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Research Article

Selective Iterative Waterfilling for Digital Subscriber Lines

Yang Xu, Tho Le-Ngoc, and Saswat Panigrahi

Department of Electrical and Computer Engineering, McGill University, 3480 University Street, Montréal, Québec, Canada H3A 2A7

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This paper presents a high-performance, low-complexity, quasi-distributed dynamic spectrum management (DSM) algorithm suitable for DSL systems. We analytically demonstrate that the rate degradation of the distributed iterative waterfilling (IW) algorithm in near-far scenarios is caused by the insufficient utilization of all available frequency and power resources due to its nature of noncooperative game theoretic formulation. Inspired by this observation, we propose the selective IW (SIW) algorithm that can considerably alleviate the performance degradation of IW by applying IW selectively to different groups of users over different frequency bands so that all the available resources can be fully utilized. For N users, the proposed SIW algorithm needs at most N times the complexity of the IW algorithm, and is much simpler than the centralized optimal spectrum balancing (OSB), while it can offer a rate performance much better than that of the IW and close to the maximum possible rate region computed by the OSB in realistic near-far DSL scenarios. Furthermore, its predominantly distributed structure makes it suitable for DSL implementation.

1. INTRODUCTION

Crosstalk is the dominant source of performance degradation in digital subscriber lines (DSLs) systems where multiple users coexist in a binder and cause crosstalk interference into each other due to close physical proximity of twisted pairs within the same binder. Crosstalk is typically 10–20 dB larger than the background noise, and can severely limit system performance if left unmitigated.

Crosstalk cancellation can be performed by exploiting the crosstalk structure through signal level coordination [1] and leads to spectacular performance gain. However, crosstalk cancellation techniques generally require tremendous computation complexity, and thus render them unsuitable for deployment in many scenarios. In this case, the effects of crosstalk must be mitigated through spectrum management in interference-limited DSL systems.

The detrimental effects of crosstalk can be mitigated through spectrum management in interference-limited DSL systems. Traditional *static spectrum management* (SSM) techniques employ identical spectral masks based on the worst-case scenarios [2] for all modems. Consequently, these spectral masks are unduly restrictive and lead to conservative performance. Recently, *dynamic spectrum management* (DSM) [3, 4] is gaining popularity as a new paradigm, which jointly adapts power spectral densities (PSDs) of each modem based on physical channel characteristics to achieve the required rates while minimizing crosstalk, and has demonstrated significant rates enhancement.

In general, DSM techniques can be categorized as either distributed or centralized, depending on the required amount of coordination and centralized control. For a distributed DSM scheme, only macroparameters such as data rates, total transmit power are reported and controlled centrally but other microparameters such as actual subcarrier-specific power and rate allocation are autonomously managed by each individual modem in a distributed manner; while centralized DSM performs spectral and rate allocations for all modems within the network and then assigns the computed PSDs to each individual modem by a centralized spectrum management center (SMC).

Distributed DSM schemes are desired for their low requirements of coordination and centralized control. Among distributed DSM techniques, *iterative waterfilling* (IW) [5] is possibly the most popular [4, 6], due to its predominantly distributed nature and significant rate enhancement over existing SSM techniques. IW formulates the spectrum management problem in DSL as a noncooperative game, in which each user performs greedy "power waterfilling" iteratively to maximize its own rate with respect to the interference and noise until achieving convergence. Under a broad range of conditions [5, 7–9], this noncooperative DSL game converges to a competitively optimal Nash equilibrium. Yet, due to its nature of noncooperative game theoretic formulation, IW does not necessarily converge to the Pareto optimal solution. Particularly, simulation results in realistic DSL environments indicate that IW performance is highly degraded in near-far scenarios compared to the maximum possible rate region achieved by centralized OSB [10], for example, mixed CO/RT ADSL [11] and upstream VDSL [12] deployment. Its severe performance degradation in near-far scenarios was also analytically shown in [8] for a simplified two-user, two-band, near-far case.

If all the direct and crosstalk channel transfer functions are known to a centralized agent, more sophisticated centralized DSM schemes can be implemented to achieve better performance than distributed IW. More specifically, an OSB approach based on dual decomposition was presented in [13] with computational complexity *linearly*[1] proportional to the number of tones, K. Unfortunately, it is still computationally intractable for practical implementation because its complexity grows exponentially in the number of lines in a DSL binder, N. To circumvent the exponential complexity bottleneck due to exhaustive search over all possible of power allocation tuples in OSB, two heuristic near-optimal low-complexity centralized algorithms [13, 14] were developed, while another approach [15] based on a global difference of convex (D.C.) optimization technique was proposed to find the global optimum solution efficiently. But all these approaches are centralized DSM requiring knowledge of all the direct and crosstalk channel responses, and hence are less favorable for practical implementation than distributed DSM in terms of simplicity. The simplicity of distributed IW and the optimality of centralized OSB are two very desirable properties of any DSM techniques.

This paper proposes a low-complexity, quasi-distributed DSM algorithm that can achieve performance close to the optimal OSB. We will first analytically show the rate degradation of the IW in near-far scenarios for a simple two-band, two-user, near-far case by highlighting the inefficiency inherent in its user's total power allocation at outer stage. We then propose *selective IW* (SIW) to alleviate the performance degradation of IW by applying IW selectively to different groups of users over different frequency bands so that all the available frequency and power resources can be fully utilized. Consequently, considerable performance improvement can be achieved at the expense of very little central coordination.

The SIW scheme is more like a distributed DSM scheme, as it requires only minimal coordination and communication with a central agent. It can be regarded as almost distributed as the original IW. In fact, the SIW is completely distributed in the case of two users. Simulation results in realistic DSL scenarios indicate that the rate region achieved by the proposed SIW approaches closely to the maximum possible rate region computed by the centralized OSB algorithm. Moreover, the SIW enjoys low complexity, at most N times that of the IW algorithm, and hence is suitable for practical deployment where N is typically 25–100.

The remainder of this paper is organized as follows. Section 2 introduces system model and presents spectrum management problem in DSL. Section 3 illustrates the suboptimal behavior of the IW algorithm in a near-far scenario by emphasizing the inefficiency inherent in its outer-stage power allocation, and then characterizes the data rate loss of the IW algorithm by employing a simple two-user two-band near-far case. To fully utilize all available frequency and power resources, we propose the SIW algorithm that selectively applies IW in different frequency bands until all frequency and power are fully utilized in Section 4. Section 5 shows the performance comparison of the proposed SIW, IW, and OSB algorithms in several realistic ADSL and VDSL-DMT scenarios. Finally, concluding remarks are made in Section 6.

2. SPECTRUM MANAGEMENT PROBLEM FORMULATION

Discrete multitone (DMT) modulation [16] has been adopted as standard in various xDSL applications such as ADSL [11] by American National Standards Institute (ANSI) and European Telecommunications Standard Institute (ETSI) and more recently for VDSL [12] by ANSI.

For a sufficiently large number of subcarriers, DMT transmission [16] over a frequency-selective fading channel can be modeled as a set of K parallel independent flat fading AWGN subcarrier channels. Under Gaussian channel assumption, the achievable bit-loading rate of user n on tone k is

$$\begin{aligned} r_k^n &\triangleq \log_2\left(1 + \frac{1}{\Gamma}\frac{\left|g_k^{n,n}\right|^2 p_k^n}{\sum_{m\neq n}\left|g_k^{n,m}\right|^2 p_k^m + \sigma_k^n}\right) \\ &= \log_2\left(1 + \frac{1}{\Gamma}\frac{h_k^{n,n} p_k^n}{\sum_{m\neq n} h_k^{n,m} p_k^m + \sigma_k^n}\right), \end{aligned} \tag{1}$$

where p_k^n, σ_k^n denote user n's transmit PSD and noise power on tone k, respectively; $g_k^{n,m}$ is the channel path gain from user m to n on tone k. Define $\mathbf{H}_k$ as the $N \times N$ channel power gain matrix on tone k and its component $h_k^{n,m} \triangleq |g_k^{n,m}|^2$ denotes the interference power gain from user m to n on tone k. The diagonal elements of $\mathbf{H}_k$ are the direct channel path gains, and the off-diagonal elements are the path gains of crosstalk channels. Γ denotes the SNR-gap to capacity, which depends on the desired BER, coding gain, and noise margin [16].

For a DMT symbol rate of f_s, the total bit rate of user n is $R_n = f_s \sum_k r_k^n$.

In practice, modems in DSL systems are generally subject to total transmission power constraint

$$\Delta_f \sum_k p_k^n \leq P_n^{\max}, \quad \forall n, \tag{2}$$

where $P_n^{\max}$ denotes the maximum total transmission power for modem n and Δ_f denotes the tone spacing.

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[1] Instead of *exponentially* as in previous approaches.

The optimization problem for spectrum management in DSL can be formulated as

$$\begin{aligned} &\max_{\mathbf{P}_1,\ldots,\mathbf{P}_N} R_{n^*} \quad \text{subject to } \left(R_n \geq T_n, \forall n \neq n^*\right), \\ &\left(\sum_{\forall k} p_k^n \leq P_n^{\max},\ p_k^n \leq p_k^{n,\text{mask}}, \forall n\right), \end{aligned} \tag{3}$$

for a user of interest n^*, where T_n and $P_n^{\max}$ are the required minimum target rate and maximum total transmission power of user n. The K-dimensional vector $\mathbf{P}_n \triangleq (p_1^n, \ldots, p_K^n)$ denotes the transmission power vector of user n over all K tones. Spectral mask constraints $p_k^{n,\text{mask}}$ may also be applied.

The rate region of a particular DSM technique is defined as the union of all the supportable rate sets $(R_1, \ldots, R_N)$ that can be simultaneously provided to users while satisfying the total transmission power constraints specified by (2). Operating point on the boundary of the rate region is the maximum achievable rate pairs. In this paper, the rate region boundary is used to evaluate and compare the performance of different DSM algorithms.

3. BEHAVIOR OF IW IN NEAR-FAR SCENARIOS

IW views multiuser interference channel as a noncooperative game and takes a game theoretic approach to derive power allocation algorithm that achieves the competitive optimal Nash equilibrium [5]. To achieve a set of target rates for the users, the IW algorithm performs repeatedly a two-stage power allocation procedure until the PSDs of all users converge to constant values at each frequency tone and the target rates of all users are satisfied. More specifically, the two-stage IW algorithm works as follows: at each iteration, the outer stage adjusts each user's total power constraint based on the comparison of its target rate and the rate achieved in the last iteration, and the inner stage optimizes the power allocation of each user over all frequency tones by performing greedy "power waterfilling" iteratively to maximize its own rate with respect to the interference and noise until reaching convergence. This two-stage power allocation scheme of IW algorithm implies that each set of total power constraints corresponds to a unique set of achievable user rates.

We illustrate the behavior of two-stage power allocation of IW algorithm in a near-far environment by considering a scenario of four 1500 ft lines and four 3000 ft lines in a typical VDSL 988 FDD with two separate upstream bands: 3.75–5.2 MHz and 8.5–12 MHz and a transmit power constraint of 11.5 dBm for each modem as depicted in Figure 1. The near-far problem in DSL occurs when two users located at different distances communicate with the central office (CO) simultaneously. As a result, the near user, CP1, inflicts overwhelming interference upon the signal of the far user, CP2, and can completely block the successful transmission of the far user. The cause of the near-far problem in DSL is the asymmetry of crosstalk channels between the near and far users. Their direct and crosstalk channel responses plotted in Figure 2 clearly show that the far user, CP2, is subject to very strong interference from the near user, CP1 (i.e., the crosstalk response h^{21} is even stronger than the direct response h^{22} at frequencies higher than 8 MHz), whereas the near user is quite immune from the interference from the far user (i.e., the crosstalk response h^{12} is more than 80 dB below the direct response h^{11} over the entire frequency range). From this viewpoint, the far user 2 can be regarded as the weak user, and the near user 1 as the dominant user.

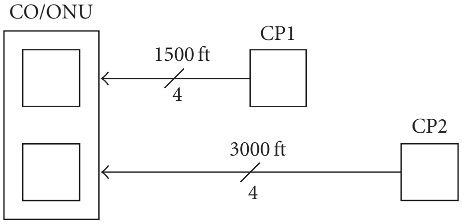


Figure 1: An example of VDSL upstream scenario.

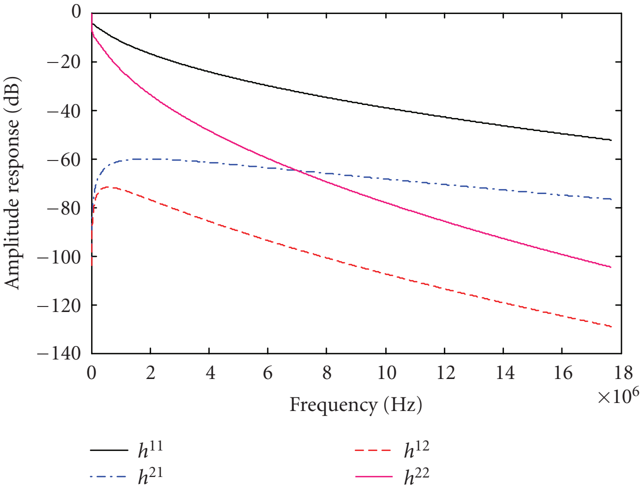


Figure 2: Typical channel profiles in VDSL upstream.

Using the two-stage power allocation IW algorithm, in order to meet the target rates of the weak user, the dominant user has to set its total power budget sufficiently low so as not to cause excessive interference to the weak user. Consequently, the waterfilling level $1/\lambda_1$ of the dominant user is decreased significantly to ensure not exceeding its total power constraint.

Mathematically, the rate-maximizing waterfilling strategy yields the PSD of the dominant user 1 and the weak user 2 as

$$\begin{aligned} p_k^1 &= \left[\frac{1}{\lambda_1} - \frac{\Gamma\left(h_k^{1,2} p_k^2 + \sigma_k^1\right)}{h_k^{1,1}}\right]^+, \\ p_k^2 &= \left[\frac{1}{\lambda_2} - \frac{\Gamma\left(h_k^{2,1} p_k^1 + \sigma_k^2\right)}{h_k^{2,2}}\right]^+. \end{aligned} \tag{4}$$

Note that the weak user 2 cannot utilize the high-frequency band due to two properties of the waterfilling nature of power allocation and their channel characteristics.

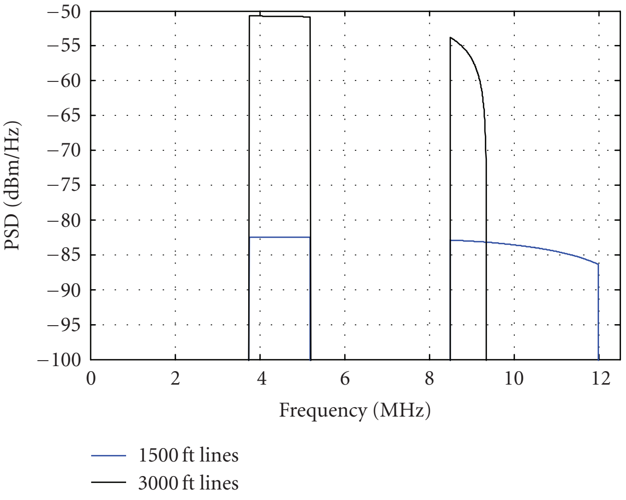


Figure 3: VDSL upstream PSDs obtained from IW. 1500 ft line @ 11.5 Mbps, 3000 ft lines @ 7 Mbps.

First, the direct channel response of the weak user 2 is generally much poorer than that of the dominant user 1 and its magnitude decreases rapidly with respect to frequency. Secondly, the total power budget of the weak user is not large enough for its PSDs to span over all available frequency bands.

On the other hand, the waterfilling level of the dominant user is sufficiently low so as not to cause excessive interference to the weak user, and p_k^1 decreases with respect to frequency as well. Thus, the dominant user also cannot utilize high-frequency band effectively due to the very low protective waterfilling level.

As a result, the high-frequency band is unused since the weak user does not have sufficient power while the dominant user is effectively “blocked” due to the low protective waterfilling level even if the dominant user still has a significant portion of unused power.

The results obtained by the IW algorithm indicate that the 3000 ft group utilizes all its power resource of 11.5 dBm to achieve 7.0017 Mbps, while the transmitted power of the 1500 ft group is only −16.5 dBm for 11.5 Mbps. Figure 3 illustrates the PSDs in dBm/Hz in the upstream bands obtained by IW algorithm. The PSD of 3000 ft line (the weak user) is quite flat in the first upstream band, but drops very sharply in the second upstream band as the direct channel response deteriorates dramatically. On the other hand, the PSD of 1500 ft (the dominant user) spans the whole frequency band at very low level, quite flat in the first upstream band and decreases slowly in the second upstream band. Clearly, with IW, the dominant 1500 ft group fails in efficiently using the large part of the high-frequency band (8.5–12 MHz), which cannot be used by the weak 3000 ft group.

In other words, the dominant user can allocate its large amount of unused power for transmission in high-frequency band to achieve higher rate without causing any harm to the weak user.

For a better understanding of the problem inherent in the two-stage power allocation of IW, consider a simple two-user, near-far scenario with two equal-bandwidth bands. The channel matrices of the first and second bands are $\mathbf{H}_1$, $\mathbf{H}_2$, respectively. This two-user, two-band channel model is also used in [8] to illustrate near-far problem. More specifically, these two channel matrices are

$$\mathbf{H}_1 = \begin{vmatrix} h_1^{1,1} & h_1^{1,2} \\ h_1^{2,1} & h_1^{2,2} \end{vmatrix}, \qquad \mathbf{H}_2 = \begin{vmatrix} h_2^{1,1} & h_2^{1,2} \\ h_2^{2,1} & 0 \end{vmatrix}. \tag{5}$$

In a near-far scenario in DSL, the direct channel response of near user 1 is typically much larger than that of far user 2, that is, $h_1^{2,2} \ll h_1^{1,1}$. Furthermore, $h_1^{2,1} \gg h_1^{1,2}$, indicating that user 1 is dominant and can generate significant crosstalk interference to the weak user 2 while the inference from the weak user 2 to user 1 is very small. The channel profiles of a VDSL upstream case depicted in Figure 3 provide justifications for this simple two-user, two-band, near-far channel model.

Note that band 2 can only be used by user 1 but not by user 2, because the direct channel gain for user 2, $h_2^{2,2}$, is zero. Given that user 2 can only use band 1, the data rate of user 2 is given by

$$R_2 = \log_2\left(1 + \frac{h_1^{2,2} p_1^2}{\Gamma(\sigma_1^2 + h_1^{2,1} p_1^1)}\right). \tag{6}$$

For the spectrum management problem defined in (3), the target rate constraint of user 2 has to be satisfied. This means that the rate of user 2 should satisfy $R_2 \geq T_2$ where T_2 is its target rate. Using IW, the outer stage iteratively adjusts the total power constraints of users until the target rate of user 2 is met. From (6) and the inequality $R_2 \geq T_2$, we can obtain the following upper bound on p_1^1:

$$p_1^1 \leq \frac{1}{h_1^{2,1}}\left(\frac{h_1^{2,2} p_1^2}{\Gamma(2^{T_2} - 1)} - \sigma_1^2\right). \tag{7}$$

The above upper bound on p_1^1 can be interpreted as the maximum possible power that user 1 can allocate to band 1 so that the crosstalk level from user 1 to user 2 is sufficiently low to support the target rate of user 2.

Due to the waterfilling structure of user power allocation, that is, a constant waterfilling level $1/\lambda_1$ for both bands, the power allocation pair (p_1^1, p_2^1) of user 1 satisfies

$$p_1^1 + h_1^{1,2} p_1^2 + \sigma_1^1 = p_2^1 + \sigma_2^1. \tag{8}$$

Since the additive Gaussian noise is the same for both users in both bands, (8) can be simplified to

$$p_1^1 + h_1^{1,2} p_1^2 = p_2^1. \tag{9}$$

Hence, using IW, the rate achieved by user 1 over two bands is

$$R_1 = \log_2\left(1 + \frac{h_1^{1,1} p_1^1}{\Gamma(\sigma_1^1 + h_1^{1,2} p_1^2)}\right) + \log_2\left(1 + \frac{h_2^{1,1} p_2^1}{\Gamma\sigma_2^1}\right), \tag{10}$$

in which p_1^1 is bound by (7) and p_2^1 is given by (9). Recall that the two-stage power allocation of IW implies the existence of a one-to-one mapping between a set of total power constraints and its corresponding set of achievable user rates. Hence, there is one and only one point on the rate region boundary of IW algorithm that corresponds to the case, in which both users fully utilize their available power, that is, ($P_1 = P_1^{\max}$, $P_2 = P_2^{\max}$). For all other points on the rate region boundary, it is either ($P_1 < P_1^{\max}$, $P_2 = P_2^{\max}$) or ($P_1 = P_1^{\max}$, $P_2 < P_2^{\max}$), that is, one of users has unused power. Note that total power $p_1^1 + p_2^1$ used by user 1 is generally much smaller than the total amount of power $P_1^{\max}$ available to user 1 in a near-far scenario. This is simply due to the fact that user 1 has to lower its transmission power significantly to reduce possible interference to user 2 so that the target rate of user 2 can be met.

The unused power of user 1, ΔP, is

$$\Delta P = P_1^{\max} - P_1 = P_1^{\max} - p_1^1 - p_2^1 = P_1^{\max} - 2p_1^1 - h_1^{1,2}p_1^2. \tag{11}$$

Since user 2 cannot use the second band, another power allocation strategy achieving higher rate for user 1 while still guaranteeing the target rate of user 2 is to allocate all the unused power ΔP of user 1 to band 2 to maximize its rate. It is evident that this strategy poses no threat to user 2 as user 2 does not transmit on band 2, and the achievable rate of user 2 remains essentially unchanged.

The rate gain of user 1 employing the new strategy of pouring all unused power on band 2 over IW algorithm can now be calculated as

$$\begin{aligned}\Delta R &= \log_2\left(1 + \frac{h_2^{1,1}(p_2^1 + \Delta P)}{\Gamma\sigma_2^1}\right) - \log_2\left(1 + \frac{h_2^{1,1}p_2^1}{\Gamma\sigma_2^1}\right)\\ &= \log_2\left(1 + \frac{h_2^{1,1}\Delta P}{\Gamma\sigma_2^1 + h_2^{1,1}p_2^1}\right).\end{aligned} \tag{12}$$

Let us now simplify (12) in a near-far DSL case with some reasonable approximations. In an interference-limited DSL system, it is reasonable to assume $\Gamma\sigma_2^1 \ll h_2^{1,1}p_2^1$. Consider the case that user 2 allocates all its available power in band 1, that is, $p_1^2 = P_2^{\max}$. Ignoring $h_1^{1,2}p_1^2$ in (9) (since the crosstalk from user 2 to user 1 is very small), the power allocation of user 1 in both bands is approximately the same, that is, $p_1^1 = p_2^1$. Using the above approximations, the expression in (12) can be simplified to

$$\Delta R \approx \log_2\left(1 + \frac{P_1^{\max} - 2p_1^1}{p_1^1}\right). \tag{13}$$

When $p_1^1 \ll P_1^{\max}$ (which is typical because the dominant user 1 has to reduce its waterfilling level sufficiently low to guarantee the target rate of the weak user 2), substituting p_1^1 in (7) into (13) yields

$$\begin{aligned}\Delta R &\approx \log_2\left(\frac{\Gamma 2^{T_2} h_1^{2,1} P_1^{\max}}{h_1^{2,2} P_2^{\max}}\right)\\ &= T_2 + \log_2\left(\frac{\Gamma h_1^{2,1}}{h_1^{2,2}}\right) + \log_2\left(\frac{P_1^{\max}}{P_2^{\max}}\right).\end{aligned} \tag{14}$$

Equation (14) reveals the rate loss of user 1 incurred by employing IW (as compared to the strategy of pouring all unused power of user 1 into band 2 to increase the rate of user 1). Furthermore, the dominant user 1 suffers significant rate loss in a near-far scenario if the rate requirement of the weak user 2 is high, that is, the rate loss of the dominant user increases with the required rate of the weak user.

4. SELECTIVE WATERFILLING ALGORITHM

Aiming to solve the spectrum management problem (3), the basic idea of the proposed selective IW algorithm is that users should allocate their remaining power over tones that are not fully utilized, so that the drawback inherent in the out-stage power allocation of IW algorithm as discussed in Section 3 can be avoided. The SIW *selectively* applies the IW algorithm in different frequency bands until all the users consume all their total power or no more underutilized frequency bands left.

Consider U, the group of users participating in the IW game, and S, the set of tones upon which the IW game is played. $\{\overline{R}_{n\neq n^*}\}$ and $\{\overline{P}_n\}$, $n \in U$ are the sets of user rate requirements and maximum power constraints, respectively. In each round, with the inputs $(n^*, U, S, \{\overline{R}_{n\neq n^*}\}, \{\overline{P}_n\})$, the IW game aims to maximize the rate of a user of interest n^* while satisfying the target rates of other users. As shown in Algorithm 1, the IW game, $(\mathbf{P}, \mathbf{R}) = \text{IW Alg}(n^*, U, S, \{\overline{R}_{n\neq n^*}\}, \{\overline{P}_n\})$, converges to the Nash equilibrium, resulting in the user's competitive optimal power allocation matrices: $\mathbf{P}$ (for optimal power with elements p_k^n) and $\mathbf{R}$ (for rates with elements r_k^n) where $(n, k) \in U \times S$.

Note that the IW algorithm described in Algorithm 1 is slightly different from its original version presented in [5] for using as a subroutine in the SIW algorithm. This IW subroutine maximizes the rate of a user of interest while satisfying the rate requirements of all other users as defined in (3), while the IW in [5] minimizes the total power needed while satisfying the rate requirements of all users.

In [5], the IW algorithm was used with $\Delta P = 3\,\text{dB}$ and $\Delta R = 10\%$ of the target rate. To achieve higher precision in date rate, smaller step sizes with $\Delta P = 0.5\,\text{dB}$ and $\Delta R = 2\%$ of the target rate were employed in all simulation runs in this paper.

The proposed SIW algorithm is presented in Algorithm 2. In *each* round of the IW game, based on the resulting power allocation matrix $\mathbf{P}$, we identify and store the users that already *fully* utilized all their available power in the set $\widetilde{U}$, and the *fully* utilized tones in the set $\widetilde{S}$. Subsequently, the sets of *remaining* users and tones permitted to participate in the *next* round of IW game are reestablished by simply removing the elements of $\widetilde{U}$ and $\widetilde{S}$ (of the *current* IW game) from U and S, respectively, that is, $U = U - \widetilde{U}$, and $S = S - \widetilde{S}$. The SIW algorithm also updates the rate requirements $\{\overline{R}_n\}$ and the power constraints $\{\overline{P}_n\}$ for the sets of *remaining* users and tones, U and S, based on the output power and rate allocation matrices $\mathbf{P}$, $\mathbf{R}$ of the *current* IW game. The SIW terminates when all users have fully utilized their maximum power

```
Iterative waterfilling $(\mathbf{P},\mathbf{R}) = \text{IW Alg}(n^*, U, S, \{\overline{R}_n\}, \{\overline{P}_n\})$
Inputs: set of users $U$, set of tones $S$, a user of interest $n^* \in U$, sets of rate
constraints $\{\overline{R}_{n\neq n^*},\ n \in U\}$, set of power constraints $\{\overline{P}_n,\ n \in U\}$.
Outputs: allocation matrices $\mathbf{P}$ (power) and $\mathbf{R}$ (rate)
initialize: $P_n = \overline{P}_n$, $p_k^n = 0$, $n \in U$, $k \in S$;
repeat
    repeat
        for $n \in U$
            $\varepsilon_k^n = \sum_{m\in U, m\neq n} h_k^{n,m} p_k^m + \sigma_k^n$;
            Set and store $\{p_k^n\}_{k\in S}$ computed by the waterfilling algorithm with
            respect to noise spectrum $\{\varepsilon_k^n\}_{k\in S}$ and total power $P_n = \sum_{k\in S} p_k^n$;
            $R_n = \sum_{k\in S} r_k^n$;
        end
    until power allocation profile $p_k^n$, $n \in U$, $k \in S$ converges
    for $n \in U$, $n \neq n^*$
        If $R_n > \overline{R}_n + \Delta R$, $P_n = P_n - \Delta P$; If $R_n < \overline{R}_n - \Delta R$, $P_n = P_n + \Delta P$.
        If $P_n > \overline{P}_n$, set $P_n = \overline{P}_n$;
    end
    if $R_n$ stays the same for every $n$, $P_{n^*} = P_{n^*} - \Delta P$;
until desired accuracy is achieved
```

Algorithm 1: Iterative waterfilling algorithm.

```
SIW algorithm
Initialize: $\overline{R}_n = T_n$, $\overline{P}_n = P_n^{\max}$, $U = \{1,\ldots,N\}$, $S = \{1,\ldots,K\}$,
    while ($U \neq \emptyset$ and $S \neq \emptyset$ and $n^* \in U$)
        $(\mathbf{P},\mathbf{R}) = \text{IW Alg}(n^*, U, S, \{\overline{R}_{n\neq n^*}\}, \{\overline{P}_n\})$;
        $\widetilde{S} = \emptyset$; $\widetilde{U} = \emptyset$;
        for every $n \in U$
            $P_n^{\text{used}} = \sum_{k\in S} p_k^n$;
            if $P_n^{\text{used}} = \overline{P}_n$
                $\widetilde{U} = \widetilde{U} + \{n\}$;
                for every $k \in S$
                    if $p_k^n > 0$, $\widetilde{S} = \widetilde{S} + \{k\}$;
                end for
            end if
        end for
        $U = U - \widetilde{U}$; $S = S - \widetilde{S}$;
        for every $n \in U$
            $\overline{P}_n = \overline{P}_n - \sum_{k\in\widetilde{S}} p_k^n$; If $n \neq n^*$, $\overline{R}_n = \overline{R}_n - \sum_{k\in\widetilde{S}} r_k^n$;
        end for
    end while
```

Algorithm 2: Multiple-user selective IW algorithm.

constraints (i.e., the *updated* $U = \emptyset$), or there are no under-utilized tones (i.e., the *updated* $S = \emptyset$).

SIW can work in a completely distributed manner for two users as follows. After each round of IW game, each user autonomously checks its power availability and determines the frequency bands unused by the other user (by comparing its current experienced interference plus noise level with its noise profile). Then, the user with remaining power can maximize its rate by applying "power waterfilling" procedure to allocate all its remaining power in frequency bands unused by the other user.

For a multiple-user case, a central agent is required to collect PSDs and rate allocation information from users after each round of IW game. Based on the power and rate allocation results of the last round of IW game, the central agent decides the allowable frequency bands (not used by users that already used all their available power) and users (with remaining power) that can participate in the next round of IW game. Since only the information of the allowable user group, frequency band, remaining power, and target rates for the next IW game is communicated between the central agent and users, the increased communication overhead is low. Note that central office (CO) always knows the tone-specific power and rate allocation for every modem even in the case of distributed IW, because each modem has to feedback its tone-specific power and rate allocation to CO so that proper bit loading can be performed at CO. Moreover, unlike centralized OSB, SIW does not require knowledge of crosstalk channel transfer functions and hence avoids the burden for accurate estimation of all the crosstalk channels in a bundle typical of 25–100 lines. Thus, the SIW scheme is more like a distributed DSM scheme.

The proposed SIW algorithm is suboptimal with respect to the achievable rate region. It selectively applies the IW subalgorithm to different groups of users over different frequency bands. In each IW round, at *least* one user completely uses its total power and would be eliminated. Theoretically, the IW algorithm can converge with complexity of $O(KN)$ to a competitively optimal Nash equilibrium under a wide range of conditions [5, 7–9] but these conditions are still restrictive and do not count for all the realistic xDSL scenarios where extensive simulations have shown the convergence of IW. Hence, the proposed SIW algorithm terminates within at most N IW rounds with complexity upper bounded by $O(KN^2)$, as verified in hundreds of simulations conducted in realistic ADSL and VDSL scenarios. On the other hand, the complexity of optimal OSB is $O(KN(P_n/\Delta_p)^N)$ where Δ_p is the granularity in the transmit PSD defined in [13] for tone-specific exhaustive search of the best power allocation configuration. Current standard [17] specifies Δ_p to be 0.5 dBm/Hz. Clearly, for large N, the exponential complexity OSB is intractable, while the polynomial complexity of the proposed SIW is more manageable for practical implementation.

5. PERFORMANCE EVALUATION

In this section, the performance of proposed SIW is evaluated in various realistic mixed CO/RT ADSL downstream and upstream VDSL scenarios [18] with 26-gauge (0.4 mm) lines, tone spacing $\Delta f = 4.3125$ kHz, DMT symbol rate $f_s = 4$ kHz, and target symbol error probability of 10^{-7} or less. The coding gain and noise margin are set to 3 dB and 6 dB, respectively. The performance of SIW is compared with that of the distributed IW algorithm [5] and centralized optimal OSB [13].

We first consider VDSL upstream transmission scenarios in presence of noise and disturbance. ETSI noise model A [19] is implemented to model non-VDSL disturbers, consisting of 10 ADSL, 4 HDSL, and 10 ISDN disturbers. In all our simulations, we adopted the FDD band plan 998 [20], which specifies two separate bands reserved for upstream transmission: 3.75–5.2 MHz and 8.5–12 MHz. The optional 30–138 kHz band is not used. For the example of 8-user case illustrated in Figure 1, the rate regions of SIW, IW, and OSB algorithms plotted in Figure 4 indicate significant rate gains offered by the proposed SIW algorithm. The rate region SIW is very close to the maximum possible rate region computed by the centralized optimal OSB. For instance, when a minimum service of 7 Mbps must be provided for 3000 ft lines, Figure 4 shows that, with IW algorithm the maximum achievable rate for 1500 ft lines is 10 Mbps, while the proposed SIW can increase the maximum achievable rate for 1500 ft lines to 16 Mbps without sacrificing the performance of 3000 ft lines. This is a rate gain of over 60% for 1500 ft lines.

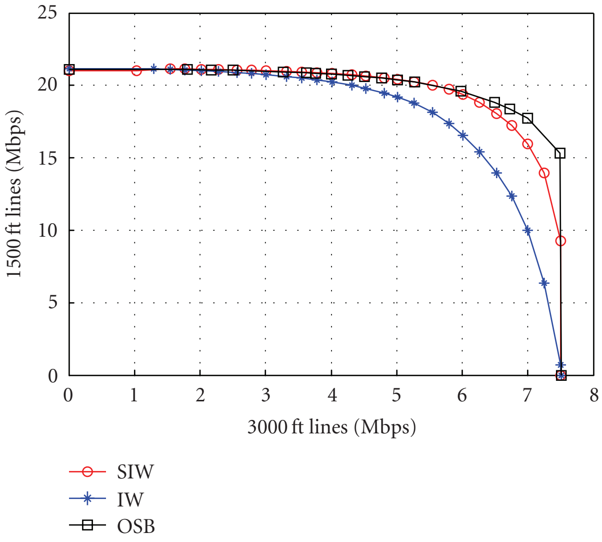


FIGURE 4: Rate region—8-user VDSL upstream scenario.

The enhancement of achievable rate of SIW algorithm results from the intelligent use of underutilized frequency band by 1500 ft lines. In contrast to IW, 1500 ft lines in SIW recognize that the high-frequency band is not used by 3000 ft lines and protective low waterfilling level is not necessary to ensure the performance of 3000 ft lines on the high-frequency band. Therefore, for 1500 ft lines, allocating all the remaining power over the high-frequency band is a smart strategy to enhance their performance without causing any harm to 3000 ft lines.

The PSDs on 1500 ft lines corresponding to 3000 ft lines transmitting at 7 Mbps are shown in Figure 5 for IW, SIW, and OSB. Figure 5 shows that the PSDs computed by the proposed SIW algorithm are very similar to those calculated by the centralized OSB. Note that both SIW and OSB exploit the fact that 3000 ft lines are inactive in the second upstream band, and allocate high PSDs level in this upstream band to achieve higher data rate than IW algorithm.

Figure 6 depicts a scenario of 16-user VDSL upstream: four 1500 ft lines, four 2000 ft lines, four 2400 ft lines and four 3000 ft lines. The target rates of 2000 ft lines, and 2500 ft lines are set to be 4 Mbps.

Figure 7 shows the rate region of 1500 ft lines and 3000 ft lines, indicating substantial gains achieved by SIW algorithm over IW algorithm. For example, when a minimum service of 6.5 Mbps must be provided for 3000 ft lines, the IW algorithm can only support 6 Mbps while SIW algorithm can provide 12 Mbps for 1500 ft lines or a gain of 100%. Again the SIW allows the 1500 ft lines to exploit effectively the high-frequency band, which is not used by all other 2000 ft, 2500 ft, and 3000 ft lines. Therefore, 1500 ft lines can increase

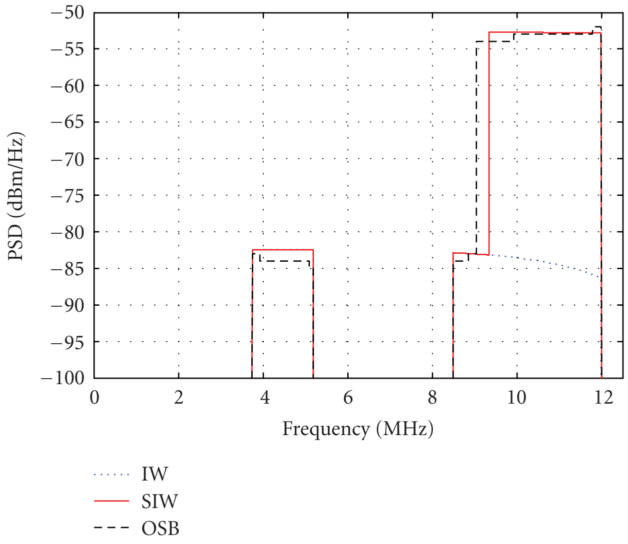


Figure 5: PSDs on 1500 ft lines (3000 ft lines @ 7 Mbps).

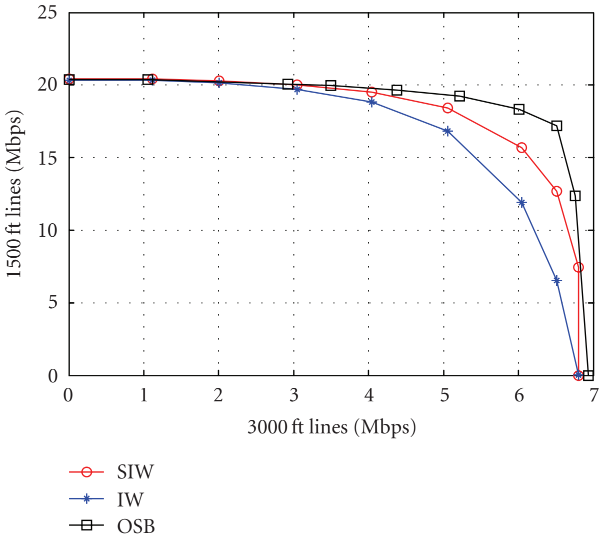


Figure 7: Rate region—16-user VDSL upstream scenario. 2000 ft lines @ 4 Mbps, 2500 ft lines @ 4 Mbps.

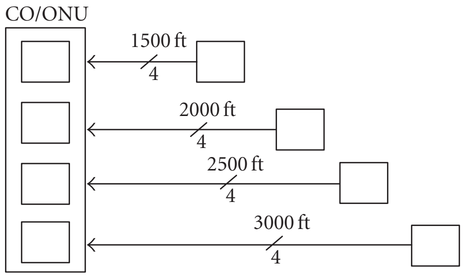


Figure 6: VDSL upstream—16-user scenario.

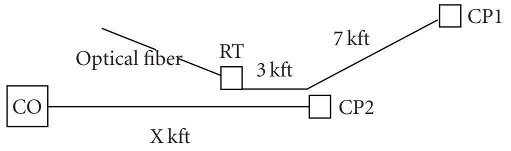


Figure 8: Two-user ADSL downstream mixed CO/RT with unequal line length.

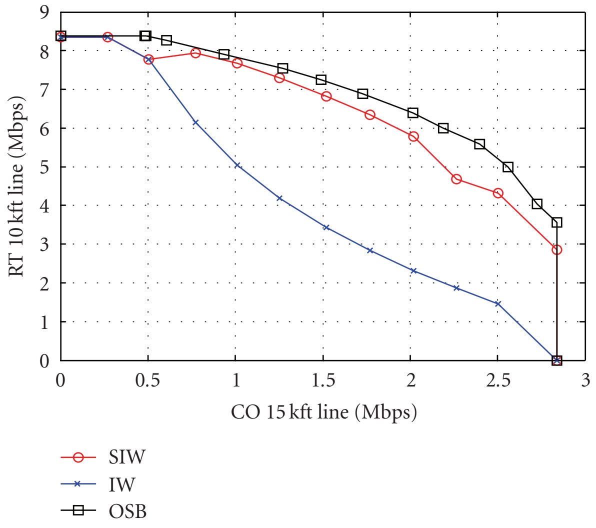


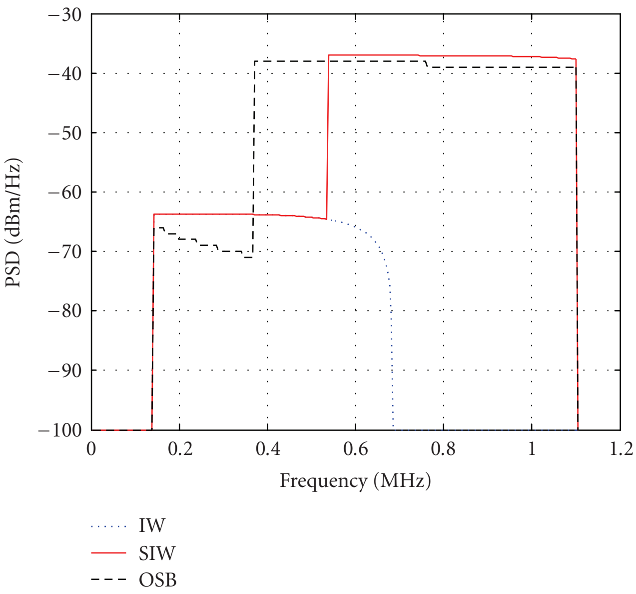
Figure 9: Rate region—2-user ADSL with unequal line lengths.

data rates without harming any other line by allocating all the remaining power over the high-frequency band to maximize their data rates.

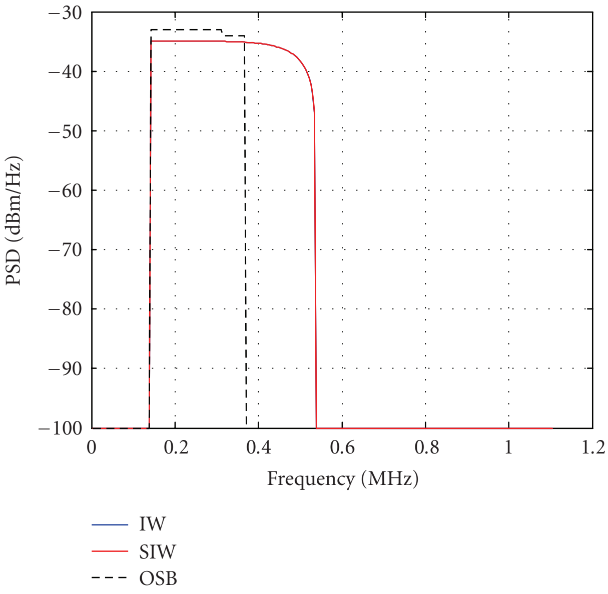
Figure 8 illustrates an example of 2-user ADSL mixed CO/RT downstream with severe near-far problem caused by highly unbalanced crosstalk channels. The 10 kft line from RT to user CP1 (called RT line) has the first 3 kft segment in the same bundle with the line from CO to user CP2 (called CO line). A maximum transmit power of 20.4 dBm is applied to each modem as defined in [21]. It can be expected that the crosstalk over the 3 kft distance from RT to CO lines is much higher than that from CO to RT lines.

Figure 9 shows the rate regions of SIW, IW, and OSB algorithms for an unequal-length case: RT line of 10 kft and CO line of 15 kft. The SIW very closely approaches the centralized optimal OSB and outperforms the IW in terms of rate region. For example, when a minimum service of 2 Mbps must be provided for CO line, with IW, the maximum achievable rate for RT line is 2.3 Mbps, while SIW can boost the maximum achievable rate to 5.8 Mbps without sacrificing the performance of CO line. This corresponds to rate gain over 250%.

The PSDs corresponding to CO line transmitting at 2 Mbps are plotted in Figure 10. Both SIW and OSB exploit the fact that CO line is inactive in high frequency band, and allocate high PSDs level in high-frequency band to achieve higher data rate than IW algorithm. The rate enhancement of SIW algorithm results from intelligent use of underutilized high-frequency band (above 550 kHz) by RT line. Unlike IW,



(a) PSDs on the RT line



(b) PSDs on the CO line

FIGURE 10: PSDs in downstream ADSL (CO line @ 2 Mbps).

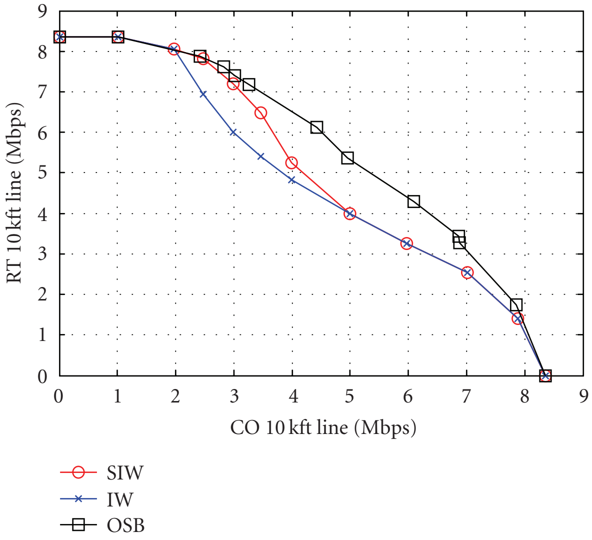


FIGURE 11: Rate region—2-user ADSL with equal line lengths.

RT line in SIW recognizes that the high frequency band is not used by CO line and protective low waterfilling level is not necessary to ensure the performance of CO line on the high-frequency band. Therefore, for RT line, allocating all the remaining power over the high-frequency band is a smart strategy to enhance its performance without causing any harm to CO line. Figure 10 also illustrates subtle difference between the PSDs of SIW and OSB, which contributes to the superior performance of OSB. Besides intelligent use of the inactive high-frequency band in RT line, OSB reduces the PSDs of RT line in the low-frequency band where RT can exert strong interference upon CO line; while SIW acts exactly as its underlying IW, failing to reduce PSDs of RT line in low-frequency band where RT line can cause strong interference to CO line. Consequently, this leads to further rate enhancement of OSB over SIW. Yet, in this ADSL downstream mixed CO-RT scenario with unequal line length, the primary reason of IW's rate degradation is due to underutilized frequency bands, and hence, SIW can successfully recover most of the rate loss of IW and approaches the maximum rate achieved by OSB.

We now consider the 2-user ADSL downstream mixed CO-RT scenario illustrated in Figure 8 when the CO and RT lines have equal length of 10 kft. Figure 11 shows that IW has smaller rate loss as compared to OSB. However, the performance gain of SIW is reduced. For the CO-line rates up to 3 Mbps, the SIW closely approaches the OSB and outperforms the IW in terms of rate region. For CO-line rates greater than 3 Mbps, the rate region of the SIW is degraded and merges to that of the IW for CO-line rates greater than 5 Mbps. The simulation results indicate that the underutilized band is not the primary reason of IW's rate loss in this case. Rather, the rate loss is due to the inability of IW to reduce the PSDs of RT line where it can exert strong crosstalk interference to the CO line. Thus, this limits the capability of SIW to boost the data rate over IW.

6. CONCLUSIONS

When the two-stage power allocation IW algorithm is used in a near-far scenario, the near user has to set its total power budgets sufficiently low to avoid excessive interference to the weak user so that the latter can achieve its target rates. As a result, the frequency band with high attenuation is unused since the far user does not have sufficient power while the near user is effectively "blocked" due to the low protective waterfilling level even if the near user still has a significant portion of unused power. Inspired by this observation, we proposed a low-complexity, high-performance DSM algorithm that selectively applies IW to different frequency bands until all the available frequency and power resources are exhausted in order to achieve higher data rate.

Simulation results in various realistic ADSL downstream and VDSL upstream scenarios indicate that the rate region achieved by the proposed SIW approaches closely the maximum possible rate region computed by the centralized OSB algorithm with significant rate enhancement compared to IW. Moreover, unlike highly complicated centralized OSB, the computational complexity of the proposed SIW is at most N times that of the IW algorithm, and its predominantly distributed nature is amenable for practically distributed DSM implementation with very little coordination and communication with a central agent.

ACKNOWLEDGMENT

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Yang Xu obtained his B.E. degree from the Department of Telecommunication Engineering, Chongqing University of Posts and Telecommunications, Chongqing, and M.E. degree from Faculty of Information Engineering, Beijing University of Posts and Telecommunications, Beijing, China, in 1998 and 2001, respectively. He is currently pursuing the Ph.D. degree at McGill University, Montréal, Canada. His research interests include multicarrier systems, resource allocation, and MIMO interference channel.

Tho Le-Ngoc obtained his B.Eng. degree (with distinction) in electrical engineering in 1976, his M.Eng. degree in microprocessor applications in 1978 from McGill University, Montréal, and his Ph.D. degree in digital communications in 1983 from the University of Ottawa, Canada. During 1977–1982, he was with Spar Aerospace Limited, involved in the development and design of satellite communications systems. During 1982–1985, he was an Engineering Manager of the Radio Group in the Department of Development Engineering of SRTelecom Inc., developed the new point-to-multipoint subscriber radio system SR500. During 1985–2000, he was a Professor in the Department of Electrical and Computer Engineering of Concordia University. Since 2000, he has been with the Department of Electrical and Computer Engineering of McGill University. His research interest is in the area of broadband digital communications with a special emphasis on modulation, coding, and multiple-access techniques. He is a Senior Member of the Ordre des Ingénieur du Québec, a Fellow of the Institute of Electrical and Electronics Engineers (IEEE), a Fellow of the Engineering Institute of Canada (EIC), and a Fellow of the Canadian Academy of

Engineering (CAE). He is the recipient of the 2004 Canadian Award in Telecommunications Research and recipient of the IEEE Canada Fessenden Award 2005.

Saswat Panigrahi received his B.Tech. degree (with National Academic Excellence Award) in electrical engineering from the Indian Institute of Technology (IIT), Kanpur, India, in 2003, and his M.Eng. degree (Dean's Honour List) in communications from McGill University, Montréal, Québec, Canada, in 2005. Since October 2005, he has been working on R&D at Ericsson Canada. His current research interests include multicarrier systems, coding theory, and cross-layer optimization.