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Letter to the Editor

A Further Result about "On the Channel Capacity of Multiantenna Systems with Nakagami Fading"

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Explicit expressions are derived for the channel capacity of multiantenna systems with the Nakagami fading channel.

1. INTRODUCTION

The recent paper by Zheng and Kaiser [1] derived various expressions for the channel capacity of multiantenna systems with the Nakagami fading channel. Most of these are expressed in terms of the integral

$$J(k,\beta)=\int_0^{\infty}\log\left(1+\frac{u}{\beta}\right)u^{k/2-1}\exp(-u)\,du, \tag{1}$$

see, for example, [1, equation (14)]. The paper provided a recurrence relation (see [1, equation (18)]) for calculating (1). Here, we show that one can derive explicit expressions for (1) in terms of well-known functions.

2. EXPLICIT EXPRESSIONS FOR (1)

We calculate (1) by direct application of certain formulas in [2]. For $k>0$, application of [2, equation (2.6.23.4)] yields

$$\begin{aligned}J(k,\beta)={}&\frac{2\pi\beta^{k/2}}{k\sin(k\pi/2)}\,{}_1F_1\left(\frac{k}{2};1+\frac{k}{2};\beta\right)\\&-\Gamma\left(\frac{k}{2}\right)\left[\left\{\log\beta-\Psi\left(\frac{k}{2}\right)\right\}\right.\\&\left.-\frac{2\beta}{2-k}\,{}_2F_2\left(1,1;2,2-\frac{k}{2};\beta\right)\right],\end{aligned} \tag{2}$$

where $\Psi(\cdot)$ denotes the digamma function defined by

$$\Psi(x)=\frac{d\log\Gamma(x)}{dx}, \tag{3}$$

and ${}_1F_1$ and ${}_2F_2$ are the hypergeometric functions defined by

$$\begin{aligned}{}_1F_1(a;b;x)&=\sum_{k=0}^{\infty}\frac{(a)_k}{(b)_k}\frac{x^k}{k!},\\{}_2F_2(a,b;c,d;x)&=\sum_{k=0}^{\infty}\frac{(a)_k(b)_k}{(c)_k(d)_k}\frac{x^k}{k!},\end{aligned} \tag{4}$$

respectively, where $(f)_k=f(f+1)\cdots(f+k-1)$ denotes the ascending factorial. If $k=2$, then by [2, equation (2.6.23.5)] one can reduce (2) to

$$J(2,\beta)=-\exp(\beta)\mathrm{Ei}(-\beta), \tag{5}$$

where $\mathrm{Ei}(\cdot)$ denotes the exponential integral defined by

$$\mathrm{Ei}(x)=\int_{-\infty}^{x}\frac{\exp(t)}{t}\,dt. \tag{6}$$

If $k=1$, then by using the facts that

$$\begin{aligned}\Psi\left(\frac{1}{2}\right)&=-\gamma-2\log 2,\\{}_1F_1\left(\frac{1}{2};\frac{3}{2};\beta\right)&=\frac{\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{\beta}\right)}{2\sqrt{\beta}},\end{aligned} \tag{7}$$

where $\gamma=0.5772\cdots$ is the Euler's constant and $\mathrm{erfi}(\cdot)$ denotes the imaginary error function defined by

$$\mathrm{erfi}(x)=\frac{2}{\sqrt{\pi}}\int_0^{x}\exp\left(t^2\right)dt, \tag{8}$$

one can reduce (2) to

$$J(1,\beta) = \pi^{3/2}\operatorname{erfi}\left(\sqrt{\beta}\right) - \sqrt{\pi}\left[\log\beta + \gamma + 2\log 2 - 2\beta\, {}_2F_2\left(1,1;2,\frac{3}{2};\beta\right)\right]. \quad (9)$$

If $k = 3$, then by using the facts that

$$\Psi\left(\frac{3}{2}\right) = 2 - \gamma - 2\log 2,$$
$${}_1F_1\left(\frac{3}{2};\frac{5}{2};\beta\right) = \frac{3\exp(\beta)}{2\beta} - \frac{3\sqrt{\pi}\operatorname{erfi}\left(\sqrt{\beta}\right)}{4\beta^{3/2}}, \quad (10)$$

one can reduce (2) to

$$J(3,\beta) = -\pi\beta^{1/2}\exp(\beta) + \frac{\pi^{3/2}\operatorname{erfi}\left(\sqrt{\beta}\right)}{2} - \frac{\sqrt{\pi}}{2}\left[\log\beta - 2 + \gamma + 2\log 2 + 2\beta\, {}_2F_2\left(1,1;2,\frac{1}{2};\beta\right)\right]. \quad (11)$$

3. DISCUSSION

We expect that the expression given by (2) and its particular cases could be useful with respect to channel capacity modeling of multiantenna systems with Nakagami fading. The given expressions involve the digamma, exponential integral, imaginary error, and the hypergeometric functions and these functions are well known and well established (see [3, Sections 8.17, 8.21, 8.36, and 9.23]). Numerical routines for computing these functions are widely available, see, for example, Maple and Mathematica.

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