(\alpha+\beta+2n)},\\
\beta_2 &= \sqrt{\frac{(\alpha+\beta+2n)^4-(\alpha+\beta+2n)^2}{n(N-n)(\alpha+n)(\beta+n)(\alpha+\beta+n+N)(\alpha+\beta+n)}},\\
\beta_3 &= \frac{\alpha+\beta+2n}{\alpha+\beta+2n-2} \times \sqrt{\frac{(n-1)(\alpha+n-1)(\beta+n-1)(N-n+1)}{n(\alpha+n)(\beta+n)(N-n)(\alpha+\beta+n)}}\\
&\quad \times \sqrt{\frac{(\alpha+\beta+n-1)(\alpha+\beta+2n+1)(\alpha+\beta+N+n-1)}{(\alpha+\beta+2n-3)(\alpha+\beta+n+N)}}.
\end{aligned} \tag{17}$$

where $\alpha, \beta > 0$.

2.5 Meixner polynomials

Meixner polynomials $m_n^{(a,b)}(x)$ of order n are defined by hypergeometric function as the follows

$$m_n^{(a,b)}(x) = (a)_n {}_2F_1(-n,-x;a;1-1/b),\ n,x = 0,1,2,\ldots,N-1. \tag{18}$$

From Eq. (1.e) and Table 1, we obtain the recursive relation of discrete orthogonal Meixner polynomials as follows:

$$m_n^{(a,b)}(x) = (x\beta_1+\beta_2)m_{n-1}^{(a,b)}(x) - \beta_3 m_{n-2}^{(a,b)}(x) \tag{19}$$

with

$$\begin{aligned}
m_0^{(a,b)}(x) &= \sqrt{\frac{b^x(a+x-1)!}{x!(a-1)!}(1-b)^a},\\
m_1^{(a,b)}(x) &= \left(a+x-\frac{x}{b}\right) \times \sqrt{\frac{b^x(a+x-1)!}{x!(a-1)!}\frac{b(1-b)^a}{a}}.
\end{aligned} \tag{20}$$

where

$$\begin{aligned}
\beta_1 &= (b-1)\sqrt{\frac{1}{n(a+n-1)b}},\\
\beta_2 &= (n-1+bn-b+ab)\sqrt{\frac{1}{n(a+n-1)b}},\\
\beta_3 &= \sqrt{\frac{(n-1)(n-2+a)}{n(a+n-1)}},
\end{aligned} \tag{21}$$

where $0 < b < 1$ and $a > 0$.

3 Discrete orthogonal moments (DOMs)

The discrete orthogonal moments are a set of moments calculated by discrete orthogonal polynomials. The set of discrete orthogonal one-dimensional (1D) moments are defined as follows [11]:

$$M_n = \sum_{x=0}^{N-1} p_n(x)s(x), \quad n = 0, 1 \ldots, N-1. \tag{22}$$

where $s(x)$ is a one-dimensional signal of size $1 \times N$, M_n is a set of moment coefficients of the signal $s(x)$ and $p(x)$ is orthogonal polynomials of order n (Tchebichef $t_n(x; N)$, Krawtchouk $k_n(x; P, N)$, Charlier $c_n(x)$, Hahn $h_n^{(\alpha,\beta)}(x)$ and Meixner $m_n^{(a,b)}(x)$).

The reconstructed signal $S(x)$ is calculated from the inverse transformation of the orthogonal moment as follows:

$$S(x) = \sum_{n=0}^{N-1} M_n p_n(x), \quad x = 0, 1, 2, \ldots, N-1 \tag{23}$$

Using the following matrix form decreases the time and complexity of 1D orthogonal moment computations significantly:

$$M_n = \begin{bmatrix} p_0(0) & p_0(1) & \ldots & p_0(N-1) \\ p_1(0) & p_1(1) & \ldots & p_1(N-1) \\ \vdots & \vdots & \vdots & \vdots \\ p_n(0) & p_n(1) & \ldots & p_n(N-1) \end{bmatrix} \times \begin{bmatrix} s(0) \\ s(1) \\ \vdots \\ s(N-1) \end{bmatrix} \tag{24}$$

where M_n indicates orthogonal polynomials of order n , s denotes $1 \times N$signal vector.

4 Ensuring the orthogonality property of discrete polynomials

In this section, we propose a procedure for ensuring the orthogonality property of discrete polynomials. According to the orthogonality property, polynomials matrix $(p_n(x))$ satisfies the following relation:

$$p_n(x)^T p_n(x) = I_n \tag{25}$$

where I_n denotes the identity matrix.

To avoid numerical errors propagation and preserve the orthogonality property of DOPs, we present an efficient method for re-orthonormalizing $p_n(x)$ matrix columns using QR decomposition methods. In these methods, a matrix $A = [u_1 ,u_2, \ldots, u_{n-1}, u_n]$ of size $n \times m$ factored as$A = QR$, where Q is an $n \times m$ matrix with orthogonal columns ($Q^T Q = I$) and R is an $m \times m$ upper triangular matrix [29]. In our situation, R matrix contains just recursive computation errors. The primary purpose of these ways is to generate the orthogonal $Q(n \times m)$ matrix from $p_n(x)$ that contains round-off errors. Many ways are used in QR decomposition, such as the Gram–Schmidt method, the Householder method, and the Given Rotations method [30].

4.1 Computation DOPs with modified Gram–Schmidt method (MGSM)

One of the most common algorithms for applying QR decomposition is the Gram–Schmidt (GS) method. It is a simple procedure for generating an orthogonal or orthonormal basis for any nonzero R^n subspace [31]. Although the modified Gram–Schmidt method is always preferred because it avoids potentially costly cancellation errors, it is not as good numerically as the Givens or Householder approaches [29]. Algorithm 1 summarizes the proposed implementation of DOPs using MGSM.

Algorithm 1: The Proposed Algorithm for computing DOPs using MGSM.

Inputs: N: the maximum value of the variable x; N_{max} : the order of polynomials.
Output: Q : the orthonormalized DOPs matrix using MGSM.

Step 1	**for** $x \leftarrow 0$ to $N-1$ **do**
	Compute $p_0(x)$ and $p_1(x)$ using Eqs. (4), (8), (12), (16), and (20) according to types of polynomials used.
	for $n \leftarrow 2$ to $N_{max}-1$ **do**
	Compute $p_n(x)$ using Eqs. (3), (7), (11), (15), and (19) according to the types of polynomials used.
	end for
	$p = p_n(x)$

Step 2	**for** $i \leftarrow 1$ to N **do**
	$Q_{1:N,i} = p_{1:N,i}$
	for $j \leftarrow 0$ to $i-1$ **do**
	$Q_{1:N,i} = Q_{1:N,i} - Q_{1:N,j}{}^{T} Q_{1:N,i} Q_{1:N,j}$
	end for
	$Q_{1:N,i} = Q_{1:N,i}/\lVert Q_{1:N,i}\rVert_2$
	end for
	end for

4.2 Computation DOPs with Householder method (HM)

The main way to apply QR decomposition is with the Householder method. [29]. This approach is regarded to be more numerically stable than the Gram–Schmidt orthogonalization method for QR matrix decomposition. The proposed computation of DOPs with HM is illustrated in Algorithm 2.

Algorithm 2: The Proposed Algorithm for computing DOPs using HM.

Inputs: N: the maximum value of the variable x; N_{max} : the order of polynomials.
Output: Q : the orthonormalized DOPs matrix using HM.

Step 1	**for** $x \leftarrow 0$ to $N-1$ **do**
	Compute $p_0(x)$ and $p_1(x)$ using Eqs. (4), (8), (12), (16), and (20) according to the types of polynomials used.
	for $n \leftarrow 2$ to $N_{max}-1$ **do**
	Compute $p_n(x)$ using Eqs. (3), (7), (11), (15), and (19) according to the types of polynomials used.
	end for
	$p = p_n(x)$

Step 2	**for** $i \leftarrow 1$ to N **do**
	$u = p_{i:N,i}$
	$v_i = sign(u_1)\lVert u\rVert_2 I_1 + u$
	$v_i = v_i / \lVert v_i\rVert_2$
	$Q_{i:N,i:N} = Q_{i:N,i:N} - 2v_i(v_i{}^{T} Q_{i:N,i:N})$
	end for
	end for

4.3 Computation DOPs with Given Rotations method (GRM)

The Given Rotations method is an alternative to the Modified Gram–Schmidt method and Householder method for calculating QR decomposition [29]. The Proposed Algorithm for computing DOPs using GRM is reported in Algorithm 3.

Algorithm 3: The Proposed Algorithm for computing DOPs using GRM.

Inputs: N: the maximum value of the variable x; N_{max} : the order of polynomials. **Output:** Q : the orthonormalized DOPs matrix using GRM.	

Step 1	**for** $x \leftarrow 0$ to $N-1$ **do**
	Compute $p_0(x)$ and $p_1(x)$ using Eqs. (4), (8), (12), (16), and (20) according to the types of polynomials used.
	for $n \leftarrow 2$ to $N_{max}-1$ **do**
	Compute $p_n(x)$ using Eqs. (3), (7), (11), (15), and (19) according to the types of polynomials used.
	end for
	$p = p_n(x)$

Step 2	$Q = I_N$
	for $j \leftarrow 1$ to N **do**
	for $i \leftarrow 1: (-1): j+1$ **do**
	$u = p_{1:N,j}$
	if $\sqrt{u_{i-1}^2 + u_i^2} \neq 0$
	$x = \frac{u_{i-1}}{\sqrt{u_{i-1}^2+u_i^2}}; \; y = \frac{-u_i}{\sqrt{u_{i-1}^2+u_i^2}}$
	$G_{(i-1):i,(i-1):i} = \begin{pmatrix} x & y \\ -y & x \end{pmatrix}$
	$Q = Q \times G$
	end if
	end for
	end for
	end for

5 Results

The experiments of this study are performed on a personal computer using Matlab Software (version R2014a) on Microsoft Windows 7, 32-bit Edition, Intel Core i3 processor, and 4 GB RAM machine. Performance evaluation has been done by ECG signals from MIT-BIH arrhythmia dataset [32], which contain cardiac information from large numbers of patients. These recordings were obtained at a sampling frequency of 360 Hz (360 samples per second) with 11-bit resolution. Our numerical simulations are presented in three sections: the first is to evaluate the performance of the proposed re-orthonormalization methods in the quality of reconstruction signals. The second compares the three proposed re-orthogonalization methods (Gram–Schmidt, Householder, and Given Rotations) in signal reconstruction quality. The third is a comparative study on the performance of Discrete Orthogonal Moments in signal reconstruction. The quality of the reconstructed signal is evaluated based on the following criteria:

Table 2 Comparison of reconstruction errors (PSNR and MSE) using Tchebichef moments with and without Householder method

Signal	Tchebichef		Tchebichef + HM	
	PSNR	MSE	PSNR	MSE
MIT-BIH Rec. 100	96.240	0.0810	101.685	0.0432
MIT-BIH Rec. 101	81.467	0.4601	106.854	0.0249
MIT-BIH Rec. 103	72.23	1.485	93.21	0.132
MIT-BIH Rec. 105	86.795	0.269	108.477	0.022
MIT-BIH Rec. 107	92.450	0.0810	106.437	0.160
MIT-BIH Rec. 108	68.234	1.9340	90.329	0.152
MIT-BIH Rec. 111	83.444	0.1618	100.245	0.0234
MIT-BIH Rec. 112	96.549	0.5013	118.381	0.0406
MIT-BIH Rec. 115	105.346	0.0740	116.436	0.0194
MIT-BIH Rec. 117	89.419	1.0391	109.422	0.1039
MIT-BIH Rec. 118	96.432	0.6840	113.102	0.1004
MIT-BIH Rec. 119	108.504	0.1434	126.726	0.0176
MIT-BIH Rec. 121	106.844	0.1150	118.520	0.0300
MIT-BIH Rec. 122	108.330	0.1217	119.917	0.0320
MIT-BIH Rec. 124	84.110	1.8815	121.8111	0.0245
MIT-BIH Rec. 200	77.677	1.1309	94.876	0.1561
MIT-BIH Rec. 201	85.309	0.1576	113.945	0.0058
MIT-BIH Rec. 202	90.079	0.0644	105.621	0.0108
MIT-BIH Rec. 205	118.605	0.0080	87.028	0.3050
MIT-BIH Rec. 208	103.437	0.0784	126.499	0.0055
MIT-BIH Rec. 209	65.139	1.6773	100.652	0.0281
MIT-BIH Rec. 213	82.214	1.6419	109.599	0.0702
MIT-BIH Rec. 214	78.031	1.2031	104.048	0.0602
MIT-BIH Rec. 219	106.807	0.1093	121.013	0.0213
MIT-BIH Rec. 222	83.8151	0.0987	103.385	0.0104
MIT-BIH Rec. 228	89.1591	0.1367	108.908	0.0141
MIT-BIH Rec. 230	75.499	1.444	112.965	0.0193
MIT-BIH Rec. 231	88.234	0.1435	115.863	0.0060
MIT-BIH Rec. 233	80.685	1.5180	114.101	0.0324
MIT-BIH Rec. 234	71.922	0.9895	104.380	0.0236
Average	89.100	0.6477	109.147	0.0564

- Peak signal to noise ratio (PSNR):

  PSNR is the highest possible signal power ratio to the corrupting noise power. It is presented as follows:

$$\mathrm{PSNR} = 20 \times log_{10} \frac{max|s(x)|}{\sqrt{MSE}}$$

- Mean-Square Error (MSE): the reconstruction error between the original and reconstructed signals.

$$MSE = \frac{1}{N} \sum_{x=0}^{N-1} (s(x) - S(x))^2$$

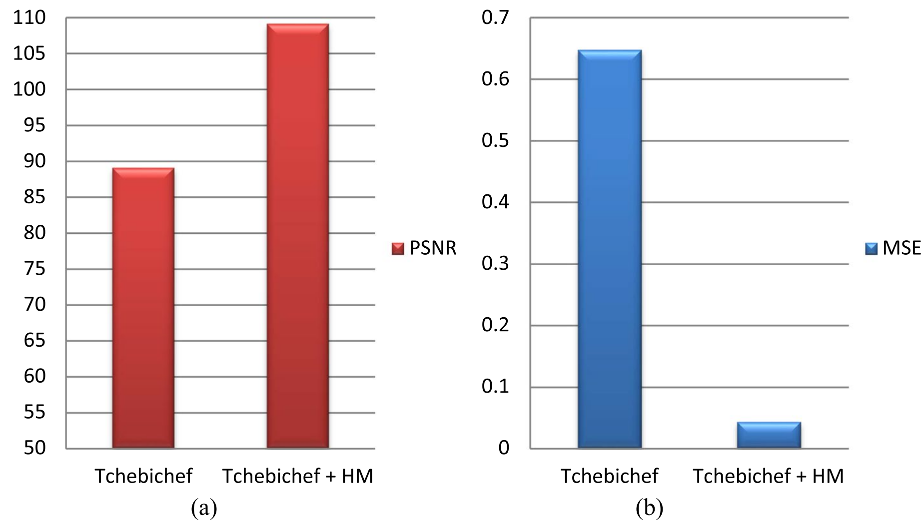

Fig. 1 The average values **a** PSNR and **b** MSE of the reconstructed signals using Tchebichef moments with and without Householder method

where $s(x)$ and $S(x)$ are the original signal and reconstructed signal, respectively.

In this experiment, the parameters of polynomials are set as $p = 0.5$ for Krawtchouk, $a_1 = 140$ for Charlier,$\alpha, \beta = 100$ for Hahn, and $a = 512, b = 0.5$ for Meixner. The signal size is N = 3600, and the order of the DOMs used is 200.

5.1 Reconstruction quality of DOMs computed using the proposed re-orthonormalization method

We started by investigating the superiority of the proposed re-orthogonalization methods with the discrete orthogonal moments in reconstruction quality signals. As shown in Table 2, we test the Tchebichef moments with and without the Householder method as one of the proposed methods for reconstructing the signals. The results obtained in Table 2 show that using the Householder method significantly improves the reconstruction quality of all records used. Tchebichef moments with Householder provide a high Peak signal to noise ratio (PSNR) with very low Mean-Square Error (MSE) values compared to Tchebichef moments. The average reconstruction errors PSNR and MSE of the proposed methods are 109.147 and 0.0564, respectively, as reported in Table 2. Figure 1 presents the reconstructed signal's reconstruction errors (PSNR, MSE) using Tchebichef moments with and without Householder method. It confirms the superiority of the proposed methods in reconstructed signals. Figure 2 shows the reconstructed "Rec. 107" signal by Tchebichef moments with and without the Householder method.

5.2 Comparison of reconstruction quality for the proposed re-orthonormalization methods

In the previous section, we investigated the ability of the proposed procedure to maintain the orthogonality property of the discrete polynomials in the reconstruction of the signal. There are three methods in the proposed procedure mentioned, and they are the Gram–Schmidt method (MGSM), the Householder method (HM), and the Given

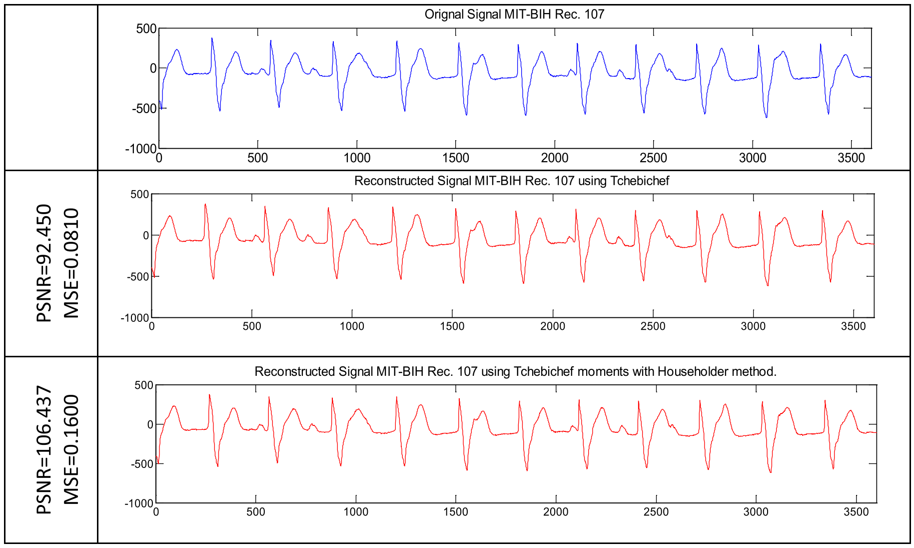

Fig. 2 Set of reconstructed "Rec. 107" signal using Tchebichef moments with and without the Householder method

Rotations method (GRM). This section will investigate which of the three methods is preferable in signal reconstruction quality and execution time. We have used Tchebichef moments with the three proposed methods (MGSM, HM, and GRM) to reconstruct the signals and summarized the results in Table 3. Figure 3 also illustrates the reconstruction errors (PSNR, MSE) of the three proposed methods using Tchebichef moments. The results displayed in Table 3 and Fig. 3 demonstrate outperforming HM on MGSM and GRM in PSNR and MSE on all records used. The reconstructed "Rec. 115" signal by using Tchebichef moments with Gram–Schmidt, Householder, and Given Rotations methods are shown in Fig. 4.

We compare the execution time of HM on MGSM and GRM to discover which of the three methods is best in terms of execution time, as shown in Fig. 5. The visual inspection from Fig. 5 indicates that HM is faster than MGSM and GRM.

5.3 Comparison of reconstruction quality for DOMs

This section determines which of the different types of moments is the best in the quality of the reconstructed signals. The compression performance of the discrete orthogonal moments (Tchebichef, Krawtchouk, Charlier, Hahn, and Meixner) in signal reconstruction is presented in Table 4. In these experiments, the Householder method is used to preserve the orthogonality property methods in discrete orthogonal moments. Table 4 illustrates the resulting PSNR and MSE as reconstruction error metrics for 30 records from MIT-BIH arrhythmia dataset. The obtained results generally indicate that Tchebichef, Krawtchouk, and Charlier are superior to Hahn and Meixner in terms of PSNR and MSE. As for the three methods, Tchebichef, Krawtchouk, and Charlier, Tchebichef is relatively superior to Krawtchouk and Charlier. The average performance of the Tchebichef in terms of PSNR and MSE is 108.924 and 0.0580, respectively. Figures 6 and 7 depict the compression of the average PSNR and MSE of discrete orthogonal moments (Tchebichef, Krawtchouk, Charlier, Hahn,

Table 3 Comparison of reconstruction errors (PSNR and MSE) for re-orthogonalization methods (MGSM, HM, and GRM) using Tchebichef moments

Signal	Algorithm 1 (MGSM)		Algorithm 2 (HM)		Algorithm 3 (GRM)	
	PSNR	MSE	PSNR	MSE	PSNR	MSE
MIT-BIH Rec. 100	96.841	0.0756	101.685	0.0432	96.240	0.0810
MIT-BIH Rec. 101	103.562	0.0364	106.854	0.0249	100.552	0.0514
MIT-BIH Rec. 103	91.995	0.1525	93.21	0.132	91.122	0.1708
MIT-BIH Rec. 105	104.0367	0.0370	108.477	0.022	100.156	0.0579
MIT-BIH Rec. 107	100.457	0.3192	106.437	0.160	96.624	0.4962
MIT-BIH Rec. 108	84.566	0.2952	90.329	0.152	82.168	0.3890
MIT-BIH Rec. 111	84.323	0.1462	100.245	0.0234	85.359	0.1298
MIT-BIH Rec. 112	111.686	0.0877	118.381	0.0406	102.311	0.2582
MIT-BIH Rec. 115	112.918	0.0290	116.436	0.0194	109.591	0.0425
MIT-BIH Rec. 117	104.355	0.0186	109.422	0.1039	98.417	0.3688
MIT-BIH Rec. 118	105.657	0.2365	113.102	0.1004	99.204	0.4971
MIT-BIH Rec. 119	115.504	0.0641	126.726	0.0176	112.437	0.0912
MIT-BIH Rec. 121	117.742	0.0328	118.520	0.0300	113.930	0.0508
MIT-BIH Rec. 122	116.414	0.0480	119.917	0.0320	115.032	0.0562
MIT-BIH Rec. 124	117.471	0.0404	121.8111	0.0245	117.060	0.424
MIT-BIH Rec. 200	88.881	0.3114	94.876	0.1561	81.134	0.7596
MIT-BIH Rec. 201	109.217	0.0101	113.945	0.0058	105.735	0.0150
MIT-BIH Rec. 202	103.154	0.0143	105.621	0.0108	102.776	0.0149
MIT-BIH Rec. 205	86.9403	0.3081	87.028	0.3050	84.342	0.4156
MIT-BIH Rec. 208	119.919	0.0118	126.499	0.0055	118.348	0.0141
MIT-BIH Rec. 209	100.061	0.0301	100.652	0.0281	97.036	0.0426
MIT-BIH Rec. 213	108.536	0.0814	109.599	0.0702	106.866	0.0961
MIT-BIH Rec. 214	102.401	0.0727	104.048	0.0602	100.998	0.0855
MIT-BIH Rec. 219	118.673	0.0279	121.013	0.0213	118.279	0.0292
MIT-BIH Rec. 222	103.708	0.0100	103.385	0.0104	102.656	0.0113
MIT-BIH Rec. 228	107.213	0.0171	108.908	0.0141	100.598	0.0366
MIT-BIH Rec. 230	112.481	0.0204	112.965	0.0193	111.332	0.0233
MIT-BIH Rec. 231	111.154	0.0103	115.863	0.0060	105.185	0.0204
MIT-BIH Rec. 233	107.528	0.0690	114.101	0.0324	100.313	0.1584
MIT-BIH Rec. 234	101.909	0.0313	104.380	0.0236	99.352	0.0421
Average	104.976	0.0881	109.147	0.0564	101.838	0.1643

and Meixner) in signal reconstruction. The reconstructed “Rec. 234” signal by using Tchebichef, Krawtchouk, Charlier, Hahn, and Meixner moment with Householder method is depicted in Fig. 8.

To further validate the efficiency of DOMs, reconstruction is conducted using Tchebichef, Krawtchouk, Charlier, Hahn, and Meixner with orders ranging between 50 and 200. Table 5 compares the quality of the signals reconstructed for the five moments in terms of PSNR and MSE in different moment orders. Figures 9 and 10 depict the curves of PSNR and MSE values corresponding to the reconstructed MIT-BIH Rec. 101 in different moments, respectively. As can be seen from the results in Table 5 and Fig. 9, The PSNR values improve appropriately with moment order increases, indicating an improvement in the reconstructed signal quality. The best quality of the reconstructed signal (lower MSE) is likewise obtained at the last moment order, as shown in Table 5 and Fig. 10.

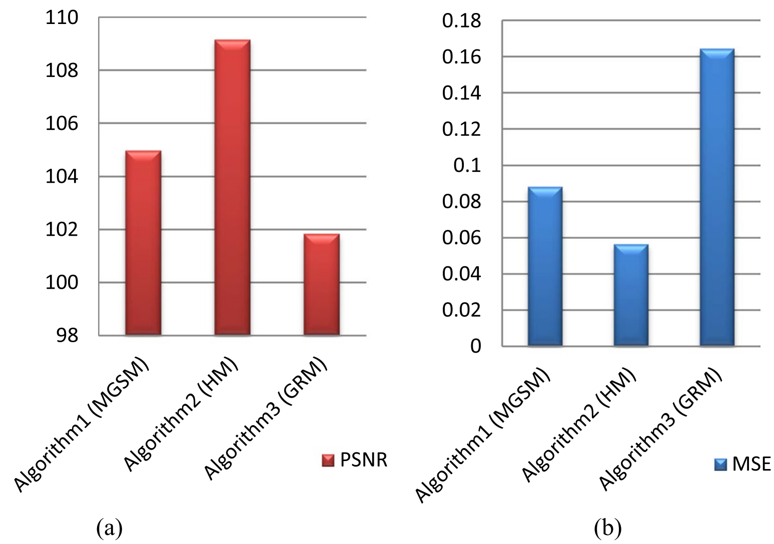

Fig. 3 The average values **a** PSNR and **b** MSE of the reconstructed signals using Tchebichef moments with Gram–Schmidt, Householder, and Given Rotations methods

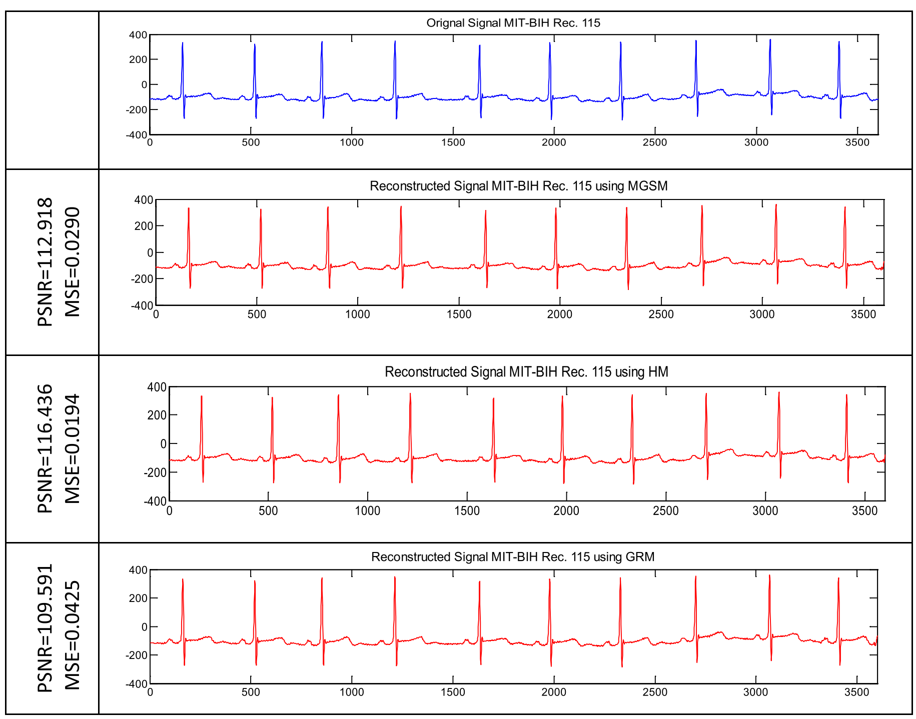

Fig. 4 Set of reconstructed "Rec. 115" signal using Tchebichef moments with Gram–Schmidt, Householder, and Given Rotations methods

6 Discussion

This paper contributes to the ongoing discussions about using DOMs in analyzing one-dimensional bio-signals. In addition, it also introduces an algorithm to overcome the propagation of numerical errors problem faces high-order computation of DOPs. The comparative experiments shown in the above tabular and graphical results assure the

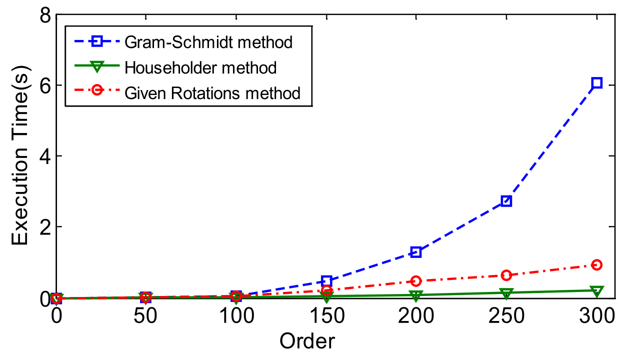

Fig. 5 Average execution time using Tchebichef moments with re-orthonormalization methods (Gram–Schmidt, Householder, and Given Rotations)

superiority of DOMs in reconstruction biosignals. It also demonstrates the advantages of the proposed re-orthonormalization methods (Gram–Schmidt, Householder, and Given Rotations).

Generally, the increase in polynomial order, the increase in error propagation. Therefore, many researchers used QR decomposition methods to overcome these errors. In this work, we present Discrete Orthogonal Moments (DOMs) in bio-signals analysis and reconstruction, which are gaining popularity in analyzing one-dimensional signals due to their effectiveness in capturing digital information without redundancy. The works addressed by others faced propagation errors at high order polynomials which destroy the orthogonality property of these polynomials. While in our work, the problem of error propagation at high order polynomials is solved using QR decomposition methods. Hence, the OP reconstructs the bio-signals efficiently. Moreover, to highlight the efficiency of the different forms of QR decomposition like the Gram–Schmidt method, the Householder method, and the Given Rotations method in different situations, we compare the different methods to each other to show the differences between them. Consequently, we introduce a road map to the interested researchers. Additionally, we compare five common types of moments (Tchebichef, Krawtchouk, Charlier, Hahn, and Meixner) to estimate the best moment in analyzing and reconstructing bio-signals which gives a clear guide to researchers in this area.

The results discussion obtained can be divided into three sections. Section one is the performance of the DOMs in biosignal reconstruction and the effect of using re-orthonormalization methods in maintaining the orthogonality property at the high-order computation of DOPs. In general, the superiority of DOMs in the reconstruction of bio-signals can be attributed to the following worthwhile factors:

- DOMs are orthogonal moments with orthogonal basis functions. Each moment coefficient can capture the signal's distinct and unique components with no information redundancy.
- According to the order value, orthogonal moments' basis functions can extract various distinct types of information from the signals.
- Moments generated from discrete orthogonal polynomials are effective at compressing signals. This is because they have a higher efficiency of energy compression for

Table 4 Reconstructions errors (PSNR and MSE) of biosignal by Tchebichef, Krawtchouk, Charlier, Hahn, and Meixner moments with the Householder method

Signal	Tchebichef + HM		Krawtchouk + HM		Charlier + HM		Hahn + HM		Meixner + HM	
	PSNR	MSE	PSNR	MSE	PSNR	MSE	PSNR	MSE	PSNR	MSE
MIT-BIH Rec. 100	101.685	0.0432	100.587	0.0781	98.183	0.0884	84.598	0.3541	80.041	0.0436
MIT-BIH Rec. 101	106.854	0.0249	104.067	0.0343	103.751	0.0448	85.811	0.2807	83.309	0.3744
MIT-BIH Rec. 103	93.210	0.132	88.584	0.2087	87.840	0.2108	80.804	0.4078	79.304	0.4801
MIT-BIH Rec. 105	108.477	0.022	108.017	0.0245	103.014	0.0407	100.950	0.1401	98.018	0.2084
MIT-BIH Rec. 107	106.437	0.1600	104.501	0.2980	103.486	0.3010	97.210	0.4017	97.841	0.4120
MIT-BIH Rec. 108	90.329	0.1520	87.897	0.2011	84.929	0.3952	70.646	1.4657	70.213	1.6172
MIT-BIH Rec. 111	100.245	0.0234	100.866	0.0247	96.501	0.0801	92.098	0.2150	91.874	0.3047
MIT-BIHRec. 112	111.686	0.0877	109.548	0.0899	108.584	0.0910	104.854	0.2857	101.014	0.4014
MIT-BIH Rec. 115	116.436	0.0194	98.514	0.1524	113.573	0.0339	83.303	0.8780	80.837	1.1661
MIT-BIH Rec. 117	109.422	0.1039	110.548	0.0741	107.580	0.2041	103.685	0.4180	99.5014	0.5018
MIT-BIH Rec. 118	113.102	0.1004	108.017	0.2048	107.150	0.2310	102.985	0.4740	100.301	0.6014
MIT-BIH Rec. 119	126.726	0.0176	124.014	0.1901	123.047	0.2204	118.014	0.3041	117.104	0.3380
MIT-BIH Rec. 121	118.520	0.0300	119.947	0.0297	117.501	0.03701	113.980	0.1148	110.018	0.1930
MIT-BIH Rec. 122	119.917	0.0320	117.504	0.0901	115.048	0.1408	105.500	0.3401	102.402	0.5014
MIT-BIH Rec. 124	121.8111	0.0245	120.010	0.0289	118.507	0.0403	111.012	0.5041	109.500	0.6140
MIT-BIH Rec. 200	94.876	0.1561	93.014	0.2014	90.301	0.2230	85.509	0.6011	82.417	0.7710
MIT-BIH Rec. 201	113.945	0.0058	114.018	0.0030	113.541	0.0070	105.037	0.0580	102.650	0.1086
MIT-BIH Rec. 202	105.621	0.0108	102.028	0.0188	101.690	0.0201	96.510	0.0813	95.054	0.1047
MIT-BIH Rec. 205	87.028	0.3050	86.510	0.0300	85.014	0.0401	80.5410	0.7089	78.014	0.9093
MIT-BIH Rec. 208	126.499	0.0055	125.847	0.0080	120.008	0.0207	114.801	0.0409	111.985	0.0804
MIT-BIH Rec. 209	100.652	0.0281	111.607	0.0080	97.126	0.0442	88.077	0.1196	87.183	0.1326
MIT-BIH Rec. 213	109.599	0.0702	106.041	0.0901	104.180	0.1470	99.548	0.4407	96.471	0.7010
MIT-BIH Rec. 214	104.048	0.0602	118.719	0.0111	102.350	0.0732	92.381	0.2306	91.497	0.2553
MIT-BIH Rec. 219	121.013	0.0213	103.675	0.1568	118.805	0.0548	87.457	1.0142	84.968	1.3510
MIT-BIH Rec. 222	103.385	0.0104	105.147	0.0258	103.014	0.0158	95.540	0.1076	94.6030	0.3058

Table 4 (continued)

Signal	Tchebichef + HM		Krawtchouk + HM		Charlier + HM		Hahn + HM		Meixner + HM	
	PSNR	**MSE**	**PSNR**	**MSE**	**PSNR**	**MSE**	**PSNR**	**MSE**	**PSNR**	**MSE**
MIT-BIH Rec. 228	108.908	0.0141	107.400	0.0201	105.048	0.0341	100.654	0.0901	99.659	0.2087
MIT-BIH Rec. 230	112.965	0.0193	106.817	0.0392	110.373	0.0261	96.191	0.1334	94.561	0.1609
MIT-BIH Rec. 231	115.863	0.0060	114.841	0.0090	111.854	0.0174	103.840	0.0998	100.360	0.2500
MIT-BIH Rec. 233	114.101	0.0324	112.890	0.0804	109.594	0.1089	102.698	0.2333	98.890	0.3017
MIT-BIH Rec. 234	104.380	0.0236	108.661	0.0144	102.879	0.0250	92.248	0.0956	89.576	0.1296
Average	108.924	0.0580	107.327	0.0815	105.482	0.1005	96.549	0.3546	94.305	0.4509

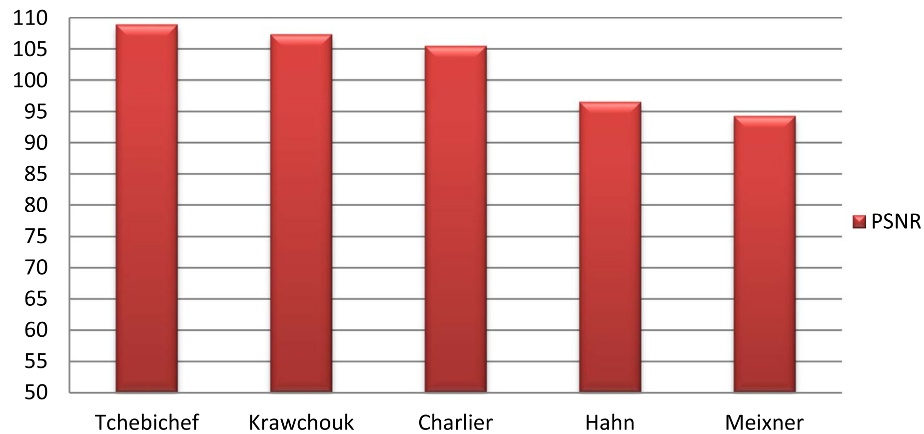

Fig. 6 The average PSNR of the reconstructed signals using Tchebichef, Krawtchouk, Charlier, Hahn, and Meixner moment with Householder method

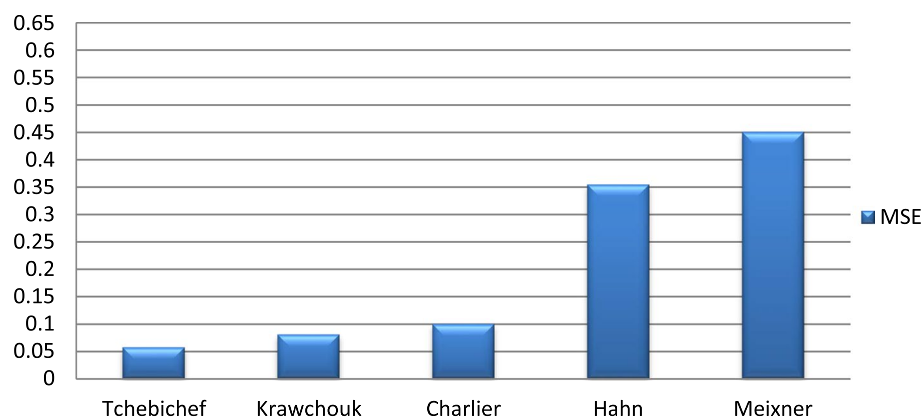

Fig. 7 The average MSE of the reconstructed signals using Tchebichef, Krawtchouk, Charlier, Hahn, and Meixner moment with Householder method

common signals. If the discrete orthogonal moment is chosen correctly, the energy in the signal is concentrated on a small fraction of the moment coefficients; these coefficients are then stored and used to generate the reconstructed signal.

- The ability of DOMs on local and global feature extraction.
- Using recursive formulas to compute polynomial values by using lower polynomial orders instead of directly computing them causes computational efficiency in the computation of the moments.

Section two determines which of the three re-orthogonalization methods (Gram–Schmidt, Householder, and Given Rotations) best preserves the orthogonality property. The comparative results indicate that the Householder method is the best in signal reconstruction in terms of reconstruction errors (PSNR, MSE) and execution time. The most likely explanation of the result has explained the fact that using Gram–Schmidt after computation of each nth Polynomial order minimizes the numerical error propagation considerably. Therefore, the Gram–Schmidt method is not stable when used in a re-orthogonalization matrix with large size. To this end, the

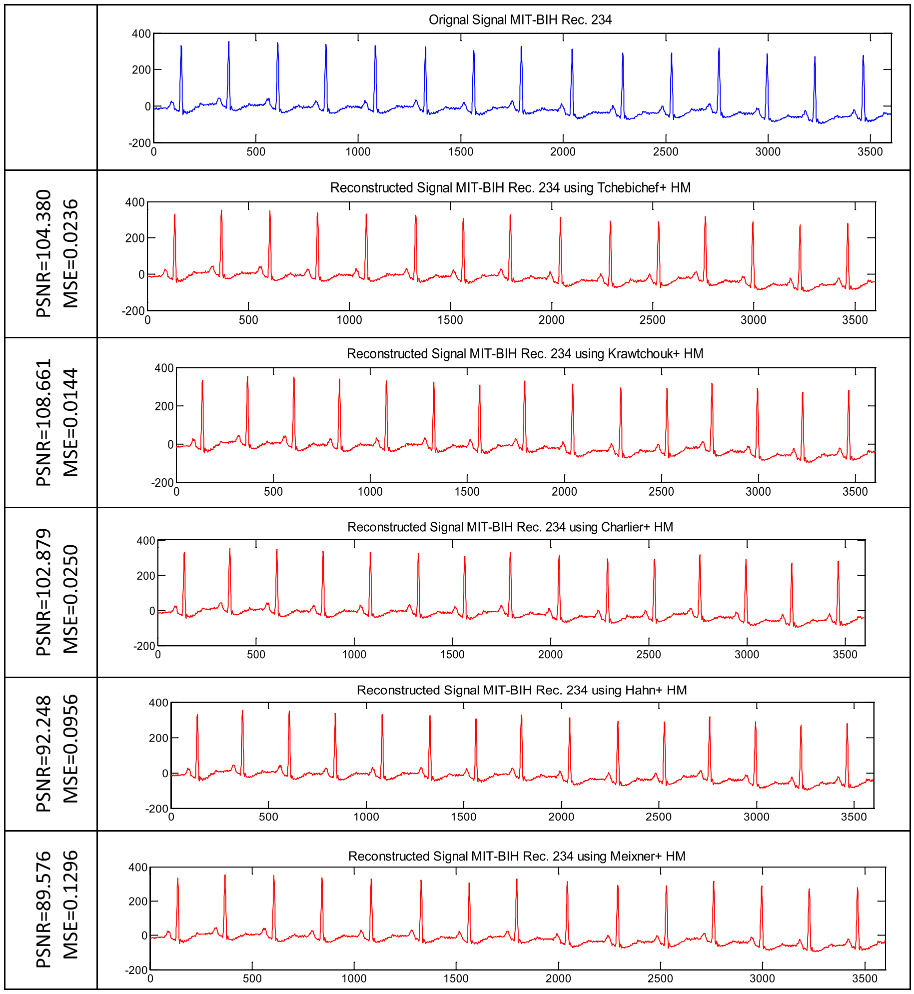

Fig. 8 Set of reconstructed "Rec. 234" signal using Tchebichef, Krawtchouk, Charlier, Hahn, and Meixner moment with Householder method

Householder method outperforms the Gram–Schmidt and Givens rotation methods in numerical stability in the *QR* decomposition of a matrix with large size. In addition to Householder method is faster compared to the Gram–Schmidt and Givens rotation methods. Because of this, the Householder method is better for real-time applications. The last section investigates which type of discrete orthogonal moments (Tchebichef, Krawtchouk, Charlier, Hahn, and Meixner) provides better-reconstructed signals. Besides that, tracking the reconstructed signal quality for DOMs at various orders of moments. The obtained results demonstrate that all of the used moment types are stable since they enabled the reconstruction of the signals until the high moment order. It reflects the effectiveness and numerical stability of the orthogonal moment for large-size signal reconstruction. This numerical stability is ensured by re-orthogonalization methods (Gram–Schmidt, Householder, and Given Rotations), especially the Householder method.

Despite the development of several reconstruction methods, substantial limitations must still be addressed. In the design of reconstruction methods, computational

Table 5 Performance of Tchebichef, Krawtchouk, Charlier, Hahn, and Meixner moments with Householder method at different orders

Signal	Order	Tchebichef + HM		Krawtchouk + HM		Charlier + HM		Hahn + HM		Meixner + HM	
		PSNR	MSE	PSNR	MSE	PSNR	MSE	PSNR	MSE	PSNR	MSE
MIT-BIH Rec. 101	50	75.232	0.9490	73.048	1.2201	75.203	0.9120	70.941	1.5553	68.381	2.088
	100	80.335	0.5273	77.213	0.7554	80.335	0.5164	75.046	0.9695	71.32	1.4872
	150	103.562	0.0364	96.533	0.1630	101.746	0.0502	83.056	0.3855	75.877	0.8815
	200	106.854	0.0249	104.067	0.0343	103.751	0.0448	85.881	0.2807	83.309	0.3744
MIT-BIH Rec. 200	50	74.015	0.9901	81.047	0.9748	74.801	1.1410	72.369	1.4047	65.841	1.8020
	100	79.541	0.6010	85.501	0.5084	78.514	0.7410	77.550	1.1470	69.147	1.5014
	150	91.854	0.1950	92.014	0.2400	88.581	0.3014	82.541	0.8040	76.540	1.0011
	200	94.876	0.1561	93.014	0.2014	90.301	0.2230	85.509	0.6011	82.417	0.7710
MIT-BIH Rec. 230	50	87.722	0.3536	75.424	1.4567	75.499	1.4442	69.167	2.0412	73.316	1.8569
	100	92.878	0.1953	85.554	0.4538	87.669	0.3557	84.705	0.5004	80.259	0.8348
	150	111.332	0.0233	94.019	0.1752	107.394	0.0394	92.393	0.2065	85.132	0.4764
	200	112.965	0.0193	106.817	0.0392	110.373	0.0261	96.191	0.1334	94.560	0.1609

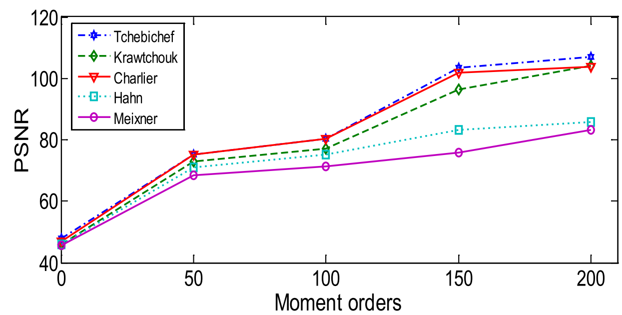

Fig. 9 PSNR of the reconstructed signal "MIT-BIH Rec. 101" using Tchebichef, Krawtchouk, Charlier, Hahn, and Meixner moment with Householder methods

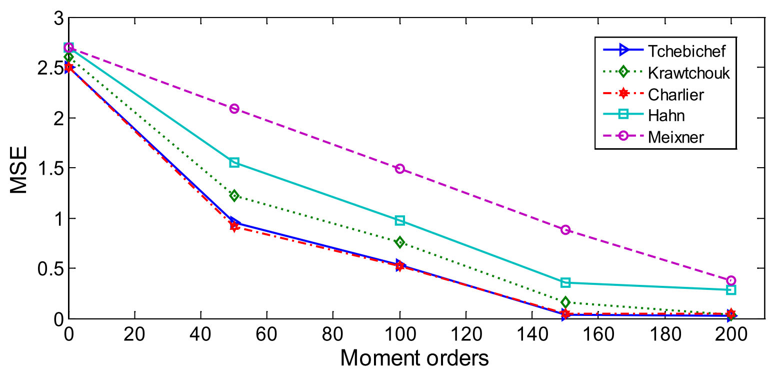

Fig. 10 MSE of the reconstructed signal "MIT-BIH Rec. 101" using Tchebichef, Krawtchouk, Charlier, Hahn, and Meixner moment with Householder methods

complexity and memory management play a crucial role, particularly in real-time applications like Remote Monitoring Systems. Reconstruction techniques increase the complexity of memory management. When the memory required to conduct the compression technique exceeds the available device memory, efficient reconstruction cannot be accomplished. Even though some reconstruction techniques achieve higher reconstruction quality, they do not manage memory effectively. Consequently, memory management and computational complexity in reconstruction techniques are interesting future research directions.

7 Conclusion

This article presents a method for bio-signal reconstruction based on Discrete Orthogonal Moments (DOMs). It also proposes a modified version of DOPs using the QR decomposition methods such as the Gram–Schmidt, Householder, and Given Rotations methods. The purpose of the proposed modification is to preserve the orthogonality property in the computation of high polynomials order. Based on the results, it can be concluded that the research into DOPs has been very successful. DOMs of various types: Tchebichef, Krawtchouk, Charlier, Hahn, and Meixner moments provide good results in reconstruction quality (PSNR, MSE). The comparative experiments demonstrate the superiority of the proposed modification of DOMs in reconstruction quality. This improvement in DOMs performance is due to QR decomposition methods to preserve the orthogonality property and then overcome the propagation of numerical errors. We also conclude that Tchebichef, Krawtchouk, and Charlier moments are better than Hahn and Meixner moments in reconstruction quality, and generally, Tchebichef has the best performance in signal reconstruction. The experiments of performance DOMs in reconstruction quality at a high order of moments are performed. We have noticed that the reconstruction quality improvement (PSNR highest, MSE lower) with moment orders increases. It means that the DOMs used in the proposed modification are efficient in large-size signal reconstruction. We could use the proposed method for large-size signal compression and classification in our future work and research direction. In addition, other applications will be used instead of bio-signal, such as volumetric medical images, Galaxies images, and Retrieval systems for Biomedical Images. The proposed method's ability in reconstruction could be improved by using a new version of DOMs as fractional DOMs, Radial DOMs.

Abbreviations

DOMs	Discrete orthogonal moments
MSE	Mean square error
PSNR	Peak signal to noise ratio
DOPs	Discrete orthogonal polynomials
MGSM	Gram–Schmidt method
HM	Householder method
GRM	Given Rotations method

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Author contributions
I.S. conceived of the problem, gathered data, conducted the analysis, and produced the first draft of the paper. M.A. reorganized the paper and provided the related work. M.M. evaluated the arguments critically, reviewed the versions, and provided helpful comments. All authors provided general feedback and helped to revisions. All authors read and approved the final manuscript.

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The data that used this study belong to Physionet database; fPCG signals from the fPCG Dataset are used.

Declarations

Ethics approval and consent to participate
We declare that the studies in this work do not involve any human participants, human data, human tissue and animal.

Consent for publication
We declare that the manuscript does not contain any individual person's data in any form (including individual details, images or videos).

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