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RESEARCH

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Chaos detection scheme for multiple variable-frequency signals with overlapping frequencies

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Abstract

As one of the most critical technologies in signal detection, weak signal detection has received a great deal of attention in many areas. Traditional methods perform well when the number of signal is single and the form of signal is simple, which are quite different from the situation in some real applications (e.g., the marine communication scenario). In this paper, we consider the problem of high-precision frequency detection of multiple variable-frequency signals with overlapping frequencies (MVFS-O) in the low signal-to-noise ratio scenario, whose key point is how to detect the parameters (frequency range and frequency modulation rate) of each signal. To solve the problem, a novel chaos detection scheme based on Duffing difference oscillator is proposed. In this scheme, the wide frequency detection range of the critical state is used to determine the frequency range of the signals quickly, and the Lyapunov exponent is used to discriminate the output state of the system qualitatively. Furthermore, combining with the time points of the periodic state of the system, we can obtain the FMR of the corresponding signal by analyzing overlapping frequency ratio. Numerical results are provided to verify the practical effectiveness of the proposed scheme.

Keywords: Chaotic system, Weak signal detection, Variable-frequency signal, Overlapping frequency, Duffing system

1 Introduction

Weak signal detection (WSD) is long standing yet still one of the most important problems in signal detection. It relies on the fact that the signal detection environment is becoming more and more challenging [1]. WSD has a variety of applications such as deep-space communications [2], disaster identification [3], radiofrequency identification [4, 5], multiple-input multiple-output [6], unmanned aerial vehicle [7], and sonar navigation [8], to name just a few. Recently, as the different types of noises and various forms of signals exist widely, the high-accuracy WSD technique has received much attention. Among various WSD schemes, the high-precision frequency detection of single non-variable-frequency signal (SNVFS), single variable-frequency signal (SVFS), multiple non-variable-frequency signals (MNVFS), and multiple variable-frequency signals (MVFS) have been widely studied in low SNR regime [9]. Due to MVFS with far-away frequencies from each other (MVFS-F) can be decomposed into many SVFS with a filter bank that consists of non-overlapping band-pass filters, the detection of MVFS-F has not been a serious concern. However, when these frequency ranges are overlapped, the detection accuracy is severely degraded. Thus, it is of importance to come up with solutions to the high-precision detection of MVFS-O.

Over the years, various WSD schemes targeted for the noisy and reverberant environments have been proposed. Roughly speaking, these schemes can be divided into two categories. The first category is the linear detection methods where the target signal is detected by the linear filtering. Popular approaches in this class include correlation detection, time-domain averaging, and adaptive noise cancellation technology [10]. However, it is challenging to achieve satisfying performance under low SNR since the target signal is also filtered. The second category is nonlinear detection methods, such as higher-order statistics, stochastic resonance, and chaos theory. Higher-order statistics methods can obtain much more useful information about the target signal than the second-order statistics by a high computation complexity [11]. Stochastic resonance methods convert a part of noise energy into signal energy, which improves the SNR of the system output under certain condition [12]. Chaos theory detects the frequency of the weak single periodic signal by changing the system output state [13]. In general, above schemes may face either one or a combination of the following problems: suppressed weak target signal, unsatisfied performance in low SNR, high computational cost, the unreliable performance in practical, and non-realistic assumptions on signal or noise models.

Benefited from the sensitivity to the initial condition and the immunity to the AWGN (additive white Gaussion noise), the chaotic system can detect the weak signal with the same frequency as the oscillators without considering the strong background noise. However, existing researches mainly focus on the detection of SNVFS in the low SNR scenario. In [14], the chaos detection scheme for MNVFS has been proposed, which is only a simple extension of SNVFS. In [15], a chaos-based detection scheme for SVFS has been proposed, which extends the chaos theory to application in radar communication. There are many challenges in multiple signals detection based on chaos theory, which is shown in Fig. 1. In Fig. 1, we can decompose MVFS-F into many SVFS [15] with a filter bank that consists of identical non-overlapping band-pass filters. However, it can not be applicable in the situation of MVFS-O.

In this paper, we propose a chaos-based scheme to detect the frequencies of MVFS-O in the low SNR regime. Specifically, utilizing the characteristics of the critical state, we

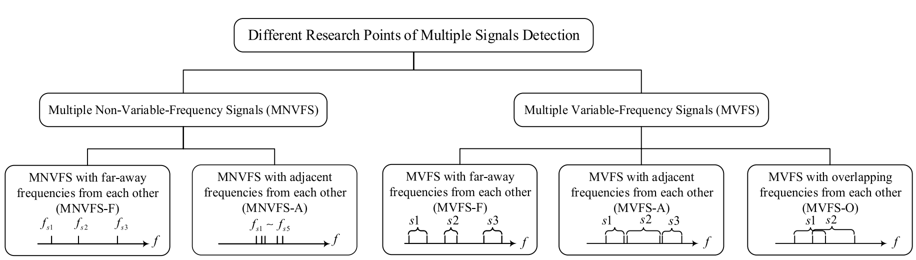


Fig. 1 Detailed architecture of challenges of multiple signals detection

can quickly determine the frequency range of the target signals, then obtain the periodic state's time points of the system based on the Lyapunov exponent (LE). To analyze MVFS-O, we introduce the overlapping frequency ratio (OFR) of the signals. When OFR is small, the scheme can achieve accurate detection of MVFS-O based on the satisfying performance for SVFS. As OFR increases, if the overlapping frequencies of signals are different, our scheme still has high detection accuracy. Furthermore, when the frequency range of a signal is contained totally in that of the other signal (OFR equals 1), we can still obtain the related parameters of the signal that has a wider frequency range. As such, the chaotic-based scheme for MVFS-O shows strong robustness in the complex real communication environment.

The rest of this paper is organized as follows. Section 2 describes the signal model of multiple variable-frequency and chaotic systems. Section 3 presents the proposed chaotic scheme based on chaos theory. The detection scheme for MVFS-O is shown in Sect. 4. Simulation results are presented in Sect. 5. Finally, we conclude the paper in Sect. 6.

2 Problem formulation and analysis

2.1 Multiple variable-frequency signals model

We consider the chirp signal given by:

$$s_m(t) = A_m(t) \cdot \exp\left[j \cdot 2\pi \left(f_m t + k_m t^2/2\right)\right] \tag{1}$$

where A_m and k_m are the amplitude and frequency modulation rate (FMR) of s_m, the frequency f_m is time-variant.

Multiple chirp signals with the number M are shown as follows:

$$\begin{aligned}\sum_{m=1}^{M} s_m(t) &= \sum_{i=1}^{M} A_m(t) \cdot \exp\left[j \cdot 2\pi \left(f_m t + \frac{1}{2} k_m t^2\right)\right] \\ &= \sum_{m=1}^{M} A_m(t)[\cos\left(2\pi f_m t + \pi k_m t^2\right) \\ &\quad + j \cdot \sin\left(2\pi f_m t + \pi k_m t^2\right)]\end{aligned} \tag{2}$$

Considering the rapid development of communication technology, the detection of several signals with overlapping frequencies widely exists. Here, we set $M = 2$, $\omega_m = 2\pi f_m$, consider the real part in the practical application, (2) can be expressed as follows:

$$\begin{aligned}\sum_{m=1}^{2} s_m(t) &= s_1(t) + s_2(t) \\ &= A_1(t)\cos\left(\omega_1 t + \pi k_1 t^2\right) + A_2(t)\cos\left(\omega_2 t + \pi k_2 t^2\right)\end{aligned} \tag{3}$$

Denote $\left[f_{s_1}^1, f_{s_1}^2\right]$ and $\left[f_{s_2}^1, f_{s_2}^2\right]$ as the frequency range of signal $s_1(t)$ and $s_2(t)$, respectively. Here, $\left(f_{s_1}^2 - f_{s_1}^1\right) > \left(f_{s_2}^2 - f_{s_2}^1\right)$. When the two signals overlap each other, $k_1 \neq k_2$, we define p_1 as the ratio of the overlapping frequency to the total frequency of signal s_1, which is shown as

$$p_1 = \frac{f_{\text{overlapping}}}{f_{\text{total}}} \tag{4}$$

where $f_{\text{overlapping}} = (f_{s_1}^2 - f_{s_2}^1)$ and $f_{\text{total}} = (f_{s_1}^2 - f_{s_1}^1)$ of s_1, respectively. Similarly, we can obtain p_2 of s_2.

2.2 Chaotic system

The Holmes–Duffing oscillator is expressed as follows:

$$\ddot{x}(t) + a\dot{x}(t) + bx(t) + cx^3(t) = \zeta \cos(\omega t) \tag{5}$$

where $\ddot{x}(t)$ and $\dot{x}(t)$ are the second-order and first-order derivatives of $x(t)$, $bx(t) + cx^3(t)$ is nonlinear restoring force, a is damping ratio, and ζ and ω are the amplitude and frequency of the driving force $\zeta \cos(\omega t)$, respectively.

Let $t = \omega\tau$, we can obtain the dynamic equation:

$$\begin{cases} \dot{x}_\tau = \omega y \\ \dot{y}_\tau = \omega(-ay - bx + cx^3 + \zeta \cos(\omega\tau)) \end{cases} \tag{6}$$

The Duffing oscillator system is illustrated in Fig. 2.

In Fig. 2, *Wave* is the driving force, *Fcn* is the nonlinear restoring force, and *Integ1* and *Integ2* are the integrals. *G1* and *G2* are the coefficients of the integrator, and *G3* is the damping ratio. Moreover, the time-domain waveform and phase diagram will be displayed on the *Scope* and *XY Graph*, respectively. When $a = 0.5$, $b = -1$, and $c = 1$, as the amplitude ζ increases, the phase diagram will change from the chaotic state to the critical state and then the large-scale periodic state. In the critical state, both chaotic state and large-scale period state occur alternatively. The time-domain waveforms and the corresponding phase diagrams are shown in Fig. 3.

Also, since the inputs of the target weak signal and oscillator have the same frequency components, the amplitude will increase, and the output state will be changed.

In phase space, Lyapunov exponents (LE) can be used to discriminate the output state of system qualitatively [16], which denotes the average exponential rates of divergence or convergence of error $|\delta r|$ between two adjacent points on the orbit through time t.

The definition of LE is expressed as follows:

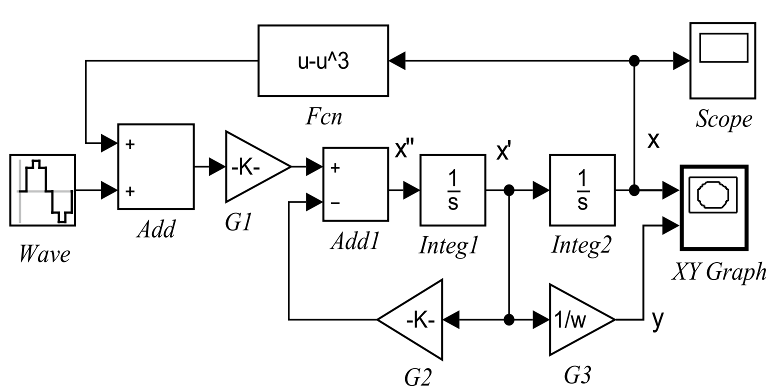


Fig. 2 Duffing oscillator system

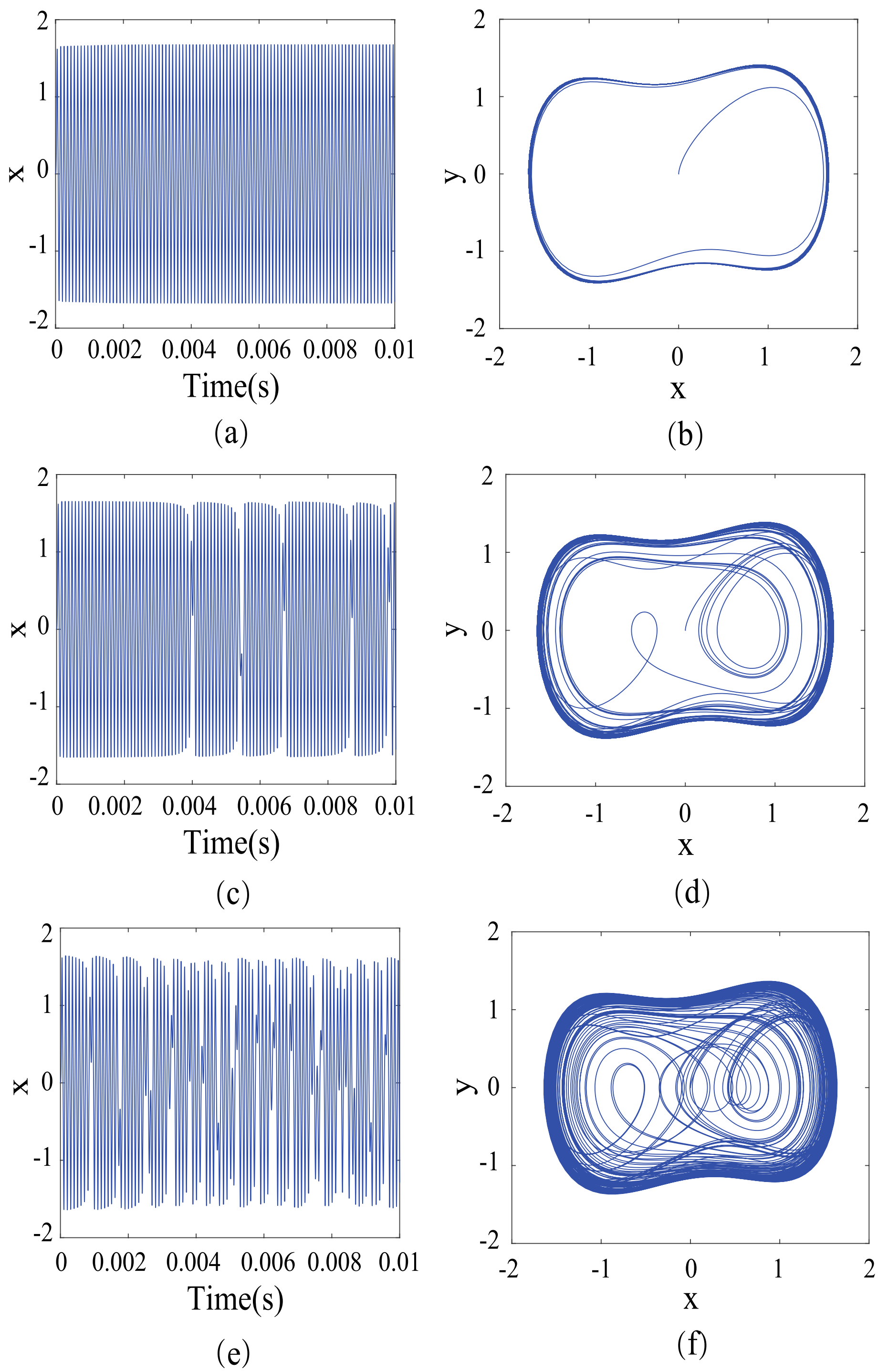


Fig. 3 The time-domain waveforms and the phase diagrams of the Duffing oscillator: **a**, **b** the periodic state, **c**, **d** the critical state, and **e**, **f** the chaotic state

$$LE(r_0, \delta r) = \lim_{\substack{t \to \infty \\ \delta r(0) \to -\infty}} \frac{1}{t} \ln \frac{|\delta r(r_0, t)|}{|\delta r(r_0, 0)|} \tag{7}$$

where $r_0 \in \mathbf{R}$, $\mathbf{R}$ is the space orbit of phase space.

From (7), we can get conclusions as follows: (1) If the nearby orbits converge, $|\delta r(r_0, t)|/|\delta r(r_0, 0)| < 1$ and LE < 0, then the system is stable, and the output state is large-scale periodic state. (2) If the nearby orbits diverge, $|\delta r(r_0, t)|/|\delta r(r_0, 0)| > 1$ and LE > 0, then the system becomes unstable, and the output state is changed into chaotic state. Based on the system in Fig. 2, the relationship between Lyapunov exponents (LE) and the amplitude ζ is illustrated in Fig. 4.

From Fig. 4, we can discriminate the output state as a chaotic state or periodic state according to that LE is positive or negative.

3 Detection scheme based on chaos theory

3.1 Duffing difference system

In the critical state, to get a better performance to discriminate between the chaotic state and the large-scale periodic state, we build a chaotic system consist of two Duffing oscillators, named Duffing difference system. Its model established is shown in Fig. 5. The parameters setting are similar to those in Fig. 2, which are not repeated here. And the time-domain waveform of the critical state in the Duffing difference system is shown in Fig. 6.

From Fig. 6, we can see that it has more apparent discrimination between the chaotic state and the large-scale periodic state in a critical state. Comparing to the time-domain waveform of critical state in Fig. 3c, the Duffing difference system has more strong robustness in the low SNR scenario.

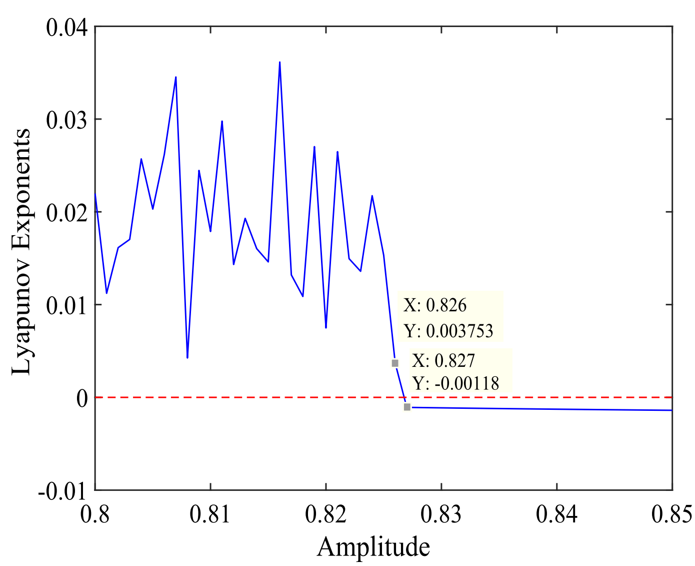


Fig. 4 The relationship between Lyapunov exponents (LE) and the amplitude ζ

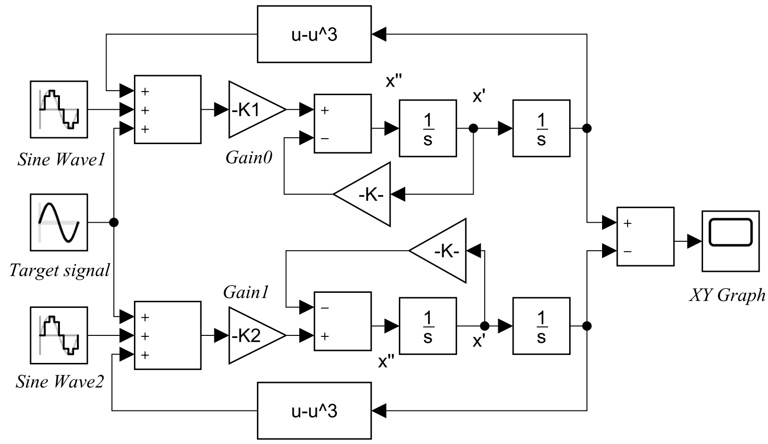


Fig. 5 The model of Duffing difference system

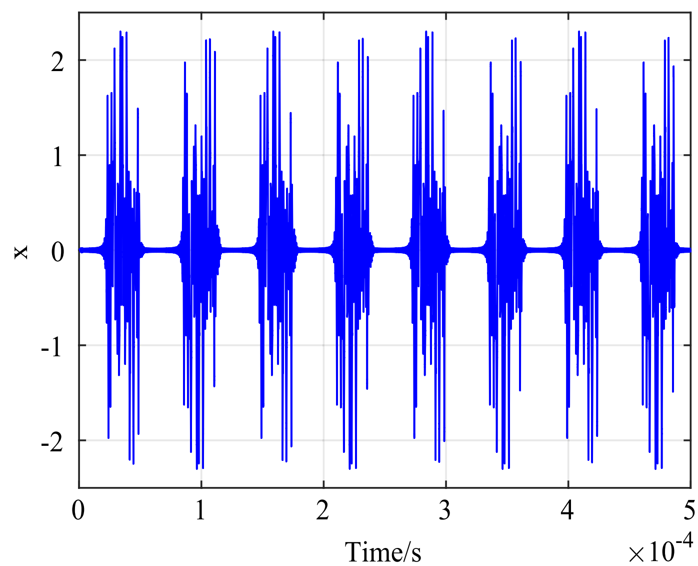
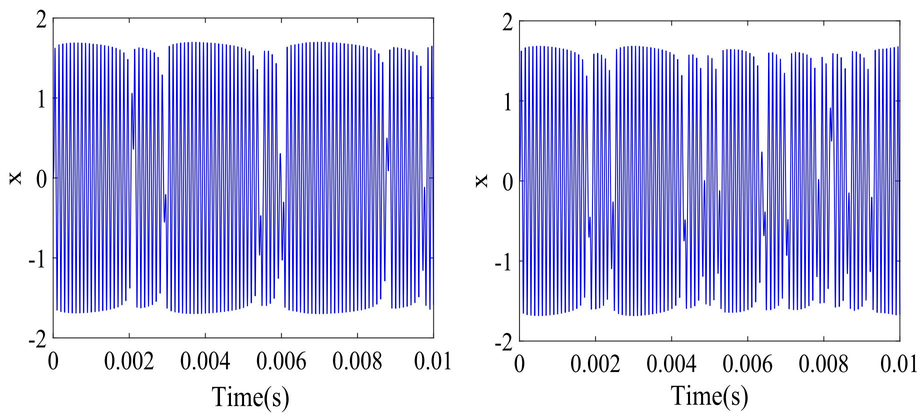


Fig. 6 Time-domain waveform of the critical state in Duffing difference system

3.2 Frequency detection range

During the period of the critical state, there are infinite combinations of chaotic state and large-scale period state. Setting the frequency of the target signal and system driving force is f_1 and f, respectively, and the frequency difference is $\Delta f = |f - f_1|$. Experimental results show that there is a visible critical state or large-scale periodic state when $\Delta f/f \leq 0.03$, which leads to the inaccurate in the determination of the period of the critical state T. Otherwise, $\Delta f/f > 0.03$, the output state becomes chaotic gradually [17]. For example, setting $f = 1 \times 10^4$Hz, when $f_1 = 1.03 \times 10^4$Hz and 1.04×10^4Hz, respectively, the output time-domain waveforms are shown in Fig. 7.

Benefiting from the frequency detection range of the critical state is $2\left|f_{t\arg et} - f\right|$, we can expand the frequency search range of target signal and determine whether the frequency is in $2\left|f_{t\arg et} - f\right|$ or not quickly.



(a) $f_1 = 1.03 \times 10^4$Hz (b) $f_1 = 1.04 \times 10^4$Hz

Fig. 7 The time-domain waveforms of the different signal frequencies

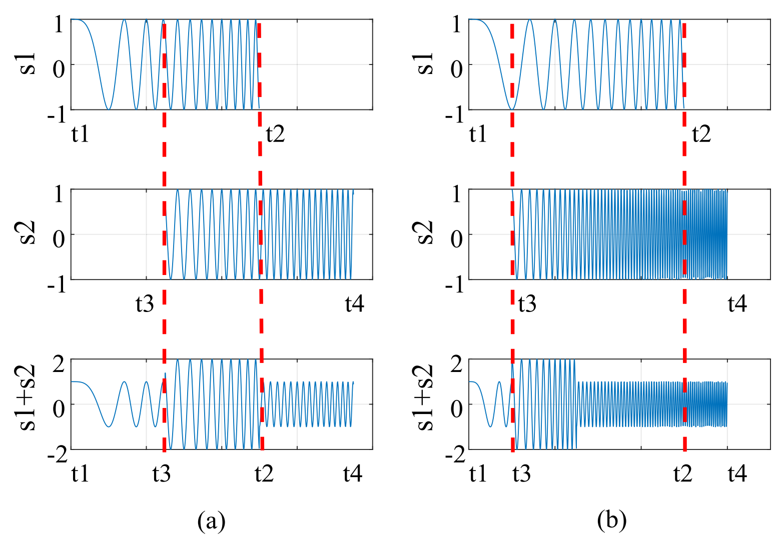


Fig. 8 The time-domain waveforms of two chirp signals with different values of k: **a** $k_1 = k_2$ and **b** $k_1 \neq k_2$

Table 1 Detection results of different k

Description	$k_1 = k_2$	$k_1 \neq k_2$
Diagram	See in Fig. 7a	See in Fig. 7b
Measured parameters	$f_1, f_4, k_1(k_2)$	$f_1, f_4, (k_1 + k_2)/2$
Unmeasured parameters	m	m

4 The detection scheme for MVFS-O

According to the different values of k_m, the time-domain waveforms of two signals are shown in Fig. 8, and the other parameters are shown in Table 1, where f_i is the instantaneous frequency of the time t_i, and k_m is the FMR of s_m. In the case of MVFSD-O, although the amplitude of the overlapping frequency increases, it is not useful for the weak signal detection of the chaotic system. Here, we can obtain the relevant parameters of the superimposed chirp signal, but the number of signals contained in it is not obtained, then we cannot get a satisfying performance. It can be explained by the basic idea of the chaotic detection system, which can detect the target signal with the same frequency as oscillator's in a signal set, that is, the frequency selecting essentially, and it is impossible to detect the frequency range of each signal in MVFS-O further.

An aim of this paper is the frequency detection of each signal in MVFS-O when SNR is low. Firstly, a chaotic system based on Duffing differential array with a random frequency value is built. Then, we input the target signals to the system to determine which oscillator's frequency f is in the frequency range of the target signals. Secondly, we set f as the center frequency of the array and select several frequencies with the same interval to further obtain the signals' frequency range. After that, the periodic state's time points t_i are obtained utilizing the LE method. Combining with the corresponding oscillator's frequency f_i, we can get the FMR by fitting the dataset (f_i, t_i). Finally, by analyzing the FMR values, we can obtain the number of signals, FMR, and the frequency range of each signal.

In the scheme above, we face two challenging points:

1. How to determine the frequency range of the target signals quickly.
2. How to get the parameters of each signal according to the dataset (f_i, t_i).

For the first challenging point, by the analysis of the characteristic of the critical state of a chaotic system, we know that it has a wide frequency detection range (0.97f–1.03f with f is the frequency of oscillator) rather than a single-frequency value [17], which results in inaccurate detection of SNVFS. However, for MVFS-O, this characteristic can be used to determine which oscillator's frequency is in the range of the target signals quickly, which increases the detection speed. The chaotic difference system based on Duffing oscillator for chirp signals is shown in Fig. 9.

For the second challenging point, we introduce the OFR of multiple signals to analyze the detection performance of variable-frequency signals. The proposed scheme can obtain high detection accuracy of MVFS-O in the following cases:

1. All the p_m are small.
2. All the p_m are large while the overlapping frequencies are different.

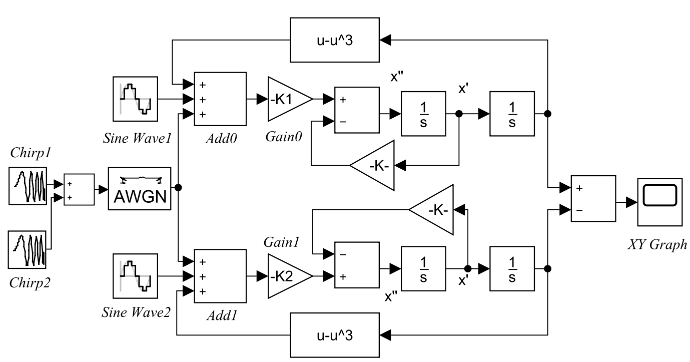


Fig. 9 The chaotic difference system based on Duffing oscillator

3. One of the p_m equals 1.

These cases are shown in Table 2 in detail. The algorithm is involved and consists of more than ten steps. Here, we summarize the key steps.

Step 1: The frequency range of the background environment and multiple chirp signals are $[f_A, f_B]$ and $\left[f_{s_1}^1, f_{s_2}^2\right]$, respectively. We add the oscillator with frequency $f_0 = \left(f_B + f_A\right)/2$ by the median detection method (MDM), and then, the LE method is used to determine the output state. The output state is chaotic state when LE is positive, the frequencies of new oscillators $\left(f_{11} = \left(f_0 + f_A\right)/2,\ f_{12} = \left(f_0 + f_B\right)/2\right)$ will be added and then repeat (7) until LE becomes negative. If we denote $\Phi = \left\{f_A, f_0, f_B\right\}$, we add the new oscillator's frequency f_{ij} to set Φ and then update the elements in ascending order, which is shown as:

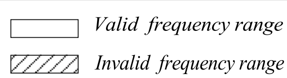
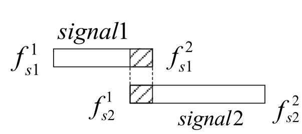
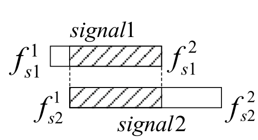
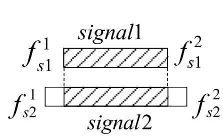
$$\begin{array}{ll} f_{11} = \text{floor}\left(\frac{\Phi[1]+\Phi[2]}{2}\right), & f_{12} = \text{floor}\left(\frac{\Phi[2]+\Phi[3]}{2}\right), \\ \cdots & , f_{ij} = \text{floor}\left(\frac{\Phi[j]+\Phi[j+1]}{2}\right) \end{array} \tag{8}$$

where $\Phi[1] = f_A$, $\Phi[2] = f_0$, $\Phi[3] = f_B$, $i = 1, 2, 3, \cdots$, $N_0 = 3$, $N_i = 2N_{i-1} - 1$, $n_i = N_i - N_{i-1}$, and $j = 1, 2, 3, \cdots, n_i$, *floor* is down integral function. We can obtain the frequency f when the output state of the system is periodic state and the corresponding LE is negative. The diagram of the frequency selection of the new Duffing oscillator is shown in Fig. 10.

Step 2: We set f as the center frequency, the Duffing array is built to detect the target signals. Based on (7), we can obtain the frequency and time point when the LE value is negative, which is dataset $\left(f_i, t_i\right)$.

Step 3: The FMR can be obtained by linear fitting based on the dataset $\left(f_i, t_i\right)$. Three cases are analyzed as follows:

Table 2 Detection results of chaotic system with p

Case	Diagram
	
p_{s1}, p_{s2} are low	
p_{s1}, p_{s2} are increase	
$p_{s1} = 1$, p_{s2} is high	

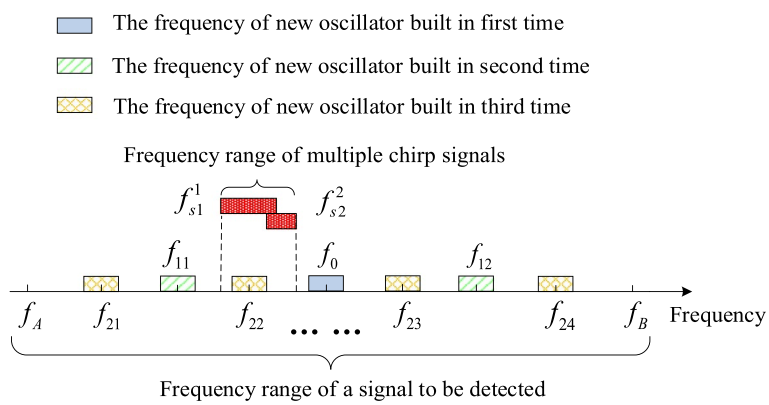


Fig. 10 The frequency selection of the new Duffing oscillator in array by MDM

Table 3 Simulation parameters and values of duffing system

Description	Parameters	Values
Number of signals	m	2
Frequency difference	Δf	0.2×10^4Hz
Number of oscillators	v	8
Sampling frequency	f_s	1×10^6Hz
Signal-to-noise ratio	SNR	5dB

Case 1: There are two intersecting straight line segments with an inflection point, which note as k_1, k_2.

Case 2: There are two non-intersecting straight line segments, which are the k_1, k_2 of signal s_1, s_2. Note that if the overlapping frequencies of the two signals are the same, which will reduce the detection accuracy greatly due to the overlapping frequencies are useless. Otherwise, the proposed scheme can obtain satisfying performance when the overlapping frequencies of the two signals are different.

Case 3: There are three straight line segments, two of which intersect with an inflection point, and the third line segment does not intersect, so there is a break point. Considering the frequency range of signal $\left(f_{s_1}^2 - f_{s_1}^1\right) > \left(f_{s_2}^2 - f_{s_2}^1\right)$, we can obtain the k_1, k_2 of signal s_1, s_2, and p_2 equals to 1.

When the number of signals is more than two, it is necessary to analyze the overlapping frequency further, which will be studied in detail in the future.

5 Simulation results

In this section, we present the detection of MVFS-O by the chaotic system consisted of Duffing difference oscillator array, and then, we analyze the simulation results. Simulation parameters of chaotic system are shown in Table 3. Considering three cases of two chirp signals with overlapping frequencies, the detailed parameters are shown in Table 4.

Table 4 Simulation parameters and values of three cases

Description	Case 1	Case 2	Case 3
Frequency range of s_1	$[1 \times 10^4, 2 \times 10^4]$Hz	$[1.5 \times 10^4, 2.4 \times 10^4]$Hz	$[1.2 \times 10^4, 2.4 \times 10^4]$Hz
Duration of s_1	0–0.01 s	0.005–0.014 s	0.002–0.014 s
Frequency range of s_2	$[1.8 \times 10^4, 2.4 \times 10^4]$Hz	$[1.0 \times 10^4, 3.0 \times 10^4]$Hz	$[1.6 \times 10^4, 2.2 \times 10^4]$Hz
Duration of s_2	0.009–0.013 s	0.001–0.011 s	0.006–0.009 s
Frequency range of array	$[1.0 \times 10^4, 2.4 \times 10^4]$Hz	$[1.0 \times 10^4, 2.4 \times 10^4]$Hz	$[1.2 \times 10^4, 2.6 \times 10^4]$Hz
OFR	$p_1 = 1/3, p_2 = 1/5$	$p_1 = 2/3, p_2 = 3/5$	$p_1 = 1/4, p_2 = 1$

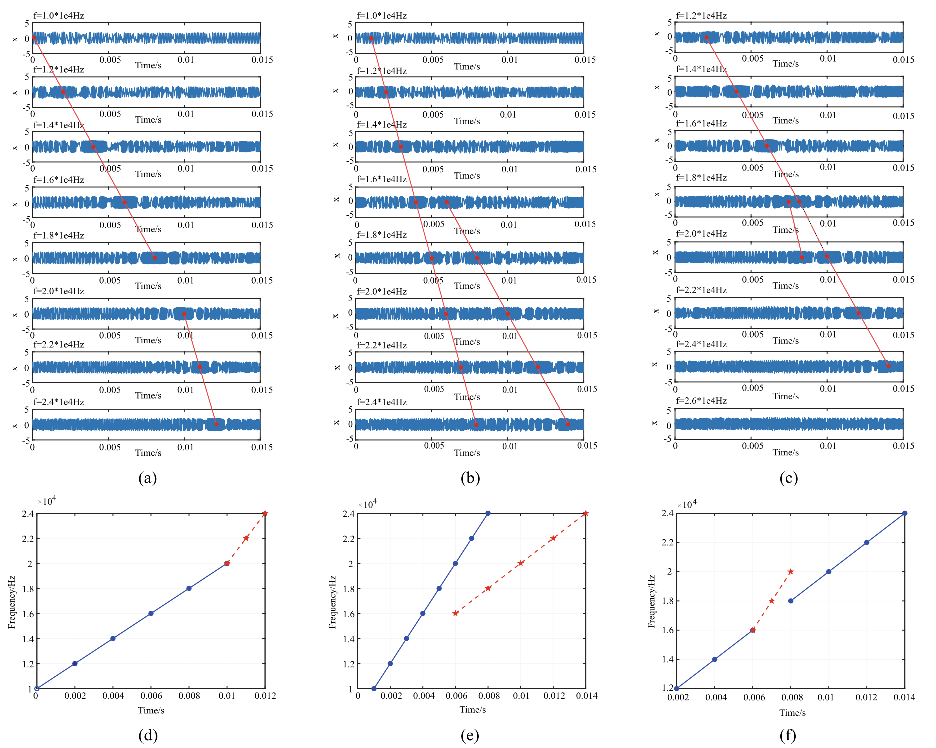


Fig. 11 The time-domain waveform of the system and corresponding FMR in different OFRs: **a**, **d** Case 1, **b**, **e** Case 2, and **c**, **f** Case 3

Case 1: Fig. 11a shows the time-domain waveform of the output state of the system when $p_1 = 1/3$ and $p_2 = 1/5$. The FMR is obtained by linear fitting of the dataset (f_i, t_i), which is shown in Fig. 11d. Since the overlapping frequency range is small, we can obtain the FMR accurately by the non-overlapping frequency.

Case 2: When OFR increases ($p_1 = 2/3$ and $p_2 = 3/5$), we should consider whether the overlapping frequencies of the two signals are same or not. If they are the same, the frequency characteristics of each signal no longer exist in the overlapping part, which leads to the output state of the system becoming chaotic. Otherwise, if they are different, benefiting from the sensitivity to the specific frequency and the immunity to interference, the proposed scheme can still obtain the high detection accuracy of each signal in MVFS-O. Figure 11b shows the time-domain waveform of the system output state when the overlapping frequencies of the two signals are different, and the FMRs are shown in Fig. 11e.

Case 3: When one of the OFRs increases to 1, we consider $p_1 = 1/4$ and $p_2 = 1$. From Fig. 11c, the FMR of s_1 can be obtained by the non-overlapping frequency range of the signal. Although we can detect several frequencies of s_2, the detection accuracy is low, which exhibits in Fig. 11f.

Generally, the proposed scheme in this paper can get satisfying detection performance in MVFS-O, which demonstrates that the chaos theory has potential advantages in MVFS. Although the target signals are multiple and the environment is complex, we still can obtain the high-precision detection by rational system model, which proves the great potential research value of chaos theory in WSD.

Considering that there are seldom detection schemes for MVFS-O scenarios, we choose the detection performance of SVFS for comparison, which is a subset of MVFS-O. Figure 12 shows the performance comparison between the literature [18] and the proposed scheme of SVFS. Benefited from the Duffing difference system and algorithm based on LE in the proposed scheme, we can achieve high-precision detection of SVFS in ultra-low SNR regime. In other words, our detection scheme for MVFS-O has significant advantages over the other solutions.

6 Conclusions and future work

In this paper, we proposed a new chaotic detection scheme to detect the frequency of each signal in MVFS-O. This scheme is different from the usual WSD methods since it takes advantage of Duffing difference array to improve the precision. We have shown that the performance of our scheme is satisfied in low SNR. Another advantage of our scheme is that it introduces the overlapping frequency ratio (OFR) to analyze the performance of detection of MVFS-O. To guarantee the detection speed, we use the Lyapunov exponent to discriminate the output state of the system qualitatively. Meanwhile, using the wide frequency detection range of the critical state, we can obtain

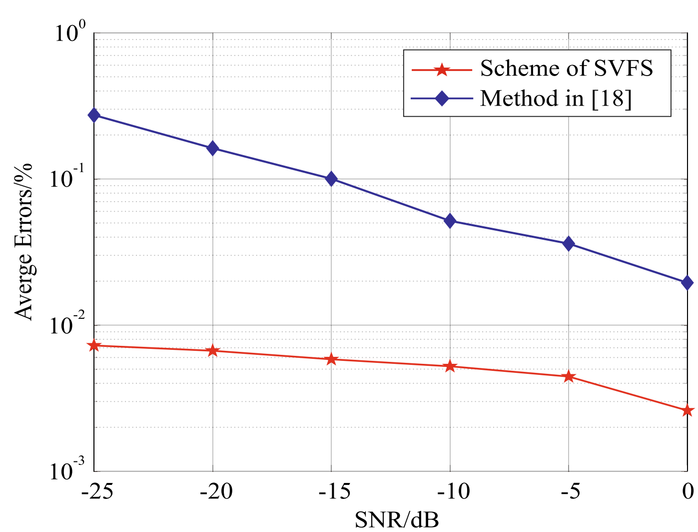


Fig. 12 The performance comparison between the literature [18] and the scheme of SVFS

the high-accuracy frequency detection of MVFS-O in the complex communication environment. Simulations are carried out to show the effectiveness of the proposed detection scheme in the situation of MVFS-O while ensuring the detection accuracy of FMR. We believe that our detection scheme represents a new way of multi-signal detection and would be able to improve WSD not only for under the strong noise but also for a complex signal having multiple harmonics.

Nowadays, there are two typical application scenarios of MVFS-O:

1. Multiple motion targets transmit signals with a fixed frequency. The detection of MVFS-O can obtain the motion parameters of multiple targets (e.g., speed and distance).
2. Multiple fixed targets transmit signals with variable frequencies. The detection of MVFS-O can obtain more information on multiple targets (e.g., emergency and encrypted message).

Although the detection scheme of MVFS-O has obtained a satisfying performance in theory, there is a lot of research on the application. The impact of different noise forms on accuracy, power consumption, and time consumption is worthy of further study.

Abbreviations

WSD	Weak signal detection
SNVFS	Single non-variable-frequency signal
SVFS	Single variable-frequency signal
MNVFS	Multiple non-variable-frequency signals
MVFS	Multiple variable-frequency signals
MVFS-O	MVFS with overlapping frequencies
MVFS-F	MVFS with far-away frequencies
SNR	Signal-to-noise ratio
FMR	Frequency modulation rate
AWGN	Additive white Gaussion noise
LE	Lyapunov exponent
OFR	Overlapping frequency ratio
MDM	Median detection method

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Author contributions
In this paper, we proposed a new chaotic detection scheme to detect the frequency of each signal in multiple variable-frequency signals with overlapping frequencies (MVFS-O). This scheme is different from the usual weak signal detection (WSD) methods since it takes advantage of Duffing difference array to improve the precision. We have shown that the performance of our scheme is satisfied in low SNR. Another advantage of our scheme is that it introduces the overlapping frequency ratio to analyze the performance of detection of MVFS-O. To guarantee the detection speed, we use the Lyapunov exponent to discriminate the output state of the system qualitatively. Meanwhile, using the wide frequency detection range of the critical state, we can obtain the high-accuracy frequency detection of MVFS-O in the complex communication environment. Simulations are carried out to show the effectiveness of the proposed detection scheme in the situation of MVFS-O while ensuring the detection accuracy of frequency modulation rate. We believe that our detection scheme represents a new way of multi-signal detection and would be able to improve WSD not only for under the strong noise but also for a complex signal having multiple harmonics.

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Publisher's Note

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