Hindawi Publishing Corporation
EURASIP Journal on Advances in Signal Processing
Volume 2009, Article ID 295739, 12 pages
doi:10.1155/2009/295739

*Research Article*

# Inter-Operator Spectrum Sharing from a Game Theoretical Perspective

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Received 15 February 2009; Revised 1 June 2009; Accepted 8 July 2009

Recommended by K. Subbalakshmi

We address the problem of spectrum sharing where competitive operators coexist in the same frequency band. First, we model this problem as a *strategic* non-cooperative game where operators *simultaneously* share the spectrum according to the Nash Equilibrium (NE). Given a set of channel realizations, several Nash equilibria exist which renders the outcome of the game unpredictable. Then, in a cognitive context with the presence of primary and secondary operators, the inter-operator spectrum sharing problem is reformulated as a *Stackelberg* game using hierarchy where the primary operator is the leader. The Stackelberg Equilibrium (SE) is reached where the best response of the secondary operator is taken into account upon maximizing the primary operator's utility function. Moreover, an extension to the multiple operators spectrum sharing problem is given. It is shown that the Stackelberg approach yields better payoffs for operators compared to the classical *water-filling* approach. Finally, we assess the goodness of the proposed distributed approach by comparing its performance to the centralized approach.



## 1. Introduction

Spectrum sharing between wireless networks improves the efficiency of spectrum usage where a migration toward flexible spectrum management is paramount to alleviate spectrum scarcity and its underutilization. In this respect and motivated by the ever-increasing demands for wireless services, several works have appeared in literature ([1–9] among many others) wherein interestingly both theoretical and practical (system-level) contributions, stemming from pricing [1, 7], opportunistic power control [10] to resource sharing [8], and others have been made.

In this paper, we study spectrum sharing between two *competing* operators operating in the same frequency band in which base stations communicate with their mobile terminals. In this case, a transmitter $T_1$ wants to send information to its mobile $R_1$, while at the same time another base station $T_2$ (from a competitive operator) wants to transmit information to its mobile $R_2$. These systems, therefore, share the same medium where the communication pairs $(T_1, R_1)$ and $(T_2, R_2)$ take place simultaneously and on the same frequency band. This setup is known as the interference channel (IFC) ([11–15] to mention a few).

There is a great deal of work on the IFC channel using game theory. In [13], the problem of power allocation in a frequency-selective multiuser interference channel is studied. An iterative Water-Filling (WF) algorithm is proposed to efficiently reach the Nash equilibrium. Moreover, it is found that under suitable conditions, the iterative WF algorithm for the two-user Gaussian interference game converges to the unique Nash equilibrium from any starting point. In their scenario, the Nash equilibria lead to nonefficient and non pareto-optimal solutions. Similarly, in [11], the authors consider the problem of spectrum sharing on the IFC for *flat-fading* channels. The interference channel is viewed as a noncooperative game and the Nash equilibrium is characterized under a set of sufficient conditions. In [16], the authors investigate the problem of simultaneous water-filling solution for the gaussian IFC under *weak* interference. Motivated by the pareto-inefficiency of the water-filling approach, the authors propose a distributed algorithm to transform a symmetric system from simultaneously waterfilled to a fair orthogonal signal space partitions.

In [17], the problem of two wireless networks operating on the same frequency band was considered. Pairs within a given network cooperate to schedule transmissions according to a random-access protocol where each network chooses an access probability for its users. In [18], the authors consider the problem of coordinating two competing multiple-antenna wireless systems in the Multiple Input Single Output (MISO) IFC. It turns out that if the systems do not cooperate, then the corresponding equilibrium rates are bounded regardless of how much transmit power the base stations have available. Also, Nash bargaining solutions were found to be close to the sum-rate bound. On the other hand, in [19–21], the authors study the problem of maximizing mutual information subject to mask constraints and transmit power, for both simultaneous and asynchronous cases. (Under this setup, some users are allowed to update their strategy more frequently than the others. And, they might even perform these updates using outdated information on the interference caused by others.) The existence of the Nash equilibrium is proven and sufficient conditions are given for the uniqueness. Finally, in [20], distributed iterative algorithms are proposed to reach the Nash equilibrium.

In most of these works, the existence of the Nash equilibrium is easily demonstrated, whereas the uniqueness is generally more complicated for which only sufficient conditions are given. Because of the very hard problem of the uniqueness of the Nash equilibrium points in the WF game, Nash bargaining (NBS) solutions were considered in [14]. However, NBS requires the knowledge of all channel state information which is not always possible in practice.

Within the same framework of spectrum sharing but under a different scenario, Stackelberg games [22] have been applied in the context of cognitive radios where the desirability of outcomes depends not only on their own actions but also on other cognitive radios. It is worth pointing out that the Stackelberg formulation naturally arises in some contexts of practical interest: (a) when primary and secondary systems share the spectrum, (b) when user have access to the medium in an asynchronous manner, (c) when operators deploy their networks at different times, and (d) when some nodes have more power than others such as the base station. Stackelberg is based on a *leader-follower* approach in which the leader plays his strategy before the follower, and then enforces it. In [23], a game theoretic framework has been proposed in the context of fading multiple-access channel where a Stackelberg formulation is proposed in which the base station is the designated game leader with the purpose to have a distributed allocation strategy approaching all corners of the capacity region. In [24], a two-level Stackelberg game is proposed for distributed relay selection and power control for multiuser cooperative networks. The objective is to jointly consider the benefits of source and relay nodes in which the source node is modeled as a buyer and the relay nodes as the sellers. Also, the energy-efficient power control problem is investigated in [25]. Moreover, in [26], the authors investigate a similar power allocation problem but *solely* focus on channel realizations in which the Nash equilibrium of the game is unique. ( The case with multiple Nash equilibria was not treated.) However, this work differs in that the Stackelberg approach is mainly motivated by the nonuniqueness of the Nash equilibrium and unpredictability of the game.

In essence, the fundamental questions we address in this paper are the following.

(i) In the first operators' deployment scenario, if both operators *simultaneously* operate in a noncooperative (i.e., *selfish*) manner: what are their power allocation strategies across their carriers? Clearly, there is a conflict situation where a good strategy for the link $(T_1, R_1)$ will generate interference for $R_2$ and viceversa. Hence, an equilibrium has to be found.

(ii) Given any set of channel realizations, is it possible to predict the outcome of the game?, and if so, how to characterize the Nash equilibria regions? Is the Nash equilibrium *unique*?

(iii) In the second scenario of operators' deployment in which primary and secondary operators coexist in the same spectral band, what is the outcome of the game when a hierarchy exists between operators? How does this approach compare with the selfish approach (classical water-filling)?

(iv) How close is the distributed approach from the centralized (*sum-rate*) power allocation?

The paper is organized as follows: the system model is introduced in Section 2. In Section 3, the spectrum sharing game between operators is formulated using noncooperative game theory. In Section 4, a special case with two operators transmitting on two carriers is investigated to gain insights into the Nash equilibria regions. In Section 5, we formulate the interoperator spectrum sharing problem as a Stackelberg game where a hierarchy exists between operators as well as an extension to the multiple operators case. Section 6 provides a comparison between the distributed (selfish) and centralized approach. Finally, we conclude this work in Section 7.

## 2. System Model

We suppose that $K$ transmitters share a frequency band composed of $N$ carriers where each transmitter transmits in any combination of channels and at any time. ( The terms transmitter and operator are interchangeably used throughout the paper.) On each carrier $n = 1 \cdots N$, transmitter $k = 1 \cdots K$ sends the information $x_k^n = \sqrt{p_k^n} s_k^n$, where $s_k^n$ represents the transmitted data and $p_k^n$ denotes the corresponding transmitted power of user $k$ on carrier $n$. The received signal at the receiver $i$ in carrier $n$ can be expressed as

$$r_i^n = \sum_{j=1}^{K} h_{ji}^n x_j^n + w_i^n, \quad i = 1, \dots, K,\ n = 1, \dots, N, \tag{1}$$

where $h_{ji}^n$ is the fading channel gain on the $n$th carrier between the pair $(T_i, R_j)$. In addition, the noise process $w_i^n$

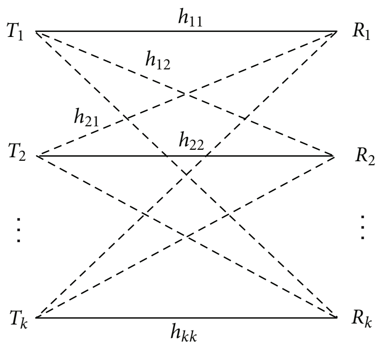


Figure 1: $K$-user $N$-carriers interference channel under study.

is characterized by its received noise power on each carrier $n$, by $\sigma_n^2$.

For transmitter $i$, the transmit power $p_i^n$ is subject to its power constraint:

$$\sum_{n=1}^{N} p_i^n \leq \overline{P}_i, \quad i = 1, \ldots, K. \tag{2}$$

At the receiver $i$, the signal to interference plus noise ratio (SINR) on carrier $n$ is given by

$$\mathrm{SINR}_i^n = \frac{p_i^n \left| h_{ii}^n \right|^2}{\sigma_n^2 + \sum_{j=1, j\neq i}^{K} p_j^n \left| h_{ji}^n \right|^2}. \tag{3}$$

Furthermore, assuming Gaussian codebook, the maximum achievable rate at receiver $i$ is given by

$$R_i = \sum_{n=1}^{N} \log_2 \left(1 + \mathrm{SINR}_i^n\right). \tag{4}$$

## 3. Noncooperative Spectrum Sharing Game

In this section, we model the interoperator spectrum sharing problem from a noncooperative standpoint [27]. Figure 1 illustrates the spectrum sharing scenario under study for $K$ operators and $N$ carriers.

*3.1. Game Formulation.* The noncooperative spectrum sharing game is defined as $\Gamma^{\mathrm{NCG}} \triangleq [\mathcal{K}, \{\mathcal{P}_i\}_{i\in\mathcal{K}}, \{\mathcal{U}_i\}_{i\in\mathcal{K}}]$. The players (from the set $\mathcal{K} \triangleq \{1, 2, \ldots, K\}$) are defined as the different links with a strategy $p_i^n \in \mathcal{P}_i$ and the payoffs are the achievable rates on each link $u_i(p_i^n, p_{-i}^n) = R_i(p_i^n, p_{-i}^n) \in \mathcal{U}_i$, for $i = 1, \ldots, K$ and $n = 1, \ldots, N$. Each player competes against the others by choosing its transmit power (i.e., strategy) to maximize its own utility subject to some power constraints $\overline{P}_i$. In this work, we assume full channel state information in which operators know their fading channel gains as well as other's fading cross-channels.

Since the operators do not cooperate, the only reasonable outcome of the spectrum conflict is an operating point which constitutes a Nash equilibrium (NE) [28]. This is a point where none of the players can improve their utilities by unilaterally changing their strategies. One should note that a Nash equilibrium is not an optimal or even desirable outcome. However, it is an insightful point where one is likely to end up operating at, if players are not willing to cooperate.

In a noncooperative approach, operator $i$ selfishly maximizes his utility function subject to the power constraint $\overline{P}_i$:

$$\begin{aligned} \max_{p_i^n} R_i &= \max_{p_i^n} \sum_{n=1}^{N} \log_2 \left(1 + \frac{\left| h_{i,i}^n \right|^2 p_i^n}{\sigma_n^2 + \sum_{j\neq i}^{K} \left| h_{j,i}^n \right|^2 p_j^n}\right) \\ \text{s.t. } & \sum_{n=1}^{N} p_i^n \leq \overline{P}_i, \\ & p_i^n \geq 0. \end{aligned} \tag{5}$$

Furthermore, the channel realization set $\underline{h}$ is defined as

$$\underline{h} = \left\{ h_{ij}^n : i, j = 1, \ldots, K,\ n = 1, \ldots, N \right\}. \tag{6}$$

The solutions to (5) are given by the water-filling power allocation solutions:

$$\begin{aligned} p_i^n &= \left( \frac{1}{\mu_i} - \frac{\sigma_n^2 + \sum_i \left| h_{-i,i}^n \right|^2 p_{-i}^n}{\left| h_{i,i}^n \right|^2} \right)^+ \\ & i = 1, \ldots, K, \quad n = 1, \ldots, N, \end{aligned} \tag{7}$$

where $(x)^+ = \max\{x, 0\}$ and $\mu_i > 0$ is the Lagrangian multiplier chosen to satisfy the power constraint: $\sum_{n=1}^{N} p_i^n = \overline{P}_i$. Note that the equality follows from the concavity of the objective function in $p_i$.

## 4. A Special Case of Two Operators and Two Carriers

In order to gain insight into the properties of the Nash equilibria for our interoperator spectrum game, let us focus on the case where two operators transmit over two carriers (i.e., $K = N = 2$).

*4.1. Notations.* (i) For the ease of notation and readability that will prove helpful in the sequel, we introduce the following notations: $g_{ij}^n = \overline{P}_i |h_{ij}^n|^2/\sigma_n^2$, $c_1 = g_{11}^1/g_{11}^2$, and $c_2 = g_{22}^1/g_{22}^2$.

(ii) The pair $(\alpha_1, \alpha_2)$ means that user 1 transmits with power $(p_1^1, p_1^2) = (\alpha_1 \overline{P}_1, (1-\alpha_1)\overline{P}_1)$ on carrier 1 and 2 while user 2 transmits with power $(p_2^1, p_2^2) = (\alpha_2 \overline{P}_2, (1-\alpha_2)\overline{P}_2)$ on carriers 1 and 2, respectively.

Figure 2 depicts the space of the 9 Nash equilibria of the game obtained upon solving (7), the details of which are given in Appendix A. Given a set of channel realizations $\underline{h}$,

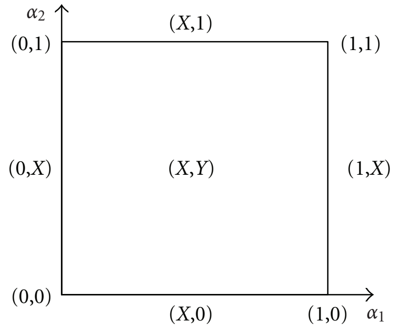


FIGURE 2: Illustration of the Nash equilibria space where $(\alpha_1, \alpha_2)$ denotes the power allocation strategy for both operators 1 and 2, in the first carrier.

the game converges to different equilibrium points. Figure 3 illustrates one possible representation of the Nash equilibria space. Depending on the quantities $(1 + g_{21}^1)/(1 + g_{11}^2)$, $(1 + g_{11}^1)/(1 + g_{21}^2)$, and $(1 + g_{22}^1)/(1 + g_{12}^2)$, $(1 + g_{12}^1)/(1 + g_{22}^2)$ (see Appendix A), four different representation of the regions are possible. These regions are depicted in Figure 4 whose purpose is to reflect the 8-dimensional problem related to the channel realization set $\underline{h}$.

It turns out that given certain channel realizations, the Nash equilibrium is *unique* (white rectangle areas) while some of the grayish rectangle regions exhibit at least one Nash equilibria.

*4.2. Existence of the Nash Equilibria.* The existence of the Nash equilibria is proven using the theorem in [29] within the context of noncooperative concave games. Hence, the game defined in (5) admits at least one Nash equilibrium.

*4.3. Uniqueness of the Nash Equilibria.* In [13], the authors give sufficient conditions for the uniqueness of the NE but do not precisely state which NE are obtained for any given channel realization set $\underline{h}$. Therefore, building on these results, a full characterization of the Nash equilibria region for the 2-operators 2-carriers case is herein given. Beside, the proof of the uniqueness when both operators transmit in both carriers (full-spread) is given in Appendix B.

*Remark 4.1.* The operators were assumed to be noncooperative hence operating at the Nash Equilibrium was their best response in a selfish context (one-shot game). It was also shown that under certain channel realizations, the spectrum sharing game is predictable with a *unique* Nash equilibrium. However, in other regions and given a set of channel realizations, *nonunique* Nash equilibria exist. In this case, the spectrum sharing game is *no longer* predictable.

*Remark 4.2.* We note that the sufficient conditions given for the *flat-fading* case studied in [11] are depicted in Figure 3 for the low-interference regime $(X, Y)$ where $g_{2,1}^1/g_{1,1}^1 < 1$ and $g_{1,2}^1/g_{2,2}^1 < 1$.

*Remark 4.3.* In the multioperators case, the results for the 2 operator and 2 carriers case carry over where the sufficient conditions for the uniqueness are given by

$$\forall n \in \{1, \ldots, N\}, \qquad \sum_{i=1, j\neq i}^{K} \frac{\left|h_{ji}^n\right|^2}{\left|h_{ii}^n\right|^2} < 1. \tag{8}$$

This result comes from the Karush-Kuhn-Tucker conditions of the optimization problem of the set of data rates. Additionally, the physical meaning of (8) is that the uniqueness of the Nash equilibrium is ensured if the links are sufficiently far from each other.

*Remark 4.4.* We note that when one of the cross-gain $|h_{-i,i}|^2 = 0$, the IFC becomes a Z-channel [30] where the NE exists and is *unique* (the characterization of the Nash equilibria region for the Z-channel follows the same lines as the IFC).

## 5. Introducing Hierarchy (Stackelberg Game Approach)

In this section, we look at the interoperator spectrum sharing problem where the concept of hierarchy is accounted for. This situation is inherent in situations where primary and secondary operators share the same spectrum. In what follows, we formulate and solve the interoperator spectrum sharing problem (with hierarchy) for the 2-operators 2-carriers case then provide insights for the case with more than 2 operators.

A *Stackelberg* game $\Gamma^{\mathrm{SG}} \triangleq [\mathcal{K}, \{\mathcal{P}_i\}_{i\in\mathcal{K}}, \{\mathcal{U}_i\}_{i\in\mathcal{K}}]$ is proposed to model the spectrum sharing problem where one of the two operators is chosen to be the leader (primary operator). The *Stackelberg* Equilibrium (SE) [22] is the best response where a hierarchy of actions exists between players. *Backward induction* [27] is applied assuming that players can reliably forecast the behavior of other players and that they believe that the other player can do the same. For this reason, the key point in this setup is the capability of the follower of sensing the environment and, therefore, the power level of operator 1 (the leader).

*5.1. Problem Formulation.* Without loss of generality, we assume that primary operator 1 is the leader and secondary operator 2 is the follower. First, we give a definition of the Stackelberg equilibrium as follows.

*Definition 5.1* (Stackelberg Equilibrium [27]). A strategy profile $(p_1^{\mathrm{SE}}, p_2^{\mathrm{SE}})$ is called a Stackelberg Equilibrium if $p_1^{\mathrm{SE}}$ maximizes the utility of the leader (operator 1) and $p_2^{\mathrm{SE}}$ is the best response of operator 2 to operator 1.

The Stackelberg spectrum sharing game can be formulated as follows. First, in the high-level problem (9), primary operator 1 maximizes his own utility function. Then, in the low-level problem (10), secondary operator 2 (follower) maximizes his own utility taking into account the optimal power allocation of operator 1 $(p_1^{\mathrm{SE}})$. By denoting $(p_1^{\mathrm{SE}}, p_2^{\mathrm{SE}})$

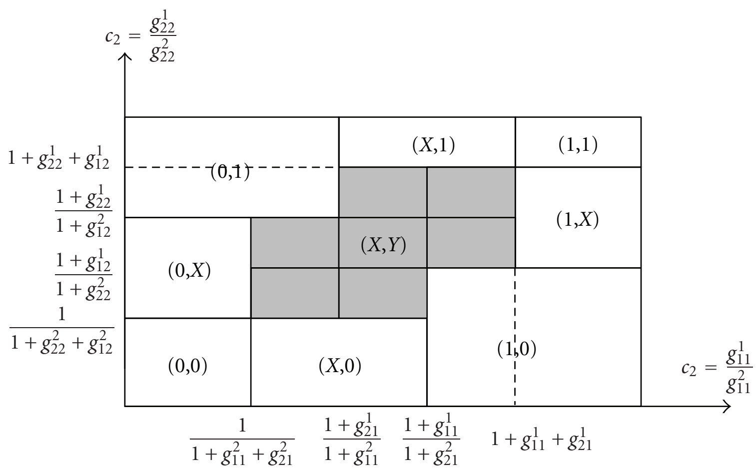


Figure 3: Characterization of the Nash equilibria regions given a set of channel realizations $\overline{h}$.

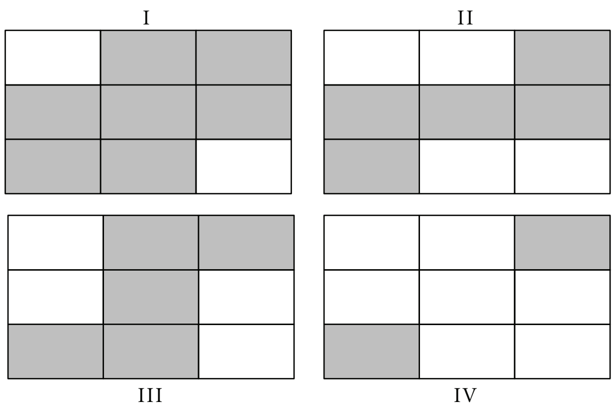


Figure 4: All four cases are depicted: (I) when $(1+g_{11}^1)/(1+g_{21}^2) > (1+g_{21}^1)/(1+g_{11}^2)$ and $(1+g_{22}^1)/(1+g_{12}^2) > (1+g_{12}^1)/(1+g_{22}^2)$, (II) when $(1+g_{11}^1)/(1+g_{21}^2) < (1+g_{21}^1)/(1+g_{11}^2)$ and $(1+g_{22}^1)/(1+g_{12}^2) > (1+g_{12}^1)/(1+g_{22}^2)$, (III) when $(1+g_{11}^1)/(1+g_{21}^2) > (1+g_{21}^1)/(1+g_{11}^2)$ and $(1+g_{22}^1)/(1+g_{12}^2) < (1+g_{12}^1)/(1+g_{22}^2)$, and finally when (IV) $(1+g_{11}^1)/(1+g_{21}^2) < (1+g_{21}^1)/(1+g_{11}^2)$ and $(1+g_{22}^1)/(1+g_{12}^2) < (1+g_{12}^1)/(1+g_{22}^2)$.

as the Stackelberg Equilibrium, the rate optimization problem for operator 1 (leader) is written as

$$\begin{aligned}
&\max_{p_1^n} \sum_{n=1}^{N} \log_2\left(1+\frac{|h_{11}^n|^2 p_1^n}{\sigma_n^2+|h_{21}^n|^2 p_2^n(p_1^n)}\right)\\
&\sum_{n=1}^{N} p_1^n \le \overline{P}_1,\\
&p_1^n \ge 0.
\end{aligned} \tag{9}$$

The rate optimization problem for operator 2 (follower) is written as

$$\begin{aligned}
&\max_{p_2^n} \sum_{n=1}^{N} \log_2\left(1+\frac{|h_{22}^n|^2 p_2^n}{\sigma_n^2+|h_{12}^n|^2 (p_1^n)^{\mathrm{SE}}}\right)\\
&\sum_{n=1}^{N} p_2^n \le \overline{P}_2,\\
&p_2^n \ge 0,
\end{aligned} \tag{10}$$

where $p_2^{\mathrm{SE}} = \mathrm{BR}_2(p_1^{\mathrm{SE}})$.

Using backward induction and given the best response of operator 2 (the follower), (10) can be rewritten as

$$\begin{aligned}
&\max_{p_1^n} \sum_{n=1}^{N} \log_2\left(1+\frac{|h_{11}^n|^2 p_1^n}{\sigma_n^2+|h_{21}^n|^2\left(1/\mu_2-(\sigma_n^2+|h_{12}^n|^2 p_1^n)/|h_{22}^n|^2\right)^+}\right)\\
&\sum_{n=1}^{N} p_1^n \le \overline{P}_1,\\
&p_1^n \ge 0.
\end{aligned} \tag{11}$$

The Stackelberg sharing game, therefore, boils down to solving (11) where several cases are considered. In our spectrum sharing problem ($K = N = 2$), the power strategies of operator 2 take 3 values. In the first case, operator 2 transmits with maximum power $\overline{P}_2$ in carrier 1 ($p_2^1 = \overline{P}_2, p_2^2 = 0$). In the second case, operator 2 transmits with $\overline{P}_2$ in carrier 2 ($p_2^1 = 0, p_2^2 = \overline{P}_2$), and finally in the third case, operator 2 transmits on both carriers (full spread) with ($p_2^1 = x, p_2^2 = \overline{P}_2 - x$), $0 < x < \overline{P}_2$. Therefore, the leader maximizes his utility function given the best response of the follower. In the following, all of the *three* cases are investigated. For simplicity sake, we assume $\overline{P}_1 = \overline{P}_2 = 1$.

*5.1.1. Operator 2 Transmits Only in Carrier 2* ($p_2^1 = 0, p_2^2 = 1$). Under this setup, $p_2^2 > 0 \Rightarrow p_1^1 \geq \beta_1$ where

$$\beta_1 = \frac{\sigma_2^2/|h_{22}^2|^2 + |h_{12}^2|^2/|h_{22}^2|^2 - \sigma_1^2/|h_{22}^1|^2 + 1}{|h_{12}^1|^2/|h_{22}^1|^2 + |h_{12}^2|^2/|h_{22}^2|^2}, \quad (12)$$

where $\beta_1$ depends on the set of channel realizations.

Furthermore, the maximization problem for the leader is written as

$$\max_{p_1^1} \log_2\left(1 + \frac{|h_{11}^1|^2 p_1^1}{\sigma_1^2}\right) + \log_2\left(1 + \frac{|h_{11}^2|^2\left(\overline{P}_1 - p_1^1\right)}{\sigma_2^2 + |h_{21}^2|^2}\right),$$
$$\max(\beta_1, 0) \leq p_1^1 \leq \overline{P}_1, \quad (13)$$

the KKT conditions are given such that

$$\begin{gathered}\lambda_1^*\left((p_1^1)^* - \max(\beta_1, 0)\right) = 0, \quad \lambda_1^* \geq 0,\\ \lambda_2^*\left((p_1^1)^* - \overline{P}_1\right) = 0, \quad \lambda_2^* \geq 0,\\ \frac{\partial \mathcal{L}_1}{\partial p_1^1} = 0,\\ (p_1^1)^* \leq \overline{P}_1,\\ (p_1^1)^* \geq \max(\beta_1, 0),\end{gathered} \quad (14)$$

where $\lambda_1^*$, $\lambda_2^*$ are the Lagrangian multipliers associated with the constraints

$$\mathcal{L}_1 = \log_2\left(1 + \frac{|h_{11}^1|^2 p_1^1}{\sigma_1^2}\right) + \log_2\left(1 + \frac{|h_{11}^2|^2\left(\overline{P}_1 - p_1^1\right)}{\sigma_2^2 + |h_{21}^2|^2}\right)$$
$$- \lambda_1(p_1^1 - \max(\beta_1, 0)) + \lambda_2\left(p_1^1 - \overline{P}_1\right),$$
$$\frac{\partial \mathcal{L}_1}{\partial p_1^1} = 0$$
$$\Longrightarrow \frac{h_{11}^1}{\sigma_1^2 + h_{11}^1 p_1^1} - \frac{h_{11}^2}{\sigma_2^2 + h_{21}^2 + h_{11}^2\left(\overline{P}_1 - p_1^1\right)} = \lambda_1 - \lambda_2. \quad (15)$$

Assume that $p_1^1 = \max(\beta_1, 0.)$, then, $\lambda_1^* \geq 0$, $\lambda_2^* = 0$ and

$$\frac{h_{11}^1}{\sigma_1^2 + h_{11}^1 \max(\beta_1, 0)} - \frac{h_{11}^2}{\sigma_2^2 + h_{21}^2 + h_{11}^2\left(\overline{P}_1 - \max(\beta_1, 0)\right)} \geq 0. \quad (16)$$

Now assuming that $p_1^1 = \overline{P}_1$, then $\lambda_1^* = 0$, $\lambda_2^* \geq 0$ and

$$\frac{h_{11}^1}{\sigma_1^2 + h_{11}^1 \overline{P}_1} - \frac{h_{11}^2}{\sigma_2^2 + h_{21}^2} \leq 0. \quad (17)$$

Finally, assuming that $\max(\beta_1, 0) < p_1^1 < \overline{P}_1$, then $\lambda_1^* = \lambda_2^* = 0$ and

$$p_1^1 = \frac{\sigma_1^2|h_{11}^1|^2 - \sigma_2^2|h_{11}^2|^2 + |h_{11}^1|^2\left(|h_{21}^2|^2 + |h_{11}^2|^2\overline{P}_1\right)}{2|h_{11}^1|^2|h_{11}^2|^2}. \quad (18)$$

*5.1.2. Operator 2 Transmits Only in Carrier 1* ($p_2^1 = 1, p_2^2 = 0$). Under this setup, $p_2^1 > 0 \Rightarrow p_1^1 \leq \beta_2$, where

$$\beta_2 = \frac{\sigma_2^2/|h_{22}^2|^2 + |h_{12}^2|^2/|h_{22}^2|^2 - \sigma_1^2/|h_{22}^1|^2 - 1}{|h_{12}^1|^2/|h_{22}^1|^2 + |h_{12}^2|^2/|h_{22}^2|^2}. \quad (19)$$

Furthermore, the maximization problem for the leader is written as

$$\max_{p_1^1} \log_2\left(1 + \frac{|h_{11}^1|^2 p_1^1}{\sigma_1^2 + |h_{21}^1|^2}\right) + \log_2\left(1 + \frac{|h_{11}^2|^2\left(\overline{P}_1 - p_1^1\right)}{\sigma_2^2}\right)$$
$$0 \leq p_1^1 \leq \min\left(\beta_2, \overline{P}_1\right). \quad (20)$$

Likewise, to derive the KKT conditions, form the Lagrangian denoted as $\mathcal{L}_2$ :

$$\begin{aligned}\mathcal{L}_2 = {} & \log_2\left(1 + \frac{|h_{11}^1|^2 p_1^1}{\sigma_1^2 + |h_{21}^1|^2}\right)\\ & + \log_2\left(1 + \frac{|h_{11}^2|^2\left(\overline{P}_1 - p_1^1\right)}{\sigma_2^2}\right)\\ & - \lambda_1 p_1^1 + \lambda_2\left(p_1^1 - \min\left(\beta_2, \overline{P}_1\right)\right)\end{aligned} \quad (21)$$

the KKT conditions are

$$\begin{gathered}\lambda_2^*\left((p_1^1)^* - \min\left(\beta_2, \overline{P}_1\right)\right) = 0, \quad \lambda_2^* \geq 0,\\ \lambda_1^*\left((p_1^1)^*\right) = 0, \quad \lambda_1^* \geq 0,\\ \frac{\partial \mathcal{L}_2}{\partial p_1^1} = 0,\\ (p_1^1)^* \geq 0,\\ (p_1^1)^* \leq \min\left(\beta_2, \overline{P}_1\right),\end{gathered} \quad (22)$$

where $\lambda_1^*$, $\lambda_2^*$ are the Lagrangian multipliers associated with the constraints

$$\frac{\partial \mathcal{L}_2}{\partial p_1^1} = 0$$
$$\Longrightarrow \frac{h_{11}^1}{\sigma_1^2 + h_{21}^2 + h_{11}^1 p_1^1} - \frac{h_{11}^2}{\sigma_2^2 + h_{11}^2\left(\overline{P}_1 - p_1^1\right)} = \lambda_1^* - \lambda_2^*. \quad (23)$$

Assume that $p_1^1 = 0$, $\lambda_1^* \geq 0$, then $\lambda_2^* = 0$ and, furthermore,

$$\frac{h_{11}^1}{\sigma_2^2 + h_{21}^2} - \frac{h_{11}^2}{\sigma_1^2 + h_{11}^2\overline{P}_1} \geq 0. \quad (24)$$

Assuming that $p_1^1 = \min(\beta_2, \overline{P}_1)$, $\lambda_2^* \geq 0$, then $\lambda_1^* = 0$ and, furthermore,

$$\frac{h_{11}^1}{\sigma_1^2 + h_{21}^2 + h_{11}^1 \min\left(\beta_2, \overline{P}_1\right)} - \frac{h_{11}^2}{\sigma_2^2 + h_{11}^2\left(\overline{P}_1 - \min\left(\beta_2, \overline{P}_1\right)\right)}$$
$$= \lambda_1^* - \lambda_2^*. \quad (25)$$

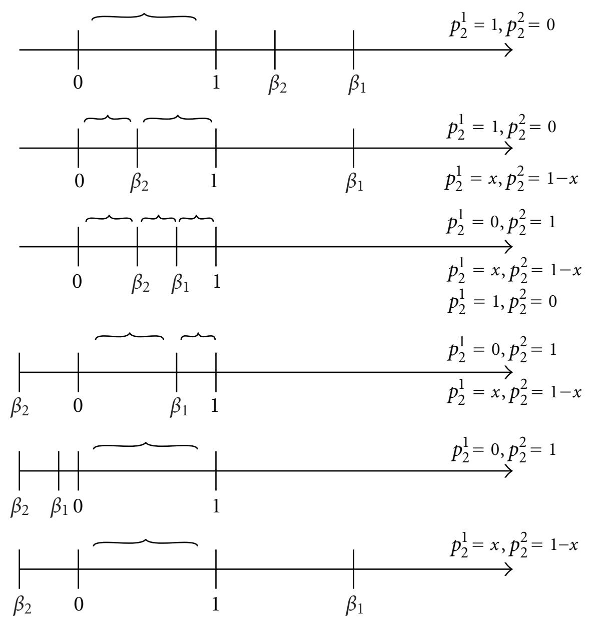


Figure 5: Power allocation strategies of the Stackelberg game in which 6 cases exist depending on the variables $\beta_1$ and $\beta_2$. The $X$-axis depicts the strategy space for the leader ($p_1^1$).

Finally, assume that $0 < p_1^1 < \min(\beta_2, \overline{P}_1)$, then $\lambda_1^* = \lambda_2^* = 0$ and

$$p_1^1 = \frac{h_{11}^1\left(\sigma_1^2 + h_{11}^2\overline{P}_1\right) - h_{11}^2(\sigma_2^2 + h_{21}^1)}{2h_{11}^1 h_{11}^2}. \tag{26}$$

*5.1.3. Operator 2 Transmits in Both Carriers ($p_2^1 = x, p_2^2 = 1 - x$).*

$$\begin{aligned}
&\max_{p_1^1} \log_2\left(1 + \frac{|h_{11}^1|^2 p_1^1}{\sigma_1^2 + |h_{21}^1|^2 x}\right) \\
&\qquad + \log_2\left(1 + \frac{|h_{11}^2|^2 p_1^2}{\sigma_2^2 + |h_{22}^2|^2 (1-x)}\right), \\
&p_1^1 + p_1^2 \le \overline{P}_1, \\
&p_1^1, p_1^2 \ge 0, \\
&\beta_2 < p_1^1 < \beta_1.
\end{aligned} \tag{27}$$

Since $p_2^1 = x = 1/\mu_2 - (\sigma_1^2 + |h_{12}^1|^2 p_1^1)/|h_{22}^1|^2 > 0$ depends on $p_1^1$, the objective function (27) of operator 1 is nonconvex in $p_1^1$ (the KKT conditions can be written in the same way as done for the previous cases and the problem is solved numerically). Figure 5 depicts all the 6 different cases depending on the values of $\beta_1$ and $\beta_2$ (note that $\beta_2 < \beta_1$). The $X$-axis depicts the strategy space for the leader ($p_1^1$).

As can be seen, in the first case ($\beta_2 > 1, \beta_1 > 1$) the leader has to perform one maximization over the interval $[0, 1]$. In the *second* case ($0 < \beta_2 < 1, \beta_1 > 1$), the leader has to perform 2 maximizations ($[0, \beta_2]$ and $[\beta_2, 1]$) and pick the power allocation that maximizes his payoff. Similarly, the leader has 3 maximizations to perform in the *third* case ($[0, \beta_2]$, $[\beta_2, \beta_1]$, and $[\beta_1, 1]$), where $0 < \beta_1 < 1$, $0 < \beta_2 < 1$ and likewise for the remaining cases. In essence, in all these cases, the leader (operator 1) forces the follower to adopt a strategy that maximizes the leader's payoff. In this way, using backward induction, the Stackelberg equilibrium is *unique*, solving thereby the problem of nonuniqueness encountered in the noncooperative approach of Section 4. Additionally, one should note that there exist Stackelberg solutions that are *non-Nash* equilibria of the noncooperative game.

Another similar approach for solving the Stackelberg equilibrium is used in [31] where an explicit expression of $p_2^n$ (function of $p_1^n$) needed to analytically find the SE is given as

$$p_2^n = \begin{cases} \left(\overline{P}_2 + \sum_{i=1}^{k}\left(\sigma_{\pi^{-1}(i)}^2 + \left|h_{12}^{\pi^{-1}(i)}\right|^2 p_1^{\pi^{-1}(i)}\right)\right) \\ \quad -\sigma_2^2 - |h_{12}^n|^2 p_1^n, & \pi(i) \le k, \\ 0, & \pi(i) > k, \end{cases} \tag{28}$$

where $\pi$ is a permutation function ranking all channels according to their noise plus interference and $k$ can be found from the following condition: $\varphi_k < \overline{P}_2 \le \varphi_{k+1}$ and $\varphi_t = \sum_{i=1}^{k}(\sigma_{\pi^{-1}(i)}^2 + |h_{12}^{\pi^{-1}(i)}|^2 p_1^{\pi^{-1}(i)})$, $\forall t \in \{1, \ldots, n\}$.

*5.2. Extension to the Multiple Operators' Case.* The inter-operator spectrum sharing in the context of two operators can be extended to the more general case with $K$ operators sharing the same spectrum. The problem is formulated in the same way where the leader's optimization problem is written as

$$\begin{aligned}
&\max_{p_1^n} \sum_{n=1}^{N} \log_2\left(1 + \frac{|h_{11}^n|^2 p_1^n}{\sigma_n^2 + \sum_{j\neq 1}^{K}\left|h_{j1}^n\right|^2 p_j^n(p_1^n)}\right), \\
&\sum_{n=1}^{N} p_1^n \le \overline{P}_1, \\
&p_1^n \ge 0,
\end{aligned} \tag{29}$$

and $p_j^{\mathrm{SE}} = \mathrm{BR}_j(p_1^{\mathrm{SE}}, \ldots, p_{-j}^{\mathrm{SE}})$ is a function of $p_1^n$.

Solving (29) becomes much more involved in the general case in which the utility function of the primary operator is nonconvex ($p_j^n$ is function of $p_1^n$). Nevertheless, there exist suboptimal and low-complexity methods to solve the problem. To this end and motivated by the work of [32], we use lagrangian duality theory wherein the *duality gap* [33] provides a nice tool for solving nonconvex optimization problem.

initialize $\lambda, \overline{P}_1, \overline{P}_2, \dots, \overline{P}_K$
repeat
for $n = 1, \dots, N$
set $p_1^n = \underset{p_1^n}{\operatorname{argmax}} \sum_{n=1}^{N} \log_2 \left(1 + \frac{|h_{11}^n|^2 p_1^n}{\sigma_n^2 + \sum_{j\neq 1}^{K} |h_{j1}^n|^2 p_j^n(p_1^n)}\right) + \lambda(\overline{P}_1 - \sum_{n=1}^{N} p_1^n)$
by keeping $p_1^1, \dots, p_1^{n-1}, p_1^{n+1}, p_1^N$ fixed.
end
until $(p_1^1, \dots, p_1^N)$ converges
update $\lambda$ using subgradient [32] method until it converges.

Algorithm 1

The lagrangian of (29) is given by

$$\begin{aligned} g(\lambda) &= \max_{p_1^n} \mathcal{L}(p_1^n, \lambda) \\ &= \max_{p_1^n} \sum_{n=1}^{N} \log_2 \left(1 + \frac{\left|h_{11}^n\right|^2 p_1^n}{\sigma_n^2 + \sum_{j\neq 1}^{K} \left|h_{j1}^n\right|^2 p_j^n}\right) \\ &\quad + \lambda\left(\overline{P}_1 - \sum_{n=1}^{N} p_1^n\right), \end{aligned} \tag{30}$$

where $\lambda$ is the lagrangian dual variable associated with the power constaint.

Consequently, solving the Stackelberg problem is done by locally optimizing the lagrangian function (30) via coordinate descent [33]. For each fixed set of $\lambda$, we find the optimal $p_1^1$ while keeping $p_1^2, \dots, p_1^N$ fixed, then find the optimal $p_1^2$ keeping the other $p_1^n$ $(n \neq 2)$ fixed, and so on. Such process is guaranteed to converge because each iteration strictly increases the objective function. Finally, $\lambda$ is found using subgradient [32] method. The algorithm is depicted in Algorithm 1.

Finally, in the case of multiple primary and secondary operators, the optimization problem in (29) becomes a multiobjective optimization problem. In this case, the issue of cooperation is at stake where primary operators $i$ can operate at feasible points yielding rates that dominate the noncooperative equilibrium rate ($R_i \geq R^{\mathrm{NE}}$). On the other hand, the secondary operators will adopt the same noncooperative and selfish approach (iterative water-filling).

## 6. Numerical Evaluation

In this section, numerical results are presented to validate the theoretical claims. Figure 6 depicts the average achievable rate of both operators for the Stackelberg approach. In the simulations, we let the individual power constraint $\overline{P}_1 = \overline{P}_2 = \overline{P} = 1$, SNR $= \overline{P}/\sigma^2$ and channel fading realizations are independent and identically distributed (i.i.d) Rayleigh distributed.

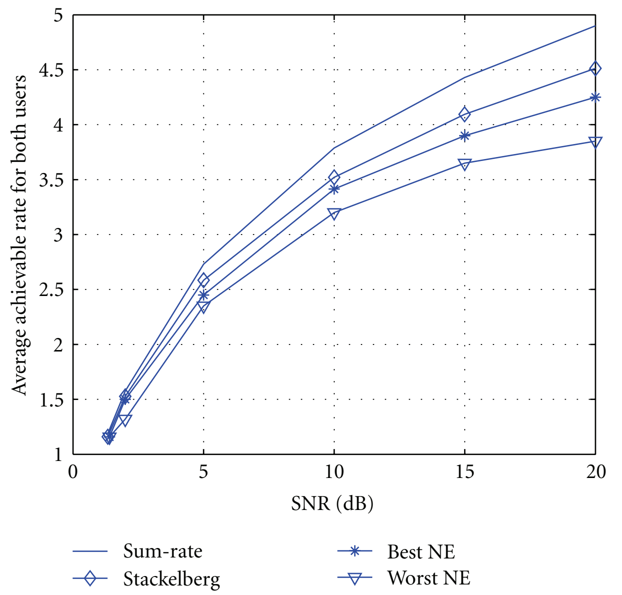


Figure 6: Average achievable rate for both users versus the signal-to-noise ratio for the centralized and Stackelberg approach. Moreover, the best and worst Nash equilibria for the non-cooperative game are illustrated.

It is important to quantify the performance loss from the optimal solution provided by the centralized strategy. To this end, we compare the Stackelberg rates with the rates obtained by sum-rate maximization (which are Pareto-optimal):

$$\begin{gathered} \max_{p_1^n, p_2^n} \sum_{i=1}^{K} \sum_{n=1}^{N} \log_2 \left(1 + \frac{\left|h_{11}^n\right|^2 p_1^n}{\sigma_n^2 + \left|h_{21}^n\right|^2 p_2^n}\right), \\ \sum_{n=1}^{N} p_1^n \leq \overline{P}_1, \qquad \sum_{n=1}^{N} p_2^n \leq \overline{P}_2, \\ p_1^n \geq 0, \quad p_2^n \geq 0, \quad n = 1, \dots, N. \end{gathered} \tag{31}$$

The objective function is nonconvex in the power variables $p_1^n$ and $p_2^n$. To solve (31), the maximization problem is transformed into a convex optimization problem using Geometric Programming [33]. Additionally, Figure 6 depicts the best and worst NE where the best NE refers to the equilibrium maximizing the sum-rate of both operators whereas the

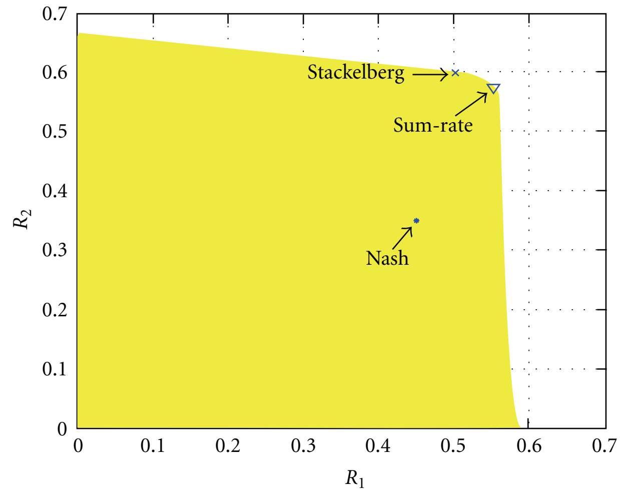


FIGURE 7: Achievable rate region for the inter-operator spectrum sharing game. Both operators achieve better payoffs when adopting the hierarchical (Stackelbeg) approach.

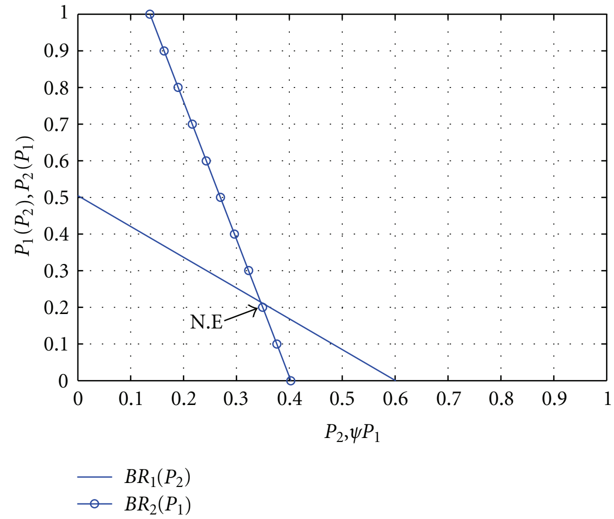


FIGURE 9: Best response functions illustrating the unique Nash equilibrium point where both operators transmit in both carriers.

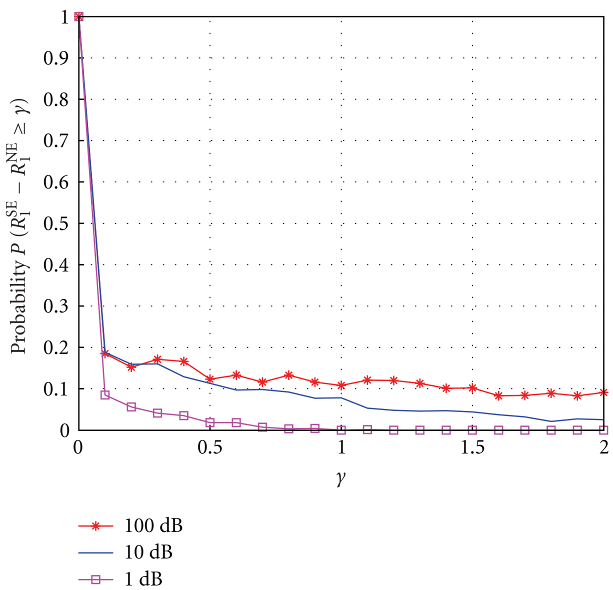


FIGURE 8: Probability when the NE approach is worse than the Stackelberg approach for the leader, for several SNR values.

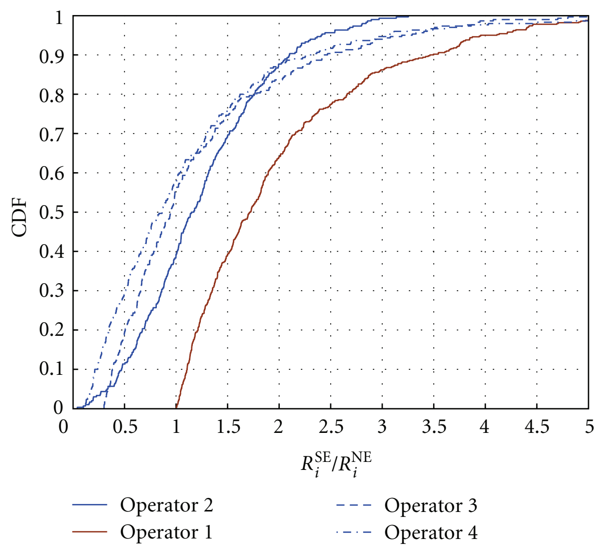


FIGURE 10: Cumulative distribution function (CDF) of the ratio of the rates between the hierarchical (Stackelberg) and noncooperative (selfish) approaches $R_i^{SE}/R_i^{NE}$ for operator $i$, $i = 1, 2, 3, 4$.

worst NE case minimizes it. It is also worth noting that the worst Nash equilibrium acts like a lower-bound for the Nash equilibrium. Furthermore, the Stackelberg approach is closer to the centralized approach as compared to the selfish case. This is due to the fact that in the Stackelberg approach, operators take into account other operators' strategies whereas in the selfish case, operators behave carelessly by using water-filling. Figure 7 shows the achievable rate region for both operators in which the Nash and Stackelberg equilibria are illustrated. Since primary operator 1 is the leader, his rate is higher with the Stackelberg approach. Also, interestingly, the rate of operator 2 is also better off with the Stackelberg approach. As a result, both operators have strong incentives in adopting the hierarchical (Stackelberg) approach. Finally, Figure 8 depicts the probability $P\ (R_1^{SE} - R_1^{NE} \geq \gamma)$, that is, when the Nash equilibrium approach is worse than the Stackelberg's for operator 1, for several SNR values where it is seen that the Stackelberg approach outperforms the classical water-filling approach.

On the other hand, Figure 10 depicts the cumulative distribution function (cdf) of the ratio between the achievable rates of the hierarchical and noncooperative approaches. In this scenario, we assume $K = 4$ operators with 1 primary operator and 3 secondary wireless operators sharing the same spectrum composed of $N = 5$ carriers. As can be seen, the primary operator (operator 1) always improves his achievable rate compared to the selfish approach. The cumulative distribution function of the secondary operators also provides insights on their achievable rates.

## 7. Conclusion

In this paper, we studied the problem of spectrum sharing between operators from two different perspectives. First, a *one-shot* game was studied where operators play simultaneously, operating at the NE point which exhibits different behaviors according to the set of channel realizations. Then, in a second approach, a *hierarchical* game is examined where the primary operator is the *leader* and the secondary operator is the *follower*, wherein their sum-rates are further improved compared to the water-filling approach. Simulation results show incentives for operators to behave cleverly by adopting the hierarchical (Stackelberg) approach. In our future work, we will use the concept of hierarchy to investigate power control schemes for femtocells networks [34].

## Appendices

### A.

We derive the a set of 9 inequalities for the Nash equilibria when $K = 2$ operators transmit over $N = 2$ carriers, for the noncooperative game $\Gamma^{\mathrm{NC}}$.

Recall that $g_{ij}^n = \overline{P}_i |h_{ij}^n|^2/\sigma^2$, $c_1 = g_{11}^1/g_{11}^2$, and $c_2 = g_{22}^1/g_{22}^2$.

It holds that $(\alpha_1, \alpha_2) = (0,0)$ is a Nash Equilibrium

$$\Longleftrightarrow \begin{cases} c_1 \leq \dfrac{1}{1+g_{11}^2+g_{21}^2}, \\ c_2 \leq \dfrac{1}{1+g_{22}^2+g_{12}^2}, \end{cases}$$

$(\alpha_1, \alpha_2) = (1,0)$ is a Nash Equilibrium

$$\Longleftrightarrow \begin{cases} c_1 \geq \dfrac{1+g_{11}^1}{1+g_{21}^2}, \\ c_2 \leq \dfrac{1+g_{12}^1}{1+g_{22}^2}, \end{cases}$$

$(\alpha_1, \alpha_2) = (0,1)$ is a Nash Equilibrium

$$\Longleftrightarrow \begin{cases} c_1 \leq \dfrac{1+g_{21}^1}{1+g_{11}^2}, \\ c_2 \geq \dfrac{1+g_{22}^1}{1+g_{12}^2}, \end{cases}$$

$(\alpha_1, \alpha_2) = (1,1)$ is a Nash Equilibrium

$$\Longleftrightarrow \begin{cases} c_1 \geq 1+g_{11}^1+g_{21}^1, \\ c_2 \geq 1+g_{22}^1+g_{12}^1, \end{cases}$$

$(\alpha_1, \alpha_2) = (x,1)$ is a Nash Equilibrium

$$\Longleftrightarrow \begin{cases} 2g_{22}^1 g_{11}^1 \leq c_2 g_{11}^1 (1+g_{12}^2) - 2g_{11}^1 - (c_2 g_{12}^2 + g_{12}^1) \\ \qquad \times (c_1 - 1 + c_1 g_{11}^2 - g_{21}^1), \\ \dfrac{1+g_{21}^1}{1+g_{11}^2} \leq c_1 \leq 1+g_{11}^1+g_{21}^1, \end{cases} \tag{A.1}$$

where $x = 1/2 - 1/2g_{11}^1 - g_{21}^1/2g_{11}^1 + 1/2g_{11}^2$,

$(\alpha_1, \alpha_2) = (1,x)$ is a Nash Equilibrium

$$\Longleftrightarrow \begin{cases} 2g_{11}^1 g_{22}^1 \leq c_1 g_{22}^1 (1+g_{21}^2) - 2g_{22}^1 - (c_1 g_{21}^2 + g_{21}^1) \\ \qquad \times (c_2 - 1 + c_2 g_{22}^2 - g_{12}^1), \\ \dfrac{1+g_{12}^1}{1+g_{22}^2} \leq c_2 \leq 1+g_{22}^1+g_{12}^1, \end{cases} \tag{A.2}$$

where $x = 1/2 - 1/2g_{22}^1 - g_{12}^1/2g_{22}^1 + 1/2g_{22}^2$,

$(\alpha_1, \alpha_2) = (0,x)$ is a Nash Equilibrium

$$\Longleftrightarrow \begin{cases} \dfrac{c_2(1+g_{12}^2)+g_{22}^1-1}{2g_{22}^1} \geq \dfrac{c_1(1+g_{21}^2+g_{11}^2)-1}{g_{21}^1+c_1 g_{21}^2}, \\ \dfrac{1}{1+g_{22}^2+g_{12}^2} \leq c_2 \leq \dfrac{1+g_{22}^1}{1+g_{12}^2}, \end{cases} \tag{A.3}$$

where $x = 1/2 + g_{12}^2/2g_{22}^2 - 1/2g_{22}^1 + 1/2g_{22}^2$,

$(\alpha_1, \alpha_2) = (x,0)$ is a Nash Equilibrium

$$\Longleftrightarrow \begin{cases} \dfrac{c_1(1+g_{21}^2)+g_{11}^1-1}{2g_{11}^1} \geq \dfrac{c_2(1+g_{12}^2+g_{22}^2)-1}{g_{12}^1+c_2 g_{12}^2}, \\ \dfrac{1}{1+g_{11}^2+g_{21}^2} \leq c_1 \leq \dfrac{1+g_{11}^1}{1+g_{21}^2}, \end{cases} \tag{A.4}$$

where $x = 1/2 + g_{21}^2/2g_{11}^2 - 1/2g_{11}^1 + 1/2g_{11}^2$,

$(\alpha_1, \alpha_2) = (x,y)$ is a Nash Equilibrium

$$\begin{gathered} \Longleftrightarrow \begin{cases} 0 < x < 1, \\ 0 < y < 1, \end{cases} \\ x = \frac{(2g_{21}^1(c1(1+g_{12}^2+g_{21}^2)-1)-\mathfrak{A})}{(4g_{11}^1 g_{22}^1 - (c_2 g_{12}^2 + g_{12}^1)(c_1 g_{21}^2 + g_{21}^1))}, \\ y = \frac{(1/(2g_{22}^2)) - 1/(2g_{22}^1) + ((1-x)g_{12}^2)}{(2g_{22}^1) - ((g_{12}^1 x)/2g_{22}^1 + 1/2)}, \end{gathered} \tag{A.5}$$

where $\mathfrak{A}$ denotes $(c_1 g_{21}^2 + g_{21}^1)(c_2(1+g_{12}^2)+g_{22}^1-1)$.

## B.

In this setup, the utility functions become

$$R_1(\alpha_1,\alpha_2) = \log_2\left(1+\frac{\left|h_{1,1}^1\right|^2\alpha_1}{\sigma_1^2+\left|h_{2,1}^1\right|^2\alpha_2}\right) + \log_2\left(1+\frac{\left|h_{1,1}^2\right|^2(1-\alpha_1)}{\sigma_2^2+\left|h_{2,1}^2\right|^2(1-\alpha_2)}\right),$$
$$R_2(\alpha_1,\alpha_2) = \log_2\left(1+\frac{\left|h_{2,2}^1\right|^2\alpha_2}{\sigma_1^2+\left|h_{1,2}^1\right|^2\alpha_1}\right) + \log_2\left(1+\frac{\left|h_{2,2}^2\right|^2(1-\alpha_2)}{\sigma_2^2+\left|h_{1,2}^2\right|^2(1-\alpha_1)}\right). \tag{B.1}$$

We will give now sufficient conditions that guarantee the uniqueness of the NE. By analyzing the first-order derivatives of the payoff functions, we can find explicit relations for the best response functions (BR):

$$\mathrm{BR}_1(\alpha_2) = \frac{-\mathfrak{B}-|h_{11}^2|^2+|h_{11}^1|^2\left(1+|h_{21}^2|^2+|h_{11}^2|^2\right)}{2|h_{11}^1|^2|h_{11}^2|^2},$$
$$\mathrm{BR}_2(\alpha_1) = \frac{-\mathfrak{C}-|h_{22}^2|^2+|h_{22}^1|^2\left(1+|h_{12}^2|^2+|h_{22}^2|^2\right)}{2|h_{22}^1|^2|h_{22}^2|^2}, \tag{B.2}$$

where $\mathfrak{B}$ denotes $[|h_{11}^2|^2|h_{21}^1|^2+|h_{11}^1|^2|h_{21}^2|^2]\alpha_2$ and $\mathfrak{C}$ denotes $[|h_{22}^2|^2|h_{12}^1|^2+|h_{22}^1|^2|h_{12}^2|^2]\alpha_1$.

We observe that the functions $\mathrm{BR}_i(\alpha_{-i})$ are linear with respect to $\alpha_{-i}$. Thus, the intersection of the BR functions is either a unique point or an infinity of points. Therefore, the sufficient conditions that ensure the uniqueness of the NE are the following:

$$\frac{|h_{11}^2|^2|h_{21}^1|^2+|h_{11}^1|^2|h_{21}^2|^2}{2|h_{11}^1|^2|h_{11}^2|^2} \neq \frac{2|h_{22}^1|^2|h_{22}^2|^2}{|h_{22}^2|^2|h_{12}^1|^2+|h_{22}^1|^2|h_{12}^2|^2},$$
$$\frac{-|h_{11}^2|^2+|h_{11}^1|^2\left(1+|h_{21}^2|^2+|h_{11}^2|^2\right)}{2|h_{11}^1|^2|h_{11}^2|^2} \neq \frac{-|h_{22}^2|^2+|h_{22}^1|^2\left(1+|h_{12}^2|^2+|h_{22}^2|^2\right)}{|h_{22}^2|^2|h_{12}^1|^2+|h_{22}^1|^2|h_{12}^2|^2}. \tag{B.3}$$

If these conditions are met, the unique point at the intersection of the BRs describes the Nash equilibrium. This is illustrated in Figure 9.

## Acknowledgments

This work has been supported by the Finnish Funding Agency for Technology and Innovation (Tekes), Nokia, Nokia Siemens Networks, Elektrobit and Tauno Tonning Foundation. This work has been performed in part in the framework of the CELTIC project CP5-026 WINNER+ and the french project TERROP. The valuable comments of Dr. Jorma Lilleberg are also very much appreciated.

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