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An improved multi-source information fusion method for IMU compensation of missile



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Abstract

The conventional SINS/CNS integrated navigation system equipped on ballistic missiles is typically equipped with attitude measurement, which can only estimate the gyro drift and has no effect on accelerometer bias. To address the issue, an improved multi-source information fusion method containing a new nonlinear framework called SINS/RKCNS with the indirect horizon reference and kinematic constraint is proposed, and a MAP-based modified iterated CKF is involved to increase positioning accuracy and system robustness. Furthermore, to reduce the influence of correlated noise, state augmentation is employed in the iterative process. Eventually, experiments are conducted, and the results confirm the effectiveness of the proposed approach.

Keywords: Multi-source information fusion, Kinematic constraint, SINS/CNS integrated navigation systems, Cubature Kalman filter, Maximum a posteriori

1 Introduction

In today's conflict, missiles are a crucial tool and a vital component of military and national security. The navigation and positioning system makes or breaks a missile's ability to carry out precision strike missions. Therefore, it is crucial that the navigation and positioning systems are highly reliable and accurate. Generally speaking, multi-source information fusion can be employed to increase the positioning accuracy of navigation systems. Currently, research on multi-source information fusion methods is separated into two areas: multi-sensor system modeling and information fusion filter design.

For multi-sensor system modeling, the Celestial Navigation System (CNS) is a fully autonomous approach that produces astronomical data from stellar sensors with excellent accuracy but is subject to weather and other variables, and the output of it is discontinuous [1]. Although the Strapdown Inertial Navigation System (SINS) is a fully autonomous navigation system with outstanding short-term accuracy, the inaccuracies of Inertial Measurement Unit (IMU) build up over time [2, 3]. As a consequence, accurate estimations of accelerometer bias and gyroscope drifts in the IMU are necessary. The most widely navigation system, with great positioning precision and mature technology, is the global navigation satellite system (GNSS). However, owing to the accelerating development of electronic countermeasures and other technologies, the satellite signals are easily blocked, and the initiative is easily lost in implementation [4].



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As a result, the SINS/CNS integrated systems, which are extensively utilized in various vehicles and have the outstanding benefits of strong autonomy and high precision, have become a popular research topic for missile navigation system development [5, 6]. The CNS can efficiently aid the SINS in correcting the gyroscope drifts in the IMU with the starlight caught by the stellar sensors in it, resulting in missile attitude optimization. These methods, however, focus on gyroscope drift estimates and attitude correction frequently but do little on accelerometer bias [7, 8], which makes it difficult to optimize position information. Gou put out a technique that allowed for the correction of infrared Earth measurements using a specific force followed by the application of the modified values to take precise measurements with CNS, enabling the improvement of the position accuracy [9]. However, this method necessitates the use of an infrared Earth sensor, which is costly and has limited accuracy due to the device. Scholars are therefore considering ways to increase system positioning precision without the aid of peripherals. It is still a challenge to fully utilize the CNS in a limited amount of time. Nowadays, a wide range of projects have been conducted by scholars throughout the SINS/CNS.

Ning presented a quick calibration technique based on movements for star sensors in SINS/CNS, which helps to increase navigational accuracy while catching one star at a time and enables incorrect of the CNS equipment mistakes quickly [10]. However, the constant deviation of the accelerometer is incorporated in the error matrix, which still impacts the performance. These tasks are completed during the initial calibration phase, but accurate real-time estimation and correction of the accelerometer bias are still required during missile flight. A low-cost navigation and positioning technique called the indirect sensitive horizon by starlight refraction was developed in the 1980s [11]. It makes use of star sensors to monitor the refracted starlight and accomplish high-precision navigation and positioning. Subsequently, the navigation system made up of starlight refraction navigation-assisted SINS has been a popular topic for researchers [12, 13] due to the extensive study connected to the atmospheric refraction model [14-16]and star identification algorithm [17, 18]. According to the optimal correction overall, Zhu offered a SINS/CNS strategy, as well as an analytical way to determine the position and horizontal reference, which has a marginally positive impact on position accuracy but omitted to mention the estimation of the accelerometer bias [19]. Thereby, the system model improvement of SINS/CNS is still an issue that requires attention. Chen's team conducted a preliminary study for the method of correcting accelerometer bias and made experimental comparisons for various misalignment angles; however, the high-precision IMU employed was costly [20].

For the design of information fusion filters, Kalman filtering (KF) based on Bayesian architecture is a successful and widely used technique. Regular KF, however, can only handle linear systems [21], despite the fact that there are no perfect linear systems in reality. Particularly, SINS/CNS is a typical nonlinear system. In this situation, the KF based on the minimum variance criterion is unable to achieve the optimal solution. Further, a variety of nonlinear filtering strategies have been developed based on the KF that approximate the state posterior probability density distribution using various numerical integration or linear approximation methods and estimate the system state using various optimality criteria, such as the extended Kalman filter (EKF), unscented Kalman filter (UKF), cubature Kalman filter (CKF). A linearized approximation to the measurement function is used in EKF [22]. Julier and Uhlmann recommended the UKF, which approximates the Gaussian-weighted integral using a trace-free transform [23, 24], to address the issue of poor EKF estimation in strongly nonlinear systems [25]. Arasaratnam et al. constructed the integral function using the spherical-radial transform to increase the accuracy and numerical stability of the UKF in state estimation applications [26]. Due to its excellent numerical stability and simplicity of use, the CKF has garnered a lot of attention. Unfortunately, the linear minimization mean square error (LMMSE)-based Gaussian approximation filter performs poorly when the measurement equation is significantly nonlinear, especially when the measurement noise is modest [27]. Thus, iterative filtering algorithms started to draw researchers' attention. Frameworks like the iterative KF [28], iterative EKF [29] and the iterative UKF [30] were explored one after the other. An iterative CKF algorithm using Gauss--Newton iterative method is proposed and verified to be effective in reducing the state estimation errors, where the measurement equations are linearized [31]. Ghorbani added the sigma points of CKF to those of UKF, making more sigma points and incorporating the concept of iteration, which is performed better in large degrees-of-freedom systems with high noise [32]. A series of studies were also done on topics like the correlation between the measurement iterations and state estimates [33]. Cui continued by confirming that maximum a posteriori (MAP)based iterative filters produce better estimation results with good quality measurements [33]. Additionally, how to make sure that the state vector is not connected with the measurement vector in the iterative process following is a problem that needs to be taken into account. According to [34], state estimation with state-correlated noise is implemented using higher-order terms; however, this method requires a significant amount of computation since Jacobian and Hessian matrices must be calculated for each iterative process.

No matter how the information fusion filtering techniques mentioned above perform, for conventional ballistic navigation systems without GNSS, SINS/CNS might ignore the accelerometer bias and only work with gyroscope drifts. As a result, in this paper, an innovative framework for the integrated navigation named SINS/Refraction and Kinematic CNS (SINS/RKCNS) is employed, together with the kinematic constraints of the missile, to account for the refracted height information provided by the indirect sensitive horizon. So, the accelerometer bias of the IMU is able to be estimated, which will reduce the position errors. Due to the nonlinearity of the measurement model in the information fusion process, the most widely used filtering algorithms exhibit inefficient linear innovation updates. We provide a modified iterative CKF (MICKF) algorithm based on the MAP framework to fully utilize the quantitative data. In the iterative stage, the state covariance is only updated once per filtering cycle, and the updated state estimates are not fed back to the prediction stage. To reduce the effect of state-dependent noise that emerges during the iterative process, we employ a state augmentation strategy that blends state estimation with measurement noise. It fully exploits the computational advantages of the CKF algorithm with sampling point approximation to increase state estimation accuracy.

In general, the major contributions of the paper are as follows:

- 1. A MAP-based algorithm called MICKF is developed, relying on the nonlinear navigation system model, which fully utilizes the measurement data and enhances the accuracy of state estimation.
- Combining the starlight indirect sensitive horizon theory with missile kinematic restrictions, a completely autonomous SINS/RKCNS is created that can estimate the accelerometer bias in the IMU.
- 3. The effectiveness of the suggested strategy is verified in the missile-borne SINS/CNS experiments.

The remainder of the paper is structured as follows: The MICKF is derived in Sect. 2. Starlight refraction principle is reviewed, and the optimized integrated navigation framework SINS/RKCNS is established in Sect. 3. In Sect. 4, the experiments are implemented. After that, conclusions are drawn in Sect. 5.

2 Modified iterated cubature Kalman filter algorithm

2.1 Motivation of the MICKF

As a widely used tool for information fusion, Kalman filter is successful method. However, it can only be used for first-order linear equations. EKF uses a linear approximation to solve the nonlinear problems that exist in the model, while CKF is usually used when the system nonlinear function is more complex. Based on the spherical-radial transform, the positive definiteness during the transfer of sampling points through the nonlinear function can be guaranteed, so the stability and accuracy are better than EKF.

What's more, Gaussian approximation filter approximates the state a posterior probability density function (PDF) poorly when the measurement function is nonlinear, especially when the measurement noise is small. Nevertheless, the iterated filter based on MAP framework can obtain better estimation results. Furthermore, to eliminate the correlated noise influence in the iterative update process, state augmentation technique is applied in the iterative measurement update. Moreover, the convergence and stability of filtering need to be further considered when the above filtering methods are used for integrated navigation.

2.2 Derivation of MICKF

In order to make the system with nonlinear measurement equations obtain better estimation result in the information fusion process and to consider the effect on the system when the measurement noise is small, the MICKF algorithm with state augmentation is derived.

Consider a discrete time nonlinear dynamic system:

$$\begin{cases} X_k = f(X_{k-1}) + W_{k-1} \\ Z_k = h(X_k) + V_k \end{cases}$$
(1)

where $f(\cdot)$ and $h(\cdot)$ represent the state and measurement functions, respectively; X_k is the state vector with dimension n at time k, and W_{k-1} is process Gaussian noise with zero means and covariance Q_{k-1} ; Z_k is the measurement vector with dimension m, and V_k is measurement noise with zero means and covariance R_k . Let the posterior PDF of X_k be:

$$p(\boldsymbol{X}_k|\boldsymbol{Z}_{1:k}) = N\left(\boldsymbol{X}_k|\hat{\boldsymbol{X}}_{k|k}, \boldsymbol{P}_{k|k}\right)$$
(2)

where $N\left(X_k|\hat{X}_k, P_{k|k}\right)$ denotes a Gaussian distribution upon such a mean of $\hat{X}_{k|k}$ and a variance of $P_{k|k}$ is satisfied by X_k . Similarly, $p(X_{k-1}|Z_{1:k-1}) = N\left(X_{k-1}|\hat{X}_{k-1|k-1}, P_{k-1|k-1}\right)$ is the posterior PDF of the X_{k-1} .

Furthermore, depending on the state function, the prior PDF of X_k is given by:

$$p(\boldsymbol{X}_k|\boldsymbol{Z}_{1:k-1}) = N(\boldsymbol{X}_k|\hat{\boldsymbol{X}}_{k|k-1}, \boldsymbol{P}_{k|k-1})$$
(3)

The moments included in the approximation can be described as follows:

$$\hat{X}_{k|k-1} = \int f(X_{k-1}) p(X_{k-1}|Z_{1:k-1}) dX_{k-1}$$
(4)

$$\boldsymbol{P}_{k|k-1} = \int \left(f\left(\boldsymbol{X}_{k-1}\right) - \hat{\boldsymbol{X}}_{k|k-1} \right) \left(f\left(\boldsymbol{X}_{k-1}\right) - \hat{\boldsymbol{X}}_{k|k-1} \right)^T p\left(\boldsymbol{X}_{k-1} | \boldsymbol{Z}_{1:k-1} \right) \mathrm{d}\boldsymbol{X}_{k-1} + \boldsymbol{Q}_{k-1}$$
(5)

Similarly, the moments in the measurement phase are given by:

$$\hat{\boldsymbol{Z}}_{k|k-1} = \int h(\boldsymbol{X}_k) p\big(\boldsymbol{X}_k | \boldsymbol{Z}_{1:k-1}\big) \mathrm{d}\boldsymbol{X}_k \tag{6}$$

$$\boldsymbol{P}_{k|k-1}^{zz} = \int \left(h(\boldsymbol{X}_k) - \hat{\boldsymbol{Z}}_{k|k-1} \right) \left(h(\boldsymbol{X}_k) - \hat{\boldsymbol{Z}}_{k|k-1} \right)^T p\left(\boldsymbol{X}_k | \boldsymbol{Z}_{1:k-1} \right) d\boldsymbol{X}_k + \boldsymbol{R}_k$$
(7)

$$\boldsymbol{P}_{k|k-1}^{xz} = \int \left(\boldsymbol{X}_k - \hat{\boldsymbol{X}}_{k|k-1} \right) \left(h(\boldsymbol{X}_k) - \hat{\boldsymbol{Z}}_{k|k-1} \right)^T p\left(\boldsymbol{X}_k | \boldsymbol{Z}_{1:k-1} \right) \mathrm{d} \boldsymbol{X}_k \tag{8}$$

Based on Bayesian theory, the posterior PDF of X_k is calculated as follows:

$$p(X_{k}|Z_{1:k}) = \frac{p(X_{k}|Z_{1:k-1})p(Z_{k}|X_{k})}{\int p(X_{k}|Z_{1:k-1})p(Z_{k}|X_{k})dX_{k}}$$

$$= a \cdot p(X_{k}|Z_{1:k-1})p(Z_{k}|X_{k})$$
(9)

where *a* denotes a constant.

According to the above formulations, solve for $\hat{X}_{k|k}$ based on the MAP framework, ignoring the constant term unrelated to the state vector:

$$\hat{X}_{k|k} = \arg \max \ln [p(X_k | Z_{1:k})]$$

$$= \arg \min [\ln \left| \boldsymbol{P}_{k|k-1}^{zz} \right| + (\boldsymbol{Z}_k - \hat{\boldsymbol{Z}}_{k|k-1})^T (\boldsymbol{P}_{k|k-1}^{zz})^{-1} (\boldsymbol{Z}_k - \hat{\boldsymbol{Z}}_{k|k-1})$$

$$+ (\boldsymbol{X}_k - \hat{\boldsymbol{X}}_{k|k-1})^T \boldsymbol{P}_{k|k-1}^{-1} (\boldsymbol{X}_k - \hat{\boldsymbol{X}}_{k|k-1}) \Big]$$

$$= \arg \min T(\boldsymbol{X}_k)$$
(10)

Newton--Raphson methodology is adopted to deal with it:

$$\boldsymbol{X}_{k|k}^{j+1} = \boldsymbol{X}_{k|k}^{j} - \left(\nabla^{2} T\left(\boldsymbol{X}_{k|k}^{j}\right)\right)^{-1} \nabla T\left(\boldsymbol{X}_{k|k}^{j}\right)$$
(11)

where $j \ge 0$ represents the iteration number; $\nabla^2 T\left(\mathbf{X}_{k|k}^j\right)$ denotes the Hessian matrix of $T\left(\mathbf{X}_{k|k}\right)$ and $\nabla T\left(\mathbf{X}_{k|k}^j\right)$ denotes the Jacobian matrix of $T\left(\mathbf{X}_{k|k}\right)$.

Define $H_i = \partial h \left(X_{k|k}^j \right) / \partial X_{k|k}^j$, then $\hat{X}_{k|k}$ can be expressed as

$$X_{k|k}^{j+1} = X_{k|k}^{j} - \left(P_{k|k-1}^{-1} + H_{i}^{T}(P_{k|k-1}^{zz})^{-1}H_{i}\right)^{-1} \times \left[P_{k|k-1}^{-1}(X_{k} - \hat{X}_{k|k-1}) - H_{i}^{T}(P_{k|k-1}^{zz})^{-1}(Z_{k} - \hat{Z}_{k|k-1})\right]$$
(12)

Combining the solution of the covariance matrix and the matrix inverse derivation, the iterative factor α_i is introduced, and the iterative solution result can be calculated:

$$\begin{aligned} \boldsymbol{X}_{k|k}^{j+1} &= \boldsymbol{X}_{k|k}^{j} + \alpha_{j} \Big\{ \Big(\boldsymbol{X}_{k|k}^{j} - \hat{\boldsymbol{X}}_{k|k-1} \Big) + \boldsymbol{K}_{k} \Big[\Big(\boldsymbol{Z}_{k} - \hat{\boldsymbol{Z}}_{k|k-1} \Big) + \Big(\boldsymbol{P}_{k|k-1}^{xz} \Big)^{T} \boldsymbol{P}_{k|k-1}^{-T} \left(\boldsymbol{X}_{k|k}^{j} - \hat{\boldsymbol{X}}_{k|k-1} \right) \Big] \Big\} \\ &= \boldsymbol{X}_{k|k}^{j} + \alpha_{j} \Xi \end{aligned}$$
(13)

MAP-based measurement updates can reduce the impact of a priori estimation errors on the results, and the iterative CKF described in the above equation can further reduce the approximation errors and thus the loss of volume measurement information. It allows the measurement information to be fully utilized and the results to be more accurate.

In order to use the volume point transfer function and make a weighted approximation to the PDF, it is necessary to generate a set of equally weighted cubature points according to the cubature rule:

$$I(f) \approx \sum_{i=1}^{2n} \omega_i f(\xi_i) \tag{14}$$

where $\xi = \sqrt{2n/2}[1]_i$ represents the cubature point, $\omega_i = 1/2n$ denotes the weight of cubature point and $i = 1, 2, \dots 2n, [1]_i$ is the i – th column vector of set [1].

Similar to the standard CKF, the prior state estimation $\hat{X}_{k|k-1}$ and prior covariance matrix $P_{k|k-1}$ at time k can be obtained:

$$X_{i,k-1|k-1} = \sqrt{\mathbf{P}_{k-1|k-1}} \xi_i + \hat{X}_{k-1|k-1}$$
(15)

$$X_{i,k|k-1} = f(X_{i,k-1|k-1})$$
(16)

$$\hat{X}_{k|k-1} = \sum_{i=1}^{2n} \omega_i X_{i,k|k-1} \tag{17}$$

$$\boldsymbol{P}_{k|k-1} = \sum_{i=1}^{2n} \omega_i X_{i,k|k-1} X_{i,k|k-1}^T - \hat{\boldsymbol{X}}_{k|k-1} \hat{\boldsymbol{X}}_{k|k-1}^T + \boldsymbol{Q}_{k-1}$$
(18)

To eliminate the correlated noise influence in the iterative update process, state augmentation technique is applied as follows [33]:

$$\widetilde{\boldsymbol{X}}_{k|k-1} = \begin{bmatrix} \widehat{\boldsymbol{X}}_{k|k-1} & \boldsymbol{O}_{1\times m} \end{bmatrix}^{T} \\
\widetilde{\boldsymbol{P}}_{k|k-1} = \begin{bmatrix} \boldsymbol{P}_{k|k-1} & \boldsymbol{O}_{m\times m} \\ \boldsymbol{O}_{m\times n} & \boldsymbol{R}_{k} \end{bmatrix}$$
(19)

where $O_{1 \times m}$ represents the zero vector with dimension *m*. Then, the final result will take the state vector of the first *n* dimensions and the covariance matrix of the first $n \times n$ dimensions.

After this, iterated algorithm is executed throughout the measurement update process with the iteration number *L*:

$$\chi_{i,k|k-1} = \sqrt{P_{k|k-1}}\xi_i + \breve{X}_{k|k-1}$$
(20)

$$Z_{i,k|k-1} = h(\chi_{i,k|k-1})$$
(21)

$$\hat{Z}_{k|k-1} = \sum_{i=1}^{2n} \omega_i Z_{i,k|k-1}$$
(22)

Once the new measurement has been obtained, the updated correlation covariance is determined using:

$$\boldsymbol{P}_{k|k-1}^{zz} = \sum_{i=1}^{2n} \omega_i (Z_{i,k|k-1} - \hat{\boldsymbol{Z}}_{k|k-1}) (Z_{i,k|k-1} - \hat{\boldsymbol{Z}}_{k|k-1})^T + \boldsymbol{R}_k$$
(23)

$$\boldsymbol{P}_{k|k-1}^{xz} = \sum_{i=1}^{2n} \omega_i (\chi_{i,k|k-1} - \boldsymbol{\breve{X}}_{k|k-1}) (\boldsymbol{Z}_{i,k|k-1}^T - \boldsymbol{\hat{Z}}_{k|k-1})^T$$
(24)

$$\boldsymbol{K}_{k} = \boldsymbol{P}_{k|k-1}^{xz} \left(\boldsymbol{P}_{k|k-1}^{zz} \right)^{-1}$$
(25)

$$X_{k|k}^{j+1} = X_{k|k}^{j} + \alpha_{j} \left\{ \left(X_{k|k}^{j} - \breve{X}_{k|k-1} \right) + K_{k} \left[\left(z_{k} - \hat{z}_{k|k-1} \right) + \left(P_{k|k-1}^{xz} \right)^{T} \left(\breve{P}_{k|k-1} \right)^{-1} \left(X_{k|k}^{j} - \breve{X}_{k|k-1} \right) \right] \right\}$$
(26)

where j = 0, 1, ...L represents the iteration number.

In the conventional iterative Kalman filtering algorithm, α_j is usually set to 1. When there is a strong nonlinearity of the system model, the large value of Ξ in Eq. (13) leads to the fact that the posterior estimates do not necessarily converge completely after a finite number of iterations. Therefore, $\alpha_j < 1$ is set so that a smaller amount of correction is gradually introduced in Ξ . Bertsekas pointed out that in the EKF based on the Gauss--Newton algorithm, the system converges faster when $\alpha_j < 1$ than when $\alpha_j = 1$ [35]. Therefore, $\alpha_j < 1$ is chosen here to speed up the iterative process. Back-tracking line search is performed to choose α_j here.

Eventually, the posterior state vector $\hat{X}_{k|k}$ can be calculated after iteration:

$$\hat{X}_{k|k} = X_{k|k}^{j+1} \tag{27}$$

The error covariance $P_{k|k}$ is updated by:

$$\boldsymbol{P}_{k|k} = \boldsymbol{P}_{k|k-1}^{a} - \boldsymbol{K}_{k} \boldsymbol{P}_{k|k-1}^{zz} \boldsymbol{K}_{k}^{T}$$

$$\tag{28}$$

2.3 Computational complexity

Computational complexity of the conventional CKF is calculated according to floatingpoint operations (flops) [8, 36]:

$$T_{CKF} = \frac{20}{3}n^3 + 10n^2 + 10n^2m + 8nm^2 + 2nm + m^3 + 3m^2 + m$$
(29)

The ICKF includes Eqs. (14)–(18) and (20)–(28), and the computational complexity of it can be calculated referring to Table 1:

$$T_{\rm ICKF} = \frac{19 + 4L}{3}n^3 + (7 + 2L + 2m + 10Lm)n^2 + (2 + 6L)m^2n + (4Lm - m + 2L)n + Lm^3 + 3Lm^2 + Lm$$
(30)

The MICKF includes Eqs. (14)–(28), and the computational complexity is:

$$T_{\text{MICKF}} = \frac{19 + 4L}{3}n^3 + (7 + 3L + 2m + 14Lm)n^2 + \left[6m^2 - m + L\left(26m^2 + 14m + 2\right)\right]n + \left(\frac{49}{3}L + 4\right)m^3 + (10L - 1)m^2 + 5Lm$$
(31)

Table 1	Computational complexity of some Equations

Eq.	ICKF	MICKF	Eq.	ICKF	MICKF
(15)	$\frac{1}{3}n^3 + 3n^2$	$\frac{1}{3}n^3 + 3n^2$	(22)	2nm	$2nm + 2m^2$
(16)	$4n^3 - 2n^2$	$4n^3 - 2n^2$	(23)	$4nm^2 + 3m^2$	$4nm^2 + 4m^3 + 3m^2$
(17)	2n ²	2 <i>n</i> ²	(24)	$4n^2m + 2nm$	$4n^2m + \left(8m^2 + 2m\right)n$
					$+4m^{3}+2m^{2}$
(18)	$2n^3 + 4n^2$	$2n^3 + 4n^2$	(25)	$m^3 + 2nm^2 - nm$	$(2m^2 - m)n + 3m^3 - m^2$
(20)	$\frac{1}{3}n^3 + 3n^2$	$\frac{1}{3}n^3 + (3+m)n^2$	(26)	$n^3 + 2n^2m + 2n$	$n^3 + 5n^2m + (7m + 2)n$
		$+(m^2+6m)n+\frac{1}{3}m^3+3m^2$		+3nm + m	$+m^{3}+3m^{2}+3nm^{2}+5m$
(21)	4n ² m — 2nm	$4mn^2 + (8m^2 - 2m)n$	(28)	$2nm^2 - nm + 2n^2m$	$2n^2m + 6nm^2 - nm$
		$+4m^3 - 2m^2$			$+3m^3 - m^2$

It can be seen that the computational complexity of CKF, ICKF, and MICKF are both $O(n^3)$ and $O(m^3)$. Therefore, the computational complexity of MICKF is the same order of magnitude as that of the other two methods, just with larger coefficients than the others.

3 Optimized integrated navigation model

3.1 Starlight refraction principles

The light of a star will be bent toward the center of the earth as it travels through the atmosphere as a result of the optical propagation laws [20]. The apparent location, as seen from the missile, will thus not be the same as the actual one.

As measured from Earth in Fig. 1, according to the geometric principle and atmospheric refraction principle [37]:

$$h_a = \eta \tan R - R_e + \sqrt{r_c^2 - \eta^2 - o}$$

$$h_a \approx h_g + K(\lambda)\rho_g(h)R_e$$
(32)

where h_a is the starlight height that is being viewed; h_g denotes the height of the starlight that has been refracted; and h_a is a minor diminution of h_g ; the refraction angle R is between the observable unrefracted and refracted beams; S in Fig. 1 denotes the geographic location of the missile. η_c denotes the direction vector of the ray before refraction in geocentric inertial frame, and r_c denotes the position vector of the missile. $r_c = |\mathbf{r}_c|$ and $\eta = |\mathbf{r}_c \cdot \boldsymbol{\eta}_c|$; o is so little that can be ignored. R_e is the radius of the earth.

Let the unit vector of the beam after refraction be S'_l while the unit vector before it be S_l . Then, the refraction angle *R* can be calculated using the following formula:

$$R = \arccos(S_l \cdot S'_l) \tag{33}$$

As a consequence, the position of the missile has now become indirectly observable with the aid of the directly measurable height.



Fig. 1 Refraction illustration of starlight

3.2 SINS/RKCNS integrated navigation framework

In the missile navigation system, the state equation is established as Eq. (1). The state vector is comprised of the SINS errors $\boldsymbol{X} = \begin{bmatrix} \boldsymbol{\Phi} & \delta \boldsymbol{\nu} & \delta \boldsymbol{p} & \boldsymbol{\varepsilon} & \nabla \end{bmatrix}^T$; $\boldsymbol{\Phi} = \begin{bmatrix} \phi_x, \phi_y, \phi_z \end{bmatrix}$ represents the misalignment angle errors; $\delta \boldsymbol{\nu} = \begin{bmatrix} \delta \nu_x, \delta \nu_y, \delta \nu_z \end{bmatrix}$ denotes the velocity errors; $\delta \boldsymbol{p} = \begin{bmatrix} \delta x, \delta y, \delta z \end{bmatrix}$ denotes the position errors. What's more, $\boldsymbol{\varepsilon} = \begin{bmatrix} \varepsilon_x, \varepsilon_y, \varepsilon_z \end{bmatrix}$ represents the gyroscope constant drift, while $\boldsymbol{\nabla} = \begin{bmatrix} \nabla_x, \nabla_y, \nabla_z \end{bmatrix}$ denotes the accelerometer constant bias.

The system equation is generated by attitude, velocity and position equations, employing the SINS update technique:

$$C_i^b = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}$$
(34)

$$\dot{\boldsymbol{v}}_{ib}^{i} = f_{ib}^{i} + \boldsymbol{\gamma}_{ib}^{i}(\boldsymbol{r}_{ib}^{i}) \tag{35}$$

$$\dot{r}^i_{ib} = v^i_{ib} \tag{36}$$

where *b* denotes the body frame while *i* represents the launch inertial one; C_i^b denotes the attitude transformation matrix; $\psi = -\arctan \frac{C_{13}}{C_{33}}$ represents the yaw angle, $\varphi = \arcsin(C_{23})$ represents the pitch angle and $\gamma = -\arctan \frac{C_{21}}{C_{22}}$ represents the roll angle; v_{ib}^i represents the velocity vector, f_{ib}^i denotes the specific force, r_{ib}^i is the Cartesian position vector, and $\boldsymbol{\gamma}_{ib}^i$ is the gravitational acceleration related to r_{ib}^i .

In contrast to the conventional framework, non-directly observable position information as well as kinematic constraints are contained in the SINS/RKCNS, in addition to attitude. Therefore, more measurement information can be introduced to the correction of accelerometer bias.

For the measurement equation, φ_{ins} , ψ_{ins} , γ_{ins} are the attitude angles from the IMU, and φ_{cns} , ψ_{cns} , γ_{cns} are provided by the CNS:

$$\begin{bmatrix} \delta\varphi\\ \delta\psi\\ \delta\gamma \end{bmatrix} = \begin{bmatrix} \varphi_{ins} - \varphi_{cns}\\ \psi_{ins} - \psi_{cns}\\ \gamma_{ins} - \gamma_{cns} \end{bmatrix}$$
(37)

Then, the measurement equation involved in attitude is:

$$Z_k^{(1)} = h^{(1)}(X_k) + V_k^{(1)}$$
(38)

where $h^{(1)}(\cdot)$ denotes the measurement function, $\boldsymbol{V}_k^{(1)}$ represents the measurement noise.

Furthermore, constructed from the celestial data mentioned in Sec. 3.1 is the measurement equation for location information:

$$\delta h_a^i = h_{a.cns}^i - h_{a.ins}^i + \nu_h \tag{39}$$

where $i \in N^*$ represents the amount of observed refracted stars; ν_h denotes the measurement errors; the CNS-derived refraction height is $h_{a,cns}$ [37], while the one calculated from SINS is $h_{a,ins}$:

$$h_{a,ins} = \eta \tan R - R_e + \sqrt{r_{ins}^2 - \eta^2} - o$$

$$h_{a,cns} = 57.08 - 6.44 \ln R + K(\lambda)\rho_e(h)(57.08 - 6.44 \ln R + R_e)$$
(40)

The apparent height measurement equation may thus be described as follows:

$$Z_k^{(2)} = h^{(2)}(X_k) + V_k^{(2)}$$
(41)

where $h^{(2)}(\cdot)$ denotes measurement function, $V_k^{(2)}$ is the related measurement noise.

In addition, an active stage and a free flight stage are contained in the ballistic missile trajectory. Once the engines shut down at the end of the active period, the missile loses the propulsion provided by them and has already exited the atmosphere. After then, the ballistic missile may be thought of as constituting a two-body motion with the Earth, meaning that it is just impacted by the gravity field of the Earth. The theoretical output of the accelerometer during the free-flight stage is regarded as zero, according to kinematic theory. Hence, the accelerometer bias as well as disturbances might both be responsible for the nonzero accelerometer output.

Thus, the velocity equation is rewritten as $\dot{v}_{ib}^i = 0 + \gamma_{ib}^i (r_{ib}^i)$. Moreover, the following is the kinematic constraint measurement equation:

$$Z_{k}^{(3)} = \tilde{f}_{ib}^{b} - f^{K}$$

= $h^{(3)}(X_{k}) + V_{k}^{(3)}$ (42)

where $h^{(3)}(\cdot)$ represents the measurement function; \tilde{f}_{ib}^{b} denotes the actual specific force generated by accelerometers and $f^{K} = 0$ denotes the kinematic constraint model, $V_{k}^{(3)}$ represents the accelerometer noise related.

It is possible to create the measurement model by combining Eqs. (38), (41) and (42). The attitude and position data are simultaneously compensated when accelerometer bias and gyroscope drifts are evaluated in SINS.

The proposed SINS/RKCNS framework is illustrated in Fig. 2 in its entirety.



Fig. 2 Structure of the proposed SINS/RKCNS integrated system

4 Experiments results and discussion

In this section, experiments have been carried out to validate the effectiveness of the suggested SINS/RKCNS-MICKF and to evaluate the positioning accuracy of the system. Figure 3 depicts the whole trajectory of a simulated real ballistic missile employed in all the experiments, where the integrated navigation system begins to perform at the 160th second. The SINS is engaged throughout the entire procedure. Moreover, all of the sensor settings are listed in Table 2. For all experiments, the initial misalignment angles are set as [10'', 6', 10''].

The performance of proposed SINS/RKCNS-MICKF was evaluated by comparison with a number of different approaches, including the conventional SINS/CNS-CKF, the SINS/ RKCNS-CKF, the SINS/RKCNS-ICKF, and the SINS/RKCNS-MICKF, and the comparative results were examined using the following metrics [8]:

$$\text{TRMSE} = \frac{1}{M} \sum_{1}^{M} \left(X_k - \hat{X}_{k|k} \right)$$
(43)



Fig. 3 Ballistic trajectory: a Real three-dimensional ballistic trajectory on Earth; b Real trajectory of the missile in *i* frame

Table 2	Experimental	parameters
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Sensors	Characteristic	Value
Gyroscope	Constant drift (° /h)	1
	Random walk noise $\left(\circ/\sqrt{h}\right)$	0.5
	Output frequency (Hz)	100
Accelerometer	Constant bias (μg)	100
	Random walk noise $\left(\mu g/\sqrt{Hz}\right)$	50
	Output frequency (Hz)	100
Star sensor	Random noise (")	3
	Output frequency (Hz)	1



Fig. 4 TRMSEs of position under various approaches

Table 3 RMSEs of position under various approaches

Approaches	RMSE of <i>p_{<i>x</i>}(<i>m</i>)</i>	RMSE of <i>p</i> _{<i>y</i>} (<i>m</i>)	RMSE of <i>p</i> _{<i>z</i>} (<i>m</i>)
Conventional SINS/CNS	104.2658	506.7657	237.6792
SINS/RKCNS-CKF	50.8478	87.4702	153.8121
SINS/RKCNS-ICKF	28.2319	81.9790	97.2022
*SINS/RKCNS-MICKF	27.2984	59.8424	63.7790

$$\text{RMSE} = \frac{1}{M} \sum_{1}^{M} \sqrt{\frac{1}{\mathbb{N}} \sum_{1}^{\mathbb{N}} \left(X_k - \hat{X}_{k|k} \right)^2}$$
(44)

where *M* denotes the experiment number, while \mathbb{N} denotes the time steps of experiments; TRMSE represents the mean signal error of *M* Monte Carlo experiments at each epoch; RMSE means the root-mean-square error.

For each approach, 100 Monte Carlo experiments were carried out under the aforementioned parameters and metrics. Figure 4 illustrates the TRMSEs of the positions for the four approaches. The corresponding RMSEs from 160th second during the experiments are indicated in Table 3. And the TRMSEs of the accompanying accelerometer bias ∇_x , ∇_y , ∇_z estimated by the four approaches are demonstrated in Fig. 5.

Similarly, Fig. 6 displays the TRMSEs of the attitude angles worked by different approaches. The associated RMSEs of the attitude angles are revealed in Table 4. Figure 7 reports the TRMSEs of corresponding gyroscope drifts ε_{x} , ε_{y} , ε_{z} . The reference values for accelerometer bias and gyroscope drifts are represented by green dotted lines in Figs. 5 and 7, respectively.



Fig. 5 TRMSEs of accelerometer bias estimated by various approaches



Fig. 6 TRMSEs of attitude under various approaches

Approaches	RMSE of yaw angle (")	RMSE of pitch angle(")	RMSE of roll angle (")	
Conventional SINS/CNS	0.8731	0.9265	0.8880	
SINS/RKCNS-CKF	0.8897	0.8218	0.9105	
SINS/RKCNS-ICKF	0.8282	0.7805	0.7638	
*SINS/RKCNS-MICKF	0.8479	0.7525	0.8081	

 Table 4
 RMSEs of attitude angles of various approaches



Fig. 7 TRMSEs of gyroscope drifts estimated by various approaches

5 Results and Discussion 1

As shown in Figs. 4, 5 and Table 3, the precision of the missile position gets improved by employing the proposed SINS/RKCNS framework with the same devices and movement trajectory as the conventional SINS/CNS. As mentioned above, Fig. 5 shows that the proposed SINS/RKCNS framework allows for accurate estimation of accelerometer bias, whereas the SINS/CNS framework does not. The RMSEs of the positions for 100 Monte Carlo runs are expressed in Table 3.

Additionally, the state characteristics are non-directly observable when the refractive apparent height is adopted as a measurement in the proposed SINS/RKCNS framework. ICKF plays a role in estimating the nonlinear measurement equations more accurately owing to the incorporation of the iterative technique. It can be observed from the Fig. 4 that the system positions have a superior steady-state accuracy and a better convergence trend than utilizing the regular CKF. Table 3 indicates that, in comparison with the CKF approach, the RMEs of positions are decreased by 44.48%, 6.28%, and 36.80% with ICKF, respectively, when SINS/RKCNS frameworks are employed. Working with the MICKF, the RMEs of positions decreased by 46.31%, 31.59%, and 58.53%, respectively. In ICKF, the iterations are transmitted only for the first moment, according to the design, and the state variables are updated after the iterations are finished because the CKF is substantially better at predicting the state vector than the covariances. Consequently, the mean and covariances are better synchronized, and the computational complexity is decreased. The innovation is fully employed and the ultimate steady-state bias is smaller through slowing down the rate of decay of the error covariance matrix. Furthermore, the MICKF has a greater advantage of steady-state error, because the first moment of the likelihood function can be approximated during update with varying accuracy owing to the iteration factor and the solutions for the state-dependent noise during the iterative process. The excellent steady-state performance and the rapid approximation of the accelerometer bias support this.

6 Results and Discussion 2.

The attitude angles statistics are evaluated in accordance with Figs. 6, 7, and Table 4. Conventional SINS/CNS can obtain high-precision attitude angles by a star sensor. As indicated by the RMSEs in Table 4, few noticeable differences occur when the SINS/ RKCNS framework is employed, since the attitude angles may be seen as independent. Additionally, the iterative approach approximates the first moment with varying accuracy, as Discussion 1, which results in a minor improvement in the steady-state accuracy of attitude angles. The proposed method, however, does not improve attitude accuracy significantly, as is clearly observed.

In summary, the proposed SINS/RKCNS-MICKF can estimate the acceleration bias and enhance the position accuracy of the nonlinear system. It works well on the state variables with weak observability, while doing less work on the state variables that are observable directly.

7 Conclusions

In order to estimate the accelerometer bias as well as enhance the position precision of the system, the SINS/RKCNS-MICKF algorithm is presented in this work. The SINS/RKCNS refers to a framework that is concentrated on attitude, apparent height, and kinematic constraints. The MAP-based MICKF is equipped with an iteration technique for states with weak observability and a state augmentation technique for the elimination of correlated noise in the nonlinear system. Experiments reveal that it is effective in estimating accelerometer bias and improving the position precision of the nonlinear navigation system. Later, further theoretical justification and comparative experiments will be included.

Abbreviations

CKF	Cubature Kalman filter
CNS	Celestial navigation system
EKF	Extended Kalman filter
GNSS	Global navigation satellite system
ICKF	Iterated cubature Kalman filter
IMU	Inertial measurement unit
KF	Kalman filtering
LMMSE	Linear minimization mean square error
MAP	Maximum a posteriori
MICKF	Modified iterated CKF
PDF	Probability density function
RKCNS	Refraction and kinematic celestial navigation system
SINS	Strapdown inertial navigation system
UKE	Unscented Kalman filter

Author contributions

All authors contributed to this study, including the methodology, experiments, and analysis. All authors read and approved the final manuscript.

Availability of data and materials

The data used in this paper are available from the corresponding author on reasonable request.

Declarations

Competing interests

The authors declare that they have no competing interests.

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