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Research Article

On the Asymptotic Optimality of Opportunistic Norm-Based User Selection with Hard SINR Constraint

Xi Zhang,[1] Eduard A. Jorswieck,[2] Björn Ottersten (EURASIP Member),[1] and Arogyasvsami Paulraj[3]

[1] *Signal Processing Laboratory, ACCESS Linnaeus Center, Royal Institute of Technology (KTH), 10044 Stockholm, SE, Sweden*
[2] *Communications Laboratory, Faculty of Electrical Engineering and Information Technology, Dresden University of Technology, 01062 Dresden, Germany*
[3] *Information Systems Laboratory, Stanford University, CA 94305, USA*

Correspondence should be addressed to Xi Zhang, xi.zhang@ee.kth.se

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Recently, user selection algorithms in combination with linear precoding have been proposed that achieve the same scaling as the sum capacity of the MIMO broadcast channel. Robust opportunistic beamforming, which only requires partial channel state information for user selection, further reduces feedback requirements. In this work, we study the optimality of the opportunistic norm-based user selection system in conjunction with hard SINR requirements under max-min fair beamforming transmit power minimization. It is shown that opportunistic norm-based user selection is asymptotically optimal, as the number of transmit antennas goes to infinity when only two users are selected in high SNR regime. The asymptotic performance of opportunistic norm-based user selection is also studied when the number of users goes to infinity. When a limited number of transmit antennas and/or median range of users are available, only insignificant performance degradation is observed in simulations with an ideal channel model or based on measurement data.

1. Introduction

The application of multiple antennas in multiuser communications leads to the need for cross-layer design and channel-aware scheduling [1, 2]. Furthermore, the number of antennas at the mobile user terminal is typically quite limited due to the weight and power requirements. The restrictions at the access point are fortunately less severe. To fully exploit the potential of multiple-input multiple-output (MIMO) systems, multiple users should be served simultaneously in the same spectrum.

The sum capacity and the capacity region of the MIMO broadcast channel with perfect channel state information (CSI) and nonlinear dirty-paper precoding (DPC) are studied in [3]. DPC presubstracts multiuser interference to precode multiple users simultaneously, and is shown to be a capacity achieving strategy in MIMO broadcast channel, although its complexity is prohibitively high. Even its suboptimal greedy variant [4] is difficult to implement in practice, especially when the number of user terminals is large.

One simpler alternative to serve multiple users simultaneously is the so called random unitary beamforming [5], which is based upon the widely used principle of opportunistic beamforming [6]. A set of randomly generated but mutually orthogonal beamformers serve several users in an opportunistic fashion. For the sum capacity of the MIMO broadcast channel, random unitary beamforming achieves the same scaling with the number of users as DPC [7]. Unfortunately, the performance of random unitary beamforming degrades quickly with decreasing number of users or increasing number of transmit antennas.

Recently, user selection schemes combined with different linear precoding strategies have been proposed that also achieve the same scaling as the sum capacity of the MIMO broadcast channel, such as the work in [8, 9]. Simplified variants of opportunistic scheduling have been proposed in [10, 11]. These schemes achieve a significant fraction of the sum capacity, especially for large number of users. At the same time, they also maintain relatively low computational complexity, except for those based on greedy user selection [9, 12].

In this work, we consider the performance of several different user selection schemes combined with a particular linear precoding strategy: the so-called max-min fair beamformer [13, 14]. The max-min fair beamformer jointly maximizes each user's SINR to meet their individual hard SINR target for nonelastic traffic, and at the same time minimizes the total transmit power at the access point. This type of linear precoding has different objectives compared to precoders maximizing achievable sum rate (such as [15]), or simpler versions of the zero-forcing beamformer achieving the same sum rate asymptotically (such as [1, 8]).

The main contributions of this paper are as follows:

(1) characterizing in closed form the minimum power needed for max-min fair beamformer when two users are selected by different user selection schemes in high SNR regime,

(2) establishing the asymptotic optimality of opportunistic norm-based user selection when two users are selected, in the context of hard SINR requirements. This complements the well known result of sum rate,

(3) verifying the asymptotic optimality by simulations based on real measurement data. Only insignificant performance degradation is observed comparing to the promised asymptotic optimality.

The paper is structured as follows. In Section 2, a brief description of the considered max-min fair beamforming system and the related system parameters is given. In Section 3, different user selection methods are revisited, including the opportunistic norm-based user selection. System performance as well as asymptotic optimality of opportunistic norm-based user selection are analyzed in Section 4, and the results are illustrated with experimental data in Section 5. Conclusions are drawn in Section 6.

Uppercase and lowercase boldface denote matrices and vectors respectively. The operator $(\cdot)^H$ is the Hermitian transpose, $(\cdot)^c$ is the complex conjugate, $\|\cdot\|$ and $|\cdot|$ denotes the ℓ_2 norm and the Cardinality, respectively. The scaling notation $x(N) \sim y(N)$ indicates that $\lim_{N\to\infty} x(N)/y(N)$ is a finite constant.

2. System Model and Problem Statement

2.1. System Setup. For simplicity, we consider a single carrier downlink system with a single access point and K users. The system feedback load is expected to be low, so only a fixed modulation and coding scheme (MCS) is used together with power control. The system is homogeneous, which means the long term average SNR for each user is the same. Each user also has a hard SINR requirement.

The system has an opportunistic norm-based user selection design. First, the access point sends out common pilot sequences. All K user terminals feed back their own channel norms. The access point selects K_s users opportunistically with regard to their channel norms. Then, the K_s selected users feed back full CSI. (In practice, limited feedback should be considered for the selected users because full feedback even for a small number of selected users is difficult to implement. The detailed discussion of limited feedback schemes is out of the scope of this paper. A comprehensive overview can be found in [16], especially for reduced-feedback opportunistic schemes such as [17].) The access point optimizes max-min fair beamformer for each user to alleviate multiuser interference. This setup is similar to the system discussed in [10, 18, 19] and can be summarized as follows

(1) the access point sends out common pilot sequences. The user terminals feed back their own channel norms,

(2) the access point selects users opportunistically with the K_s largest channel norms,

(3) the selected users feed back full CSI,

(4) the access point optimizes max-min fair beamformers.

In this setup, the opportunistic norm-based user selection is based on channel norms instead of SINR, because optimized max-min fair beamformers in the last step will significantly reduce the interference, and the *a priori* SINR information will be obsolete. This is different from the random opportunistic beamforming [5] where the beamformers remain the same, and user selection by *a priori* SINR is reasonable.

No fairness is explicitly considered in scheduling users for the sake of simplicity, but it is an important issue especially for nonelastic traffic or heterogeneous channels. Just as [8], this system setup can also be readily modified with the proportional fair scheduling [6] or the score-based scheduling [20].

2.2. Signal Model. The access point is assumed to have N transmit antennas ($N \geq 3$) (This assumption will be justified later in the analysis section.) while each user has only a single receive antenna. The carrier is modeled as a narrow band quasi static channel and the corresponding baseband received signal for user i is

$$y_i = \mathbf{h}_i^H \sum_{j\in\mathcal{S}} \mathbf{w}_j x_j + n_i, \tag{1}$$

where for user i, $\mathbf{h}_i \in \mathbb{C}^{N\times 1}$ is the baseband channel from the access point and has independent and identically-distributed (i.i.d.) elements distributed as $\mathcal{CN}(0,1)$; the transmitted signal is a zero-mean unit-energy uncorrelated scalar x_i, that is, $\mathbb{E}\{x_i x_i^c\} = 1$, and the beamformer at the access point is $\mathbf{w}_i \in \mathbb{C}^{N\times 1}$. The noise n_i is modeled as an additive white Gaussian noise with variance σ^2. Only a portion out of the K users is selected to access the channel at any transmission burst. Also $\mathcal{S}$ is the set of indices of those selected users, and $|\mathcal{S}| = K_s$. Furthermore, it is assumed that each user terminal knows its own channel perfectly. It follows directly that the SINR for user i is

$$\mathrm{SINR}_i(\{\mathbf{w}_i\}) = \frac{\left\|\mathbf{w}_i^H \mathbf{h}_i\right\|^2}{\sum_{i\in\mathcal{S}\setminus\{i\}}\left\|\mathbf{w}_j^H \mathbf{h}_i\right\|^2 + \sigma^2}. \quad (2)$$

The max-min fair beamformers are optimized to jointly maximize each user's SINR to meet the same SINR target ρ and minimize the total transmit power, $\sum_{i\in\mathcal{S}}\|\mathbf{w}_i\|^2$, at the access point [13]. More exactly, for fixed MCS, the max-min fair beamforming is defined as the solution to the following optimization problem:

$$\begin{aligned} \underset{\mathbf{w}}{\text{minimize}} \quad & \sum_{i\in\mathcal{S}}\|\mathbf{w}_i\|^2 \\ \text{subject to} \quad & \mathrm{SINR}_i \geq \rho, \quad \forall i \in \mathcal{S}. \end{aligned} \quad (3)$$

This type of beamforming is of interest to operators as it minimizes the interference and reduces the radiation power while maintaining targeted data rate. If the selected users' indices set $\mathcal{S}$ is determined, the max-min fair beamformer optimization problem (3) can be solved efficiently [21], although it is nonlinear and nonconvex.

The problem studied in this paper is user selections combined with such max-min fair beamforming. However, it is not trivial to optimally and efficiently select K_s users from the K candidates for each transmission burst. Such optimization problem can be generally described without limiting to any specific user selection method as

$$\begin{aligned} \underset{\mathcal{S},\mathbf{w}}{\text{minimize}} \quad & \sum_{i\in\mathcal{S}}\|\mathbf{w}_i\|^2 \\ \text{subject to} \quad & |\mathcal{S}| = K_s \\ & \mathrm{SINR}_i(\{\mathbf{w}_i\}) \geq \rho, \quad \forall i \in \mathcal{S}. \end{aligned} \quad (4)$$

In some cases, even the number of selected users are also optimized in user selection problems, but K_s is assumed to be a predefined system parameter in this paper.

Problem (4) is hard to solve in general as it requires combinatorial optimization over the user set, in additional to the simpler max-min fair beamformer optimization. The following section reviews four different user selection methods. The asymptotic optimality of the opportunistic norm-based user selection will be established by the comparisons of different methods and proper bounds.

3. Review of User Selection Methods

The simple but nonefficient optimal method to solve the user selection part in Problem (4) is exhaustive search, which tries all the possible combinatorial subsets of users and chooses the optimal subset to minimize the total transmit power. Obviously, its complexity is too high for any practical implementation even when the number of users is moderate. Thus, many other heuristic approaches have been proposed to solve the user selection problem suboptimally but very efficiently. Four methods are reviewed in the sequel: semiorthogonal user selection (SUS), angle-based user selection (AUS), opportunistic norm-based user selection (NUS), and random user selection (RUS).

3.1. Semiorthogonal User Selection (SUS). One way to ease the beamforming optimization is to select the users whose channels are as orthogonal as possible and also maintaining as large channel gains as possible. More exactly, it selects users one at a time, and each time it tries to maximize the channel projection to the orthogonal subspace spanned by the channels of all the users already selected. This is called semiorthogonal user selection and has different variants, such as [4, 8, 9, 22]. The uplink equivalent is discussed in [23].

If the extra angle threshold parameter is ignored, a simplified version of semiorthogonal user selection can be described as in the following pseudocode description (*cf.* [8]).

The SUS is a suboptimal user selection method and some special properties can be observed:

(i) full CSI for all users has to be available at the access point, which implies a high feedback requirement.

(ii) because projections are used during the selection, SUS is sensitive to imperfect CSI, such as estimation errors or even quantization errors.

(iii) even with the simplification in [8], projections have to be calculated for several iterations for a large fraction of users. This contributes to the computational complexity of SUS. In recent work of [24], a very nice technique was used to ease the complexity of SUS, which makes SUS more attractive in practical systems.

3.2. Angle-Based User Selection (AUS). When the channel norms are ignored and only the orthogonality,that is, the angle between the channels, is considered, the user selection method simply chooses the strongest user first and then selects other users one by one to maximize the angle between the user's channel and the subspace spanned by the channels of all the users already selected. This is named angle-based user selection, and can be described as in the following pseudocode description.

The AUS shows similar properties as the SUS:

(i) full CSI for all users has to be available at the access point to be able to calculate all the angles.

(ii) AUS is also sensitive to imperfect CSI.

(iii) similar to SUS, iterations are required in the selection process.

(iv) because the channel norm is ignored, a user with very poor SNR can potentially be selected. The performance of AUS is therefore expected to be inferior to SUS.

(1) Initialization

$$\mathcal{T} = \{1,\dots,K\}, \qquad \mathcal{S} = \emptyset$$

(2) Select the first user

$$\pi = \arg\max_{k\in\mathcal{T}} \|\mathbf{h}_k\|$$

$$\mathcal{S} \Longleftarrow \mathcal{S} \cup \{\pi\}, \qquad \mathcal{T} \Longleftarrow \mathcal{T} \setminus \{\pi\}, \qquad \mathbf{g}_\pi = \mathbf{h}_\pi$$

(3) For each $k \in \mathcal{T}$, calculate

$$\mathbf{g}_k = \Big(\mathbf{I} - \sum_{j\in\mathcal{S}} \frac{\mathbf{g}_j\mathbf{g}_j^H}{\|\mathbf{g}_j\|^2}\Big)\mathbf{h}_k$$

(4) Select the additional user as

$$\pi = \arg\max_{k\in\mathcal{T}} \|\mathbf{g}_k\|$$

$$\mathcal{S} \Longleftarrow \mathcal{S} \cup \{\pi\}, \qquad \mathcal{T} \Longleftarrow \mathcal{T} \setminus \{\pi\}$$

(5) Repeat Steps 3 and 4 until $|\mathcal{S}| = K_s$.

Algorithm 1: Semiorthogonal.

(1) Initialization

$$\mathcal{T} = \{1,\dots,K\}, \qquad \mathcal{S} = \emptyset$$

(2) Select the first user

$$\pi = \arg\max_{k\in\mathcal{T}} \|\mathbf{h}_k\|$$

$$\mathcal{S} \Longleftarrow \mathcal{S} \cup \{\pi\}, \qquad \mathcal{T} \Longleftarrow \mathcal{T} \setminus \{\pi\}$$

(3) For each $k \in \mathcal{T}$, calculate

$$g_k = \sum_{j\in\mathcal{S}} \frac{|\mathbf{h}_k^H\mathbf{h}_j|}{\|\mathbf{h}_k\|\|\mathbf{h}_j\|}$$

(4) Select the additional user as

$$\pi = \arg\min_{k\in\mathcal{T}} g_k$$

$$\mathcal{S} \Longleftarrow \mathcal{S} \cup \{\pi\}, \qquad \mathcal{T} \Longleftarrow \mathcal{T} \setminus \{\pi\}$$

(5) Repeat Steps 3 and 4 until $|\mathcal{S}| = K_s$.

Algorithm 2: Angle-based.

3.3. Opportunistic Norm-Based User Selection (NUS). When the orthogonality is fully ignored and only the channel norm is considered as the merit function to select users, the user selection is much simpler: order the channel norms in a descending order and pick the first K_s users in an opportunistic fashion. We name it opportunistic norm-based user selection as opposed to the SUS and AUS. This method can be described as in the following pseudocode description.

Unlike the SUS or AUS, NUS has very distinctive properties:

(i) no full CSI is required at the access point, the only information required is the channel norm, which is one quantized real number, from each user,

(ii) compared to SUS and AUS, which rely on computing projections, NUS is relatively insensitive to channel estimation errors or the quantization error,

(iii) no iterations are required during the selection,

(iv) contrary to AUS, user orthogonality is fully ignored during the selection, so two highly collinear users may be scheduled when their channel norms are large. This indicates potential performance degradation.

3.4. Random User Selection (RUS). Another totally different approach is to select users randomly. This simplest user selection method can be described in a similar way as in the following pseudocode description.

The RUS is the simplest selection method:

(i) no CSI is required at the access point,

(ii) as no CSI is considered during the user selection, inferior performance can be expected compared to the previous three selection methods,

(iii) RUS is a fair scheduling strategy.

4. Performance Analysis

The performance of different user selection methods is compared in this section to establish the asymptotic optimality of opportunistic norm-based user selection in connection with max-min fair beamformer. The cases when $K_s = 2$ are considered first and closed form solutions of the average beamforming powers are obtained. (Note that a capacity-achieving beamforming scheme will simultaneously transmit N beams, which means N users should be selected ($K_s = N$). In our setup with max-min fair beamformer, the main objective is to minimize transmit power while satisfying

(1) For $k = 1, \ldots, K$, sort all $\|\mathbf{h}_k\|$, such that
$\|\mathbf{h}_{\pi(1)}\| \geq \|\mathbf{h}_{\pi(2)}\| \geq \cdots \geq \|\mathbf{h}_{\pi(K)}\| \geq 0$
(2) $\mathcal{S} = \{\pi(1), \pi(2), \ldots, \pi(K_s)\}$

Algorithm 3: Opportunistic Norm-Based.

(1) Let $\{\pi(1), \pi(2), \ldots, \pi(K_s)\}$ be a set of any K_s random indices from $\{1, 2, \ldots, K\}$
(2) $\mathcal{S} = \{\pi(1), \pi(2), \ldots, \pi(K_s)\}$

Algorithm 4: Random.

SINR constraints for selected users with fixed MCS, not to maximize rum-rate or other capacity related metrics. Hence the case of $K_s = 2$ is of interest to start with.) Based on the solutions, opportunistic norm-based user selection is shown to be asymptotically optimal in high SNR regime as N goes to infinity. When $K_s \geq 3$, the corresponding analytical proof is still open due to lack of closed form solutions of average beamforming powers.

4.1. Average Beamforming Power Comparison. When only two users are selected for transmission in a single time slot, $K_s = 2$, the minimum transmit max-min fair beamforming power p_t in the solution of (3) can be expressed in closed form as suggested in [14, Chapter 4.3.2]:

$$\begin{aligned} p_t(\theta, \mathbf{h}_1, \mathbf{h}_2) &= \sum_{i \in \mathcal{S}} \|\mathbf{w}_i\|^2 \\ &= \frac{\sigma^2\left(\rho - 1 + \sqrt{1 + \rho^2 - 2\rho\cos(2\theta)}\right)}{2\sin(\theta)^2} \\ &\quad \times \left(\frac{1}{\|\mathbf{h}_1\|^2} + \frac{1}{\|\mathbf{h}_2\|^2}\right), \end{aligned} \tag{5}$$

where θ is the angle between $\mathbf{h}_1$ and $\mathbf{h}_2$ as defined in Appendix A.

Combining the beamforming power expression above with the angle and norm distributions based on the i.i.d. Gaussian channel model, the average beamforming powers for different user selection methods can be calculated.

Theorem 1. *When the SINR target is large, that is, $\rho \gg 1$, the average beamforming power for AUS is*

$$p_a = \rho\sigma^2\left(\frac{1}{N-1} + \left(1 - \frac{2}{K}\right)\alpha_{N,K}\right)\frac{(N-1)K}{(N-1)(K-1)-1}, \tag{6}$$

the average beamforming power for NUS is

$$p_n = \rho\sigma^2\left(\alpha_{N,K-1} - \left(1 - \frac{2}{K}\right)\alpha_{N,K}\right)\frac{(N-1)K}{N-2}, \tag{7}$$

and the average beamforming power for RUS is

$$p_r = \rho\sigma^2\frac{2}{N-2}, \tag{8}$$

where $\alpha_{N,K}$ is a constant decided by N and K as

$$\alpha_{N,K} = \int_0^\infty K\frac{e^{-x}x^{N-2}}{\Gamma(N)}P(N,x)^{K-1}dx, \tag{9}$$

and $P(N,x)$ is the regularized gamma function [25].

Proof. See Appendix A. □

The results of Theorem 1 agree very well with the simulation results shown in Figures 1 and 2. Some key observations include the following.

(i) Although Theorem 1 refers to very large SINR constraint, the results are valid for realistic choices of smaller SINR as well, such as 10 dB used in Figures 1 and 2.

(ii) When N is reasonably large, the performance difference between SUS, NUS and AUS is small. In Figure 1, the power difference between NUS and SUS is less than 0.2 dB; and even the random RUS performs relatively well, with a performance loss of less than 1 dB.

(iii) When more users can be selected from, that is, larger K, the performance differences are more noticeable as in Figure 2.

Figure 1 also suggests that as N grows, the performance difference between SUS and NUS vanishes. This observation is explained by the asymptotic results in the following section.

4.2. Asymptotic Optimality. When $K_s = 2$, it is easy to obtain a lower bound on the average max-min fair beamforming power $\mathbb{E}\{p_t\}$ for any user selection method by considering a dummy second user, whose angle equals the maximal angle chosen by AUS and whose channel norm equals the second largest norm chosen by NUS, as illustrated in the following lemma.

Lemma 2. *When the SINR target is large, that is, $\rho \gg 1$, there exists a lower bound p_l for the average max-min fair beamforming power $\mathbb{E}\{p_t\}$ for any user selection method:*

$$p_l = \rho\sigma^2\left(\alpha_{N,K-1} - \left(1 - \frac{2}{K}\right)\alpha_{N,K}\right)\frac{(N-1)(K-1)K}{(N-1)(K-1)-1}. \tag{10}$$

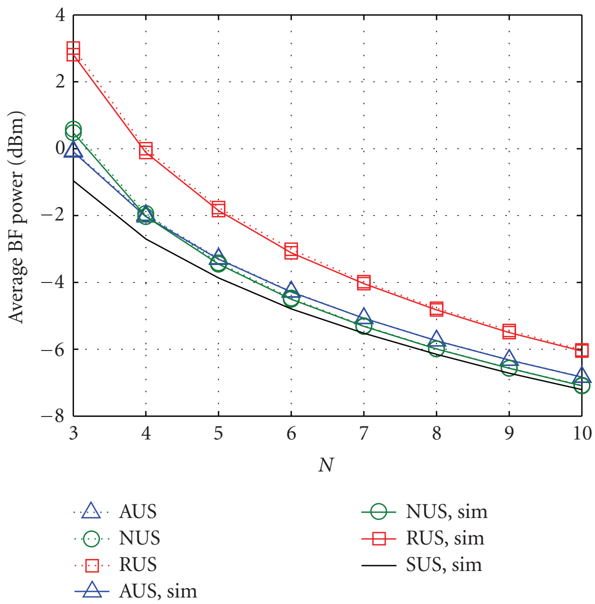

Figure 1: Average beamforming power versus N for different user selection methods, $K = 4$, $K_s = 2$, $\sigma^2 = 0.1$, $\rho = 10$ dB.

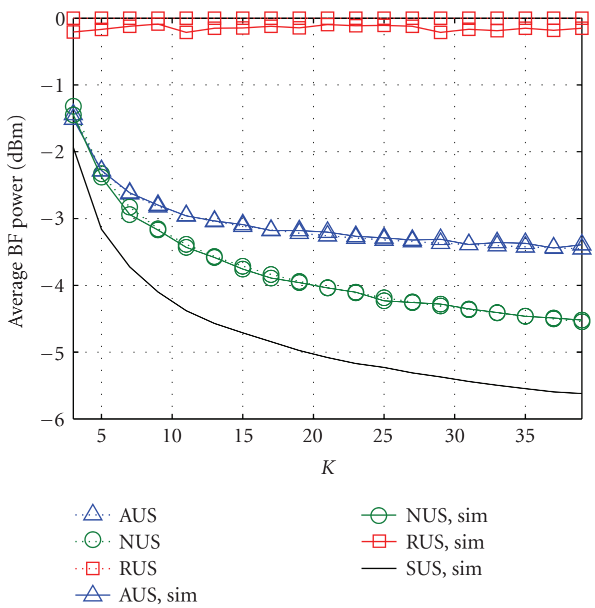

Figure 2: Average beamforming power versus K for different user selection methods, $N = 4$, $K_s = 2$, $\sigma^2 = 0.1$, $\rho = 10$ dB.

Proof. See Appendix B. □

Comparing this lower bound with the average max-min fair beamforming powers for different user selection methods in Theorem 1, and letting either the number of antennas, N, or the number of users, K, go to infinity, the asymptotic optimality of NUS and SUS can be established.

Theorem 3. *When the SINR target is large, that is, $\rho \gg 1$, and for fixed K, as N goes to infinity, the performance difference*

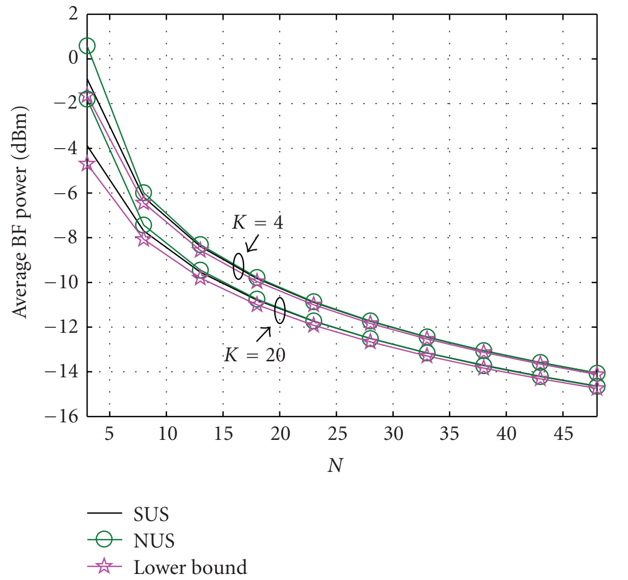

Figure 3: Asymptotic average beamforming power and lower bound versus N for different K, $K_s = 2, \sigma^2 = 0.1, \rho = 10$ dB.

between the lower bound p_l and the NUS or SUS goes to zero for max-min fair beamforming, that is,

$$\lim_{N \to \infty} \frac{p_n}{p_l} = \lim_{N \to \infty} \frac{p_s}{p_l} = 1. \tag{11}$$

However, for fixed N, NUS is bounded away from the lower bound p_l by a constant, as K goes to infinity:

$$\lim_{K \to \infty} \frac{p_n}{p_l} = \frac{N-1}{N-2}. \tag{12}$$

Proof. See Appendix B. (Note that (12) does not contradict with the results from [5, 6] because of different system models and optimization objectives used. The capacity expressions based on the strict SINR constraints are sometimes called delay-limited capacity. Therefore, the asymptotic results differ between ergodic and delay-limited capacity.) □

Figures 3 and 4 demonstrate the asymptotic behavior of NUS and SUS. The lower bound in Lemma 2 is very tight even when N is small, indicating that NUS and SUS are close to optimal even for small N. The performance of SUS is difficult to analyze, but as it is bounded between NUS and the lower bound, its performance can be roughly approximated by the behavior of the lower bound and NUS.

Furthermore, in order to characterize the benefit of adding additional transmit antennas at the access point, several scaling laws of these average beamforming powers can be established.

Lemma 4. *When the SINR target is large, that is, $\rho \gg 1$, and for a fixed K, as N grows, the average max-min fair beamforming powers for AUS and RUS scale as $1/N$, while the average max-min fair beamforming powers for NUS and SUS scale at least as fast as $1/N$.*

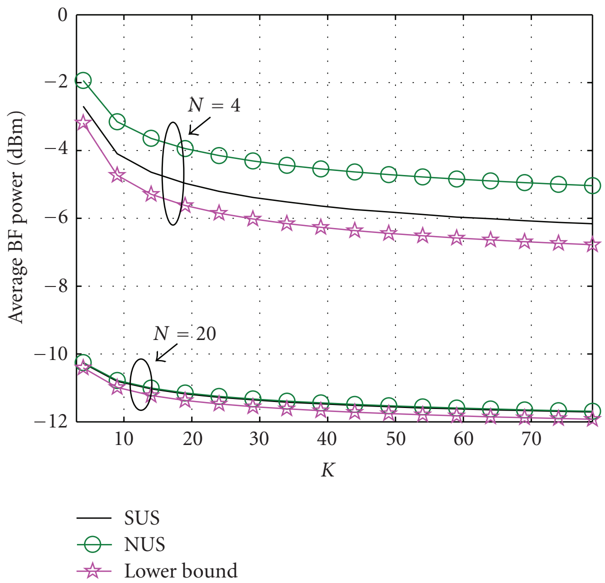

Figure 4: Asymptotic average beamforming power and lower bound versus K for different N, $K_s = 2, \sigma^2 = 0.1, \rho = 10$ dB.

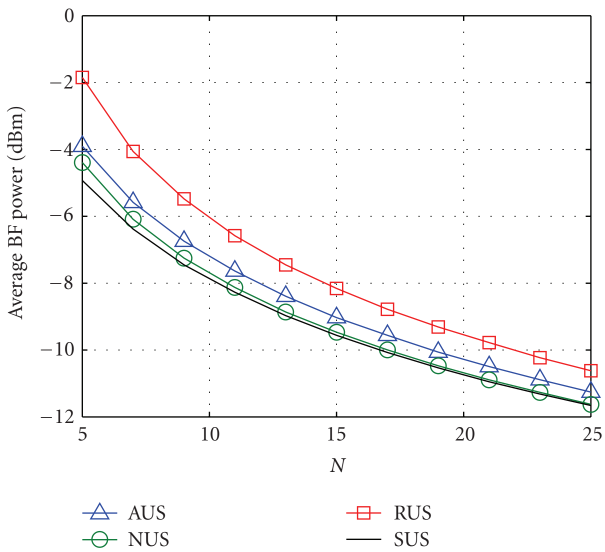

Figure 5: Average beamforming power versus N for different user selection methods, $K = 8$, $K_s = 4$, $\sigma^2 = 0.1$, $\rho = 10$ dB.

Proof. See Appendix C. □

The analysis of different user selection methods presented above holds for the case $K_s = 2$. If more than 2 users are selected, closed form solutions are difficult to obtain. However, when $K_s \geq 3$, the user selection methods have roughly similar relative performance relations, according to our various observations.As examples, simulations shown in Figures 5 and 6 demonstrate the case with $K_s = 4$. In Figure 5, NUS performs very close to SUS and the difference is negligible when $N \geq 10$. In Figure 6, the performance loss between NUS and SUS is less than 1 dB for different K, even when N is as small as 5. In the left half of the

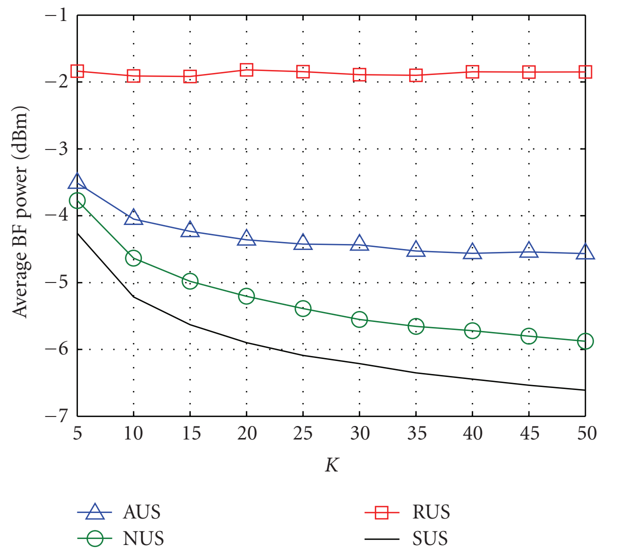

Figure 6: Average beamforming power versus K for different user. $N = 5$, $K_s = 4$, $\sigma^2 = 0.1$, $\rho = 10$ dB.

curves in Figure 6, where K is small, the performance loss is even smaller. This is of practical significance. Because the performance loss is negligible for the cases illustrated above, it suffices to use the simple NUS scheme under certain scenarios.

5. Experimental Data Verification

To further illustrate the performance of the opportunistic norm-based user selection in a max-min fair beamforming system, and to eliminate possible artifacts from the i.i.d. channel model, real measurement data is used for verification. The MIMO multiuser channel measurement at 2.45 GHz band was carried out in the David Packard Building at Stanford University, which is a typical office building scenario as shown in the map in Figure 7. The access point transmitter (marked as a star in the map) is placed in front of the main door in the hall and its antenna beams are directed to the middle of the hall. Slowly moving (or stationary) user terminals are placed in 8 different locations in the corridors. Only non-line-of-sight (NLOS) channels are measured. The system is a 3×1 MIMO system, that is, only one receive antenna at the user terminals and three transmit antennas at the access point. Detailed parameters for the measurement can be found in Table 1.

The four different user selection algorithms are evaluated on the channel measurements from this setup: SUS, NUS, RUS and the optimal exhaustive search. The maximum allowed transmit power is set to an extreme amount (As the max-min fair beamformer will minimize the total transmit power, an extreme high transmit power limit will eliminate power clipping for the beamformer. No selected users will be in outage due to insufficient transmit power available for allocation. This assumption will ensure fair comparison across different selection methods.) to avoid user outage caused by transmit power clipping. SINR target is $\rho = 5$ dB

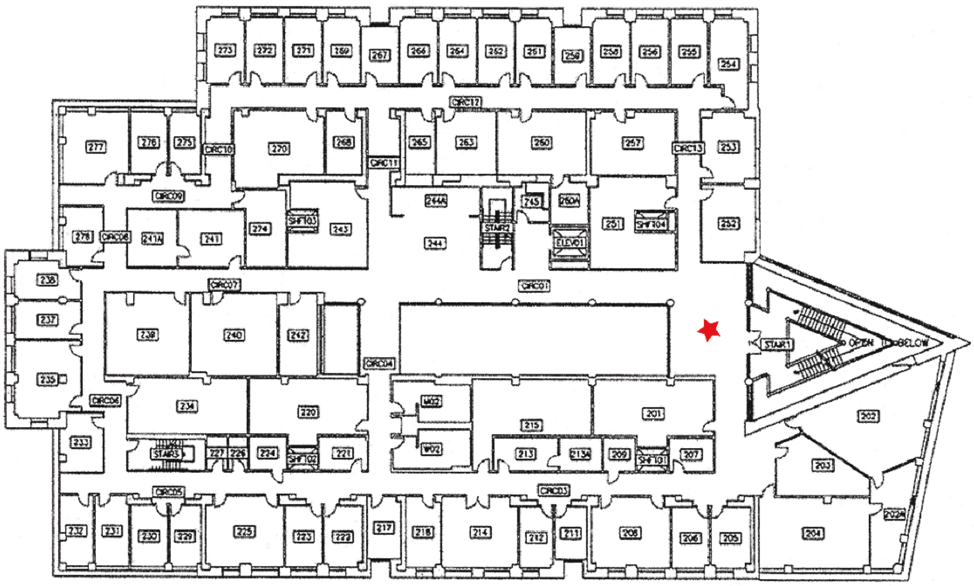

Figure 7: Illustration of measurement building floor map.

Table 1: Measurement parameters.

environment	indoor, NLOS
center frequency	2.45 GHz
bandwidth	100 MHz
nr. of subcarriers	161
nr. of users	8
nr. of antennas on AP	3
antenna spacing on AP	$0.5\,\lambda$ ULA
type of antennas on AP	sectorized (120°)
nr. of antennas on user	1
type of antennas on user	omnidirectional disc cone
position of AP	entrance hall
position of users	corridors
nr. of snaps	1000

and the background noise level $\sigma^2 = 10^{-3}$. There are 8 users sharing each subcarrier, and 2 users are selected at one time to communicate with on that frequency. For each subcarrier, the optimized max-min fair beamforming powers are averaged over 1000 channel realizations, and the same comparisons are repeated for all 161 frequencies across the 100 MHz bandwidth. The experiment is repeated after the access point is relocated into one of the long corridors to mimic highly correlated transmission scenario (also *cf.* [26]). Results are summarized in Table 2 for the beamforming powers averaged over the whole bandwidth under these two different scenarios. In such typical indoor office situation, corridors have the effect of isolating interferences for selected users. Therefore the obtained results are similar to the numerical simulations in Section 4 which is based on i.i.d. Gaussian channels. In Table 2, NUS performs very close to SUS for almost all the subcarriers. In fact, both NUS and SUS perform very close to the optimal user selection achieved by the exhaustive search. On average, NUS or SUS requires only 1.1% or 0.02% more power than the exhaustive search respectively. This agrees with our previous theoretical development. When the access point is in the long corridor, the performance difference between NUS and SUS is more visible than in the open hall setup, but still remains very small. This also essentially matches with the recent work reported in [27] based on $K_s = 2$ and correlated channels. However, in both cases, the RUS scheme requires far more power than the exhaustive search.

The whole experiment is also repeated when 3 users are selected, $K_s = 3$, and the results are summarized in Table 3. In this case, all spatial degrees of freedom are used to accommodate three users, as there are only three transmit antennas at the access point. The performance of SUS is still quite close to the exhaustive search, but NUS shows some degradation. In order to safely ignore the orthogonality between users during the user selection process, more transmit antennas are required, which is suggested by Theorem 3. In practice, as shown in Tables 2 and 3, there should be at least one more transmit antenna than the number of selected user. Such an extra antenna is enough to bring the performance gap between NUS and exhaustive search to a negligible level for max-min fair beamforming systems.

6. Conclusion

We studied the performance of four user selection algorithms for the MIMO broadcast channel, in conjunction with the max-min fair beamforming that guarantees certain SINR requirements under transmit power minimization. It is shown that both the opportunistic norm-based user selection (NUS) and the semiorthogonal user selection

Table 2: User selection comparison $K_s = 2$, average transmit power (dBm).

	SUS	NUS	RUS	Exhaustive Search
Hall TX	20.9235	21.0194	46.7641	20.9215
Corridor TX	25.6084	26.3677	49.9515	25.5762

Table 3: User selection comparison $K_s = 3$, average transmit power (dBm).

	SUS	NUS	RUS	Exhaustive Search
Hall TX	32.3481	36.9216	57.2865	31.9293
Corridor TX	34.4997	40.9564	60.3061	33.9473

(SUS) are asymptotically optimal, as the number of transmit antennas goes to infinity when only two users are selected in high SNR regime. Only insignificant performance loss is observed when limited number of transmit antennas and/or median range of users are available, which is confirmed by simulations based on both the simple channel model and real measurement data.

The good nonasymptotic performance of NUS is mainly due to the fact that finding users with small interference turns out to be easier than we expected (especially with real measurement). Intuitively, the vector angle distribution obtained in the appendix indicates that user channels tend to be close to orthogonal very quickly as the number of transmit antenna increases. (This intuitive argument is true only if the total number of selected users is fixed while the number of transmit antennas increases. When maximizing system throughput is the objective, the number of selected users will remain the same as the number of transmit antennas, and increases together. In such systems setup, the mentioned intuition might not hold.) We suggest that this finding might be of interest in distributed indoor office MIMO scenarios, where the isolation effect is beneficial. Hence channel norms or SNRs could be the dominating factor during user selection.

Appendices

A. Proof of Theorem 1

Define the angle between two complex vectors $\mathbf{u}, \mathbf{v} \in \mathbb{C}^{N\times 1}$ as

$$\cos\theta_{(\mathbf{u},\mathbf{v})} = \frac{|\mathbf{u}^H\mathbf{v}|}{\|\mathbf{u}\|\|\mathbf{v}\|}, \quad 0 \le \theta \le \frac{\pi}{2}. \tag{A.1}$$

It is easy to see the following three results.

Lemma 5. *When $\mathbf{u} \in \mathbb{C}^{N\times 1}$ and $\mathbf{v} \in \mathbb{C}^{N\times 1}$ have i.i.d. elements distributed as $\mathcal{CN}(0,1)$, the pdf of the angle $\theta(\mathbf{u},\mathbf{v})$ is*

$$f_\theta(\theta) = (2N-2)\cos(\theta)\sin(\theta)^{2N-3}. \tag{A.2}$$

Lemma 5 simply follows from the fact that the random variable $\cos^2(\theta)$ is known to have Beta $(1, N-1)$ distribution [28].

Similarly, when there are K random vectors $\mathbf{u}_1, \ldots, \mathbf{u}_K$, and they are ordered by the norm, that is, $\|\mathbf{u}_1\| \ge \|\mathbf{u}_2\| \ge \cdots \ge \|\mathbf{u}_K\|$, the maximal angle between $\mathbf{u}_1$ and the rest of the vectors can be defined as

$$\phi = \max_i \theta(\mathbf{u}_i, \mathbf{u}_1), \quad i = 2, \ldots, K. \tag{A.3}$$

By simple derivation following the pdf relation as a function of identically distributed random variables per [29], it is easy to see

Lemma 6. *When K random vectors $\mathbf{u}_i \in \mathbb{C}^{N\times 1}$ have i.i.d. elements distributed as $\mathcal{CN}(0,1)$, the maximal angle ϕ is distributed as*

$$f_\phi(\phi) = 2(N-1)(K-1)\cos(\phi)\sin(\phi)^{2(N-1)(K-1)-1}. \tag{A.4}$$

In addition to the angle distributions, the vector norm distribution is also obtained.

Lemma 7. *When K random vectors $\mathbf{u}_i \in \mathbb{C}^{N\times 1}$ have i.i.d. elements distributed as $\mathcal{CN}(0,1)$, the maximal squared ℓ_2 vector norm $\|\mathbf{u}_1\|^2$ has cdf and pdf as $(x \ge 0)$*

$$F_1(x) = P(N,x)^K, \tag{A.5}$$

$$f_1(x) = K\frac{e^{-x}x^{N-1}}{\Gamma(N)}P(N,x)^{K-1}. \tag{A.6}$$

The second maximal squared ℓ_2 vector norm $\|\mathbf{u}_2\|^2$ has pdf as $(x \ge 0)$

$$f_2(x) = K(K-1)\frac{e^{-x}x^{N-1}}{\Gamma(N)}\left(P(N,x)^{K-2} - P(N,x)^{K-1}\right), \tag{A.7}$$

where $P(N,x)$ is the regularized gamma function [25].

The proof of Lemma 7 is direct. The distribution of $\|\mathbf{u}_i\|^2$ is chi-square with $2N$ degree of freedom [25], so the cdf of $\|\mathbf{u}_1\|^2$ and $\|\mathbf{u}_2\|^2$ is easily obtained by order statistics relation [29].

When the SINR target is large, that is, $\rho \gg 1$, the beamforming power in (5) averaged over the channel realizations can be written as

$$\begin{aligned}&\mathbb{E}\{p_t(\theta, \mathbf{h}_1, \mathbf{h}_2)\}\\ &\quad = \mathbb{E}\left\{\frac{\rho\sigma^2}{\sin^2(\theta)}\right\}\left(\mathbb{E}\left\{\frac{1}{\|\mathbf{h}_1\|^2}\right\} + \mathbb{E}\left\{\frac{1}{\|\mathbf{h}_2\|^2}\right\}\right).\end{aligned} \tag{A.8}$$

For AUS, $\|\mathbf{h}_1\|$ is the largest norm in Step 2. The angle θ is the largest angel between $\mathbf{h}_1$ and other channel vectors, so its pdf is $f_\phi(\theta)$. Due to the independence between the angle and the norm, $\|\mathbf{h}_2\|$ is any channel norm from the K users except the largest one. The average beamforming power is therefore

$$p_a = \mathbb{E}\{p_t(\theta, \mathbf{h}_1, \mathbf{h}_2)\} = q_1(q_2 + q_3), \tag{A.9}$$

where

$$q_1 = \mathbb{E}\left\{\frac{\rho\sigma^2}{\sin^2(\phi)}\right\}, \tag{A.10}$$

$$q_2 = \mathbb{E}\left\{\frac{1}{\|\mathbf{h}_i\|^2} \mid \|\mathbf{h}_i\| \text{ is the largest norm}\right\}, \tag{A.11}$$

$$q_3 = \mathbb{E}\left\{\frac{1}{\|\mathbf{h}_i\|^2} \mid \|\mathbf{h}_i\| \text{ is not the larges tnorm}\right\}. \tag{A.12}$$

In what follows, we will calculate q_1, q_2, and q_3 separately. It is not difficult to see that when $N \geq 3$, $K \geq 2$, the expectation

$$\begin{aligned} q_1 &= \rho\sigma^2 \int_0^{\pi/2} \frac{1}{\sin^2(x)} f_\phi(x)dx \\ &= \rho\sigma^2 \frac{(N-1)(K-1)}{(N-1)(K-1)-1}, \end{aligned} \tag{A.13}$$

$$q_2 = \int_0^\infty \frac{1}{x} f_1(x)dx = \alpha_{N,K},$$

where $f_1(x)$ is given in (A.6).

Similarly, we have

$$q_3 = \frac{K}{(N-1)(K-1)} - \frac{1}{K-1}\alpha_{N,K}, \tag{A.14}$$

and finally

$$p_a = \rho\sigma^2\left(\frac{1}{N-1} + \left(1 - \frac{2}{K}\right)\alpha_{N,K}\right)\frac{(N-1)K}{(N-1)(K-1)-1}. \tag{A.15}$$

For NUS, it is clear that $\|\mathbf{h}_1\|$ is the largest norm, $\|\mathbf{h}_2\|$ is the second largest norm, and θ is independently distributed with $f_\theta(\theta)$. The average beamforming power is therefore

$$p_n = \mathbb{E}\{p_t(\theta, \mathbf{h}_1, \mathbf{h}_2)\} = q_5(q_2 + q_6), \tag{A.16}$$

where q_2 is defined in (A.11) and

$$q_5 = \mathbb{E}\left\{\frac{\rho\sigma^2}{\sin^2(\theta)}\right\},$$

$$q_6 = \mathbb{E}\left\{\frac{1}{\|\mathbf{h}_i\|^2} \mid \|\mathbf{h}_i\|^2 \text{ is the second largest norm}\right\}. \tag{A.17}$$

More exactly, when $N \geq 3$, the first expectation is

$$q_5 = \rho\sigma^2 \int_0^{\pi/2} \frac{1}{\sin^2(x)} f_\theta(x)dx = \rho\sigma^2 \frac{N-1}{N-2}, \tag{A.18}$$

and the second expectation is

$$q_6 = \int_0^\infty \frac{1}{x} f_2(x)dx = K\alpha_{N,K-1} - (K-1)\alpha_{N,K}, \tag{A.19}$$

where $f_2(x)$ is defined in (A.7). Hence

$$p_n = \rho\sigma^2\left(\alpha_{N,K-1} - \left(1 - \frac{2}{K}\right)\alpha_{N,K}\right)\frac{(N-1)K}{N-2}. \tag{A.20}$$

For random user selection, the squired norm $\|\mathbf{h}_i\|^2$ is distributed as $f_i(x)$ and the angle, θ, is independently distributed as $f_\theta(\theta)$. The average beamforming power is given by

$$\begin{aligned} p_r &= \mathbb{E}\{p_t(\theta, \mathbf{h}_1, \mathbf{h}_2)\} = q_5(q_4 + q_4) \\ &= \rho\sigma^2 \frac{2}{N-2}. \end{aligned} \tag{A.21}$$

B. Proof of Lemma 2 and Theorem 3

The lower bound can be constructed by assuming there exists a dummy user: it has the second largest norm, and at the same time, it also has the maximal angle to the first selected user:

$$p_l = \mathbb{E}\{p_t(\theta, \mathbf{h}_1, \mathbf{h}_2)\} = q_1(q_2 + q_6). \tag{B.22}$$

Evoking the values of q_1, q_2, and q_6 from Appendix A, Lemma 2 is proved.

With this lower bound and the expression of average power of NUS in Theorem 1, it is easy to reach

$$\frac{p_n}{p_l} = \frac{(N-1)(K-1)-1}{(N-2)(K-1)}. \tag{B.23}$$

Take the limit when $N \to \infty$ while K is fixed

$$\lim_{N\to\infty} \frac{p_n}{p_l} = 1. \tag{B.24}$$

Take the limit when $K \to \infty$ while N is fixed

$$\lim_{K\to\infty} \frac{p_n}{p_l} = \frac{N-1}{N-2} > 1. \tag{B.25}$$

From (5), when $\rho \gg 1$ and $K_s = 2$, the optimal user selection method should minimize

$$\frac{1}{\sin^2(\theta)\|\mathbf{h}_1\|^2} + \frac{1}{\sin^2(\theta)\|\mathbf{h}_2\|^2}. \tag{B.26}$$

While SUS in fact minimizes

$$\frac{1}{\|\mathbf{h}_1\|^2} + \frac{1}{\sin^2(\theta)\|\mathbf{h}_2\|^2}, \tag{B.27}$$

NUS ignores the angle between the channel vectors and only tries to minimize

$$\frac{1}{\|\mathbf{h}_1\|^2} + \frac{1}{\|\mathbf{h}_2\|^2}. \tag{B.28}$$

Thus, the performance of NUS is lower bounded also by SUS, which means $p_l \leq p_s \leq p_n$. Because of (B.24), the average power of SUS is also asymptotically optimal:

$$\lim_{N\to\infty} \frac{p_s}{p_l} = 1. \tag{B.29}$$

C. Proof of Lemma 4

For RUS, the result is direct because

$$p_r = \rho\sigma^2 \frac{2}{N-2} \sim \frac{1}{N}. \tag{C.30}$$

For AUS, because $\alpha_{N,K}$ is positive, the average beamforming power is bounded from below:

$$\begin{aligned} p_a &\geq \rho\sigma^2 \frac{1}{N-1} \frac{(N-1)K}{(N-1)(K-1)-1} \\ &= \rho\sigma^2 \frac{K}{(N-1)(K-1)-1} \sim \frac{1}{N}. \end{aligned} \tag{C.31}$$

However, because $0 \leq P(N,x) \leq 1$, $\alpha_{N,K}$ can be upper bounded:

$$\alpha_{N,K} \leq \int_0^\infty K \frac{e^{-x}x^{N-2}}{\Gamma(N)} dx = \frac{K}{N-2}. \tag{C.32}$$

Therefore p_a is also bounded from above:

$$\begin{aligned} p_a &\leq \rho\sigma^2 \left(\frac{1}{N-1} + \frac{K-2}{N-2} \right) \frac{(N-1)K}{(N-1)(K-1)-1} \\ &\sim \frac{1}{N}. \end{aligned} \tag{C.33}$$

Both the upper and lower bound scale as $1/N$, so p_a also scales as $1/N$.

For NUS, again because $\alpha_{N,K}$ is positive, the average beamforming power is bounded from above:

$$\begin{aligned} p_n &\leq \rho\sigma^2 \alpha_{N,K-1} \frac{(N-1)K}{N-2} \\ &\leq \rho\sigma^2 \frac{K-1}{N-2} \frac{(N-1)K}{N-2} \sim \frac{1}{N}. \end{aligned} \tag{C.34}$$

Hence p_n scales at least as $1/N$. As $p_s \leq p_n$, p_s scales also at least as $1/N$.

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