

## Research Article

# Ring-Based Optimal-Level Distributed Wavelet Transform with Arbitrary Filter Length for Wireless Sensor Networks

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We propose an optimal-level distributed transform for wavelet-based spatiotemporal data compression in wireless sensor networks. Although distributed wavelet processing can efficiently decrease the amount of sensory data, it introduces additional communication overhead as the sensory data needs to be exchanged in order to calculate the wavelet coefficients. This tradeoff is explored in this paper with the optimal transforming level of wavelet transform. By employing a ring topology, our scheme is capable of supporting a broad scope of wavelets rather than specific ones, and the “border effect” generally encountered by wavelet-based schemes is also eliminated naturally. Furthermore, the scheme can simultaneously explore the spatial and temporal correlations among the sensory data. For data compression in wireless sensor networks, in addition to minimizing energy and consumption, it is also important to consider the delay and the quality of reconstructed sensory data, which is measured by the ratio of signal to noise (PSNR). We capture this with  $\text{energy} \times \text{delay} / \text{PSNR}$  metric and using it to evaluate the performance of the proposed scheme. Theoretically and experimentally, we conclude that the proposed algorithm can effectively explore the spatial and temporal correlation in the sensory data and provide significant reduction in energy and delay cost while still preserving high PSNR compared to other schemes.

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## 1. INTRODUCTION

Edging toward real world deployments, wireless sensor networks have revealed vast potentials in a plethora of applications including battle field monitoring, environmental exploration, and precision agriculture [1, 2]. Owing to the severe resource constraints such as memory space, computation power, and communication bandwidth, gathering all the raw, original sensory data is not often feasible in wireless sensor networks. Motivated thereby, extensive research efforts have been focusing on wavelet data compression in wireless sensor networks, with a goal of data amount reduction and hence energy conservation. For example, the WISDEN system [3] is designed for structural monitoring. In this system, wavelet compression is first performed in a single sensor node and the wavelet coefficients are then sent for further processing at a central location. Aiming at time-series sampled by a single sensor node, RACE [4] proposes a rate adaptive Haar wavelet compression algorithm. The support of Haar wavelet is 1 and its structure is simple. As a result,

the algorithm can be executed efficiently. However, the above wavelet-based approaches do not exploit the fact that data originated from physically proximate sensors are often highly correlated. Consequently, energy can be wasted due to the transmission of redundant data. Dimensions [5, 6] propose a hierarchical routing scheme with its wavRoute protocol. This scheme exploits the temporal data redundancy at the bottom level of the routing hierarchy firstly, and then performs spatial data reduction in the middle. Still, there exists the transmission of spatially redundant data from the bottom to the middle of the hierarchy.

On the other hand, a series of papers have pioneered in wavelet-based distributed compression [7–10] recently. In [7, 8, 10], distributed wavelet transforms (WT) are implemented based on a one-dimensional chain network model. Although these schemes are simple to implement, they have ignored “border effect” of wavelet transform. Even for large scale sensor networks, border effect still can have significant impact on the quality of reconstructed sensory data. Indeed, the chain network model employed [7, 8, 10] exaggerates

the “border effect” particularly. Transforming level is another important property of wavelet transform. Although higher compression efficiency can be obtained along with increasing transforming levels, additional energy and delay cost are introduced as more sensory data need to be exchanged to perform the transform. Therefore, it is important to look for the optimal transforming levels in conjunction with proper topology. Although Haar wavelet-based adaptive level multiresolution representation is proposed in [10], the scheme is difficult to generalize to wavelet function with arbitrary length support. Moreover, the scheme has ignored network delay, which is crucial for certain applications. Performance evaluation of distributed WT algorithms is also an interesting problem. In [9], irregular WT is studied and its performance is evaluated using “mean square error” (MSE) and energy metrics separately. On the contrary, in [7–10], the metrics of evaluation involve many aspects, such as energy consumption, reconstruction quality metric, delay, and so on. However, they only use those metrics unilaterally without considering their relation. This, in turn, has also limited their performance and application scope.

Motivated thereby, in this paper, we propose a ring topology-based, optimal-level distributed transform for wavelet functions, whose support length can be arbitrary. Our scheme simultaneously exploits the spatial and temporal correlation residing in the sensor data within clusters. The ring model will naturally eliminate the “border effects” encountered by WT and hence further strengthen its support to general wavelets. Furthermore, our scheme is capable of accommodating a broad range of wavelets which can be designated by different applications. Moreover, we propose a scheme of optimal level of WT, which can explore the tradeoff between the benefit of distributed WT and the corresponding overhead. We evaluate the performance of data compression for sensor networks with *energy*  $\times$  *delay*/*PSNR* metric, which gives enough consideration in the tradeoff of energy consumption, network delay, and the quality of reconstructed sensory data. Theoretically and experimentally, we analyze the performance of our proposed algorithm and perform comparison with other schemes.

The remainder of this paper is organized as follows. In Section 2, we first study the border effect in sensor networks, then detail the ring model and describe the optimal WT thereon. In Section 3, we present the performance evaluation model for data compression in sensor networks, and then analyze the performance of the proposed framework. Experimental study is presented in Section 4 and we conclude in Section 5.

## 2. SPATIAL-TEMPORAL WAVELET COMPRESSION

In this section, we first examine the potential impact of border effect on sensory data reconstruction in sensor networks, and then present the network model and the construction of the virtual ring topology to eliminate the border effect. Subsequently, the optimal transforming-level-based algorithm for compressing spatial and temporal correlated data is detailed.

### 2.1. Border effect in sensor networks

We assume that sensory data collected are stored in each sensor node in a distributed fashion. While these data can be compressed employing wavelet based on one-dimensional network model [2], border effect will induce errors when reconstructing the sensing field. If the reconstructed data are different from the original data, the results are considered to be distortive.

For general wavelet functions with arbitrary supports, let their lowpass and corresponding highpass analysis filter be

$$\begin{aligned} L_n, -i_1 \leq n \leq j_1, \quad i_1 \geq 0, j_1 > 0, \\ H_n, -i_2 \leq n \leq j_2, \quad i_2 \geq 0, j_2 > 0. \end{aligned} \quad (1)$$

Let

$$L = \max((i_1 + j_1 + 1), (i_2 + j_2 + 1)), \quad (2)$$

$$I = \max(i_1, i_2), \quad (3)$$

$$J = \max(j_1, j_2). \quad (4)$$

As a consequence of border effect, we have the following theorem.

**Theorem 1.** *Performing  $K$ -level distributed WT on sensory data stored in  $N$  sensor nodes, if  $2^K(\lceil(2^K - 1)I/2^K\rceil + \lceil(2^K - 1)(J - 1)/2^K\rceil) + (2^K - 1)(I + J - 1) < N$ , where  $\lceil \cdot \rceil$  is a operator of bounding, then the sensor nodes whose reconstructed data are distortive amount to  $2^K(\lceil(2^K - 1)I/2^K\rceil + \lceil(2^K - 1)(J - 1)/2^K\rceil) + (2^K - 1)(I + J - 1)$ , otherwise, the reconstructed sensory data in all  $N$  sensor nodes will be distortive.*

*Proof.* The sensory data stored in  $N$  sensor nodes can be regarded as a one-dimensional array with  $N$  elements. Using wavelet function as defined by (1)–(4), we perform  $K$ -level WT on the one-dimensional array. According to the decomposition steps of Mallat algorithm [11]

$$\begin{aligned} x_k^{i+1} &= \sum_n L_{n-2k} x_n^{(i)}, \\ d_k^{i+1} &= \sum_n H_{n-2k} x_n^{(i)}, \end{aligned} \quad (5)$$

where  $i \geq 0$ ,  $x_k^{i+1}$  and  $d_k^{i+1}$  is the  $k$ th approximation and detail coefficients in the  $(i+1)$ th level WT, respectively. If the border of the array is not extended, the distortive detail coefficients in the  $K$ th level WT will amount to

$$\left\lceil \frac{(2^K - 1)i_1}{2^K} \right\rceil + \left\lceil \frac{(2^K - 1)(j_1 - 1)}{2^K} \right\rceil. \quad (6)$$

The distortive approximation coefficients correspondingly are

$$\left\lceil \frac{(2^K - 1)i_2}{2^K} \right\rceil + \left\lceil \frac{(2^K - 1)(j_2 - 1)}{2^K} \right\rceil. \quad (7)$$

According to the reconstruction steps of Mallat algorithm,

$$x_n^i = \sum_k L_{n-2k} x_k^{i+1} + \sum_k H_{n-2k} d_k^{i+1}. \quad (8)$$

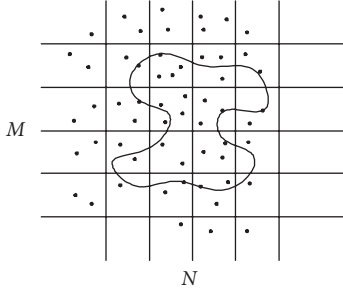


FIGURE 1: Ring topology based on virtual grid.

Along with (3) and (4), the total number of the distortive sensory data becomes

$$\text{Num} = 2^K \left( \left\lceil \frac{(2^K - 1)I}{2^K} \right\rceil + \left\lceil \frac{(2^K - 1)(J - 1)}{2^K} \right\rceil \right) + (2^K - 1)(I + J - 1). \quad (9)$$

This shows that the sensory data reconstructed are distortive as compared to those originally stored in the sensor nodes. For simplicity, we consider wavelet function to have the same analysis and synthetic filters. Obviously, if  $\text{Num} \geq N$ , all  $N$  reconstructed sensory data are distortive.  $\square$

Here, we give a simple example to illustrate the border effect. Assume that the number of nodes in a network is 400, and a 3-level WT on the sensory data employing Daubechies 9/7 wavelet is performed. There will be 105 nodes whose reconstructed data are distortive according to Theorem 1. This accounts to around a quarter of all the sensor nodes. From Theorem 1 and this example, we conclude that the border effect can potentially have significant impact on the reconstruction of sensory data.

## 2.2. Ring topology based on virtual grid

Below, we describe a ring topology based on virtual grid. As we will illustrate later, ring-topology-based WT can eliminate border effect and can fully explore the spatial correlation among sensory data.

### 2.2.1. Virtual grid and data correlation model

We assume that the sensor network is divided into different clusters, each of which is controlled by a cluster head [12]. Our focus is given to energy-efficient gathering of the sensory data from various cluster members to the cluster head. Routing the data from the cluster head to the sink is out of the scope of this paper although it may benefit from the compression algorithm presented in this paper. We assume that in a cluster, nodes are distributed in a virtual grid as illustrated in Figure 1. The distance among nodes can be estimated according to the distance among the corresponding grid cells. It can also be calculated according to the factual positions of the nodes. The division of cells in a cluster relies on the network topology and node density. We assume that one cell at

least contains one node and in each cell, one node is selected as the reporting node (for reporting the data to the cluster head).

Without confusion, we will simply use node to refer to this reporting sensor. We remark that this model is neither restrictive nor unrealistic. In the worst case, a single node can logically reside in one grid cell and can be required to report its data corresponding to every query or during every specified interval.

There exists a certain correlation for the sensory data stored in each node, which can be described using a correlation model. Let correlation coefficient  $\rho$  represent the data correlation and let  $r_s$  represent correlation scope. In correlation model,  $\rho$  will be zero if the distance between two nodes exceeds  $r_s$ . If the distance is  $d$  ( $d < r_s$ ), then  $\rho = 1 - d/r_s$ .

### 2.2.2. Ring topology based on virtual grid

The key for our construction is that we form a ring topology among the reporting sensor nodes, as illustrated in Figure 1. To do this, we initially select a node randomly as the ring head, and then determine a neighboring node as its next node. This neighbor-selection procedure will be repeated until the ring topology is completed. In order to maximize the correlation among neighboring nodes and hence the effect of compression, the ring can be computed in a centralized manner by the cluster head and broadcast to all nodes. Notice that multiple rings may be available due to node density.

In this ring topology, neighboring nodes belong to spatial adjacent grid cells. A node on the ring receives data from one of its neighbors, fuses the data with its own, and further forward the results to the other neighbor. As the nodes are relaying the sensory data, WT will be executed and certain wavelet coefficients will be actually stored locally and some others will be forwarded. Indeed, nodes in a particular grid cell can alternatively participate in the ring and hence the data-gathering procedure. This way, energy consumption can be more evenly distributed among the nodes and thus extend the network lifetime. Readers are referred to [13] for approaches of scheduling nodes within one grid, for example, power on and off, for this purpose.

Given the ring topology, in each data gathering round, a node will be chosen as the ‘‘head’’ of the ring and the nodes will be indexed accordingly as  $s_0, s_1, \dots, s_i, \dots, s_{N-1}$ , where  $N$  is the number of nodes on the ring. In addition, we assume that sensor  $i$  stores data  $c_{ji}$ ,  $j = 0, 1, \dots, M - 1$ , where  $j$  is the temporal index and  $c_{ji}$  represents the sensory data of sensor  $i$  at time index  $j$ . Evidently, dependent on  $M$ , each sensor will window out history data. Accordingly, we can arrange the sensory data on the ring according to their spatial and temporal relationship to a matrix  $C^0 = \{c_{ji}\}$ ,  $0 \leq i < N$ ,  $0 \leq j < M$ , where column  $i$  represents the data of sensor node  $i$ . For ease of notation, we will use  $\vec{C}_i$  to denote column  $i$ . Notice that  $\vec{C}_0$  and  $\vec{C}_{N-1}$  are adjacent on the ring topology and hence will possess relatively higher correlation. As we will detail later, this unique feature of ring topology is in particular adapt to WT with arbitrary supports and can effectively help us eliminate the border effects of WT.

We remark that while extensive data-gathering structures have been studied in the literature, they are usually tree-based. Undeniably, the ring construction requires careful study in order to best benefit from its special properties. While considering this as our future work, we provide here a brief discussion. First of all, due to the procedure of distributed wavelet compression, it is desirable to have higher data correlation among neighboring nodes. This way data can be better compressed while being forwarded along the ring and hence energy can be saved. Often, this can be naturally satisfied by selecting physically proximate nodes to be neighbors on the ring. At the same time, the longer the ring, the more compression can possibly be achieved. However, with increasing the length of the ring, the number of wavelet coefficients will also increase which can in turn introduce additional calculation and storage cost. Additionally, network delay for data gathering will also increase as the ring length increases. Balancing the size of the ring and the number of the rings will require careful tradeoff among all the above-mentioned factors.

### 2.3. VGRT-based optimal level spatial-temporal wavelet transform

#### 2.3.1. Spatial-temporal wavelet transform

Our goal is to employ the WT for compressing sensory data on the ring so that it can be energy efficiently transmitted to the cluster head. The approach is to simultaneously exploit the temporal and spatial correlation among the nodes' data and reduce the redundancy thereby. As the data is represented by matrix  $C^0$ , the temporal (within a node) and spatial (among multiple nodes) correlations are then captured by the columns and rows, respectively. Correspondingly, in our design, we will first perform WT on each column and then perform WT on the rows. Furthermore, these column WT and row WT can be performed recursively to achieve a  $K$ -level WT. Notice that column WT is within a single node hence no communication is required although data will be buffered. On the contrary, the row WT is among the sensor nodes and hence requires additional communications.

Our first step is to perform transform on the columns of  $C^0$  to exploit temporal correlation. Let  $L_n$  and  $H_n$  be lowpass and highpass analysis filters, respectively, we have

$$\begin{aligned} c_{m,i}^{1,L} &= \sum_n L_{(n-2m)} \vec{C}_i(n), \\ c_{m,i}^{1,H} &= \sum_n L_{(n-2m)} \vec{C}_i(n), \quad 0 \leq m \leq N/2, \end{aligned} \quad (10)$$

where  $C_{m,i}^{1,L}$  represents the  $m$ th approximation wavelet coefficient in the  $i$ th column in the first level of the column WT,  $C_{m,i}^{1,H}$  is the corresponding detail wavelet coefficient, and  $\vec{C}_i(n)$  denotes the  $n$ th element of  $\vec{C}_i$ . Notice that this transform is performed within each node on its own sensory data and thus does not require any communication among the

nodes on the ring. Subsequently, we can realign the resultant wavelet coefficients and obtain matrix

$$C_1 = \begin{Bmatrix} c_{0,0}^{1,L} & c_{0,1}^{1,L} & \cdots & c_{0,N-1}^{1,L} \\ \vdots & \vdots & \ddots & \vdots \\ c_{M/2-1,0}^{1,L} & c_{M/2-1,1}^{1,L} & \cdots & c_{M/2-1,N-1}^{1,L} \\ c_{0,0}^{1,H} & c_{0,1}^{1,H} & \cdots & c_{0,N-1}^{1,H} \\ \vdots & \vdots & \ddots & \vdots \\ c_{M/2-1,0}^{1,H} & c_{M/2-1,1}^{1,H} & \cdots & c_{M/2-1,N-1}^{1,H} \end{Bmatrix}. \quad (11)$$

Given matrix  $C_1$ , our second step is to perform WT on its rows to explore the spatial correlation among the nodes. Note that the first and the last columns are adjacent on the ring topology, and this resembles a periodic extension to the signal. Towards this end, for general wavelets with arbitrary supports whose lowpass analysis filter is  $L_n$ ,  $-i_1 \leq n < j_1$  and highpass analysis filter is  $H_n$ ,  $-i_2 \leq n < j_2$ , where  $i_1, i_2, j_1, j_2 \geq 0$ , we analyze the different cases of the row transform based on whether  $j_1$  and  $j_2$  are even or odd.

*Case 1.* If  $j_1$  is even and  $j_2$  is odd, by performing WT on the rows in a similar way to the column WT, we obtain

$$C_2 = \begin{Bmatrix} c_{0,l_0}^{1,LL} & c_{0,h_0}^{1,LH} & \cdots & c_{0,l_{N/2-1}}^{1,LL} & c_{0,h_{N/2-1}}^{1,LH} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ c_{M/2-1,l_0}^{1,LL} & c_{M/2-1,h_0}^{1,LH} & \cdots & c_{M/2-1,l_{N/2-1}}^{1,LL} & c_{M/2-1,h_{N/2-1}}^{1,LH} \\ c_{0,l_0}^{1,HL} & c_{0,h_0}^{1,HH} & \cdots & c_{0,l_{N/2-1}}^{1,HL} & c_{0,h_{N/2-1}}^{1,HH} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ c_{M/2-1,l_0}^{1,HL} & c_{M/2-1,h_0}^{1,HH} & \cdots & c_{M/2-1,l_{N/2-1}}^{1,HL} & c_{M/2-1,h_{N/2-1}}^{1,HH} \end{Bmatrix}, \quad (12)$$

where  $l_i = ((N - j_1 + 2i)/2 \bmod N/2)$ ,  $h_i = ((N - j_2 + 2i + 1)/2 \bmod N/2)$ ,  $c_{m,n}^{1,LL}$  and  $c_{m,n}^{1,HL}$  represent the approximation coefficients in the first level of the row WT, and  $c_{m,n}^{1,LH}$  and  $c_{m,n}^{1,HH}$  represent the corresponding detail coefficients. We remark that for a node with index  $i$ , if  $i$  is even, the node stores coefficients  $c_{m,(N-j_1+i)/2 \bmod N/2}^{1,LL}$  and  $c_{m,(N-j_1+i)/2 \bmod N/2}^{1,HL}$ ; if  $i$  is odd, the node stores coefficients  $c_{m,(N-j_2+i)/2 \bmod N/2}^{1,LH}$  and  $c_{m,(N-j_2+i)/2 \bmod N/2}^{1,HH}$ ,  $0 \leq m \leq M/2 - 1$ . Notice that this transform is performed among the sensor nodes on the ring to harvest the spatial correlation and hence resultant wavelet coefficients cannot be realigned as in the column WT.

Based on the approximation coefficients in  $C_2$ , we can obtain matrix  $C^1$  as

$$C^1 = \begin{Bmatrix} c_{0,l_0}^{1,LL} & c_{0,l_1}^{1,LL} & \cdots & c_{0,l_{N/2-1}}^{1,LL} \\ c_{1,l_0}^{1,LL} & c_{1,l_1}^{1,LL} & \cdots & c_{1,l_{N/2-1}}^{1,LL} \\ \vdots & \vdots & \ddots & \vdots \\ c_{M/2-1,l_0}^{1,LL} & c_{M/2-1,l_1}^{1,LL} & \cdots & c_{M/2-1,l_{N/2-1}}^{1,LL} \end{Bmatrix}. \quad (13)$$

We can perform the second-level column and row WT on matrix  $C^1$  as those to matrix  $C^0$  and extend to the  $K$ th level spatiotemporal WT similarly.

Once the  $K$ -level WT is performed, the original data gathered by the nodes on the ring is transformed to the wavelet domain. Since the spatial and temporal correlations are exploited, we can represent the original data using fewer bits. In lossless compression, all the wavelet coefficients will be encoded and sent to the cluster head; in lossy compression, according to different application-specific requirements, the wavelet coefficients can selectively be encoded and sent to the cluster head by different nodes.

*Case 2.* If  $j_1$  and  $j_2$  are both odd, while we can perform the transform following similar procedure, the matrices will be significantly different. Due to space limitation, we omit them here but remark that those nodes whose indexes are odd will not store wavelet coefficients.

When we perform row WT, the first group of approximation coefficients is calculated using the data stored in the  $((N - i_1) \bmod N)$ th node to the  $(j_1 \bmod N)$ th node and are stored in the  $(j_1 \bmod N)$ th node. The corresponding detail coefficients are calculated using the data stored in the  $(N - i_2)$ th node to the  $(j_2 \bmod N)$ th node and are stored in the  $(j_2 \bmod N)$ th node. When  $j_1$  is odd and  $j_2$  is even, it will be similar to the first case, and when  $j_1$  and  $j_2$  are both even, it will be similar to the second case discussed above.  $i_1$  and  $i_2$  will not affect the distribution of wavelet coefficients.

### 2.3.2. Optimal transforming level of wavelet function

In this subsection, we will study how many transforming levels needed to be performed to obtain optimal network performance. We evaluate network performance with energy and delay.

From data compression's point of view, WT is desired only if the average number of encoding bits of wavelet coefficients can be reduced. Let  $B_{k-1}$  and  $B_k$  be the average encoding bits of wavelet coefficients in the  $(k-1)$ th and the  $k$ th level, respectively, then we have

$$B_{k-1} - B_k = f(\text{data, wavelet}). \quad (14)$$

Since most of energy consumption is data transmission and most of delay factor is in the transmission time for wireless sensor networks, we measure energy and delay in terms of the size of data being transmitted. We might as well denote  $e(\cdot)$  and  $d(\cdot)$  are energy and delay cost function respectively. Studying the optimal transforming level of spatial-temporal WT, we have the following theorem.

**Theorem 2.** Let  $E_{k,IN}$  and  $D_{k,IN}$  be the additional energy and delay cost by distributed spatial-temporal the  $k$ th-level WT, respectively,  $K_1 = \max(k : E_{k,IN} - e(B_{k-1} - B_k) \leq 0)$ ,  $K_2 = \max(k : D_{k,IN} - d(B_{k-1} - B_k) \leq 0)$ , then the optimal transforming level of spatial-temporal WT  $K$  is  $K = \min(K_1, K_2)$ .

*Proof.* The energy consumption and delay of sending  $(B_{k-1} - B_k)$  bits data is  $e(B_{k-1} - B_k)$  and  $d(B_{k-1} - B_k)$ , respectively. If the energy and delay cost generated by the  $k$ th level WT

are all less than or equal to that of the  $(k-1)$ th level WT, then the  $k$ th level WT would be performed. So we can easily obtain Theorem 2.  $\square$

The optimal transforming level of wavelet function can be calculated distributively by the nodes on the ring. Without loss of generality, we assume that the energy function  $e(\cdot)$  and delay function  $d(\cdot)$  have been loaded to nodes in advance. The energy and delay are calculated by nodes during they perform WT. Each node forwards the value of energy and delay while the data are sent to the next node to produce wavelet coefficients. So the node, which stores the last column wavelet coefficients, knows the total energy and delay cost in the corresponding transforming level, and thus it can decide if the next transforming level will be performed. If the decision is "YES," then the new level WT will be initiated by the node that stores the first column wavelet coefficients. The decision can be easily transmitted to the node thanks to the ring topology.

## 2.4. Discussion

In the above WT, the ring head can be alternated among different nodes when performing the data-gathering procedure. Consequently, the wavelet coefficients will be distributed to different nodes accordingly which in turn will balance the energy consumption within the cluster. Furthermore, neighboring nodes on the ring belong to spatial adjacent virtual grids, so the data gathered by the neighboring nodes are more likely spatially correlated. Because the calculation of approximation and detail wavelet coefficients are for neighboring nodes within a support length, performing WT based on the ring can make full use of spatial correlation to remove the data redundancy and hence reduce transmission cost.

More importantly, performing WT based on ring topology naturally eliminates the "border effect" problem inherent in WT. It is well known that general wavelet functions are defined on the real axis  $\mathfrak{R}$  while the signal is always limited in a finite region  $\mathfrak{K}$ . Therefore, the approximate space  $L^2(\mathfrak{R})$  will not match the signal space  $L^2(\mathfrak{K})$  which will result in the "border effect" and thus introduce errors during signal reconstruction. One of the general methods to deal with "border effect" is extending border. The ring topology resembles a periodic extension to the signal that naturally dissolves the "border effect."

Before going forward, we remark here that our scheme aims at traditional wavelet transform and hence is not directly applicable to the second-generation wavelet. Moreover, due to the strict requirement of the topology for data forwarding, the scheme lacks robustness in its current form. These are considered our future work.

## 3. PERFORMANCE EVALUATION

In this section, we analyze the energy consumption and delay of the proposed scheme, and then present a model to evaluate the data compression algorithms for wireless sensor networks.

### 3.1. Energy consumption and delay analysis

We now briefly analyze the total energy consumption and delay of the proposed scheme. For this purpose, we adopt the first-order radio model described in [12]. In this model, a radio dissipates  $E_{\text{elec}}$  amount of energy at the transmitter or receiver circuitry and  $\epsilon_{\text{amp}}$  amount of energy for transmit amplifier. Signal attenuation is modeled to proportional to  $d^2$  on the channel, where  $d$  denotes distance. For  $k$  bits data and a distance  $d$ , the transmission energy consumption  $E_{\text{Tx}}$  and reception energy consumption  $E_{\text{Rx}}$  can be calculated, respectively, as

$$\begin{aligned} E_{\text{Tx}}(k, d) &= E_{\text{Tx-elec}}(k) + E_{\text{Tx-amp}}(k, d), \\ E_{\text{Tx}}(k, d) &= E_{\text{elec}} \times k + \epsilon_{\text{amp}} \times k \times d^2, \\ E_{\text{Rx}}(k) &= E_{\text{Rx-elec}}(k) = E_{\text{elec}} \times k. \end{aligned} \quad (15)$$

We further assume that the sensor nodes can transmit simultaneously and neglect the processing and propagation delay. Let the transmission time of one data unit be one unit time. Let  $E_{\text{IN}}$  and  $D_{\text{IN}}$  represent the energy consumption and delay resulting from communication among the nodes within the cluster for performing the proposed WT. We can derive the following theorem.

**Theorem 3.** For general wavelets with arbitrary supports, let the lowpass analysis filter be  $L_n$ ,  $-i_1 \leq n < j_1$ , and let the high-pass analysis filter be  $H_n$ ,  $-i_2 \leq n < j_2$ , where  $i_1, i_2, j_1, j_2 \geq 0$ . For a  $K$ -level distributed spatiotemporal WT based on the ring topology proposed above, to gather the sensory data in a cluster of  $N$  node,

$$E_{\text{IN}} = E_{\text{WAV}} + E_{\text{OPT}}, \quad (16)$$

$$\begin{aligned} D_{\text{IN}} &= \sum_{n=1}^K \left( \max_{0 \leq l \leq N/2^n - 1} \left( \sum_{i=0}^{i_1+j_1-1} q_{\text{inl}}^L + B^L \right) \right. \\ &\quad + \max_{0 \leq l \leq N/2^n - 1} \left( \sum_{i=0}^{i_2+j_2-1} q_{\text{inl}}^H + B^H \right) \\ &\quad \left. + (N - 2^n)(B^L + B^H) \right), \end{aligned} \quad (17)$$

where

$E_{\text{WAV}}$

$$\begin{aligned} &= \sum_{n=1}^K \sum_{l=0}^{N/2^n - 1} \left( E_{n,l}^P + 2E_{\text{elec}} \left( \sum_{i=0}^{i_1+j_1-1} (q_{\text{inl}}^L + B^L) + \sum_{i=0}^{i_2+j_2-1} (q_{\text{inl}}^H + B^H) \right) \right. \\ &\quad + \epsilon_{\text{amp}} \left( \sum_{i=0}^{i_1+j_1-1} ((q_{\text{inl}}^L + B^L) \cdot d_{\text{inl}}^L) \right. \\ &\quad \left. \left. + \sum_{i=0}^{i_2+j_2-1} ((q_{\text{inl}}^H + B^H) \cdot d_{\text{inl}}^H) \right) \right), \end{aligned}$$

$$\begin{aligned} E_{\text{OPT}} &= \sum_{n=1}^K \left( 2E_{\text{elec}} (B^L (N - 2^n) + B^H (N - 2^n)) \right. \\ &\quad + \epsilon_{\text{amp}} \left( \sum_{j=-i_1+N+2^{n-1}l+(2^{n-1}-1)(i_1+j_1)+N-2^{n-1}}^{-i_1+N+2(n-1)l+(2^{n-1}-1)(i_1+j_1)+N-2^{n-1}} B^L \cdot d_{j \bmod N} \right. \\ &\quad \left. + \sum_{j=-i_2+N+2^{n-1}l+(2^{n-1}-1)(i_2+j_2)+N-2^{n-1}}^{-i_2+N+2(n-1)l+(2^{n-1}-1)(i_2+j_2)+N-2^{n-1}} B^H \cdot d_{j \bmod N} \right) \Big) \\ q_{\text{inl}}^L &= q_{n,(-i_1+N+2^n l+(2^{n-1}-1)(i_1+j_1)+2^{n-1}i) \bmod N}^L, \\ q_{\text{inl}}^H &= q_{n,(-i_2+N+2^n l+(2^{n-1}-1)(i_2+j_2)+2^{n-1}i) \bmod N}^H, \\ q_{\text{inl}}^L &= \left( \sum_{j=-i_1+N+2^n l+(2^{n-1}-1)(i_1+j_1)}^{-i_1+N+2^n l+(2^{n-1}-1)(i_1+j_1)+2^{n-1}-1} d_j \bmod N \right)^2, \\ q_{\text{inl}}^H &= \left( \sum_{j=-i_2+N+2^n l+(2^{n-1}-1)(i_2+j_2)}^{-i_2+N+2^n l+(2^{n-1}-1)(i_2+j_2)+2^{n-1}-1} d_j \bmod N \right)^2. \end{aligned} \quad (18)$$

$E_{\text{WAV}}$  and  $E_{\text{OPT}}$  are the energy consumption for producing the wavelet coefficients and forwarding the values of energy and delay, respectively.  $B^L$  and  $B^H$  are the bit number storing the value of energy and delay, introduced by the production of low- and high-wavelet coefficients, respectively.  $q_{n,i}^L$  and  $q_{n,i}^H$  are the data amount transmitted by the  $i$ th node when the  $l$ th approximation coefficient and the corresponding detail coefficient in the  $n$ th level row WT are calculated, respectively,  $d_{j \bmod N}$  is the distance between the  $(j \bmod N)$ th node and the  $((j+1) \bmod N)$ th node,  $E_{n,l}^P$  is the processing energy of when the  $l$ th wavelet coefficients are calculated in the  $n$ th level WT.

*Proof.* When the  $l$ th approximation wavelet coefficient in the  $n$ th level row WT is calculated, the transmitting cost  $E_{n,l}^L$  is

$$\begin{aligned} E_{n,l}^L &= E_{\text{Tx}} + E_{\text{Rx}} = 2E_{\text{elec}} \sum_{i=0}^{i_1+j_1} q_{\text{inl}}^L \\ &\quad + \epsilon_{\text{amp}} \sum_{i=0}^{i_1+j_1} (q_{\text{inl}}^L \cdot d_{\text{inl}}^L). \end{aligned} \quad (19)$$

When the  $l$ th detail wavelet coefficient in the  $n$ th level row WT is calculated, the transmitting cost  $E_{n,l}^H$  is

$$E_{n,l}^H = 2E_{\text{elec}} \sum_{i=0}^{i_2+j_2} q_{\text{inl}}^H + \epsilon_{\text{amp}} \sum_{i=0}^{i_2+j_2} (q_{\text{inl}}^H \cdot d_{\text{inl}}^H). \quad (20)$$

When the  $n$ th level WT is performed, the processing cost  $E_p$  is

$$E_p = \sum_{l=0}^{N/2^n - 1} E_{n,l}^P. \quad (21)$$

Then, if  $K$ -level WT are performed, the energy cost  $E_{\text{IN}}$  is

$$E_{\text{IN}} = \sum_{n=1}^K \left( E_p + \sum_{l=0}^{N/2^n - 1} (E_{n,l}^L + E_{n,l}^H) \right). \quad (22)$$

Taking (19), (20), and (21) into (22), we can obtain (16). In the system of CDMA, the communication interference among nodes is little, so the wavelet coefficients can be calculated simultaneously. The network delay of the  $n$ th WT is

$$D_{IN} = \max_{0 \leq l \leq N/2^n - 1} \left( \sum_{l=0}^{i_1+j_1-1} q_{inl}^L \right) + \max_{0 \leq l \leq N/2^n - 1} \left( \sum_{l=0}^{i_2+j_2-1} q_{inl}^H \right). \quad (23)$$

Hereby, it is easy to get (17).  $\square$

Noting that  $E_{n,l}^P$  includes two parts, one is the processing cost when nodes perform column WT in a single node, and the other is the processing cost when nodes fuse data obtained from the preceding nodes. We can conclude from the theorem that, along with increasing levels of the WT, the energy cost also increases. However, the detail wavelet coefficients stored by the nodes also increase. As a result, the data can be coded using fewer bits.

For performance comparison, we employ a *nondistributed approach* for data gathering. In this approach, sensor nodes in the cluster will send their data to the cluster head directly and thus no internodes communications are required. Comparing the energy consumption and delay between our algorithm and the nondistributed approach, we have the following theorem.

**Theorem 4.** *Let the average distance between nodes and the cluster head be  $D$  meters. Let the amount of the original data that is quantized be  $Q$  bits and let the amount of data be  $Q'$  bits after  $K$ -level distributed spatiotemporal WT is performed. (1) If  $Q' \leq Q - E_{IN}/(E_{elec} + \epsilon_{amp} \cdot D^2)$ , the energy consumption by performing our algorithm is less than that of nondistributed approach; (2) if  $Q' \leq Q - D_{IN}$ , the delay by performing our algorithm is smaller than that of the nondistributed approach.*

*Proof.* Suppose that the cost of transmitting data to cluster head is  $E_T$ , and then the total energy consumption  $E_D$  by performing our algorithm is  $E_D = E_{IN} + E_T = E_{IN} + E_{elec} \cdot Q' \cdot D^2 = E_{IN} + Q'(E_{elec} + \epsilon_{amp} \cdot D^2)$ . The total energy consumption  $E_C$  for the nondistributed approach is  $E_C = E_T = Q(E_{elec} + \epsilon_{amp} \cdot D^2)$ . From  $E_D \leq E_C$ , we can get  $Q' \leq Q - E_{IN}/(E_{elec} + \epsilon_{amp} \cdot D^2)$ . Suppose that  $D_T$  is the delay of transmitting the data to the cluster head, then the total delay for our algorithm is  $D_D = D_T + D_{IN} = Q' + D_{IN}$ , and the delay for the nondistributed approach is  $D_C = D_T = Q$ ; from  $D_D \leq D_C$ , we can easily get  $Q' \leq Q - D_{IN}$ .  $\square$

Noting that the ratio of the total energy consumption of our algorithm and that of the nondistributed approach is

$$\frac{E_D}{E_C} = \frac{E_{IN} + E_{elec}Q' + \epsilon_{amp}Q'D^2}{E_{elec}Q + \epsilon_{amp}QD^2} = \frac{E_{IN}}{E_{elec}Q + \epsilon_{amp}QD^2} + \frac{Q'}{Q}. \quad (24)$$

Evidently,  $E_D/E_C$  will decrease when the distance  $D$  increases. Therefore, we can conclude that with increasing the distance between the cluster members from the cluster head, the proposed algorithm will save more energy.

### 3.2. Performance-evaluation model

We now establish a model to evaluate the performance of data compression algorithms for sensor networks.

One important goal of designing a sensor networks is to reduce energy consumption of sensor nodes and prolong its lifetime correspondingly. However, for many applications, in addition to minimizing energy cost, it is also important to consider the delay incurred in compressing sensory data. So, it is necessary to look for the tradeoff point between energy consumption and network delay. We capture this with *energy*  $\times$  *delay* metric.

In data compression, the ratio of signal to noise (*PSNR*) is often used to evaluate the algorithm efficiency. *PSNR* has some relations with the compression ratio. Generally, high *PSNR* will be subject to low compression ratio and vice versa. We pursue high *PSNR* when designing data compression algorithm for sensor networks.

Based on the above analysis, we propose the following model to evaluate the performance of data compression algorithm.

$$EP = f(\text{energy}, \text{delay}, \text{PSNR}) = \frac{\text{energy} \times \text{delay}}{\text{PSNR}}, \quad (25)$$

where *EC* and *delay* represent energy consumption and network delay, respectively, performance evaluation function *EP* is decided by *energy*, *delay*, and *PSNR*. The delay cost can be calculated as units of time, and we assume that 1 bit sensory data can be transmitted in 1 unit time.

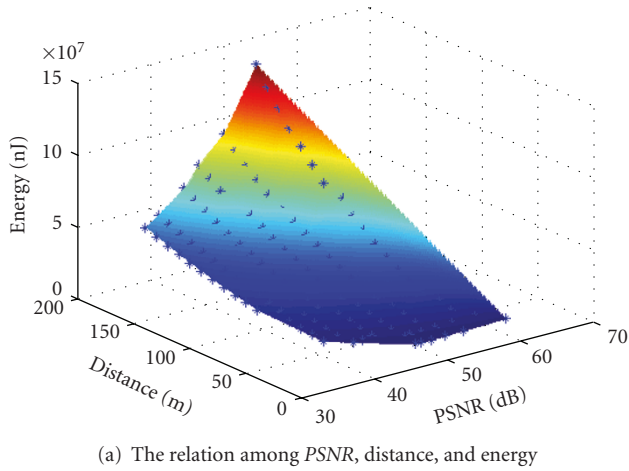
Obviously, minimizing *energy*  $\times$  *delay*/*PSNR* satisfies the requirement to energy consumption and lower network delay while obtaining high *PSNR*. So, *EP* is a reasonable model for evaluating data compression algorithm for sensor networks.

## 4. SIMULATION AND RESULTS

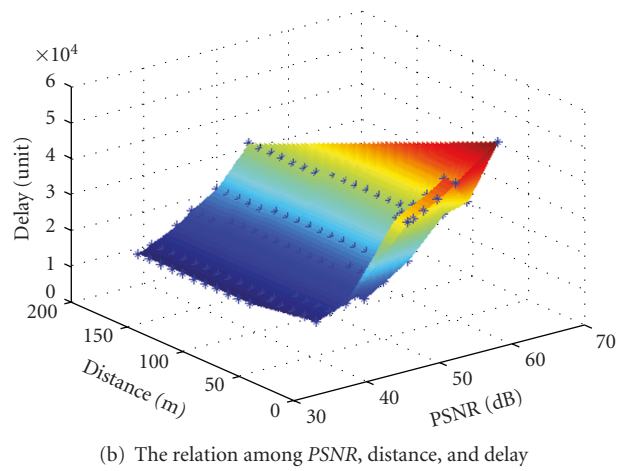
In this section, using Haar wavelet, we evaluate the performance of our algorithm and in particular compare it with the nondistributed approach.

We consider a ring composed of 128–896 nodes, assuming the average distance among the neighboring nodes is 5 meters. We use real life data obtained from the Tropical Atmosphere Ocean Project (<http://www.pmel.noaa.gov/tao>), which are the ocean temperatures sampled by 896 sensor nodes from different moorings at different depths at 12:00 pm from 1/20/2004 to 5/26/2004. In the experiment, we employ uniform quantization and no entropy coding. Three cases are compared: optimal transforming level of wavelet, nondistributed approach, and 2-level WT. The reason for choosing 2-level WT is that the appropriate level of transforming 65536 (256\*256) data is 2 based on the conclusion from standard signal processing techniques. The results are shown in Figures 2 to 6 and Table 1.

Figures 2–4 illustrate the relationship among energy consumption, delay, data reconstruction quality, and the position of cluster head for optimal level, distributed 2-level WT and nondistributed approach, respectively. Here, “distance” denotes the average distance between cluster head and sensor

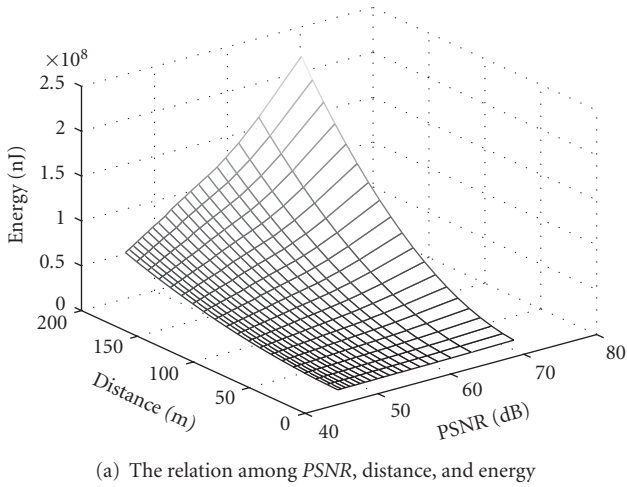


(a) The relation among PSNR, distance, and energy

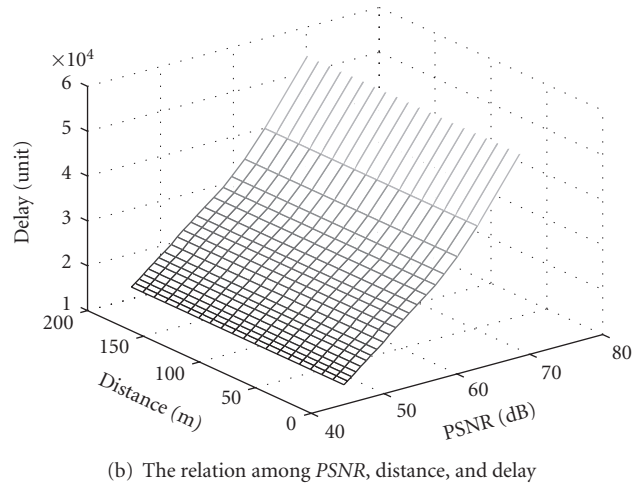


(b) The relation among PSNR, distance, and delay

FIGURE 2: Optimal level WT.

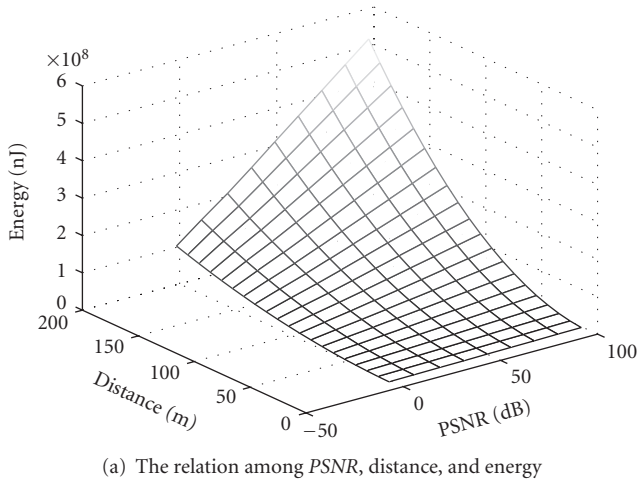


(a) The relation among PSNR, distance, and energy

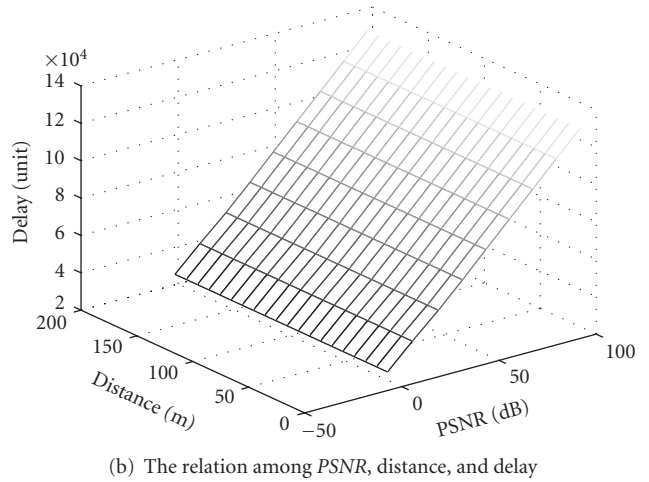


(b) The relation among PSNR, distance, and delay

FIGURE 3: Distributed 2-level WT.



(a) The relation among PSNR, distance, and energy



(b) The relation among PSNR, distance, and delay

FIGURE 4: Nondistributed approach.



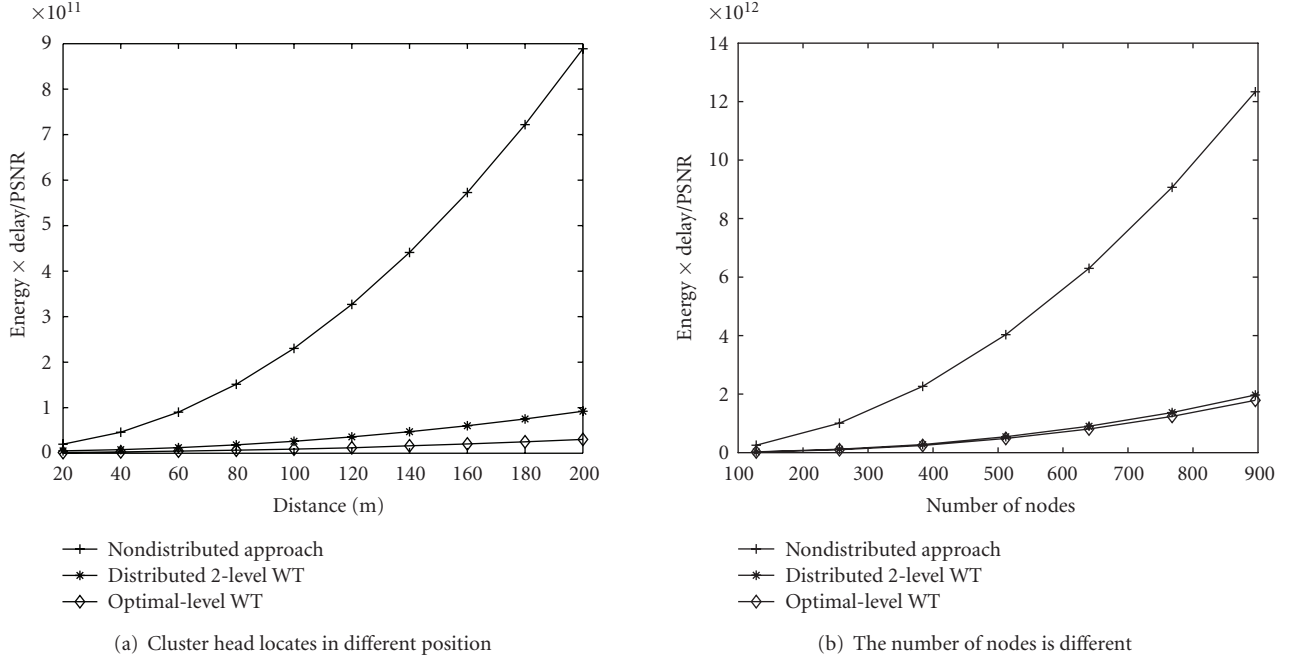


FIGURE 5: Comparison of performance.

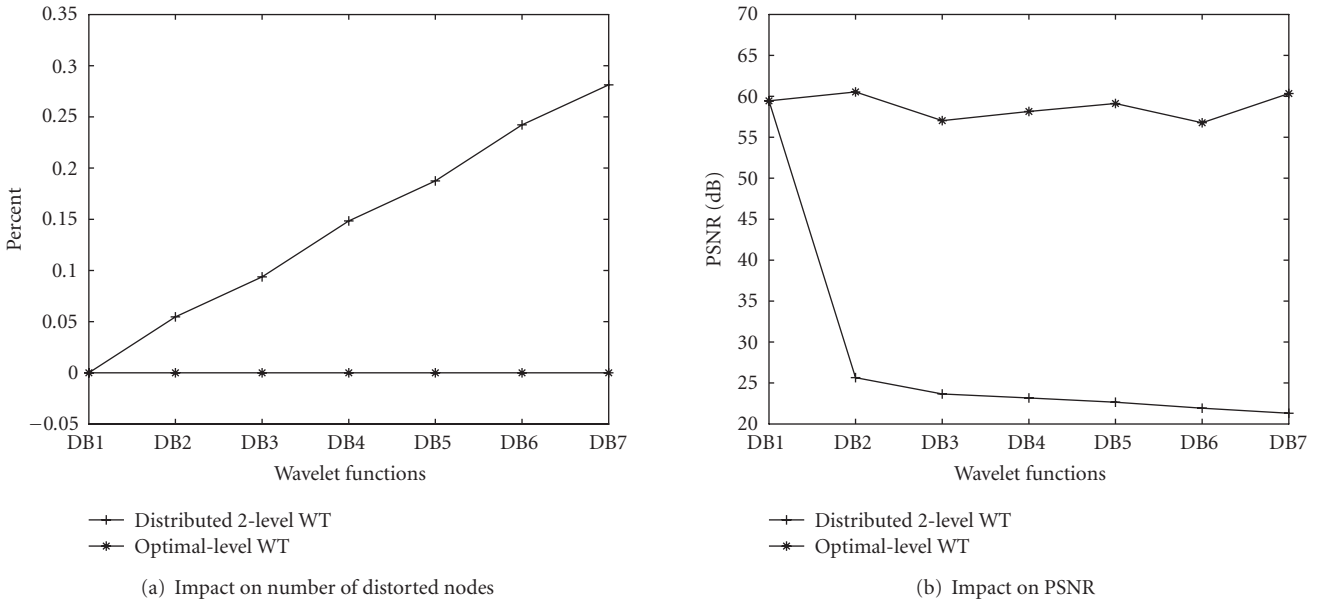


FIGURE 6: Impact of "border effect."

nodes, and "PSNR" indicates the data reconstruction quality as detailed in the previous section. As we can see, along with the increasing of PSNR and distance, the performances of distributed algorithms are better than nondistributed approach, and our proposed algorithm has the least energy consumption and delay. Notably, the shape of Figure 2 is not as regular as Figures 3 and 4. This is because our algorithm can adjust the transform level adaptively according to the distance, and thus the size of energy consumption, delay, and PSNR varies along with the transform level irregularly.

In Figure 5, we compare the performance of our proposed approach, 2-level WT, and nondistributed approach using  $energy \times delay/PSNR$  metric. Figure 5(a) shows scenario when the cluster head is located at different positions. Figure 5(b), shows the scenario where the number of nodes are varying. Again, "distance" denotes the average distance between the cluster head and sensor nodes. Figure 6(a) shows the performance comparison when the scopes of sensor networks are different. The results show that distributed algorithms outperform nondistributed approach significantly

TABLE 1: The relations among optimal transforming level, distance, the reconstructed quality, energy, and delay.

Opt-level	Distance (m)	PSNR (dB)	Energy ( $10^7$ nJ)	Delay ( $10^4$ units)
1	20	61.1	0.8	5.3
2	30	58.4	1.0	3.6
3	40	56.8	1.3	3.2
3	50	56.6	1.6	3.2
4	60	55.4	2.0	3.1
4	70	55.4	2.4	3.1
4	80	55.4	2.8	3.1
4	90	55.4	3.4	3.1
4	100	55.4	3.9	3.1
4	110	55.4	4.6	3.1
4	120	55.4	5.3	3.1
4	130	55.4	6.0	3.1
5	140	55.4	6.9	3.1
5	150	55.4	7.8	3.1
5	160	55.4	8.8	3.1
5	170	55.4	9.7	3.1
5	180	55.4	11.0	3.1
5	190	55.4	11.9	3.1
5	200	55.4	13.1	3.1

when employing the  $energy \times delay/PSNR$  metric. Our proposed algorithm also outperforms the general distributed algorithm.

Figure 6 shows the impact of border impact on data reconstruction. Figure 6(a) indicates that the percentage of nodes, in which the reconstructed data is distortive out of the total 256 nodes, increases if the border effect is not removed along with the alteration of the wavelet function (From DB1 to DB7). Accordingly, the reconstructed data quality (PSNR) deteriorates as compared with our approach. In Figure 6(b), we intentionally employ threshold and quantization to form an application scenario where the compression is lossy. It shows that, even with lossy compression, in terms of data reconstructed quality, our approach far outperforms the traditional distributed 2-level WT approach.

The relationship among optimal level of WT(Opt-level), distance between nodes and cluster head, PSNR, energy consumption, and delay is captured in Table 1. The result shows that the optimal transforming levels are different along with the variety of distance between nodes and cluster head while ensuring almost the same reconstructed quality. When the distance increases, the energy consumption increases and network delay decreases correspondingly. This is because energy consumption is dependent on the distance under first-order radio model, and network delay only relies on the average number of encoding bits. In our simulation, when the proportion of the discarding detail coefficients to total wavelet coefficients in the WT reaches 73 percent, the PSNR is still reach 49 dB. We believe that the reasons are the data used in the simulation have strong spatio-temporal correlations and our algorithm can move them efficiently.

As we can see from the simulation results, the optimal level of WT is 0 when the distance between nodes and clus-

ter head is less than 20 meters. This indicates that WT is not necessary under this case, and the non-distributed approach obtain good performance, for it has no additional energy consumption. However, with increasing distance between the nodes and the cluster head, the benefit of compression outweigh the energy consumption due to inter-node communication for performing the WT, and then the proposed algorithm will save more energy. Table 1 shows that different transforming levels needed to be performed to obtain the similar PSNR while minimizing energy and delay cost.

## 5. CONCLUSION

In this paper, we have proposed a distributed optimal-level spatiotemporal compression algorithm based on the ring model for general wavelets with arbitrary supports. Our algorithm can accommodate a broad range of wavelet functions in order to effectively exploit the temporal and spatial correlation for data compression. Furthermore, the ring topology can effectively eliminate the “border effect” by naturally extending the signal space. In particular, our algorithm can choose optimal transforming levels to obtain better performance according to the given network circumstance. The proposed  $energy \times delay/PSNR$  model is capable of effectively evaluating the data compression algorithms for wireless sensor networks. The theoretical and experimental results show that the proposed scheme can achieve significant reduction in energy consumption and delay for data gathering in a sensor cluster.

We are currently investigating the methods to effectively accept or reject the detail wavelet coefficients generated by the scheme so that constant or limited bit rate for sensor transmission can be achieved.

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