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*Research Article*

# A Dual Decomposition Approach to Partial Crosstalk Cancelation in a Multiuser DMT-xDSL Environment

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In modern DSL systems, far-end crosstalk is a major source of performance degradation. Crosstalk cancelation schemes have been proposed to mitigate the effect of crosstalk. However, the complexity of crosstalk cancelation grows with the square of the number of lines in the binder. Fortunately, most of the crosstalk originates from a limited number of lines and, for DMT-based xDSL systems, on a limited number of tones. As a result, a fraction of the complexity of full crosstalk cancelation suffices to cancel most of the crosstalk. The challenge is then to determine which crosstalk to cancel on which tones, given a complexity constraint. This paper presents an algorithm based on a dual decomposition to optimally solve this problem. The proposed algorithm naturally incorporates rate constraints and the complexity of the algorithm compares favorably to a known resource allocation algorithm, where a multiuser extension is made to incorporate the rate constraints.

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## 1. INTRODUCTION

Far-end crosstalk (FEXT), which is typically 10–15 dB larger than the background noise, is a major source of performance degradation in xDSL systems. One strategy for dealing with this crosstalk is crosstalk cancellation. Several crosstalk cancellation schemes have been proposed. Linear pre- and post filtering [1, 2] requires coordination at both the transmitters and receivers. Successive interference cancellation or precompensation [3, 4] can be used if there is only coordination available at the receivers or transmitters, respectively, for example, in the case of crosstalk cancellation in an upstream VDSL scenario. For this level of coordination, it is shown in [5, 6] that a simple linear zero-forcing canceller or linear precompensator performs near-optimally in an xDSL environment.

Even for these simple linear cancellers, the complexity grows with the square of the number of lines. For example, in a binder of 8 VDSL lines transmitting on 4096 tones at a block rate of 4000 blocks per second, the runtime complexity of crosstalk cancellation exceeds 1 billion multiplications per second.

However, crosstalk exhibits space and tone selectivity [7]. Measurements show that most of the crosstalk originates from a limited number of lines, for example, those in close proximity. Moreover, crosstalk coupling is heavily dependent on the frequency.

Because most of the crosstalk originates from a limited number of lines on a limited number of tones, a fraction of the complexity of full crosstalk cancellation suffices to cancel most of the crosstalk. This is called partial crosstalk cancellation [7, 8].

The challenge in these upstream VDSL scenarios is then to determine for every user which crosstalk to cancel on which tones. In [7], an algorithm based on resource allocation is presented to solve this single-user problem. This paper presents an alternative optimal algorithm, based on a dual decomposition. The complexity of the algorithm is found to be more favourable than the complexity of the resource allocation algorithm, where a multiuser extension is made to incorporate rate constraints.

In Section 2, the partial crosstalk cancellation problem is presented and then solved following a dual decomposition approach. A number of observations is made to reduce the complexity without losing the optimality of the solution. In Section 3, the complexity of the single-user version of the dual decomposition algorithm is compared to the complexity of the resource allocation algorithm for the single-user case, where each user has an individual complexity constraint. Section 4 then extends these results to the multiuser case where all users share a complexity constraint. A search procedure is presented to dynamically distribute the available complexity for crosstalk cancellation according to the rate constraints. Section 5 provides some simulation results and finally Section 6 concludes the paper.

## 2. DUAL DECOMPOSITION

### 2.1. System model

Most current DSL systems use discrete multitone (DMT) modulation. The available frequency band is divided in a number of parallel subchannels or tones. Each tone is capable of transmitting data independently from other tones, and so the transmit power and the number of bits can be assigned individually for each tone.

Transmission for a binder of $N$ users can be modelled on each tone $k$ by

$$\mathbf{y}_k = \mathbf{H}_k\mathbf{x}_k + \mathbf{z}_k, \quad k = 1 \cdots K. \tag{1}$$

The vector $\mathbf{x}_k = [x_k^1, x_k^2, \ldots, x_k^N]^T$ contains the transmitted signals on tone $k$ for all $N$ users. $[\mathbf{H}_k]_{n,m} = h_k^{n,m}$ is an $N \times N$ matrix containing the channel transfer functions from transmitter $m$ to receiver $n$. The diagonal elements are the direct channels, the off-diagonal elements are the crosstalk channels. $\mathbf{z}_k$ is the vector of additive noise on tone $k$, containing thermal noise, alien crosstalk, RFI, .... The vector $\mathbf{y}_k$ contains the received symbols.

The linear zero-forcing crosstalk canceller $\mathbf{W}$ cancels the crosstalk by making a linear combination of the received signals:

$$\hat{\mathbf{x}}_k = \mathbf{W}_k\mathbf{y}_k = \mathbf{W}_k\mathbf{H}_k\mathbf{x}_k + \mathbf{W}_k\mathbf{z}_k, \quad k = 1 \cdots K, \tag{2}$$

where $\mathbf{W}_k$ is chosen based on the zero-forcing criterion such that the equivalent channel $\mathbf{W}_k\mathbf{H}_k$ becomes an identity matrix. In [5, 6] it is shown that, due to the characteristics of the xDSL channel, $\mathbf{W}$ exists and does not change the statistics of the noise. In the case of partial crosstalk cancellation $\mathbf{W}_k$ is chosen to be sparse [7], thereby saving on the number of calculations that is required, such that the resulting equivalent channel also becomes sparse.

In this paper, partial crosstalk cancellation is taken into account by introducing an equivalent channel $\widetilde{\mathbf{H}}$. This is the same channel as the original channel $\mathbf{H}$, but with off-diagonal elements set to zero where the crosstalk is cancelled. If user $n$ is cancelling crosstalk originating from user $m$ on tone $k$, then $\widetilde{h}_k^{n,m} = 0$.

We denote the transmit power as $s_k^n \triangleq \Delta_f E\{|x_k^n|^2\}$, the noise power as $\sigma_k^n \triangleq \Delta_f E\{|z_k^n|^2\}$. The DMT symbol rate is denoted as $f_s$, the tone spacing as $\Delta_f$.

It is assumed that each modem treats interference from other modems as noise. When the number of interfering modems is large, the interference is well approximated by a Gaussian distribution. Under this assumption the achievable bit loading of user $n$ on tone $k$, given the transmit spectra of all modems in the system and the crosstalk cancellation configuration, is

$$b_k^n \triangleq \log_2\left(1 + \frac{1}{\Gamma}\frac{|\widetilde{h}_k^{n,n}|^2 s_k^n}{\sum_{m\neq n}|\widetilde{h}_k^{n,m}|^2 s_k^m + \sigma_k^n}\right), \tag{3}$$

where $\Gamma$ denotes the SNR-gap to capacity, which is function of the desired BER, the coding gain and noise margin. The data rate for user $n$ is

$$R^n = f_s \sum_k b_k^n. \tag{4}$$

When interference is being cancelled, the assumption of Gaussian noise becomes less valid. Under non-Gaussian noise, (3) gives a lower bound on the capacity of the channel. However, it remains the best model available for the achievable bitrate.

### 2.2. Partial crosstalk cancellation problem

Because of the runtime complexity of full crosstalk cancellation, only a limited amount of crosstalk can be cancelled. The cancellation of the crosstalk from one user on some tone is done by a cancellation tap. The number of cancellation taps that can be used is constrained by the *cancellation tap constraint* $C^{\text{tot}}$ [9]. The partial crosstalk cancellation problem amounts to finding an optimal selection of which crosstalk to cancel, thereby maximizing the capacity of the network.

Secondly, there is a *rate constraint* $R^{n,\text{target}}$ for each user. Typically, service providers offer a number of profiles to guarantee a certain quality of service. The rate constraint then indicates a minimum data rate required by the user.

The allocation of cancellation taps in partial crosstalk cancellation then results in the following maximization problem:

$$\begin{aligned}
&\text{maximize}_{\mathbf{c}} \sum_{n=1}^{N} R^n \\
&\text{subject to } C = \sum_{k=1}^{K}\sum_{m=1}^{N}\sum_{n=1}^{N} c_k^{n,m} \leq C^{\text{tot}}, \\
&\qquad\qquad R^n \geq R^{n,\text{target}} \quad n = 1 \cdots N \\
&\text{with} \qquad [\mathbf{c}_k]_{n,m} = c_k^{n,m} \quad c_k^{n,m} = \begin{cases} 0 \Longrightarrow \widetilde{h}_k^{n,m} = h_k^{n,m}, \\ 1 \Longrightarrow \widetilde{h}_k^{n,m} = 0, \end{cases}
\end{aligned} \tag{5}$$

where $\mathbf{c} = [\mathbf{c}_1, \mathbf{c}_2, \ldots, \mathbf{c}_K]$. $c_k^{n,m} = 1$ indicates that a cancellation tap is assigned on tone $k$ for cancelling crosstalk on line $n$ originating from line $m$.

To find the global optimum for this optimization problem, one has to exhaustively search through all possible cancellation tap configurations $\mathbf{c}$. Because the cancellation tap constraint and the rate constraints are coupled over the tones, this results in an exponential complexity in the number of tones. By using a dual decomposition this complexity can be made linear [9–13]. This is done by using Lagrange multipliers to move the constraints coupled over tones to the objective function of the optimization problem [10]:

$$\mathbf{c}^{\text{opt}} = \underset{\mathbf{c}}{\operatorname{argmax}} \sum_{n=1}^{N} \omega_n R^n + \lambda \left( C^{\text{tot}} - \sum_{k=1}^{K} \sum_{m=1}^{N} \sum_{n=1}^{N} c_k^{n,m} \right)$$
$$\text{subject to } \lambda \geq 0, \quad \omega_n \geq 0 \quad n = 1 \cdots N, \tag{6}$$

where $\lambda$ and $\omega_n$ are Lagrange multipliers. For a given set of $\lambda$ and $\boldsymbol{\omega} = [\omega_1, \ldots, \omega_N]^T$, (6) is a maximization of a sum over tones that can be performed by maximizing each tone individually. The optimization problem can then be solved in a per-tone fashion:

$$\text{for } k = 1 \cdots K, \quad \mathbf{c}_k^{\text{opt}} = \underset{\mathbf{c}_k}{\operatorname{argmax}} \sum_{n=1}^{N} \omega_n f_s b_k^n - \sum_{n=1}^{N} \sum_{m=1}^{N} \lambda c_k^{n,m}$$
$$\text{subject to } \lambda \geq 0, \quad \omega_n \geq 0 \quad n = 1 \cdots N. \tag{7}$$

Maximization of (7) for given Lagrange multipliers can be performed by an exhaustive search. For each tone, all possible combinations for the cancellation taps of the users should be checked. The combination giving the largest value for this expression is the optimal allocation of canceller taps for this tone.

The constraints can be enforced by choosing appropriate values for the Lagrange multipliers. The $\lambda$ can be viewed as a cost for crosstalk cancellation taps. Larger values for the Lagrange multiplier result in less cancellation taps being allocated. The data rates of the users are weighted by $\boldsymbol{\omega}$, thereby giving more importance to some users. In this way, all possible tradeoffs can be made to enforce the data rate constraints.

To solve (5) by (7), $\boldsymbol{\omega}$ and $\lambda$ should be tuned to enforce the constraints. In [10, 11], an efficient Lagrange multiplier search procedure is presented for a similar problem. This procedure can be easily adapted for this partial cancellation problem. The basis for this procedure is relation (8), which is proven in the appendix:

$$\begin{bmatrix} -(\Delta\boldsymbol{\omega})^T & \Delta\lambda \end{bmatrix} \begin{bmatrix} \Delta\mathbf{R} \\ \Delta C \end{bmatrix} \leq 0, \tag{8}$$

$\mathbf{R} = [R^1, \ldots, R^N]^T$ is a vector with the data rates and $C$ is the number of cancellation taps corresponding to the Lagrange multipliers at hand.

Following [10, 11], relation (8) leads to the following update formula for the Lagrange multipliers:

$$\begin{bmatrix} \Delta\boldsymbol{\omega} \\ \Delta\lambda \end{bmatrix} = -\mu \begin{bmatrix} \mathbf{R} - \mathbf{R}^{\text{target}} \\ C^{\text{tot}} - C \end{bmatrix}$$
$$\Longrightarrow \begin{bmatrix} \boldsymbol{\omega} \\ \lambda \end{bmatrix}^{t+1} = \left( \begin{bmatrix} \boldsymbol{\omega} \\ \lambda \end{bmatrix}^{t} - \mu \begin{bmatrix} \mathbf{R} - \mathbf{R}^{\text{target}} \\ C^{\text{tot}} - C \end{bmatrix} \right)^{+}, \tag{9}$$

```
while distance > tolerance do
    Θ = [ω, λ]^T = best [ω, λ]^T so far
    μ = 1
    while distance ≤ previousDistance do
        previousDistance = distance
        μ = μ × 2
        ΔΘ = [Δω, Δλ]^T = update formula (9)
        [R^{Θ+ΔΘ}, C^{Θ+ΔΘ}, c] = exhaustiveSearch(Θ + ΔΘ)
        distance = ||[R^{Θ+ΔΘ} − R^target, C^tot − C^{Θ+ΔΘ}]^T||
    endwhile
endwhile
```

Algorithm 1: Lagrange multiplier search algorithm.

where $(x)^+$ means $\max(0, x)$ and $\mu$ is a stepsize parameter. Note that all the Lagrange multipliers are updated in parallel. This update formula is used in Algorithm 1, adopted from [10], to converge to the Lagrange multipliers that enforce the constraints.

The partial crosstalk cancellation problem (5) is a nonconvex constrained optimization problem. Without dual decomposition, finding the global optimum requires an exhaustive search over all possible solutions. On a certain tone, a user has to decide which crosstalk of $N-1$ other users has to be cancelled. There are $2^{N-1}$ possibilities to do this. For $N$ users and $K$ tones, this results in a total complexity of $\mathcal{O}((2^{N-1})^{NK})$.

In [9] it is shown that when using a dual decomposition in multicarrier systems, the duality gap is zero. Therefore the solution for the dual problem is also the solution for the primal problem.

The dual decomposition decouples the problem over the tones, therefore reducing the exponential complexity in the number of tones $K$ to linear complexity: $\mathcal{O}(K(2^{N-1})^N)$. This amounts to $K$ exhaustive searches of complexity $\mathcal{O}((2^{N-1})^N)$. For an 8 user VDSL system, the complexity is reduced from $2^{7\times 8\times 4096}$ to $4096 \times 2^{7\times 8}$. This is an enormous reduction in complexity. Moreover, as shown in the next subsection, the complexity can be even further reduced by observing that many cancellation tap configurations can be eliminated in advance.

### 2.3. Per-tone search complexity reduction

To determine the optimal allocation of crosstalk cancellation taps on a certain tone, all of the $(2^{N-1})^N \approx 2^{N^2}$ possible allocations have to be evaluated. Even for a limited number of users this becomes complex. Fortunately, many of these possibilities can be eliminated based on two observations: user independence and line selection.

(i) *User independence*: all users have to decide on a crosstalk cancellation configuration. This leads to an exponential complexity in the number of users $N$. However, from (3) it can be seen that if user $n$ allocates a crosstalk cancellation tap to cancel crosstalk caused by user $m$ (i.e., $\widetilde{h}_k^{n,m} = 0$) this only has an influence on the capacity of user $n$. This corresponds to a per-user decoupling of (7), leading to

$$\begin{aligned}
&\text{for } k = 1 \cdots K,\\
&\text{for } n = 1 \cdots N,\\
&\mathbf{c}_k^{n,\text{opt}} = \underset{\mathbf{c}_k^n}{\text{argmax}}\, \omega_n f_s b_k^n - \sum_{m=1}^{N} \lambda c_k^{n,m}\\
&\text{subject to } \lambda \geq 0,\\
&\qquad \omega_n \geq 0 \quad n = 1 \cdots N.
\end{aligned} \tag{10}$$

As a consequence, the exponential complexity in $N$ is reduced to linear complexity. Instead of one large search over all users, there are $N$ independent searches for the users. This observation results in the following complexity reduction:

$$\left(2^{N-1}\right)^N \longrightarrow N\left(2^{N-1}\right). \tag{11}$$

(ii) *Line selection*: a user has to decide for $N-1$ other users whether or not to cancel the crosstalk originating from these other users. This leads to $2^{N-1}$ possible crosstalk cancellation configurations. However, from (3) it can be seen that to maximize the capacity, one should allocate crosstalk cancellation taps to cancel the users which are causing the largest crosstalk. Therefore, if $n$ crosstalk cancellation taps are available, these should be used to cancel the $n$ largest sources of crosstalk. As a consequence, the $2^{N-1}$ possibilities for crosstalk cancellation are reduced to $N$ possibilities: cancel no crosstalker, cancel the strongest crosstalker, cancel the 2 strongest crosstalkers,..., cancel all $N-1$ crosstalkers,

$$\begin{aligned}
&\text{for } k = 1 \cdots K,\\
&\text{for } n = 1 \cdots N,\\
&\mathbf{c}_k^{n,\text{opt}} = \underset{\mathbf{c}_k^n}{\text{argmax}}\, \omega_n f_s b_k^n(r) - \lambda r\\
&\text{subject to } \lambda \geq 0,\\
&\qquad \omega_n \geq 0 \quad n = 1 \cdots N,
\end{aligned} \tag{12}$$

where $b(r)$ is the capacity when the $r$ largest crosstalkers are cancelled.

When both observations are combined, $N$ users independently have to choose one of $N$ possible crosstalk cancellation configurations. This results in the following total complexity reduction:

$$\left(2^{N-1}\right)^N \longrightarrow NN. \tag{13}$$

In an 8-user case, these observations reduce the number of crosstalk cancellation configurations to be evaluated from $2^{56}$ to $2^6$. Note that despite drastic complexity reductions, the solution is still optimal.

## 3. SINGLE-USER ALGORITHMS AND COMPLEXITY COMPARISON

In this section, the complexity of the algorithm based on dual decomposition is analyzed and compared to the complexity of the optimal resource allocation algorithm of [7]. The resource allocation algorithm is a single-user algorithm. Therefore, a single-user formulation of the dual decomposition algorithm is used for the complexity comparison. The results will then be extended to the multiuser case in Section 4.

### 3.1. Single-user resource allocation algorithm

The resource allocation algorithm uses the average capacity increase per allocated crosstalk cancellation tap on a certain tone:

$$\nu_k(r) = \frac{b_k(r) - b_k(0)}{r}, \tag{14}$$

with $b_k(r)$ the capacity on tone $k$ when the $r$ largest crosstalkers are cancelled (cf. Section 2.3, line selection). A greedy algorithm then selects the tone $k$ and number of crosstalkers $r$ to cancel by searching the largest value of $\nu_k(r)$. The average capacity increase per allocated crosstalk cancellation tap should then be recalculated on tone $k_s$, based on the selected value $\nu_{k_s}(r_s)$, as follows:

(i) the average capacity increase for allocating less or equal crosstalk cancellation taps than $r_s$ is set to zero,

(ii) the average capacity increase for allocating more crosstalk cancellation taps than $r_s$ is recalculated as $\nu_k(r) = (b_k(r) - b_k(r_s))/(r - r_s)$, where the increase is now referenced to $b_k(r_s)$.

This is repeated until all available crosstalk cancellation taps are allocated. Note that in each iteration of the algorithm a minimum of 1 and a maximum of $N-1$ crosstalk cancellation taps are allocated. Because of this varying granularity, the crosstalk cancellation tap constraint cannot always be enforced tightly. However, the granularity is small enough to get close to the constraint.

The procedure is presented in Algorithm 2. A $K \times (N-1)$ table is initialized containing the average capacity increases per allocated crosstalk cancellation tap. For each of $K$ tones the capacity increase has to be calculated for all $N-1$ crosstalk cancellation configurations. To be able to calculate the capacity increase, the capacity without crosstalk cancellation $b_k(0)$ also has to be calculated for every tone. This results in $KN$ capacity calculations. Another $K(N-1)$ multiplications and additions are required to calculate the average capacity increase per allocated crosstalk cancellation tap. The $N-1$ crosstalk cancellation configurations are based on the line selection observation of Section 2.3. This requires a sort over the crosstalkers for each tone. This sort can be accomplished by selecting the crosstalkers one by one and placing them in the correct position of a sorted list. Because the resulting list is sorted at all times, a binary search can be used to find the correct position to place the current crosstalker. This results in a complexity of $\sum_{i=1}^{N-1} \log_2(i)$ comparisons to sort the list.

The table is then sorted to be able to efficiently find the maximum. This can be done analogous to the sorting of the crosstalkers and requires a complexity of $\sum_{i=1}^{K(N-1)} \log_2(i)$ comparisons.

| | Capacities | Multiplications | Additions | Comparisons |
|---|---|---|---|---|
| init: $v_k(r) = \dfrac{(b_k(r) - b_k(0))}{r}$ $\begin{cases} k = 1 \cdots K \\ r = 1 \cdots N-1 \end{cases}$ | $KN$ | $K(N-1)$ | $K(N-1)$ | $K\sum_{i=1}^{N-1} \log_2(i)$ |
| sort $v_k(r)$ | 0 | 0 | 0 | $\sum_{i=1}^{K(N-1)} \log_2(i)$ |
| **repeat** | | | | |
| $(k_s, r_s) = \operatorname{argmax}_{k,r} v_k(r)$ | 0 | 0 | 0 | 0 |
| $v_{k_s}(r) = 0, \quad \forall r \leq r_s$ | 0 | 0 | 0 | 0 |
| $v_{k_s}(r) = \dfrac{(b_k(r) - b_k(r_s))}{(r - r_s)}, \quad \forall r > r_s$ | $\dfrac{N-1}{2} + 1$ | $\dfrac{N-1}{2}$ | $N-1$ | 0 |
| re-sort $v_k(r)$ | 0 | 0 | 0 | $\sum_{i=K(N-1)-((N-1)/2-1)}^{K(N-1)} \log_2(i)$ |
| **while** $\sum_k r_k < C^{\text{tot}}$ | 0 | 0 | 1 | 1 |

Algorithm 2: Single-user resource allocation algorithm.

Crosstalk cancellation taps can now be allocated by selecting the element with the maximum average capacity increase of the table, located at the top of the sorted list. On average, $(N-1)/2$ crosstalk cancellation taps are thereby allocated. $(N-1)/2$ elements in the table then have to be recalculated to the new reference capacity $b_k(r_s)$. This requires $(N-1)/2 + 1$ capacity calculations, $(N-1)/2$ multiplications, and $N-1$ additions.

To keep the list sorted, $(N-1)/2$ binary searches are performed to find the new positions for the $(N-1)/2$ updated elements. This requires $\sum_{i=K(N-1)-((N-1)/2-1)}^{K(N-1)} \log_2(i)$ comparisons. The number of currently allocated cancellation taps is updated and compared to the cancellation tap constraint $C^{\text{tot}}$.

This is repeated until all available crosstalk cancellation taps are allocated. In [7] it was shown that with a runtime complexity of 30% of full crosstalk cancellation, almost all crosstalk can be cancelled. This means that approximately $K(N-1)/3$ crosstalk cancellation taps have to be allocated. Taking into account that in each iteration of the algorithm $(N-1)/2$ taps are allocated, there are $K(N-1)/(3(N-1)/2)$ iterations required on average.

### 3.2. Single-user dual decomposition algorithm

To be able to compare the algorithm based on dual decomposition to the resource allocation algorithm, a single-user formulation of the partial crosstalk cancellation problem (5) is used for user $n$:

$$\begin{aligned} &\text{maximize}_{\mathbf{c}}\ R^n \\ &\text{subject to}\ \ C^n = \sum_{k=1}^{K}\sum_{m=1}^{N} c_k^{n,m} \leq C^{n,\text{tot}} \\ &\text{with} \qquad [\mathbf{c}_k]_{n,m} = c_k^{n,m} \quad c_k^{n,m} = \begin{cases} 0 \Longrightarrow \tilde{h}_k^{n,m} = h_k^{n,m}, \\ 1 \Longrightarrow \tilde{h}_k^{n,m} = 0. \end{cases} \end{aligned} \tag{15}$$

This results in the following dual problem which is decoupled over the tones:

$$\begin{aligned} &\text{for } k = 1 \cdots K, \\ &\mathbf{c}_k^{\text{opt}} = \operatorname*{argmax}_{\mathbf{c}_k} b_k^n - \sum_{m=1}^{N} \lambda c_k^{n,m} \\ &\qquad \text{subject to } \lambda \geq 0. \end{aligned} \tag{16}$$

This can be viewed as one optimization of the multiuser problem where all users are allocated a crosstalk cancellation tap budget in advance.

Algorithm 3 presents the single-user dual decomposition algorithm. It starts by initializing a $K \times N$ table of capacities for $K$ tones and $N$ possible crosstalk cancellation configurations. To obtain the $N$ possible crosstalk cancellation configurations, the line selection observation of Section 2.3 is used. This requires sorting the crosstalkers which uses $K\sum_{i=1}^{N-1} \log_2(i)$ comparisons.

The algorithm then starts from some initial $\lambda$ and performs $K$ per-tone exhaustive searches. There are $N$ possible values for $\lambda r$, which can be calculated in advance. This requires $N$ multiplications. These precalculated values are then subtracted from the corresponding elements of the $K \times N$ table. Finally, $K$ exhaustive searches of $N$ values are performed to obtain the maximum on each tone. This requires $K(N-1)$ comparisons.

The cancellation tap constraint is then checked by summing the number of taps allocated on each tone. If the constraint is not tightly satisfied, the Lagrange multiplier $\lambda$ is updated and then the per-tone search is repeated. Because there is only one Lagrange multiplier, bisection can be used. This requires typically 10 iterations.

Table 1 summarizes the total complexity of the single-user resource allocation algorithm and the dual decomposition algorithm.

Figure 1 shows the initialization complexity as a function of the number of users for the single-user resource allocation

| | Capacities | Multiplications | Additions | Comparisons |
|---|---|---|---|---|
| init: $b_k(r) \quad \begin{cases} k = 1 \cdots K \\ r = 0 \cdots N-1 \end{cases}$ | $KN$ | 0 | 0 | $K\sum_{i=1}^{N-1} \log_2(i)$ |
| **repeat** | | | | |
| **for** $k = 1 \cdots K$ | | | | |
| $\mathbf{c}_k^{\text{opt}} = \underset{r}{\text{argmax}}\, b_k(r) - \lambda r$ | 0 | $N$ | $KN$ | $K(N-1)$ |
| **endfor** | | | | |
| update $\lambda$ based on (9) | | | | |
| **while** $\sum_k \mathbf{c}_k^{\text{opt}} \neq C^{\text{tot}}$ | 0 | 0 | $K-1$ | 1 |

Algorithm 3: Single-user dual decomposition algorithm.

Table 1: Complexity comparison single-user algorithms.

| | Resource allocation | Dual decomposition |
|---|---|---|
| Capacities | $KN + \dfrac{K(N-1)}{3((N-1)/2)}\left(\dfrac{N-1}{2}+1\right)$ | $KN$ |
| Multiplications | $K(N-1) + \dfrac{K(N-1)}{3((N-1)/2)}\,\dfrac{N-1}{2}$ | $10 \times N$ |
| Additions | $K(N-1) + \dfrac{K(N-1)}{3((N-1)/2)}N$ | $10 \times (KN + K - 1)$ |
| Comparisons | $K\sum_{i=1}^{N-1}\log_2(i) + \sum_{i=1}^{K(N-1)}\log_2(i) + \dfrac{K(N-1)}{3((N-1)/2)}\left(1 + \sum_{i=K(N-1)-((N-1)/2-1)}^{K(N-1)} \log_2(i)\right)$ | $K\sum_{i=1}^{N-1}\log_2(i) + 10 \times (K(N-1)+1)$ |

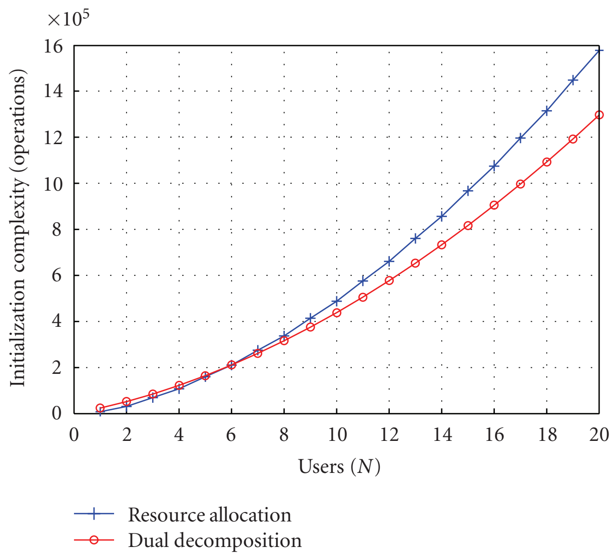


Figure 1: Complexity comparison single-user algorithms.

algorithm and the dual decomposition algorithm for $K = 1000$. It is taken into account that a capacity calculation in an $N$-user system roughly takes $N + 2$ multiplications and $N$ additions. Assuming the remaining 3 operations (multiplication, addition, and comparison) are equally resource consuming, one can see an 18% complexity reduction in the 20-user case.

## 4. MULTIUSER ALGORITHMS AND COMPLEXITY COMPARISON

The extension to the multiuser case can be made by dividing the cancellation tap budget over the users in advance. By varying the cancellation tap budget allocated to each user, various tradeoffs can be made in the data rates. This reduces the problem to multiple single-user problems. The core complexity of both the resource allocation algorithm and the dual decomposition algorithm is then increased by a factor $N$. Because of user independence and fixed individual cancellation tap budgets, optimization of the individual users also results in the optimization of the sum rate.

In this section, the single-user algorithms are extended to automatically determine the correct proportions of the cancellation tap budget to be allocated to the users such that the rate constraints are satisfied.

### 4.1. *Multiuser resource allocation algorithm*

For the resource allocation algorithm in [7], no procedure is available to automatically distribute the cancellation tap

| | Capacities | Multiplications | Additions | Comparisons |
|---|---|---|---|---|
| init: $v_k^n(r) = \dfrac{(b_k^n(r) - b_k^n(0))}{r}$ $\begin{cases} k = 1 \cdots K \\ r = 1 \cdots N-1 \\ n = 1 \cdots N \end{cases}$ | $KNN$ | $KN(N-1)$ | $KN(N-1)$ | $KN\sum_{i=1}^{N-1}\log_2(i)$ |
| **repeat** | | | | |
| $v_k^{\omega,n}(r) = \omega_n v_k^n(r)$ | 0 | $KN(N-1)$ | 0 | 0 |
| sort $v_k^{\omega,n}(r)$ | 0 | 0 | 0 | $\sum_{i=1}^{KN(N-1)}\log_2(i)$ |
| **repeat** | | | | |
| $(k_s, r_s, n_s) = \operatorname{argmax}_{k,r,n} v_k^{\omega,n}(r)$ | 0 | 0 | 0 | 0 |
| $v_{k_s}^{\omega,n_s}(r) = 0, \quad \forall r \leq r_s$ | 0 | 0 | 0 | 0 |
| $v_{k_s}^{\omega,n_s}(r) = \omega_{n_s}\dfrac{(b_k^{n_s}(r) - b_k^{n_s}(r_s))}{(r - r_s)}, \quad \forall r > r_s$ | $\dfrac{N-1}{2}+1$ | $N-1$ | $N-1$ | 0 |
| re-sort $v_k^{\omega,n}(r)$ | 0 | 0 | 0 | $\sum_{i=KN(N-1)-((N-1)/2-1)}^{KN(N-1)}\log_2(i)$ |
| **while** $\sum_{n=1}^{N}\sum_{k=1}^{K} r_k^n < C^{\text{tot}}$ | 0 | 0 | 1 | 1 |
| update $\boldsymbol{\omega}$ based on (9) | | | | |
| **while** rate constraints not satisfied | | | | |

Algorithm 4: Multiuser resource allocation algorithm.

budget over the users so that certain data rate constraints are satisfied. However, by introducing weights $\omega_n$, some lines can be emphasized to meet the rate constraints. To achieve a higher data rate for a user, more crosstalk cancellation taps should be allocated to that user. In order to do this, the average benefit of adding a crosstalk cancellation tap for that user is increased by a factor $\omega_n$. A larger weight leads to more crosstalk cancellation taps allocated and thus a higher data rate.

A given set of $\omega_n$'s implies a cancellation tap budget for each user (which is known after the optimization is done with these $\omega_n$'s). Because of the user independence, this again leads to an optimization of the sum rate. However, the rates are now weighted with $\omega_n$'s, thus a weighted rate sum is optimized.

Therefore, the following relation can be derived, analogous to the derivation in the appendix:

$$\Delta\boldsymbol{\omega}\Delta\mathbf{R} \geq 0. \tag{17}$$

This is a reduced form of (8), which leads to a simplified version of the update formula (9):

$$\Delta\boldsymbol{\omega} = -\mu(\mathbf{R} - \mathbf{R}^{\text{target}}) \Longrightarrow \boldsymbol{\omega}^{t+1} = \left[\boldsymbol{\omega}^{t} - \mu(\mathbf{R} - \mathbf{R}^{\text{target}})\right]^{+}. \tag{18}$$

During $I$ iterations, this update formula can then be used to steer the $\omega_n$'s so that the rate constraints are satisfied.

Algorithm 4 presents the resulting multiuser resource allocation algorithm with its associated complexities. Note that the table of $KN(N-1)$ average capacity increases per crosstalk cancellation tap is now globally searched instead of individually per user.

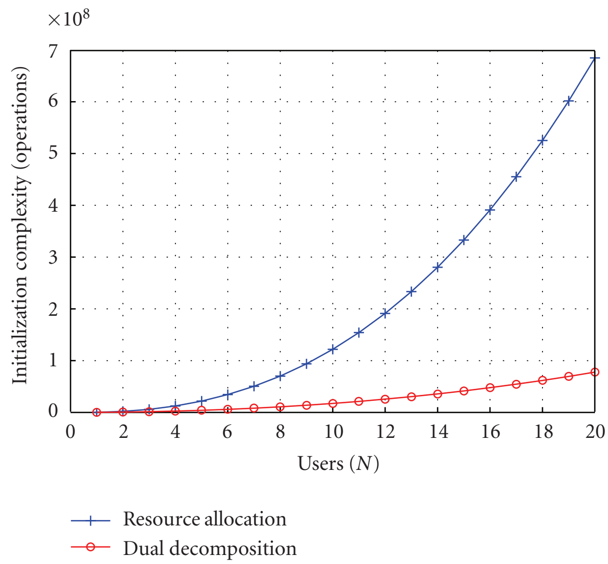


Figure 2: Complexity comparison multiuser algorithms.

#### 4.2. Multiuser dual decomposition algorithm

In the dual decomposition approach, Algorithm 1 can be used to find an appropriate distribution of the cancellation tap budget over the users, where the per-tone search is simplified based on the observations in Section 2.3. The resulting algorithm and complexities are shown in Algorithm 5. Because the updates of the Lagrange multipliers are based on the same update formula as in the resource allocation

| | Capacities | Multiplications | Additions | Comparisons |
|---|---|---|---|---|
| init: $b_k^n(r)$ $\begin{cases} k = 1 \cdots K \\ r = 0 \cdots N-1 \\ n = 1 \cdots N \end{cases}$ | $KNN$ | 0 | 0 | $KN\sum_{i=1}^{N-1}\log_2(i)$ |
| **repeat**<br>**for** $k = 1 \cdots K$<br>**for** $n = 1 \cdots N$<br>$\mathbf{c}_k^{n,\text{opt}} = \underset{r}{\text{argmax}}\, \omega_n b_k^n(r) - \lambda r$<br>**endfor**<br>**endfor**<br>update $\boldsymbol{\omega}, \lambda$ based on (9) | 0 | $N + KNN$ | $KNN$ | $KN(N-1)$ |
| **while** $\sum_{n=1}^{N}\sum_{k=1}^{K}\mathbf{c}_k^{n,\text{opt}} \neq C^{\text{tot}}$<br>and rate constraints not satisfied | 0 | 0 | $(N-1)(K-1)$ | 1 |

Algorithm 5: Multiuser dual decomposition algorithm.

Table 2: Complexity comparison multiuser algorithms.

| | Resource allocation | Dual decomposition |
|---|---|---|
| Capacities | $KNN + I \times \frac{KN(N-1)}{3((N-1)/2)}\left(\frac{N-1}{2}+1\right)$ | $KNN$ |
| Multiplications | $KN(N-1) + I \times \left(KN(N-1) + \frac{KN(N-1)}{3((N-1)/2)}(N-1)\right)$ | $I \times (N + KNN)$ |
| Additions | $KN(N-1) + I \times \frac{KN(N-1)}{3((N-1)/2)}N$ | $I \times (KNN + (N-1)(K-1))$ |
| Comparisons | $KN\sum_{i=1}^{N-1}\log_2(i) + I \times \sum_{i=1}^{KN(N-1)}\log_2(i) + \frac{KN(N-1)}{3((N-1)/2)}\left(1 + \sum_{i=KN(N-1)-((N-1)/2-1)}^{KN(N-1)}\log_2(i)\right)$ | $KN\sum_{i=1}^{N-1}\log_2(i) + I \times (KN(N-1)+1)$ |

algorithm, roughly the same number of $I$ iterations is required to enforce the constraints.

In Table 2 the total complexities of the multiuser resource allocation algorithm and the multiuser dual decomposition algorithm are compared.

Figure 2 shows the initialization complexity as function of the number of users for the resource allocation algorithm and the dual decomposition algorithm for $K = 1000$, under the assumption that $I = 50$ iterations are required to enforce the constraints. It is taken into account that a capacity calculation in an $N$-user system roughly takes $N + 2$ multiplications and $N$ additions. Assuming the remaining 3 operations (multiplication, addition, and comparison) are equally resource consuming, one can see an 88% complexity reduction in the 20-user case.

## 5. SIMULATION RESULTS

In [7] a simplified joint line/tone selection algorithm is also presented. This algorithm has a much lower complexity than the algorithms discussed in this paper and is claimed to be near-optimal. This algorithm can also be extended to the multiuser case by introducing the weights $\omega$. However, this near-optimality largely depends on the scenario. For simple scenarios with only two different line lengths, the simplified joint line/tone selection algorithm indeed performs near-optimal. However, for practical scenarios with lines of varying lengths, this simplified algorithm can be suboptimal depending on the runtime complexity that is allowed.

In Figure 3 the performance of both the optimal as well as the simplified line/tone selection is presented for different runtime complexities. This is done for an 8-user upstream VDSL scenario, with line lengths varying from 150 m to 1200 m in 150 m intervals. An empirical channel model [14] is used with line diameter of 0.5 mm (24 AWG) that generates both the direct channels and the crosstalk channels. The transmit power is set to −60 dBm on all tones. The SNR gap $\Gamma$ is set to 12.9 dB, corresponding to a target symbol error probability of $10^{-7}$, coding gain of 3 dB, and a noise margin of 6 dB. The tone spacing $\Delta_f = 4.3125$ kHz and the DMT symbol rate $f_s = 4$ kHz.

To allow for an easier comparison, cancellation taps are allocated to each line using a single-user algorithm, keeping all other lines at a fixed bitrate with no crosstalk cancellation. Note that for small runtime complexities, the optimal joint line/tone selection algorithm can increase bitrates up to 50%

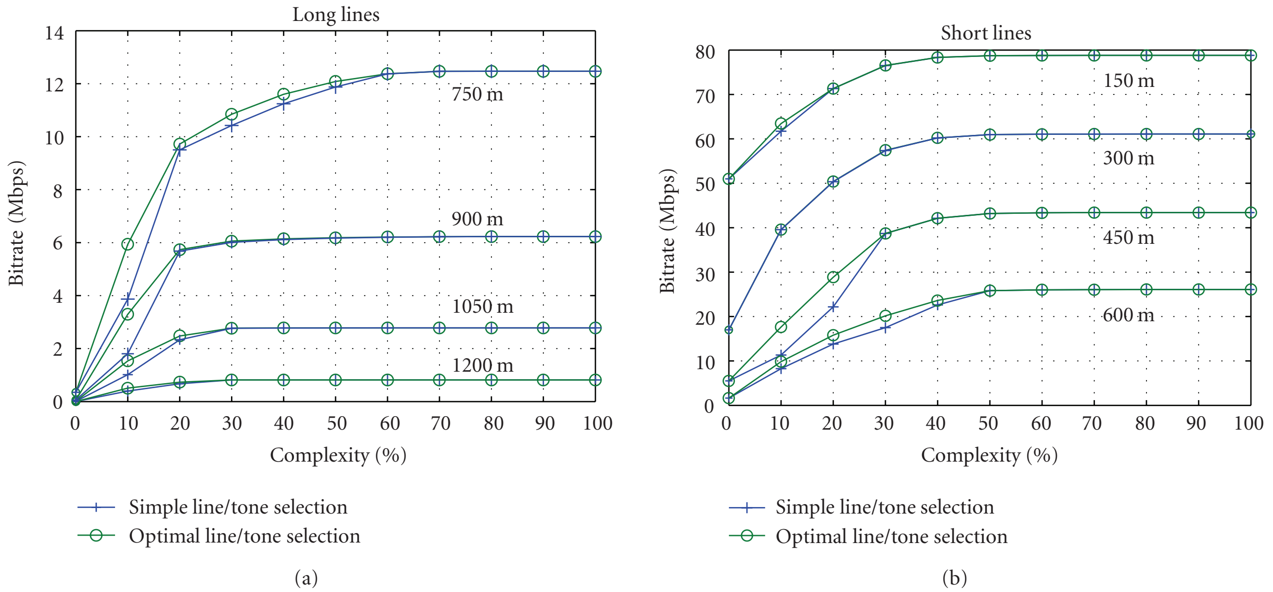


Figure 3: Performance comparison between optimal and simple line/tone selection algorithms.

of the performance of the simplified joint line/tone selection algorithm. Especially for the far-end users, which should be protected most from crosstalk, this performance difference is large.

Secondly, note the difference in runtime complexity for different lines to approach the full crosstalk cancellation performance. For long lines, 30% of full crosstalk cancellation is sufficient because only few tones carry a significant amount of bits. As the lines get shorter, up to 50–60% of full crosstalk cancellation is necessary. Therefore, multiuser algorithms are more suitable to solve the partial crosstalk cancellation problem because they can automatically distribute the cancellation tap budget over the users, in contrast to single-user algorithms where the budget has to be distributed in advance, taking into account the different line lengths.

The simplified joint line/tone selection algorithm requires a high runtime complexity before it starts performing optimal. For low runtime complexities however, the optimal algorithm reaches a much higher performance. Thus depending on the allowed runtime complexity, the optimal joint line/tone algorithm can be preferred over the simplified algorithm, trading of runtime complexity for initialization complexity when the required bitrate is fixed.

In Figure 4, rate regions are shown for a symmetric upstream VDSL scenario with two 300 m lines. Various crosstalk cancellation complexities are considered when allocating crosstalk cancellation taps optimally. One can see for, for example, a runtime complexity of 25% of the runtime complexity of full crosstalk cancellation that the available cancellation tap budget can be shifted between the users, thereby trading off the performance in terms of bitrate. If full priority is given to one user, only that user will gain the extra capacity due to the crosstalk cancellation. If the priority is divided over the users, both will gain some capacity. For small runtime complexities (almost no crosstalk can be cancelled) and large runtime complexities (all the largest crosstalk components can be cancelled) the tradeoff that can be made between the users is small.

## 6. CONCLUSION

In modern DSL systems, crosstalk is a major source of performance degradation. Crosstalk cancellation schemes have been proposed to mitigate the effect of crosstalk. However, the complexity of crosstalk cancellation grows with the square of the number of lines in the binder. Fortunately, most of the crosstalk originates from a limited number of lines on a limited number of tones. As a result, a fraction of the complexity of full crosstalk cancellation suffices to cancel most of the crosstalk, which is exploited by partial crosstalk cancellation. The challenge is then to determine which crosstalk to cancel on which tones, given a certain complexity constraint. In this paper, we have presented an algorithm to optimally solve this problem, based on a dual decomposition.

Two cases were considered: single-user and multiuser. In the single-user case, each user has an individual cancellation tap budget to be allocated. It was shown that the dual decomposition algorithm has a favourable complexity compared to the optimal resource allocation algorithm.

In the multiuser case, all users have a common cancellation tap budget. This budget has to be distributed over the users in such a way that rate constraints are satisfied. The dual decomposition approach naturally incorporates these rate constraints. The resource allocation algorithms were extended to this multiuser case to also include these rate constraints. The extension allows for the same search procedure to be used to find the distribution of the cancellation tap budget over the users as used in the dual decomposition

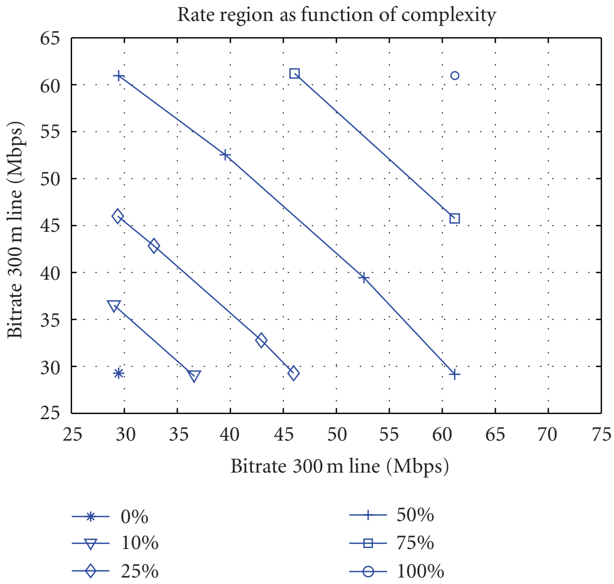


Figure 4: Rate regions for various crosstalk cancellation complexities.

algorithm. Also in this multiuser case, the complexity of the dual decomposition algorithm was found to compare favorably with the complexity of the multiuser resource allocation algorithm.

## APPENDIX

## SEARCH ALGORITHM FOR THE LAGRANGE MULTIPLIERS

The proof presented in [10, 11] can be easily adapted for partial crosstalk cancellation. Assume a two-user scenario with signal-level control. Starting from two optimal solutions $(R^{1,\boldsymbol{\omega}_A,\lambda_A}, R^{2,\boldsymbol{\omega}_A,\lambda_A}, C^{\boldsymbol{\omega}_A,\lambda_A})$ and $(R^{1,\boldsymbol{\omega}_B,\lambda_B}, R^{2,\boldsymbol{\omega}_B,\lambda_B}, C^{\boldsymbol{\omega}_B,\lambda_B})$ corresponding to $(\boldsymbol{\omega}_A, \lambda_A)$ and $(\boldsymbol{\omega}_B, \lambda_B)$, respectively, optimality for $(\boldsymbol{\omega}_A, \lambda_A)$ implies

$$\begin{aligned} &\omega_{1,A}R^{1,\boldsymbol{\omega}_B,\lambda_B} + \omega_{2,A}R^{2,\boldsymbol{\omega}_B,\lambda_B} - \lambda_A C^{\boldsymbol{\omega}_B,\lambda_B} \\ &\quad \leq \omega_{1,A}R^{1,\boldsymbol{\omega}_A,\lambda_A} + \omega_{2,A}R^{2,\boldsymbol{\omega}_A,\lambda_A} - \lambda_A C^{\boldsymbol{\omega}_A,\lambda_A}. \end{aligned} \tag{A.1}$$

Optimality for $(\boldsymbol{\omega}_B, \lambda_B)$ implies

$$\begin{aligned} &\omega_{1,B}R^{1,\boldsymbol{\omega}_A,\lambda_A} + \omega_{2,B}R^{2,\boldsymbol{\omega}_A,\lambda_A} - \lambda_B C^{\boldsymbol{\omega}_A,\lambda_A} \\ &\quad \leq \omega_{1,B}R^{1,\boldsymbol{\omega}_B,\lambda_B} + \omega_{2,B}R^{2,\boldsymbol{\omega}_B,\lambda_B} - \lambda_B C^{\boldsymbol{\omega}_B,\lambda_B}. \end{aligned} \tag{A.2}$$

Taking the sum of (A.1) and (A.2) results in

$$\begin{aligned} &-\underbrace{(\omega_{1,B} - \omega_{1,A})}_{\Delta\omega_1}\underbrace{(R^{1,\boldsymbol{\omega}_B,\lambda_B} - R^{1,\boldsymbol{\omega}_A,\lambda_A})}_{\Delta R^1} \\ &\quad -\underbrace{(\omega_{2,B} - \omega_{2,A})}_{\Delta\omega_2}\underbrace{(R^{2,\boldsymbol{\omega}_B,\lambda_B} - R^{2,\boldsymbol{\omega}_A,\lambda_A})}_{\Delta R^2} \\ &\quad +\underbrace{(\lambda_B - \lambda_A)}_{\Delta\lambda}\underbrace{(C^{\boldsymbol{\omega}_B,\lambda_B} - C^{\boldsymbol{\omega}_A,\lambda_A})}_{\Delta C} \leq 0. \end{aligned} \tag{A.3}$$

Relation (A.3) is straightforwardly extended to a multiuser scenario:

$$\begin{bmatrix} -(\Delta\boldsymbol{\omega})^T & \Delta\lambda \end{bmatrix} \begin{bmatrix} \Delta\mathbf{R} \\ \Delta C \end{bmatrix} \leq 0, \tag{A.4}$$

$\boldsymbol{\omega} = [\omega_1, \ldots, \omega_N]$ is a vector containing the Lagrange multipliers for the weights for the users, $\lambda$ is the Lagrange multiplier controlling the number of cancellation taps used. $\mathbf{R} = [R^1, \ldots, R^N]^T$ is a vector with the corresponding data rates and $C$ is the corresponding number of cancellation taps.

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