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Research Article

Cooperative Multibeamforming in Ad Hoc Networks

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We treat the problem of cooperative multiple beamforming in wireless ad hoc networks. The basic scenario is that a cluster of source nodes cooperatively forms multiple data-carrying beams toward multiple destination nodes. To resolve the *hidden node* problem, we impose a link constraint on the receive power at each unintended destination node. Then the problem becomes to optimize the transmit powers and beam weights at the source cluster subject to the maximal transmit power constraint, the minimal receive signal-to-interference-plus-noise ratio (SINR) constraints at the destination nodes, and the minimal receive power constraints at the unintended destination nodes. We first propose an iterative transmit power allocation algorithm under fixed beamformers subject to the maximal transmit power constraint, as well as the minimal receive SINR and receive power constraints. We then develop a joint optimization algorithm to iteratively optimize the powers and the beamformers based on the duality analysis. Since channel state information (CSI) is required by the sources to perform the above optimization, we further propose a cooperative scheme to implement a simple CSI estimation and feedback mechanism based on the subspace tracking principle. Simulation results are provided to demonstrate the performance of the proposed algorithms.

1. INTRODUCTION

Recently, a new approach of achieving spatial diversity gain in relay networks, namely, *cooperative diversity* or *user cooperation diversity*, has received considerable interests [1–5]. *Cooperative diversity* comes from the fact that multiple nodes in an ad hoc network can cooperatively form a virtual antenna array providing the potential of realizing spatial diversity. As an effective technique of exploiting spatial diversity in multiple-antenna systems, space-timing coding has been widely studied for cooperative ad hoc networks (e.g., see [6–9]). Beamforming is another important diversity technique in multiple-antenna systems, and several beamforming-based schemes have been developed in current literature for cooperative ad hoc networks. Specifically, distributed receive beamforming is treated in [10, 11]. The effects of phase noises in distributed beamforming schemes are analyzed in [12]. A probabilistic transmit beamforming scheme, namely, collaborative beamforming, is proposed in [13, 14]. In [15], the power optimization issue and also the beamforming at the relay side have been addressed in ad hoc wireless networks. The cooperative beamforming concept and power efficiency issues in fading channels have been treated in [16].

In existing work, one key assumption is that the neighboring nodes which form one cluster can share the data information a priori. From the viewpoint of power consumption, this assumption is reasonable in the sense that the overhead requested by intracluster information sharing is relatively small due to the short distances among intracluster nodes. Another key issue is the synchronization among multiple cooperative nodes [12], for example, carrier frequency, phase, and timing synchronization. It is worth noting that one major problem brought by beamforming applications in wireless networks is the so-called “*hidden node*” problem. In particular, carrier-sense-multiple-access (CSMA) mechanism is employed in 802.11 standards, where each node attempts to access the network and transmits only when it detects no energy from other nodes. Such a CSMA mechanism brings the problem of potential collisions among different transmissions in the case that multiple nodes cannot sense one another’s transmission. The problem of potential collision is, namely, the *hidden node* problem [17, 18]. In the wireless networks employing beamforming schemes, the *hidden node* problem becomes more severe due to the fact that a directional beam inevitably reduces the energy delivered to some unintended destination nodes in the network, and consequently, collisions happen more frequently and result in more retransmission, delay, and packet loss.

In this paper, instead of considering the beamforming problem that a cluster of nodes cooperatively forms one beam toward one destination node (e.g., [13, 14, 18]), we treat the problem of simultaneously forming multiple beams for multiple concurrent data transmissions in wireless ad hoc networks. Figure 1 shows an example of multiple beamforming. This problem resembles the multiuser beamforming problem in MIMO systems which has been studied in [19]. Moreover, different from the probabilistic approach (e.g., see [18]) to resolve the *hidden node* problem, we propose a deterministic approach which imposes a link constraint on the minimum receive power at each unintended destination node. Therefore, the cooperative multiple beamforming problem can be formulated as a multiuser beamforming problem with extra receive power constraints for unintended destination nodes. To solve this problem, we first propose an iterative power allocation algorithm to maximize the balanced SINR ratio under fixed beamformers. Then we develop a joint optimization algorithm to iteratively optimize the powers and the beamformers. Note that channel state information (CSI) is required for the source nodes to perform the above optimizations, and thus, some CSI estimation and feedback mechanism are necessary. We then present a scheme for the source and destination clusters to cooperatively implement a simple CSI tracking mechanism.

The remainder of this paper is organized as follows. In Section 2, the system model is described and the cooperative multiple beamforming problem is formulated. In Section 3, an iterative power allocation strategy is proposed under fixed beamformers. In Section 4, the joint power and beamforming optimization algorithm is developed. In Section 5, the subspace tracking based CSI feedback scheme is presented. Section 6 contains the conclusions.

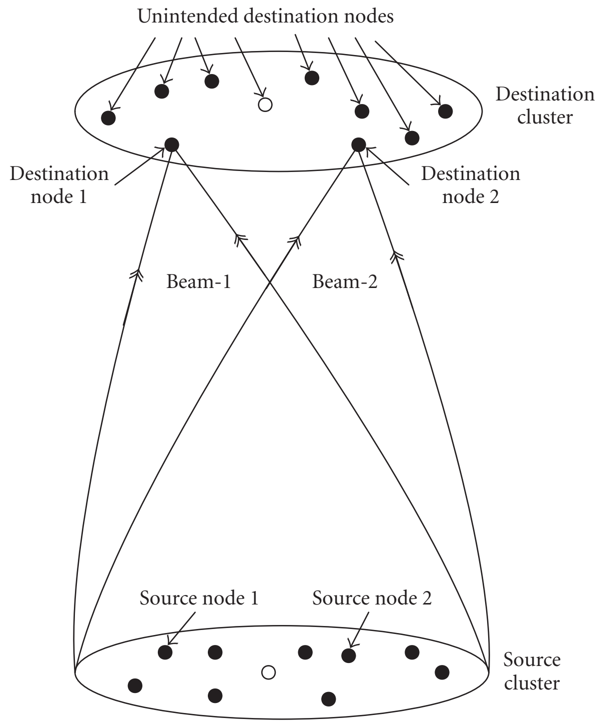

Figure 1: Cooperative multiple beamforming in wireless ad hoc networks: two concurrent beams are formed; $K = 10$ nodes in the source/destination cluster; $M = 2$ source/destination nodes; $K - M = 8$ unintended destination nodes.

2. SYSTEM MODEL AND PROBLEM FORMULATION

The basic concept of cooperative multiple beamfomring is to simultaneously transmit several data-bearing signal beams toward some destination nodes and non-data-bearing signal beams toward unintended destination nodes. As shown in Figure 1, there are K nodes in the source cluster where M ones, namely, source nodes, have information to transmit; there are totally K nodes in the destination cluster, where M of them are the destination nodes, namely, destination nodes, and the other K-M ones are the unintended destination nodes.

2.1. Cooperative multiple beamforming

Cooperative beamforming consists of two stages, local broadcasting and cooperative transmission. In particular, in local broadcasting, each source node broadcasts its data-bearing signal to the other ones in the source cluster; then in cooperative transmission, each node in the source cluster acts as a relay for the others, and the source cluster cooperatively forms multiple concurrent beams. Note that perfect synchronization is assumed in this paper.

2.1.1. Local broadcasting

In the first stage, the received signal at node j in the source cluster from source node i is

$$y_{i,j} = \sqrt{P_{i,0}}h_{i,j}s_i + n_j, \quad 1 \le i \le M,\ 1 \le j \le K,\ i \ne j, \quad (1)$$

where s_i is the data-bearing signal from source node i and $E\{|s_i|^2\} = 1$; $P_{i,0}$ is the transmit power of source node i; $n_j \sim \mathcal{CN}(0, \eta)$ denotes the AWGN at node j; $h_{i,j} \sim \mathcal{CN}(0,1)$ is the channel response between the nodes i and j. The *amplify-and-forward* scheme is employed in the source cluster, that is, each node does not attempt to decode but directly forwards the received signal. Specifically, $y_{i,j}$ at node j is first normalized by $\alpha_{i,j} := \sqrt{E\{|y_{i,j}|^2\}}$, that is,

$$s_{i,j} = \frac{y_{i,j}}{\alpha_{i,j}} = \frac{\sqrt{P_{i,0}}h_{i,j}}{\sqrt{P_{i,0}|h_{i,j}|^2 + \eta}}s_i + \frac{1}{\sqrt{P_{i,0}|h_{i,j}|^2 + \eta}}n_j, \quad (2)$$
$$1 \le i \le M, \quad 1 \le j \le K, \quad j \ne i.$$

Define the cooperative data-bearing signal vector toward each destination node D_i as $\mathbf{s}_i := [s_{i,1}, s_{i,2}, \ldots, s_{i,K}]^T$, where $s_{i,i} = s_i$, $1 \le i \le M$, and the non-data-bearing signal vector toward each unintended destination node D_j as $\mathbf{s}_j := [s_j, s_j, \ldots, s_j]^T$, $M + 1 \le j \le K$.

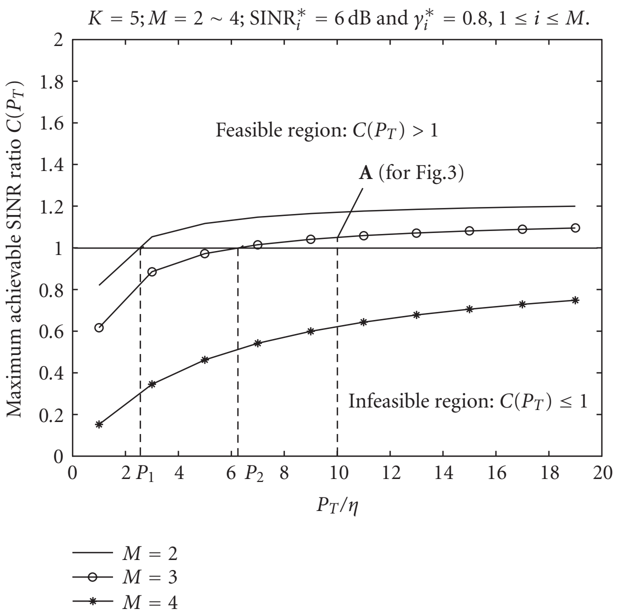

FIGURE 2: Feasible region of problem (**B**): $K = 5$; $M = 2 \sim 4$; $\mathrm{SINR}_i^* = 6$ dB and $\gamma_i^* = 0.8$, $1 \le i \le M$.

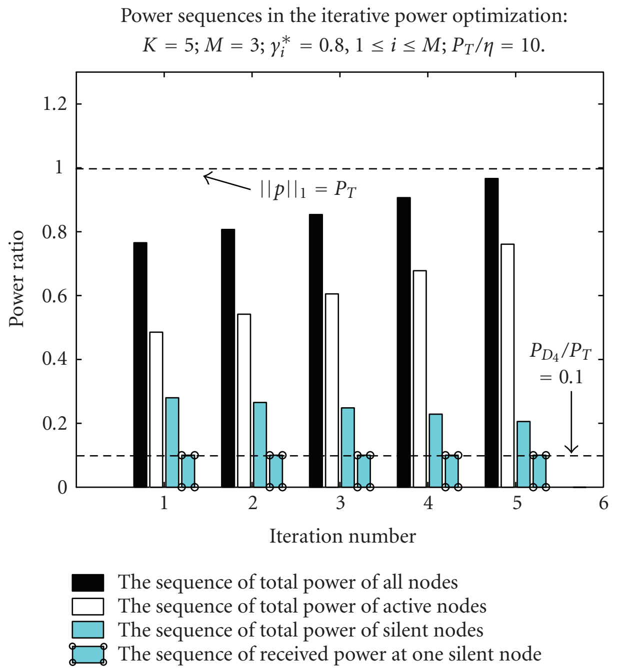

FIGURE 3: Power distribution in the iterative power optimization algorithm (Algorithm 1): $K = 5$; $M = 3$; $\gamma_i^* = 0.8$, $1 \le i \le M$; $P_T/\eta = 10$.

2.1.2. Cooperative transmission

In the second stage, each node j $(1 \le j \le K)$ in the source cluster transmits the signal $x_j = \sum_{i=1}^{K} \sqrt{P_i} u_{i,j} s_{i,j}$, where $u_{i,j}$ is the beam weight at node j for the transmission toward destination node D_i. Denote $\mathbf{u}_i := [u_{i,1}, u_{i,2}, \ldots, u_{i,K}]^T$ and $\mathbf{h}_i^H := [h_{1,D_i}, h_{2,D_i}, \ldots, h_{K,D_i}]$, $1 \le i \le K$, as the beamformer and the channel vector for the reception of $\mathbf{s}_i$ at D_i, respectively. Then the received data-bearing signal $\mathbf{s}_i$ at destination node D_i is given by

$$s_{D_i} = \sqrt{P_i} \sum_{j=1}^{K} h_{j,D_i} u_{i,j} s_{i,j} = \sqrt{P_i} \mathbf{h}_i^H \mathbf{\Lambda}_i \mathbf{u}_i s_i + \sqrt{P_i} \mathbf{h}_i^H \mathbf{\Xi}_i \mathbf{u}_i, \quad 1 \le i \le M, \tag{3}$$

where $\mathbf{\Lambda}_i := \mathrm{diag}\{\beta_{i,1}, \ldots, \beta_{i,i-1}, 1, \beta_{i,i+1}, \ldots, \beta_{i,K}\}$ with $\beta_{i,j} := \sqrt{P_{i,0}} h_{i,j} / \sqrt{P_{i,0}|h_{i,j}|^2 + \eta}$, and $\mathbf{\Xi}_i := \mathrm{diag}\{\xi_{i,1}, \ldots, \xi_{i,i-1}, 0, \xi_{i,i+1}, \ldots, \xi_{i,K}\}$ with $\xi_{i,j} := n_j / \sqrt{P_{i,0}|h_{i,j}|^2 + \eta}$, $1 \le j \le K$ and $j \ne i$. Moreover, the received data-bearing signal $\mathbf{s}_j$ at D_i $(j \ne i)$ is given by

$$I_{D_i} = \sum_{j=1, j \ne i}^{M} \sqrt{P_j} \mathbf{h}_i^H \mathbf{\Lambda}_j \mathbf{u}_j s_j + \sum_{j=1, j \ne i}^{M} \sqrt{P_j} \mathbf{h}_i^H \mathbf{\Xi}_j \mathbf{u}_j + \sum_{l=M+1}^{K} \sqrt{P_l} \mathbf{h}_i^H \mathbf{u}_l s_l, \tag{4}$$

where the first two terms come from the data-bearing signal $\mathbf{s}_j$ $(1 \le j \le M, j \ne i)$, and the last term is from the non-data-bearing signal $\mathbf{s}_l$ $(M + 1 \le l \le K)$. Then the overall received signal $y_{D_i} = s_{D_i} + I_{D_i} + n_{Di}$ at each destination node D_i can be written as

$$y_{D_i} = \sum_{j=1}^{M} \sqrt{P_j} \mathbf{h}_i^H \mathbf{\Lambda}_j \mathbf{u}_j s_j + \sum_{j=1}^{M} \sqrt{P_j} \mathbf{h}_i^H \mathbf{\Xi}_j \mathbf{u}_j + \sum_{l=M+1}^{K} \sqrt{P_l} \mathbf{h}_i^H \mathbf{u}_l s_l + n_{D_i}, \quad 1 \le i \le M. \tag{5}$$

2.1.3. Receive SINR and power

Define $\mathbf{\Omega}_i := \mathbf{h}_i \mathbf{h}_i^H$ and $\widetilde{\mathbf{\Omega}}_i := E\{\mathbf{\Lambda}_i^H \mathbf{\Omega}_i \mathbf{\Lambda}_i\}$, $1 \le i \le K$. For a given $\{\mathbf{h}_1, \mathbf{h}_2, \ldots, \mathbf{h}_K\}$, the receive SINR at each destination node D_i can be expressed as

$$\mathrm{SINR}_i = \frac{P_i \mathbf{u}_i^H \widetilde{\mathbf{\Omega}}_i \mathbf{u}_i}{\sum_{j=1}^{M} P_j \mathbf{u}_j^H \mathbf{\Delta}_i \mathbf{u}_j + \sum_{j=M+1}^{K} P_j \mathbf{u}_j^H \mathbf{\Omega}_i \mathbf{u}_j - P_i \mathbf{u}_i^H \widetilde{\mathbf{\Omega}}_i \mathbf{u}_i + \eta}, \quad 1 \le i \le M, \tag{6}$$

where $\mathbf{\Delta}_i := E\{(\mathbf{\Lambda}_j + \mathbf{\Xi}_j)^H \mathbf{\Omega}_i (\mathbf{\Lambda}_j + \mathbf{\Xi}_j)\} = E\{\mathbf{\Lambda}_j^H \mathbf{\Omega}_i \mathbf{\Lambda}_j + \mathbf{\Xi}_j^H \mathbf{\Omega}_i \mathbf{\Xi}_j\}$ and $\mathbf{\Delta}_i = \mathrm{diag}\{\mathbf{\Omega}_i\}$ for $1 \le j \le M$. Further define γ_i as an increasing function of SINR_i in (6)

$$\gamma_i := \frac{\mathrm{SINR}_i}{1 + \mathrm{SINR}_i} = \frac{P_i \mathbf{u}_i^H \widetilde{\mathbf{\Omega}}_i \mathbf{u}_i}{\sum_{j=1}^{M} P_j \mathbf{u}_j^H \mathbf{\Delta}_i \mathbf{u}_j + \sum_{j=M+1}^{K} P_j \mathbf{u}_j^H \mathbf{\Omega}_i \mathbf{u}_j + \eta}, \tag{7}$$

which is essentially equivalent to SINR_i. It should be notified that the SINR_i based analysis and optimization are quite involved in cooperative ad hoc networks, and the metric γ_i can help to make the analysis and optimization much more tractable. The optimization based on γ_i can be viewed as an approximation of the optimization based on SINR_i. Note that we will adopt γ_i as the performance metric throughout this paper. For convenience, hereafter, we call γ_i the receive SINR at D_i, though the receive SINR is in fact SINR_i given by (6). The receive power at each unintended destination node D_j is given by

$$\underbrace{\begin{bmatrix} \mathbf{u}_1^H \boldsymbol{\Delta}_{M+1}\mathbf{u}_1 & \cdots & \mathbf{u}_K^H \boldsymbol{\Omega}_{M+1}\mathbf{u}_K \\ \vdots & \ddots & \vdots \\ \mathbf{u}_1^H \boldsymbol{\Delta}_{K}\mathbf{u}_1 & \cdots & \mathbf{u}_K^H \boldsymbol{\Omega}_{K}\mathbf{u}_K \end{bmatrix}}_{\boldsymbol{\Theta}} \underbrace{\begin{bmatrix} P_1 \\ \vdots \\ P_K \end{bmatrix}}_{\mathbf{p}} = \underbrace{\begin{bmatrix} P_{D_{M+1}} \\ \vdots \\ P_{D_K} \end{bmatrix}}_{\mathbf{p}_D}. \tag{8}$$

Remark 1. One key assumption in the existing literature on distributed beamforming is that one cluster can share information *a priori*. Under this assumption, the received signal and the SINR at each D_i are given, respectively, by

$$\tilde{y}_{D_i} = \sum_{j=1}^{K} \sqrt{P_j}\mathbf{h}_i^H \mathbf{u}_j s_j + n_{D_i}, \tag{9}$$

$$\widetilde{\text{SINR}}_i = \frac{P_i \mathbf{u}_i^H \boldsymbol{\Omega}_i \mathbf{u}_i}{\sum_{j=1, j\neq i}^{K} P_j \mathbf{u}_j^H \boldsymbol{\Omega}_i \mathbf{u}_j + \eta}. \tag{10}$$

Assuming that each relay receives broadcasting signals without noises, we have $\boldsymbol{\Lambda}_i = \mathbf{I}_K$, $\boldsymbol{\Xi}_i = \mathbf{O}_K$, and $\boldsymbol{\Delta}_i = \widetilde{\boldsymbol{\Omega}}_i = \boldsymbol{\Omega}_i$. Then (5) and (6) reduce to (9) and (10), respectively. Moreover, (9) and (10) also hold for the *decode-and-forward* scheme in relay networks assuming perfect decoding at relays. Hence, the assumption of perfect *a priori* sharing among source nodes is a special case of the general relay scenarios (5) and (6), and the existing distributed beamforming approaches still fall in the cooperative relay framework treated in this paper.

2.2. Problem formulation

The cooperative beamforming problem is to find the optimal power and beamforming matrix to maximize the minimal receive SINR of destination nodes under the maximal transmit power constraint and the minimal receive power constraints for unintended destination nodes,

$$(\mathbf{A}) \quad C(\mathbf{p}^*, \mathbf{U}^*) = \max_{\mathbf{p},\mathbf{U}} \min_{1\le i\le M} \frac{\gamma_i(\mathbf{p},\mathbf{U})}{\gamma_i^*},$$
$$\text{subject to} \begin{cases} \|\mathbf{p}\|_1 = \sum_{i=1}^{K} P_i \le P_T, \\ C(\mathbf{p},\mathbf{U}) \ge 1, \\ P_{D_j}(\mathbf{p},\mathbf{U}) \ge P_j^{\min}, \quad M+1 \le j \le K, \end{cases} \tag{11}$$

where $\mathbf{U} := [\mathbf{u}_1, \mathbf{u}_2, \dots, \mathbf{u}_K]$; P_T is the maximal transmit power; γ_i^* is the minimal SINR for destination node D_i; $P_j^{\min}$ is the minimal receive power for unintended destination node D_j.

Remark 2. In problem (**A**), an assumption of $\boldsymbol{\Theta}$ in (8) is that for each j $(M+1 \le j \le K)$, $\mathbf{u}_i^H \boldsymbol{\Delta}_j \mathbf{u}_i < \mathbf{u}_k^H \boldsymbol{\Omega}_j \mathbf{u}_k$, $1 \le i \le M$, $M+1 \le k \le K$. This assumption is reasonable and necessary due to the *hidden node* problem. In particular, the *hidden node* problem exists when the receive powers at the unintended destination nodes are small, that is, $\sum_{i=1}^{M} P_i \mathbf{u}_i^H \boldsymbol{\Omega}_j \mathbf{u}_i$ in (8). Thus it is necessary to form the extra non-data-bearing beams to ensure certain receive powers. On the other hand, if $\mathbf{u}_i^H \boldsymbol{\Delta}_j \mathbf{u}_i \ge \mathbf{u}_k^H \boldsymbol{\Omega}_j \mathbf{u}_k$, the minimum receive power constraints can be guaranteed by only allocating power to those data-bearing beams (i.e., let $P_i = 0$, $1+M \le i \le K$), and thus the *hidden node* problem becomes trivial [18].

3. OPTIMAL POWER ALLOCATION STRATEGY

3.1. Optimal power allocation problem

For a given beamforming matrix $\mathbf{U}$, problem (**A**) reduces to the power allocation problem

$$(\mathbf{B}) \quad C(\mathbf{p}^*) = \max_{\mathbf{p}} \min_{1\le i\le M} \frac{\gamma_i(\mathbf{p})}{\gamma_i^*},$$
$$\text{subject to} \begin{cases} \|\mathbf{p}\|_1 = \sum_{i=1}^{K} P_i \le P_T, \\ C(\mathbf{p}) \ge 1, \\ P_{D_j}(\mathbf{p}) \ge P_j^{\min}, \quad M+1 \le j \le K. \end{cases} \tag{12}$$

Note that a similar problem but without the receive power constraints has been treated in [19, 20], where a specific structure is exploited to calculate $\mathbf{p}^*$. Such a structure, however, does not exist for problem (**B**) due to the extra constraints on receive powers $P_{D_j}(\mathbf{p})$.

To solve problem (**B**), we further treat the following total power minimization problem:

$$(\widetilde{\mathbf{B}}) \quad \rho(\mathbf{p}^*) = \min_{\mathbf{p}} \sum_{i=1}^{K} P_i,$$
$$\text{subject to} \begin{cases} \gamma_i(\mathbf{p}) \ge \gamma_i^*, & 1 \le i \le M, \\ P_{D_j}(\mathbf{p}) \ge P_j^{\min}, & M+1 \le j \le K, \end{cases} \tag{13}$$

which is to find $\mathbf{p}^*$ for a given $\mathbf{U}$ so as to minimize the total transmit power under the minimum constraints on receive powers and SINRs. Note that the problems ($\widetilde{\mathbf{B}}$) and (**B**) are closely related [19] in the sense that without the minimum receive power constraints, they are equivalent and have the same solution if and only if $\rho(\mathbf{p}^*) = P_T$. Then it can be solved by an iterative approach where in each iteration, $\mathbf{p}^*$ of problem ($\widetilde{\mathbf{B}}$) is calculated under a given target SINR set $\{\gamma_i^*\}_i$, and then increase $\{\gamma_i^*\}_i$ if $\|\mathbf{p}^*\|_1$ is less than P_T. As $\|\mathbf{p}^*\|_1$ approximates P_T, $C(\mathbf{p}^*)$ will reach the maximal achievable value. With the minimum receive power constraints, however, it is difficult to find the optimal solution, and thus we propose to find an approximation of $\mathbf{p}^*$ as follows.

3.2. Iterative power optimization algorithm

Denote $\mathbf{p}_M = [P_1, \dots, P_M]^T$ and $\mathbf{p}_{K-M} = [P_{M+1}, \dots, P_K]^T$. First, consider the optimal $\mathbf{p}_M$ under a given $\mathbf{p}_{K-M}$. Since each γ_i in (7) is monotonically increasing with respect to P_i ($1 \leq i \leq M$) and monotonically decreasing with respect to P_j ($1 \leq j \leq K$ and $j \neq i$) under a given $\mathbf{p}_{K-M}$, the optimal $\mathbf{p}_M$ of problem ($\widetilde{\mathbf{B}}$) only with the minimum receive SINR constraints can be achieved when $\gamma_i(\mathbf{p}_M, \mathbf{p}_{K-M}, \mathbf{U}) = \gamma_i^*$, $1 \leq i \leq M$. Using (7), these M linear equations can be written into the matrix representation:

$$\left[\mathbf{\Gamma}^{-1}\mathbf{\Psi} - \underbrace{\begin{bmatrix} \mathbf{u}_1^H \mathbf{\Delta}_1 \mathbf{u}_1 & \cdots & \mathbf{u}_K^H \mathbf{\Omega}_1 \mathbf{u}_K \\ \vdots & \ddots & \vdots \\ \mathbf{u}_1^H \mathbf{\Delta}_M \mathbf{u}_1 & \cdots & \mathbf{u}_K^H \mathbf{\Omega}_M \mathbf{u}_K \end{bmatrix}}_{\mathbf{\Phi}}\right] \mathbf{p} = \eta \cdot \mathbf{1}_M, \quad (14)$$

where $\mathbf{\Gamma} := \operatorname{diag}\{\gamma_1^*, \ldots, \gamma_M^*\}$; $\mathbf{1}_M := [1, \ldots, 1]^T$ has a dimension of M; $\mathbf{\Psi} := [\mathbf{\Psi}_1, \mathbf{O}_{M\times(K-M)}]$, where $\mathbf{\Psi}_1 := \operatorname{diag}\{\mathbf{u}_1^H \widetilde{\mathbf{\Omega}}_1 \mathbf{u}_1, \ldots, \mathbf{u}_M^H \widetilde{\mathbf{\Omega}}_M \mathbf{u}_M\}$. Next consider the optimal $\mathbf{p}_{K-M}$ under a given $\mathbf{p}_M$. Using (8) with a given $\mathbf{p}_M$, the optimal $\mathbf{p}_{K-M}$ of problem ($\widetilde{\mathbf{B}}$) with only the minimum receive power constraints is achieved when

$$\mathbf{\Theta}\mathbf{p} = \mathbf{p}^{\min}. \quad (15)$$

Iteratively optimizing $\mathbf{p}_M$ and $\mathbf{p}_{K-M}$ using (14) and (15) under increasing target SINRs, $\|\mathbf{p}\|_1$ will approximate P_T. The iterative power allocation is summarized in Algorithm 1.

Denote $\mathbf{p}^* = [\mathbf{p}_M^{*T}, \mathbf{p}_{K-M}^{*T}]^T$ as the optimal solution of problem ($\widetilde{\mathbf{B}}$). In step (1), $\|\mathbf{p}_{K-M}(1)\|_1 = 0 \leq \|\mathbf{p}_{K-M}^*\|_1$ and $\|\mathbf{p}_M(1)\|_1 \leq \|\mathbf{p}_M^*\|_1$, and thus $\|\mathbf{p}(1)\| \leq \|\mathbf{p}^*\|_1$. In step (2), $\|\mathbf{p}_{K-M}(2)\|_1 \geq \|\mathbf{p}_{K-M}^*\|_1$ and $\|\mathbf{p}_M(2)\|_1 \geq \|\mathbf{p}_M^*\|_1$, and thus $\|\mathbf{p}(2)\|_1 \geq \|\mathbf{p}^*\|_1$, $\|\widetilde{\mathbf{p}}(1)\| \geq \|\mathbf{p}(1)\|_1$. In step (3), $\|\mathbf{p}_{K-M}(3)\|_1 \leq \|\mathbf{p}_{K-M}^*\|_1$, and thus $\|\mathbf{p}(1)\|_1 \leq \|\widetilde{\mathbf{p}}(1)\|_1$, $\|\mathbf{p}^*\|_1$, $\|\widetilde{\mathbf{p}}(2)\|_1 \leq \|\mathbf{p}(2)\|_1$. In steps (4)–(6), we have $\|\mathbf{p}_M(n+1)\|_1 \geq \|\mathbf{p}_M(n)\|_1$ due to $\gamma_i^*(n+1) \geq \gamma_i^*(n)$ in (14), that is, $\|\mathbf{p}_M(n)\|_1$ is increasing with respect to the iteration index n. Consequently, (15) further implies that $\|\mathbf{p}_{K-M}(n+1)\|_1 \leq \|\mathbf{p}_{K-M}(n)\|_1$, that is, $\|\mathbf{p}_{K-M}(n)\|_1$ is decreasing. Then the convergence of Algorithm 1 depends on whether $\|\mathbf{p}(n)\|_1 = \|\mathbf{p}_M(n)\|_1 + \|\mathbf{p}_{K-M}(n)\|_1$ is increasing with respect to n. Remember that the assumption of $\mathbf{\Theta}$ stated in Remark 2 ensures that for each $M+1 \leq j \leq K$, $\mathbf{u}_i^H \mathbf{\Omega}_j \mathbf{u}_i < \mathbf{u}_k^H \mathbf{\Omega}_j \mathbf{u}_k$, $i \leq M < k$. Hence, we have $\sum_{i=1}^M P_i(n+1) - \sum_{i=1}^M P_i(n) \geq \sum_{k=M+1}^K P_k(n) - \sum_{k=M+1}^K P_k(n+1)$, that is,

$$\begin{aligned} ||p(n+1)||_1 &= \sum_{i=1}^M P_i(n+1) + \sum_{k=M+1}^K P_k(n+1) \\ &\geq \sum_{i=1}^M P_i(n) + \sum_{k=M+1}^K P_k(n) = ||\mathbf{p}(n)||_1. \end{aligned} \quad (16)$$

This guarantees the convergence of Algorithm 1, which is summarized as follows.

Theorem 1. *The sequence $\{\|\mathbf{p}(n)\|_1\}$ obtained in Algorithm 1 is a monotonically increasing sequence. The optimal solution to problem ($\mathbf{B}$) is achieved when $\|\mathbf{p}(n)\|_1$ reaches P_T.*

3.3. Simulation results

Figure 2 shows the achievable region of SINR ratios for problem ($\mathbf{B}$) under a fixed beamforming matrix $\mathbf{U}$. The results are the averaged performances over 1000 channel realizations. For each channel realization, $\mathbf{u}_i$ in the fixed $\mathbf{U}$ is the optimal beamforming vector for node i's single transmission, that is, the eigenvector corresponding to the largest eigenvalue of $\mathbf{\Omega}_i$. The simulation conditions in Figure 2 are as follows: $K = 5$; $M = 2\sim4$; the minimum receive SINR is $\gamma_i^* = 0.8$ (i.e., $\text{SINR}_i^* = 6\,\text{dB}$), $1 \leq i \leq M$; the minimum receive power is $\mathbf{p}^{\min} = [1, \ldots, 1]^T$. In Figure 2, the maximum achievable SINR ratio for problem ($\mathbf{B}$) $C(P_T) := C(\mathbf{p}^*)$ depends on both P_T and $\{\gamma_i^*\}_i$, and is monotonically increasing with respect to the total transmit power P_T. The feasible region corresponds to the region $C(P_T) > 1$ in Figure 2, and depends on $\{\gamma_i^*\}_i$. It is seen from Figure 2 that P_1 and P_2 ($P_1 < P_2$) are the minimum total transmit powers to guarantee feasible solutions, respectively, for the cases of $M = 2$ and $M = 3$. For the case of $M = 4$, however, there exists no possible solution in the feasible region, that is, no feasible solution exists for problem ($\mathbf{B}$) when $M = 4$. Hence, we conclude from Figure 2 that on the one hand, the more concurrent transmissions the system simultaneously supports, the higher the total transmit power required to guarantee feasible solutions is; on the other hand, under some cases, there exists no feasible solution even if $P_T \to \infty$, and this has also been pointed out in [19] for multiuser beamforming scenarios. In the latter case, beamforming optimization will play an important role which will be demonstrated later.

Figure 3 shows the sequences of total transmit power $\{\|\mathbf{p}(n)\|_1\}$ generated in Algorithm 1 under the same conditions as those in Figure 2, where $M = 3$ and $P_T/\eta = 10$. Note that the maximum achievable SINR ratio in Figure 3 corresponds to the point A in Figure 2 ($C(P_T) = 1.2$). It is observed that $\|\mathbf{p}(n)\|_1$ is increasing and reaches P_T (i.e., $\|\mathbf{p}(n)\|_1/P_T \to 1$) as n increases. Moreover, it is seen from Figure 3 that the total transmit power sequence for data-bearing transmissions $\{\sum_{i=1}^M P_i(n)\}$ is also an increasing one; in contrast, the total transmit power sequence for non-data-bearing transmissions $\{\sum_{i=M+1}^K P_i(n)\}$ is a decreasing one. Figure 3 also shows that the receive power sequence for the unintended destination node $P_{D_4}(n) = P_4^{\min} \cong 1$ is approximately fixed as the minimum value. This implies that the power consumption to guarantee the receive power constraints on the unintended destination nodes is minimized.

4. JOINT POWER AND BEAMFORMER OPTIMIZATION

4.1. Optimal beamforming and duality property

Under a given power set $\mathbf{p}$, problem ($\mathbf{A}$) is then reduced to the beamforming problem

$$(\mathbf{C1}) \quad C^* = C(\mathbf{U}^*) = \max_{\mathbf{U}} \min_{1 \leq i \leq M} \frac{\gamma_i(\mathbf{U})}{\gamma_i^*}. \quad (17)$$

It is observed from (7) that each γ_i is coupled with the entire beamforming matrix $\mathbf{U} = [\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_K]$, and thus problem ($\mathbf{C1}$) is hard to solve. Note that it has been proven in [19]

1: Given $\mathbf{p}_{K-M}(1) = \mathbf{0}_{K-M} = [0, 0, \dots, 0]^T$, calculate $\mathbf{p}_M(1)$ using (14).
If $\mathbf{p}_D(\mathbf{p}(1)) \geq \mathbf{p}^{\min}$, then stop the iteration and let $\mathbf{p}^* = \mathbf{p}(1)$, where $\mathbf{p}(1) = [\mathbf{p}_M(1)^T, \mathbf{p}_{K-M}(1)^T]^T$.
2: Given $\mathbf{p}_M(1)$, calculate $\mathbf{p}_{K-M}(2)$ using (15), and then given $\mathbf{p}_{K-M}(2)$, calculate $\mathbf{p}_M(2)$ using (14). Then $\widetilde{\mathbf{p}}(1) = [\mathbf{p}_M(1)^T, \mathbf{p}_{K-M}(2)^T]^T$, and $\mathbf{p}(2) = [\mathbf{p}_M(2)^T, \mathbf{p}_{K-M}(2)^T]^T$.
3: Given $\mathbf{p}_M(2)$, calculate $\mathbf{p}_{K-M}(3)$ using (15). Then $\widetilde{\mathbf{p}}(2) = [\mathbf{p}_M(2)^T, \mathbf{p}_{K-M}(3)^T]^T$. Let the target SINR be $\gamma_i^*(2) = \gamma_i^*$; $n \Leftarrow 3$.
4: $\gamma_i^*(n) = C(n-1)\gamma_i^*(n-1)$, $1 \leq i \leq M$, where $C(n-1) = \max_{1\leq i\leq M}(\gamma_i(\widetilde{\mathbf{p}}(n-1))/\gamma_i^*(n-1))$.
5: Given $\mathbf{p}_{K-M}(n)$, calculate $\mathbf{p}_M(n)$ using (14), and then given $\mathbf{p}_K(n)$, calculate $\mathbf{p}_{K-M}(n+1)$ using (15). Then $\mathbf{p}(n) = [\mathbf{p}_M(n)^T, \mathbf{p}_{K-M}(n)^T]^T$ and $\widetilde{\mathbf{p}}(n) = [\mathbf{p}_K(n)^T, \mathbf{p}_{K-M}(n+1)^T]^T$.
6: If$\|\widetilde{\mathbf{p}}(n)\|_1 < P_T$, then$n \Leftarrow n+1$, and go to step (4); otherwise, stop and$\mathbf{p}^* \leftarrow \mathbf{p}(n-1)$.

Algorithm 1: Iterative power allocation algorithm.

that the downlink multiuser beamforming problem can be solved by alternatively treating the dual uplink problem due to the uplink-downlink duality for multiuser beamforming scenarios without receive power constraints. Then an interesting question is whether the duality still holds under the extra receive power constraints in the problem considered in this paper.

Remark 3. In Section 3, we only assume that $\mathbf{u}_i^H \mathbf{\Omega}_j \mathbf{u}_i < \mathbf{u}_k^H \mathbf{\Omega}_j \mathbf{u}_k$, $i \leq M < k$ and $M+1 \leq j \leq K$. Hereafter, we further assume that the channels of the unintended destination nodes fall in the orthogonal space spanned by the channels of the destination nodes, that is, $\mathbf{u}_i^H \mathbf{\Omega}_j \mathbf{u}_i = 0$ for $1 \leq i \leq M$ and $M+1 \leq j \leq K$. In such a case, the extra non-data-bearing transmission (e.g., *complementary beamforming* [18]) is a must. Furthermore, under this assumption, $\mathbf{p}^*$ for problem $(\widetilde{\mathbf{B}})$ can be obtained by simultaneously solving (14) and (15), that is,

$$\underbrace{\begin{bmatrix} \mathbf{\Gamma}^{-1}\mathbf{\Psi} - \mathbf{\Phi} \\ \mathbf{\Theta} \end{bmatrix}}_{\mathbf{\Upsilon}} \mathbf{p} = \underbrace{\begin{bmatrix} \eta \mathbf{1}_M \\ \mathbf{p}^{\min} \end{bmatrix}}_{\widetilde{\boldsymbol{\eta}}}. \tag{18}$$

Then problem (**B**) can be solved via the simplified version of Algorithm 1, where $\mathbf{p}^*$ of problem $(\widetilde{\mathbf{B}})$ is obtained from (18) for given $\{\gamma_i^*\}_i$, and then $\{\gamma_i^*\}_i$ are increased if $\|\mathbf{p}^*\|_1 < P_T$.

Now consider a virtual scenario with the same P_T, $\mathbf{p}^{\min}$, $\mathbf{\Gamma}$, and $\mathbf{U}$ as those in problem (**B**). Define the receive SINR for each destination node i in this virtual scenario as

$$\widetilde{\gamma}_i = \frac{P_i \mathbf{u}_i^H \widetilde{\mathbf{\Omega}}_i \mathbf{u}_i}{\mathbf{u}_i^H \left(\sum_{j=1}^{M} P_j \mathbf{\Delta}_j + \sum_{k=M+1}^{K} \mathbf{\Omega}_k + \eta \mathbf{I} \right) \mathbf{u}_i}, \quad 1 \leq i \leq M. \tag{19}$$

Replacing γ_i in problem (**B**) and problem $(\widetilde{\mathbf{B}})$ by $\widetilde{\gamma}_i$ in (19), the power optimization problem and the total power minimization problem can then be formulated for the virtual scenario (19). The virtual power optimization problem can be solved by a similar approach as Algorithm 1, that is, iteratively solving the virtual total power minimization problem under the increasing target SINRs. In particular, under the assumption stated in Remark 3, the optimal power vector for the virtual total power minimization problem can be obtained by solving a similar equation as (18) for solving problem $(\widetilde{\mathbf{B}})$

$$\mathbf{\Upsilon}^T \mathbf{p} = \widetilde{\boldsymbol{\eta}}, \tag{20}$$

where $\mathbf{\Upsilon}$ in (18) is replaced by $\mathbf{\Upsilon}^T$. The following lemma indicates the duality between problem (**B**) and the above virtual power optimization problem under the extra constraints on receive powers. Let $\widetilde{C}$ be the maximum achievable SINR ratio of this virtual problem.

Lemma 1. *For the same* $\mathbf{U}$*,* P_T*, and* $\mathbf{p}^{\min}$*, problem (***B***) and the above virtual power optimization problem have the same achievable SINR regions, that is,* $C(\mathbf{U}, P_T) = \widetilde{C}(\mathbf{U}, P_T)$*.*

Proof. To guarantee the minimum receive power constraints in problem (**B**), the transmit powers $\mathbf{p}$ should satisfy $\mathbf{\Theta}\mathbf{p} = \mathbf{p}^{\min}$. Based on the assumption stated in Remark 3, $\mathbf{\Theta}\mathbf{p} = \mathbf{p}^{\min}$ can then be rewritten into the following one:

$$\mathbf{\Theta}_1 \mathbf{p}_{K-M} = \mathbf{p}^{\min}, \tag{21}$$

where $\mathbf{\Theta}_1$ is the $(K-M) \times (K-M)$ bottom-right submatrix in $\mathbf{\Theta}$. It is observed from (21) that the receive powers for the unintended destination nodes only depend on the extra powers of non-data-bearing transmissions $\mathbf{p}_{K-M}$. Similarly, we have the same conclusion for the transmit powers $\widetilde{\mathbf{p}} = [\widetilde{P}_1, \dots, \widetilde{P}_K]^T$ in the virtual problem, that is,

$$\mathbf{\Theta}_1^T \widetilde{\mathbf{p}}_{K-M} = \mathbf{p}^{\min}, \tag{22}$$

where $\widetilde{\mathbf{p}}_{K-M} = [\widetilde{P}_{M+1}, \dots, \widetilde{P}_K]^T$. Using (21) and (22), we have

$$\sum_{i=M+1}^{K} P_i = \mathbf{1}^T \mathbf{\Theta}_1^{-1} \mathbf{p}^{\min} = \mathbf{1}^T \left(\mathbf{\Theta}_1^T\right)^{-1} \mathbf{p}^{\min} = \sum_{i=M+1}^{K} \widetilde{P}_i. \tag{23}$$

That is, the total transmit powers for the non-data-bearing transmissions in the two problems are the same. Hence, given the same total transmit power P_T, the total transmit powers for the data-bearing transmissions are also the same in the two problems, that is,

$$\sum_{j=1}^{M} P_j = P_T - \sum_{i=M+1}^{K} P_i = P_T - \sum_{i=M+1}^{K} \widetilde{P}_i = \sum_{j=1}^{M} \widetilde{P}_j. \qquad (24)$$

Given the same total power of data-bearing transmissions, it has been proven in [19] that the two problems have the same achievable SINR region. □

A direct consequence of Lemma 1 is that problem (**A**) can be solved by iteratively optimizing the powers and the beamformers using the dual problems. In particular, replacing γ_i in problem (**C1**) by $\widetilde{\gamma}_i$ in (19), we have the virtual beamformer optimization problem

$$(\mathbf{C2}) \quad \mathbf{u}_i^* = \arg\max_{\mathbf{u}_i} \widetilde{\gamma}_i(\mathbf{u}_i) = \arg\max_{\mathbf{u}_i} \frac{\mathbf{u}_i^H \widetilde{\mathbf{R}}_i \mathbf{u}_i}{\mathbf{u}_i^H \mathbf{Q}_i \mathbf{u}_i}, \quad 1 \le i \le M, \qquad (25)$$

where $\widetilde{\mathbf{R}}_i := P_i \widetilde{\mathbf{\Omega}}_i$ and $\mathbf{Q}_i := \sum_{j=1}^{M} P_j \mathbf{\Delta}_j + \sum_{k=M+1}^{K} P_k \mathbf{\Omega}_k + \eta \mathbf{I}$. In problem (**C2**), each $\widetilde{\gamma}_i$ only depends on its own beamformer $\mathbf{u}_i$, and thus it is relatively easy to solve. The optimal beamformer $\mathbf{u}_i^*$ to problem (**C2**) is given by the dominant generalized eigenvector of the matrix pair $\{\widetilde{\mathbf{R}}_i, \mathbf{Q}_i\}$, $1 \le i \le M$ [19]. Moreover, for the non-data-bearing transmissions, the beamformer optimization problem is formulated as the receive power maximization:

$$\begin{aligned}(\mathbf{C3}) \quad \mathbf{u}_j^* &= \arg\max_{\mathbf{u}_j} P_{D_j} \\ &= \arg\max_{\mathbf{u}_j} \mathbf{u}_j^H (P_j \mathbf{\Omega}_j) \mathbf{u}_j, \quad M+1 \le j \le K.\end{aligned} \qquad (26)$$

Then the optimal solution to problem (**C3**) is given by the eigenvector corresponding to the largest eigenvalue of the matrix $\{P_j \mathbf{\Omega}_j\}$.

4.2. Joint power and beamformer optimization algorithm

In Sections 3.2 and 4.1, the power optimization algorithm under a given $\mathbf{U}$ and the beamformer optimization algorithm under a given $\mathbf{p}$ are developed, respectively. Then the algorithm for solving problem (**A**) (see Algorithm 2) is to iteratively optimize $\mathbf{p}$ using Algorithm 1 and optimize $\mathbf{U}$ using the algorithm in Section 4.1 until reaching convergence.

Furthermore, the convergence of Algorithm 2 is revealed in the following theorem.

Theorem 2. *The sequence* $\{C(\mathbf{U}(n), \mathbf{p}(n))\}$ *generated in Algorithm 2 is a monotonically increasing one, if only the optimum has not been reached. It approximates the global optimal solution of problem (**A**).*

Proof. From (25), $\mathbf{u}_i(n+1) = \arg\max_{\mathbf{u}_i} \widetilde{\gamma}_i(\mathbf{u}_i, \mathbf{p}(n))$ for given $\mathbf{p}(n)$, $1 \le i \le M$, then

$$\min_{1\le i\le M} \frac{\widetilde{\gamma}_i(\mathbf{u}_i(n+1), \mathbf{p}(n))}{\gamma_i^*} \ge \min_{1\le i\le M} \frac{\widetilde{\gamma}_i(\mathbf{u}_i(n), \mathbf{p}(n))}{\gamma_i^*}. \qquad (27)$$

As revealed by Algorithm 1, the balanced SINR ratio $C(n) := C(\mathbf{U}(n), \mathbf{p}(n))$ for given $\mathbf{U}(n)$

$$\begin{aligned} C(n) &= \max_{\mathbf{p}} \min_{1\le i\le M} \frac{\widetilde{\gamma}_i(\mathbf{u}_i(n), \mathbf{p})}{\gamma_i^*} \\ &= \min_{1\le i\le M} \frac{\widetilde{\gamma}_i(\mathbf{u}_i(n), \mathbf{p}(n))}{\gamma_i^*} = \frac{\widetilde{\gamma}_i(\mathbf{u}_i(n), \mathbf{p}(n))}{\gamma_i^*}. \end{aligned} \qquad (28)$$

Using (27) and (28), we then have

$$\min_{1\le i\le M} \frac{\widetilde{\gamma}_i(\mathbf{u}_i(n+1), \mathbf{p}(n))}{\gamma_i^*} \ge C(n). \qquad (29)$$

Similarly, for the given $\mathbf{U}(n+1)$, $C(n+1) := C(\mathbf{U}(n+1), \mathbf{p}(n+1))$ satisfies

$$\begin{aligned} C(n+1) &= \frac{\widetilde{\gamma}_i(\mathbf{u}_i(n+1), \mathbf{p}(n+1))}{\gamma_i^*} \\ &\ge \min_{1\le i\le M} \frac{\widetilde{\gamma}_i(\mathbf{u}_i(n+1), \mathbf{p}(n))}{\gamma_i^*}. \end{aligned} \qquad (30)$$

It is shown from (29) and (30) that $C(n+1) \ge C(n)$, that is, the sequence $\{C(\mathbf{U}(n), \mathbf{p}(n))\}$ is a monotonically increasing one. Since the optimal solution to problem (**A**) is nonnegative and bounded, the monotonicity property implies the existence of a limited value as the global optimum $\lim_{n\to\infty} C(n)$, that is, $\{C(n)\}$ approximates the global optimal solution. □

4.3. Simulation results

Figure 4 shows the achievable region of SINR ratios for problem (**A**). Note that different from Figure 2 where only power optimization is considered, we treat joint power and beamformer optimization in Figure 4. The simulation conditions are the same as those in Figure 2 with $M = 4$. It is also worth noting that the definition of $C(P_T, \mathbf{U})$ in Figure 4 is the same as that in Figure 2, that is, $C(P_T, \mathbf{U}) := C(\mathbf{p}^*, \mathbf{U}) = \max_{\mathbf{p}} \min_i (\gamma_i(\mathbf{p}, \mathbf{U})/\gamma_i^*)$. The quantities with index n denotes those in the nth iteration in the joint power and beamformer optimization algorithm (Algorithm 2), for example, $\mathbf{U}(n)$ denotes the optimal beamforming matrix in the nth iteration. It is seen from Figure 4 that $C(P_T, \mathbf{U}(n))$ is increasing as n increases. In particular, it is seen that the lowest curve $(C(P_T, \mathbf{U}(1)))$ corresponds to the case of $M = 4$ in Figure 2, which always falls in the infeasible region. Moreover, when $P_T/\eta \ge P_{T,0}/\eta = 10$, as n increases, $C(P_T, \mathbf{U}(n))$ is successively increasing such that the following points $C(P_T, \mathbf{U}(2))$ and $C(P_T, \mathbf{U}(3))$ fall in the feasible region. This demonstrates that the optimization of beamformers can significantly improve the system performance.

Figure 5 shows the convergence of Algorithm 2. The simulation conditions are the same as those in Figure 4. In particular, $C(n)$ denotes the balanced SINR ratio after both power and beamformer optimization in the nth

1: $n \Leftarrow 0$; $\mathbf{p}(n) = [0, \ldots, 0]^T = \mathbf{0}_K$; do the following iterative steps.
2: $n \Leftarrow n+1$; $\mathbf{u}_i(n) \Leftarrow \mathbf{v}_{\max}\{\widetilde{\mathbf{R}}_i, \mathbf{Q}(\mathbf{p}(n-1))\}$, $1 \le i \le M$; $\mathbf{u}_j(n) \Leftarrow \mathbf{v}_{\max}\{P_j(n-1)\mathbf{\Omega}_j\}$, $M+1 \le j \le K$; $\mathbf{u}_i(n) \Leftarrow \mathbf{u}_i(n)/\|\mathbf{u}_i(n)\|_2$, $1 \le i \le K$.
3: Calculate $\mathbf{p}(n)$ for the given $\mathbf{U}(n)$ using Algorithm 1, where (18) is replaced by (20).
4: If $C(\mathbf{p}(n), \mathbf{U}(n)) - C(\mathbf{p}(n-1), \mathbf{U}(n-1)) < \epsilon$, then stop; otherwise, go back to step (2).

Algorithm 2: Joint power and beamforming optimization algorithm.

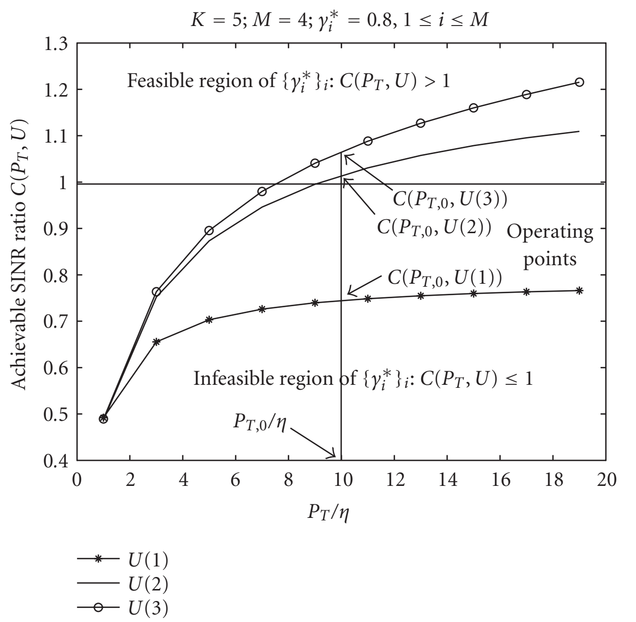

Figure 4: Feasible region of problem (**A**): $K = 5$; $M = 4$; $\gamma_i^* = 0.8$, $1 \le i \le M$; $P_T/\eta = 10$.

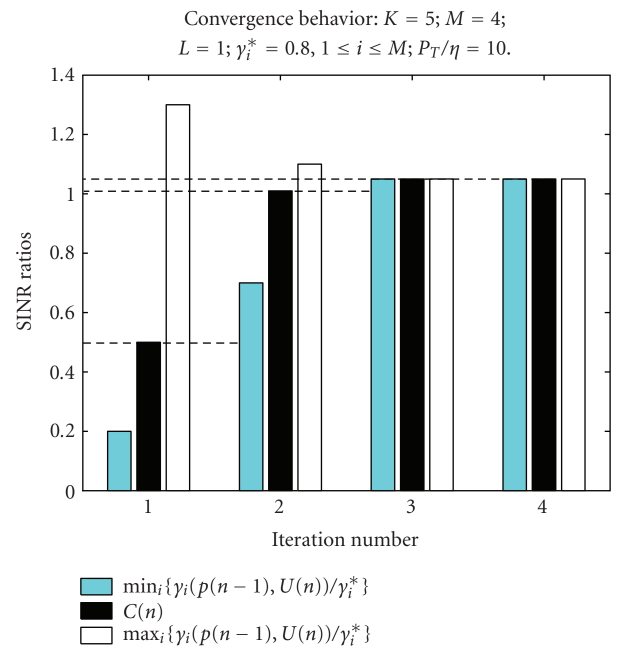

Figure 5: The convergence performance of the iterative joint power and beamformer algorithm (Algorithm 2): $K = 5$; $M = 4$; $\gamma_i^* = 0.8$, $1 \le i \le M$; $P_T/\eta = 10$.

iteration, that is, $C(n) := C(P_T, \mathbf{U}(n)) = C(\mathbf{p}(n), \mathbf{U}(n)) = \max_{\mathbf{p}} \min_i(\gamma_i(\mathbf{p}, \mathbf{U}(n))/\gamma_i^*)$; the SINR ratios after beamformer optimization and before power optimization in the nth iteration are denoted as $\{\gamma_i(\mathbf{p}(n-1), \mathbf{U}(n))/\gamma_i^*\}_i$. Note that without power optimization in each iteration, $\{\gamma_i(\mathbf{p}(n-1), \mathbf{U}(n))/\gamma_i^*\}_i$ are not necessarily balanced. Then $\min_i(\gamma_i(\mathbf{p}(n-1), \mathbf{U}(n))/\gamma_i^*) \le C(n) \le \max_i(\gamma_i(\mathbf{p}(n-1), \mathbf{U}(n))/\gamma_i^*)$ in each iteration n. It is seen from Figure 5 that the convergence is achieved until the SINR ratios of all transmissions are balanced, that is, $\min_i \gamma_i(\mathbf{p}(n-1), \mathbf{U}(n)) = \max_i \gamma_i(\mathbf{p}(n-1), \mathbf{U}(n))$. Moreover, it is seen from Figure 5 that the convergence can be quickly achieved within only a few iterations.

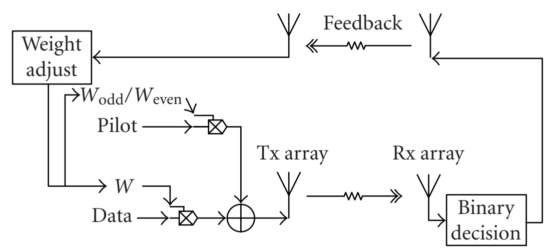

Figure 6: Subspace tracking scheme with binary feedback in multiple-antenna systems.

5. SUBSPACE TRACKING FOR COOPERATIVE BEAMFORMING

In Sections 3 and 4, we assume perfect CSI when optimizing the powers and the beamformers. In practical systems, however, only estimated CSI is available. In particular, in FDD systems, CSI has to be estimated at the destination cluster, and then fed back to the source cluster, namely, *forward estimation and feedback*. In TDD systems, CSI can be estimated either at the source cluster or at the destination cluster, and in the latter case, CSI estimates have to be further fed back to the source cluster, namely, *backward estimation*. Moreover, the data rate of the feedback channel is typically very low in practical systems. Hence, in this section, we propose to employ a simple subspace tracking scheme with only binary feedback to track channel variations [21, 22]. Note that we assume perfect feedback channels, which is reasonable because only binary feedback is required.

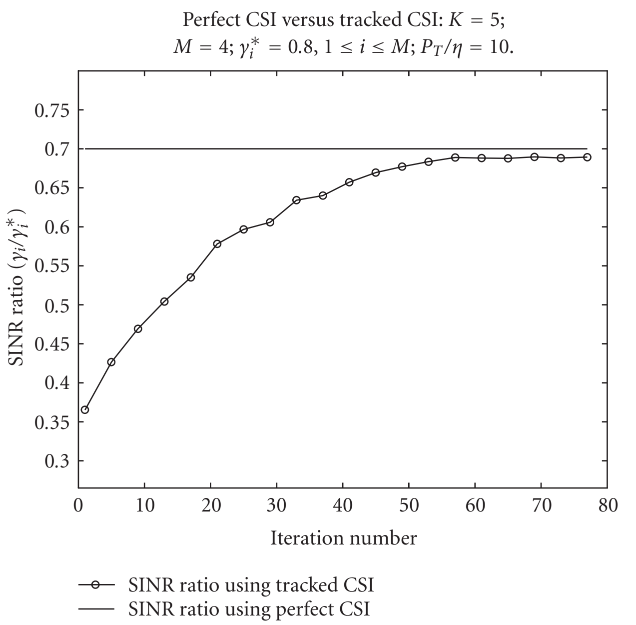

FIGURE 7: The performance of the subspace tracking based approach (Algorithm 3): the perfect CSI case versus the tracked CSI case; $K = 5$; $M = 4$; $\gamma_i^* = 0.8$, $1 \le i \le M$; $P_T/\eta = 10$.

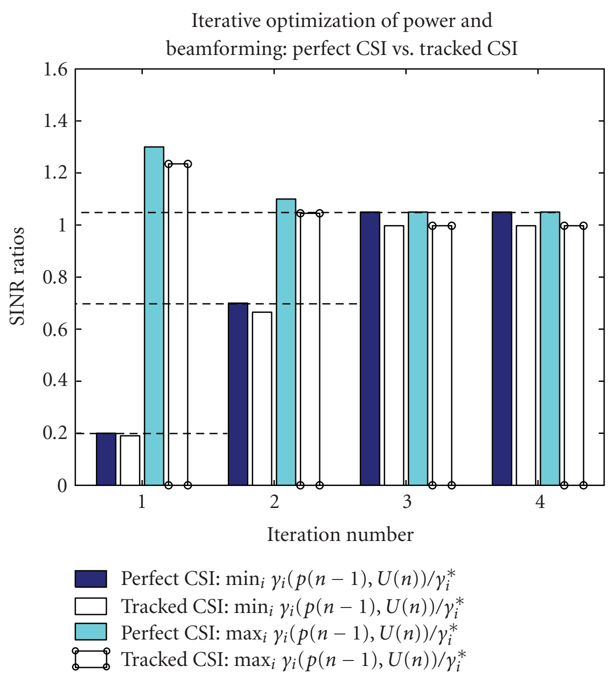

FIGURE 8: The maximum achievable SINR ratios: the perfect CSI case versus the tracked CSI case; $K = 5$; $M = 4$; $\gamma_i^* = 0.8$, $1 \le i \le M$; $P_T/\eta = 10$.

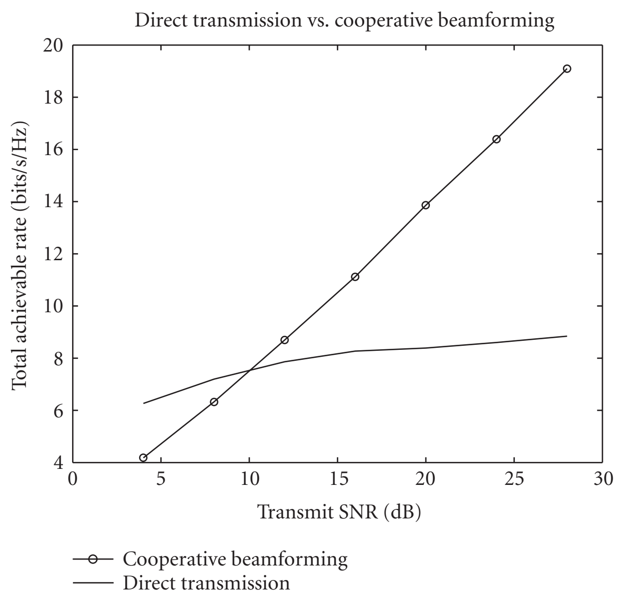

FIGURE 9: The comparison between cooperative multiple beamforming and direct transmission: $K = 4$ and $M = 2$.

5.1. *Beamformer optimization via subspace tracking*

Figure 6 shows the diagram of the subspace tracking scheme with binary feedback for multiple-antenna systems [21, 22]. Note that the source nodes in Figure 1 cooperatively form a virtual antenna array, and also, as we addressed in Sections 2 and 3.1, cooperative multiple beamforming resembles the multiuser beamforming in multiple-antenna systems. Therefore, in Figure 6, we adopt the multiple-antenna system diagram as a simplified illustration to show the subspace tracking-based scheme for the cooperative multiple beamforming system. In particular, the transmitter modulates the signals with two different but related weights ($\mathbf{u}_{i,\mathrm{e}}$ and $\mathbf{u}_{i,\mathrm{o}}$) in two consecutive time slots, even and odd time slots, respectively. Then the receiver side evaluates the two different transmit weights, and generates a binary feedback $\mathrm{sign}(T_i)$ which indicates the preferred transmit weight. For problem (**C2**) in (25), T_i is defined as the metric to maximize the receive SINR $\widetilde{\gamma}_i(\mathbf{u}_i)$ under a given power set

$$T_i := \widetilde{\gamma}_i(\mathbf{u}_{i,\mathrm{o}}) - \widetilde{\gamma}_i(\mathbf{u}_{i,\mathrm{e}}) = \frac{\mathbf{u}_{i,\mathrm{o}}^H \widetilde{\mathbf{R}}_i \mathbf{u}_{i,\mathrm{o}}}{\mathbf{u}_{i,\mathrm{o}}^H \mathbf{Q}_i \mathbf{u}_{i,\mathrm{o}}} - \frac{\mathbf{u}_{i,\mathrm{e}}^H \widetilde{\mathbf{R}}_i \mathbf{u}_{i,\mathrm{e}}}{\mathbf{u}_{i,\mathrm{e}}^H \mathbf{Q}_i \mathbf{u}_{i,\mathrm{e}}}, \quad 1 \le i \le M. \tag{31}$$

Similarly, for problem (**C3**) in (26), T_j is defined to maximize the receive power $P_{D_j}(\mathbf{u}_j)$:

$$T_j := \mathbf{u}_{j,\mathrm{even}}^H (P_j \mathbf{\Omega}_j) \mathbf{u}_{j,\mathrm{even}} - \mathbf{u}_{j,\mathrm{odd}}^H (P_j \mathbf{\Omega}_j) \mathbf{u}_{j,\mathrm{odd}}, \quad M+1 \le j \le K. \tag{32}$$

With the aid of such a binary feedback $\mathrm{sign}(T_i)$, the transmitter can iteratively adjust the transmit weights to make the transmissions more adaptive to the channels [21, 22]. Such a subspace tracking-based approach is summarized in Algorithm 3.

To compute T_i at the estimation end, pilot signals and certain cooperations are necessary. For instance, in the *forward estimation and feedback* scheme, the pilot signals ($\widetilde{\mathbf{s}}_i$) of different nodes at the source cluster are successively transmitted. That is, only $\widetilde{\mathbf{s}}_1$ is transmitted during the first time slot,

1: Given the adaptation rate β, the test perturbation vector $\boldsymbol{\mu}$, and the initial base weight $\mathbf{u}_{i,\mathrm{b}}$, $1 \le i \le M$, do the following iterative steps.
2: $\mathbf{u}_{i,\mathrm{e}} = \mathbf{u}_{i,\mathrm{b}} + \beta\|\mathbf{u}_{i,\mathrm{b}}\|\boldsymbol{\mu}$ and $\mathbf{u}_{i,\mathrm{o}} = \mathbf{u}_{i,\mathrm{b}} - \beta\|\mathbf{u}_{i,\mathrm{b}}\|\boldsymbol{\mu}$, $1 \le i \le M$.
3: Calculate T_i using (31) and (32).
4: If $\mathrm{sign}(T_i) = 1$, $\mathbf{u}_{i,\mathrm{b}} \Leftarrow \mathbf{u}_{i,\mathrm{o}}$; otherwise, $\mathbf{u}_{i,\mathrm{b}} \Leftarrow \mathbf{u}_{i,\mathrm{e}}$, $1 \le i \le M$.
5: Perform Gram-Schmidt orthogonalization on $\mathbf{u}_{i,\mathrm{b}}$, $1 \le i \le M$.

Algorithm 3: Subspace tracking algorithm for beamformer optimization.

and then, only $\widetilde{\mathbf{s}}_2$ is transmitted during the second time slot, and so on. Correspondingly, at the destination cluster, the receive powers at D_j $(1 \le j \le K)$ are simply measured during the successive time slots. After some local information sharing within the destination cluster, each node can then calculate its T_j using (31), and $\mathbf{u}_{i,\mathrm{base}}$ in Algorithm 3 will converge to the optimal $\mathbf{u}_i^* = \mathbf{v}_{\max}\{\widetilde{\mathbf{R}}_i, \mathbf{Q}_i\}$ for problem (**C2**) [21, 22]. Similar pilot signals and cooperations can be employed in the *backward estimation* scheme, and T_j in (32) can also be calculated using the local measurements in the source cluster.

Remark 4. In the above implementation of the subspace tracking-based algorithm (Algorithm 3), we assume that the local measurements can be perfectly shared at the estimation end, for example, the destination cluster in the *forward estimation and feedback* scheme and the source cluster in the *backward estimation* scheme.

5.2. Power optimization scheme

As mentioned in Sections 3 and 4, the optimal power vector $\mathbf{p}(n)$ for a given $\mathbf{U}(n)$ can be obtained by solving (18) or (20). According to the definition of $\boldsymbol{\Upsilon}$ in (18), it is necessary to know $\widehat{h}_{i,j} := \mathbf{h}_i^T\mathbf{u}_j$ $(1 \le i \le K$ and $1 \le j \le K)$ to calculate $\mathbf{p}(n)$ in step (4) of Algorithm 2. It has been pointed out by [21, 22] that the equivalent channel estimates $\widehat{h}_{i,j}$ in the system shown by Figure 6 can be simply obtained by the mean of the even and the odd time slot channel estimates, that is, $\widehat{h}_{i,j} = \widehat{\mathbf{h}}_i^T u_{i,\mathrm{b}} = (\widehat{\mathbf{h}}_i^T\mathbf{u}_{j,\mathrm{e}} + \widehat{\mathbf{h}}_i^T\mathbf{u}_{j,\mathrm{o}})/2$. In the *forward estimation and feedback* scheme, $\widehat{h}_{i,j}$ $(1 \le i, j \le K)$ are obtained at the destination cluster, and the optimal power vector $\mathbf{p}(n)$ can be calculated using (20) at the destination cluster. Then $\mathbf{p}(n)$ will be fed back to the source cluster. In the *backward estimation* scheme, both $\widehat{h}_{i,j}$ $(1 \le i, j \le K)$ and $\mathbf{p}(n)$ can be directly extracted at the source cluster. Similarly as the *forward estimation and feedback* scheme, $\mathbf{p}(n)$ will also be sent to the destination cluster.

5.3. Simulation results

Figure 7 shows the performance of the subspace tracking based approach (Algorithm 3). The simulation conditions are the same as those in Figure 5. In particular, Figure 7 demonstrates the achievable SINR of one destination node within one iteration of Algorithm 2. That is, for the given $\mathbf{p}(n)$, $\gamma_i(\mathbf{p}(n), \mathbf{u}_i(n+1)) = \max_{\mathbf{u}_i}\gamma_i(\mathbf{p}(n), \mathbf{u}_i)$. It is seen from Figure 7 that $\gamma_i(\mathbf{p}(n), \mathbf{u}_i(n+1))/\gamma_i^*$ where $\mathbf{u}_i(n+1)$ is tracked using Algorithm 3 can asymptotically approximate the optimal SINR where $\mathbf{u}_i(n+1)$ is calculated assuming perfectly CSI. Furthermore, Figure 8 shows the performance comparison between the joint power and beamformer optimization (Algorithm 2) based on the tracked CSI and that based on perfect CSI. Also, the conditions here are the same as those in Figure 5. It is seen from Figure 8 that when solving problem (**A**) using Algorithm 2, the achievable SINR ratio obtained using the tracked CSI can approximate those calculated assuming the perfect CSI. Therefore, we conclude from Figures 7 and 8 that Algorithm 3 is an efficient scheme to realize the cooperative beamforming in practice.

Figure 9 shows the comparison between the proposed cooperative multiple beamforming scheme and the conventional direct transmission scheme. In Figure 9, $K = 4$; $M = 2$; $K = 4$; $\mathbf{p}^{\min} = [1, \ldots, 1]^T$. The direct transmission is achieved by simultaneously transmitting M independent links between the source and the destination clusters. Here, we compare the total throughput of the system. Note that the transmit power and the bandwidth are both normalized to guarantee a fair comparison. In particular, given $\|\mathbf{p}\|_1 = P_T$, the rate of each cooperative transmission $\mathbf{s}_i$ is given by $r_i = \log(1 + \mathrm{SINR}_i(\mathbf{p}, \mathbf{U}))$; in contrast, the rate of each direct transmit link is given by $r_i = (M+1)\log(1 + \mathrm{SINR}_i(2\mathbf{p}))$. Note that the gains $M+1$ and 2 in the direction transmission come from the bandwidth loss in the cooperative transmission due to the local broadcasting in the source cluster and the extra local broadcasting power required in the cooperative transmission, respectively. Also note that we here assume equal transmit power for each link in the direction transmission scheme. It is seen from Figure 8 that in the low SNR region, the direct transmission outperforms the proposed cooperative multiple beamforming scheme; in contrast, in the high SNR region which it is interference-dominant, the proposed cooperative multiple beamforming scheme evidently outperforms the direct transmission scheme, because the interferences among multiple concurrent transmissions can be effectively suppressed at the receivers.

6. CONCLUSIONS

In this paper, we have analyzed the problem of cooperative multiple beamforming in wireless ad hoc networks. We have proposed the iterative power allocation algorithm for given beamformers, and studied its convergence. Then we have developed the iterative joint power and beamformer optimization algorithm to solve the problem based on the duality analysis. Moreover, we have proposed to employ the simple subspace tracking-based algorithm with only binary feedback to practically track the channel variation in the system where only bandwidth limited feedback channels are available. We further presented the cooperative scheme to implement such a subspace tracking algorithm. Simulation results have been demonstrated to verify the performances of the proposed algorithms.

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