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Research Article

Weak GPS Signal Acquisition Algorithm Based on Chaotic Oscillator

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To detect weak Global Position System (GPS) signal indoors, various high-sensitivity detection algorithms have been proposed. However, a common tradeoff between high-sensitivity and computation burden impedes development of high sensitivity GPS receiver. As another strategy, chaotic oscillator, sensitive to periodic signal and inert to noise, possesses huge advantage in weak signal acquisition. In this paper chaotic oscillator is employed in weak GPS signal acquisition. With numerical indication of Lyapunov exponents (LEs), chaotic oscillator can achieve acquisition in extremely weak GPS signal. Compared with conventional algorithm, chaotic oscillator consumes less acquisition time and is capable of detecting weak GPS signal. In the final section of paper, results from computer simulation illustrate that chaotic oscillator algorithm can acquire GPS signal at −48 dB/2MHz SNR.

1. Introduction

GPS signal indoors become extremely weak because of fading, refraction, reflection, and multipath interference [1]. Normally line-of-sight GPS signal is at 44 dBHz [2], while signal strength will degrade larger than 25 dB in bad case [3, 4]. However common commercial GPS receiver can only acquire GPS signal above 38 dBHz. To achieve successful position and navigation in such challenging environment, much work has been performed, which focused on prolonging integration duration to increase SNR.

Until now there are three typical solutions to weak GPS signal acquisition in high-sensitivity receiver, alternate half-bit accumulation [5], full-bit accumulation with estimation of bit transition time [5], and differentially coherent acquisition algorithm [6]. Alternate half-bits algorithm [5] divided raw GPS signal data into equal cells by the length of 10 milliseconds, and all cells alternately belong to two groups. As the epoch of data bit is 20 milliseconds, such division by 10 milliseconds ensured that one of the two groups avoids data bit transition. Simulation in [5] illustrated well performance of alternate half-bit accumulation in detecting weak GPS signal. However alternate half-bit algorithm demanded huge computations. Full-bit accumulation divided processing data into 20 groups, and any two adjacent groups started at 1-millisecond time difference, the epoch of code chip. As one of the 20 groups was assured to avoid data bit transition, integration duration could be theoretically prolonged without limits. In fact, hardware implementation did not allow such duration extension. Furthermore, computation burden caused by 20 groups is amazing. In differential coherent [6], perform received data correlation with replica signal at the length of 1 millisecond, multiply all the adjacent two correlation results, accumulate all products to get detection statistic, and compare the statistic with threshold to judge whether expected signal exist. As claimed in [6], differential coherent accumulation is insensitive to data bit transition and tolerates Doppler shift. However, it sacrifices square gain.

To improve high-sensitivity performance, all the three conventional algorithms pile up raw data to increase SNR. But the soaring computation burden constrains development of high-sensitivity receiver. Chaotic oscillator [7] is sensitive to periodic signal and inert to noise, which can be utilized to achieve successful acquisition in weak signal. Bo [8] used chaotic oscillator to detect weak linear frequency modulation (LFM) signal after dechirping. Chaotic oscillator can detect extremely weak LFM signal, even at −27 dBHz. Ding [9] developed a practicable method in detecting weak periodic signal with indication of Lyapunov Exponent (LE). Furthermore, Ding proposed an effective method of computing LE, which contributes a lot to signal detection based on chaotic oscillator.

Up to now (to our knowledge), chaotic oscillator has not been employed in weak GPS signal detection. In this paper, a GPS signal acquisition algorithm based on Duffing chaotic oscillator is presented. Without prolonging accumulation interval, this method is totally different from conventional acquisition algorithm. In Section 5, simulation results demonstrate that chaotic oscillator algorithm outperforms conventional ones.

2. Duffing Oscillator Acquisition Method

In this section, chaotic oscillator will be introduced. In the field of nonlinear dynamics, various chaotic oscillators have been brought up. Among the chaotic oscillators, Duffing oscillator has been extensively studied [8–10]. The application of Duffing oscillator will be exemplified in details below.

2.1. Physical Explanation of Duffing Oscillator Acquisition in Weak Signal. Duffing Oscillator was brought into nonlinear dynamics in 1918 by Duffing. D [7]. Its primal form is $\ddot{x}(t) + x(t) + \varepsilon x^3(t) = 0$. Holmes [11] modified the primal equation and reached novel one which depicts forced double-well model, as follows:

$$\ddot{x}(t) + k\dot{x}(t) - ax(t) + bx^3(t) = f\cos(t). \tag{1}$$

The equation represents a strengthened spring system with cubic restore force term. Constant k is a damping coefficient, constants a and b are real coefficients, the term $ax(t) - bx^3(t)$ is nonlinear restore force, and constant f denotes amplitude of periodic disturbing force (PEDF) $f\cos(t)$. State of oscillator system (1) changes obviously when the frequency of input signal is close to frequency of PEDF. Such characteristic is key in weak signal acquisition.

2.2. Principle of Duffing Oscillator in Acquisition GPS Signal. From (1), the state equations of Duffing oscillator can be derived, as

$$\begin{aligned} \dot{x} &= y, \\ \dot{y} &= -k \cdot y + x - x^3 + f\cos(t). \end{aligned} \tag{2}$$

States of Duffing oscillator can be mapped on phase plane. As (2) is a Poincaré map, states of the oscillator can be indicated by fixed points of (2) [7]. Solution procedure of these fixed points is described as below.

Let $\dot{x} = 0$, $\dot{y} = 0$, (2) can be written as

$$\begin{aligned} y &= 0, \\ -k \times y + x - x^3 + f\cos(t) &= 0. \end{aligned} \tag{3}$$

Firstly, consider a simplified situation without PEDF, that is, $f = 0$. State (3) can be rewritten as

$$\begin{aligned} y &= 0, \\ -k \times y + x - x^3 &= 0. \end{aligned} \tag{4}$$

Three fixed points $(0,0)$, $(1,0)$, $(-1,0)$ are gotten by solving (4).

The Jacobian matrix of (4) is

$$J = \begin{pmatrix} 0 & 1 \\ 1 - 3x^2 & -k \end{pmatrix}. \tag{5}$$

Here set k as a typical value, $k = -0.5$ [9].

At the fixed point $(x, y) = (0,0)$ in equation $\Delta = \|J\| = -1 < 0$ holds, which means that $(0,0)$ is a saddle point [7].

At fixed points $(x, y) = (\pm 1, 0)$ in equation $\Delta = \|J\| = 2 > 0$ holds, trace of J equals to -0.5, and in equation $\tau^2 - 4\Delta < 0$ holds. Therefore partial conclusion can be drawn that points $(\pm 1, 0)$ are two stable spirals. So the two points $(\pm 1, 0)$ form a strange attractor [7]. All the trajectories in phase plane are attracted by this strange attractor.

In fact, amplitude f of the PEDF is limited in the range from 0 to 1, which is small enough to be ignored in engineering application. Therefore the conclusion in the end of the former paragraph stands still, when $f \neq 0$. However the tiny disturbance from $f\cos t$ will change motion states of Duffing oscillator. It is the change which indicates whether expected signal exists in received data.

2.3. GPS Signal Model. Mark [5] proposed a typical received GPS signal model as

$$r_k = Ad(t_k)c[(1+\eta)(t_k - t_s)]\cos[\omega_{IF}t_k - (\omega_D t_k + \phi_0)] + \delta(t_k), \tag{6}$$

where r_k is output of RF front end at sample time t_k, $f_D = \omega_D/2\pi$ is Doppler shift, $\eta = \omega_D/(2\pi \times 1575.42 \times 10^6)$ accounts for Doppler shift caused chip length distortion, $f_{IF} = \omega_{IF}/2\pi$ is carrier frequency, ϕ_0 is initial carrier phase, $\delta(t_k)$ is Gaussian band-limited white noise, constant A is signal amplitude, and $d(t_k)$ stands for GPS data stream.

Front end output is at frequency $f_{IF} - f_D$. Sampling frequency f_S is 5 MHz. After sampling, output from lowpass filter with 2 MHz bandwidth is at frequency $f_G = f_{IF} - f_D - nf_S$, corresponding to angular frequency ω_G.

2.4. Deduced Original Duffing Oscillator. For application of Duffing oscillator in GPS signal acquisition, (1) can be deduced to (7) at ω_G:

$$\ddot{x}(t) + \omega_G k\dot{x}(t) - \omega_G{}^2[x(t) - x^3(t)] = \omega_G{}^2 f\cos(\omega_G t). \tag{7}$$

If the angular frequency ω of GPS signal is same to the inherent angular frequency ω_G of oscillator system (7), motion state of trajectories on phase plane will change largely, which indicates that received data contains expected GPS signal.

3. Judgment of System State Based on Lyapunov Exponent

To judge existence of expected GPS signal, observing change on phase plane is concise and instinctive. However, this method is time consuming. Human eyes detect the change after many periods. Furthermore, human judgment is inaccurate and inefficient, which cannot be implemented in engineering. Fortunately, LE can be employed in GPS signal acquisition based on chaotic oscillator [11].

Trajectories on phase plane are sensitive to initial conditions of oscillator system [7]. Two trajectories, which start very close, will rapidly diverge from each other and have totally different futures ultimately. LE is numeric indicator that describes divergence situation and motion state of trajectories.

LE can be solved out from state equation of system. Two stage nonautonomous equation (7) of Duffing oscillator system has two LEs corresponding to x- and y- dimension. The signs of LEs can indicate system states as follows: if one of two LEs is positive and the other is negative, system is at chaotic state; if both are negative, system is at big dimension circle state; if one LE equals to zero and the other is negative, the system is at critic state, the transition state between big dimension circle and chaotic ones [12].

3.1. Computation of LE. The state equation of (7) is expressed as

$$\begin{aligned} \dot{x} &= y, \\ \dot{y} &= -\omega_G k y + \omega_G{}^2 (ax - bx^3) + \omega_G{}^2 f \cos(\omega_G t). \end{aligned} \tag{8}$$

As both variables x and y are time dependent, (8) belongs to nonautonomous equation [11]. Let $z = t$, the two stage nonautonomous equation can be transformed into three stage autonomous equation as

$$\begin{aligned} \dot{x} &= y, \\ \dot{y} &= -\omega_G k y + \omega_G{}^2 (ax - bx^3) + \omega_G{}^2 f \cos(\omega_G z), \\ \dot{z} &= 1. \end{aligned} \tag{9}$$

Furthermore, (9) can be simplified as the expression $\dot{A} = J \times A$, where

$$J = \begin{bmatrix} 0 & 1 & 0 \\ \omega_G{}^2 (a - 3bx^2) & -\omega_G k & -\omega_G{}^3 f \sin(\omega_G z) \\ 0 & 0 & 0 \end{bmatrix}. \tag{10}$$

At fixed point $(x, y) = (0, 0)$ in equation $\Delta = \|J\|_{(0,0)} < 0$ holds, which means that point $(0, 0)$ is a saddle point.

At fixed points $(x, y) = (\pm 1, 0)$, in equation if $\Delta = \|J\|_{(\pm 1,0)} > 0$ holds, trace of J equals to -0.5, and in equation $\tau^2 - 4\Delta < 0$ holds. Therefore two points $(\pm 1, 0)$ also form a strange attractor.

LE can be gotten:

$$\lambda_{\mathrm{i}}(\mathbf{p}_{\mathrm{i}}) = \lim_{t \to \infty} \frac{1}{t} \log||\mathbf{A}(t)\mathbf{p}_{\mathrm{i}}||, \tag{11}$$

where $\mathbf{p}_{\mathrm{i}}$ is initial matrix. By Runge Kutta iterative method [12], LE can be easily calculated in engineering.

3.2. Flows of GPS Signal Detection Method with LE. Procedure of GPS signal acquisition based on LE can be summarized as follows.

(1) *Setting Threshold of f.* Modify amplitude f of PEDF from minute positive number to larger one smoothly and compute corresponding LEs. When the larger one of two LEs most approaches to zero from positive value, corresponding value of f is threshold f_T.

(2) *Detectting Signal.* Impose received data on oscillator system. If the larger LE becomes negative, GPS signal at frequency ω_G is assured to be comprised in received data.

Operation flows in Figure 1 illustrate succinctly the acquisition procedure.

4. Doppler and Code Phase Resolution

For implementation convenience in computer, (11) is rewritten in form of discrete time:

$$\lambda_{\mathrm{i}}(\mathbf{p}_{\mathrm{i}}) = \lim_{N \to \infty} \frac{1}{N} \sum_{k}^{N} \log||\mathbf{A}(k)\mathbf{p}_{\mathrm{i}}||. \tag{12}$$

Time interval determines accuracy of LE. With fixed time step, increasing N induces decrease in variance of calculated LEs by ratio $1/N^2$. In fact N cannot approach infinity; therefore the number N should selected at reasonable one. Also, length of iteration step h affects accuracy of LE. Ambiguity of LE increases when iteration step becomes larger, and vice versa. Unfortunately, the iteration step cannot be shortened arbitrarily considering convergence and computation burden. Doppler resolution is affected by accuracy of the LE number which is calculated at critical state. Code phase resolution is determined by sampling frequency [2]. Supposed that received data is sampled at 5 MHz in the front end, phase resolution of the oscillator system is 0.1 microsecond.

5. Simulations

In simulation, Monte Carlo method is adopted to test the performance of chaotic oscillator acquisition system. As contrast, conventional acquisition methods are used in simulation. GPS signal is based on signal model (6). Data length of conventional algorithm is 0.4 second. Referring to [2], GPS strength is at nominal level -130 dBm, and noise level is -174 dBm. From reference [4], GPS signal strength indoors would decrease in the range from 15 dB to 30 dB. Therefore, SNR of simulated indoor GPS signal ranges from -49 dB to -34 dB, and CNR from 14 dBHz to 30 dBHz.

Threshold is chosen to allow one false alarm happens among 10^6 times test on data without expected GPS signal. As for alternate half-bit algorithm, the threshold used in peak detection is 56; for differentially coherent acquisition

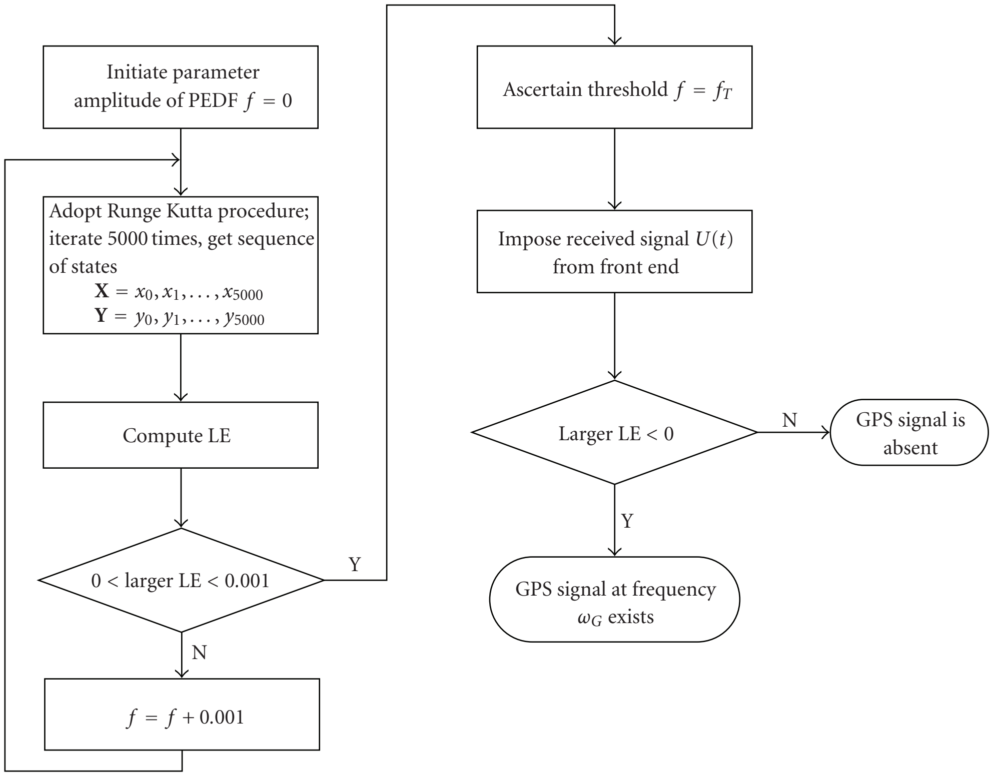

Figure 1: Operation flows of Duffing oscillator acquisition system.

algorithm threshold is 50; for chaotic oscillator, threshold value of LE is −0.001 with 200 Hz Doppler resolution.

Damping coefficient k equals to 0.5. Initial state of (9) is at point $(0, 0, 0)$; iteration time step is $1/(1.25 \times 10^6 \times 10)$ second. Figure 2 illustrates two LE curves versus amplitude of periodic disturbing force f, and iteration step is $\Delta f = 0.001$. A key point that should be mentioned is that three-stage autonomous system has three LE curves. The third LE curve corresponding to time dimension is always zero, which is neglected in Figure 2.

Figure 2 shows us that larger LE (upper curve) approaches to zero when f equals to 0.829. So $f = 0.829$ is selected as threshold f_T corresponding to critic state. Figure 3 intuitively demonstrates phase plane at critic state. With determined threshold f_T, impose data, which contains expected GPS signal, on chaotic oscillator-based acquisition system. The larger LE turns to be negative at the number -0.169, which indicates big dimension circle state, as shown in Figure 4.

To illustrate performance of the three algorithms, acquisition probability curves are listed in Figure 5.

From Figure 5, chaotic oscillator system outperforms other two conventional algorithms. Chaotic oscillator acquisition algorithm approaches 1 detection ratio when CNR is 19 dBHz; detection ratio is close to 0.9 when CNR is 16 dBHz. However, to guarantee 0.9 detection ratio, alternate half-bit method and differential coherent algorithm demands for signal strength at 21 dBHz and 20 dBHz CNR, respectively.

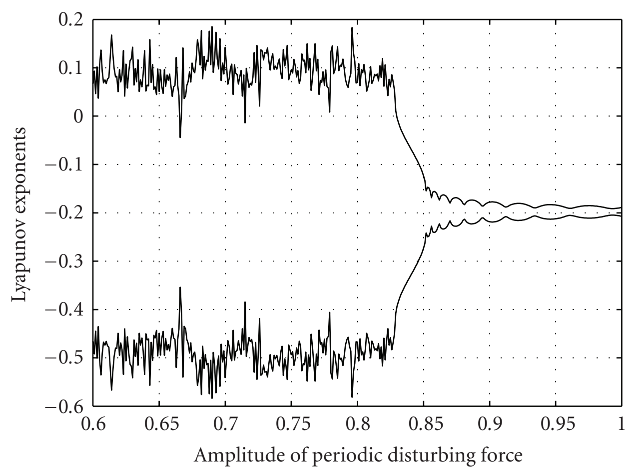

Figure 2: Nonzero Lyapunov exponent curve via the f from 0.600 to 0.999.

6. Computation Burdern Considerasion

Computation burden is a key issue in GPS receiver design, which affects tracking and decoding message.

In alternate half-bit method, received data is divided into 10 milliseconds cells. Each cell correlates with locally generated signal. FFT/IFFT is employed to implement fast correlation. In the condition of 5 MHz sampling frequency, numbers of real addition and real multiplication in each

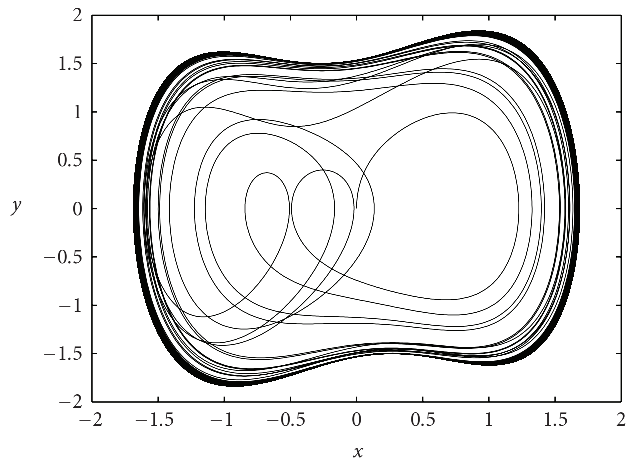

Figure 3: Phase plane at critic state.

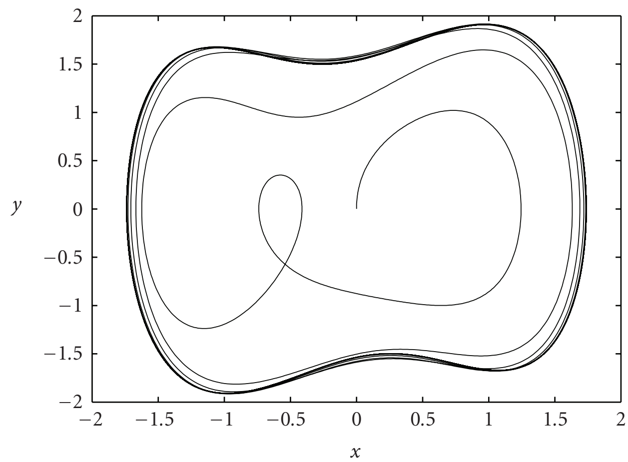

Figure 4: Phase plane at big dimension circle state.

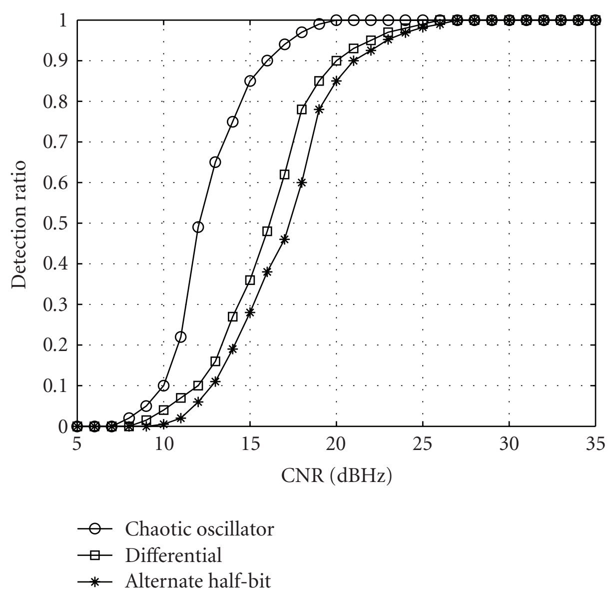

Figure 5: Acquisition probability curve: CNR from 5 dBHz to 35 dBHz.

cell are 3.12×10^6 and 0.781×10^6, respectively. After correlation, noncoherent calculation should be performed, which needs 0.221×10^6 real additions and 0.142×10^6 real multiplications. Therefore, to process 400-millisecond data, 31.2×10^6 real multiplications and 125×10^6 real additions are needed.

In full-bit method, experimental data is divided into 20 groups. $20 \times 169 \times 10^6$ real addition and $20 \times 43.9 \times 10^6$ real multiplications are needed to deal with the 400 milliseconds data. In fact, partial correlation can be used in accelerating calculation, which will divide each group into equal sections. If each section is at length 20 milliseconds, $20 \times 135 \times 10^6$ real additions and $20 \times 33.2 \times 10^6$ real multiplications are needed totally.

As for differential coherent method, after correlation between 1 milliseconds received data and local signal, two adjacent correlation results perform conjugate multiplication. For 400-millisecond data, computation burden is above 504×10^6 real additions and 186×10^6 real multiplications.

As for Duffing oscillator, whole procedure is divided into two parts. In the former part, the LE values in critical state are computed. Afterwards, received data is taken as input to test existence of expected signal. Suppose that iteration step is $1/(1.25 \times 10^6 \times 10)$ in the whole acquisition procedure. To determine threshold, state turns into critical one after 5000 times of iteration in simulation. After threshold determination, 10-millisecond data is taken as input of Duffing oscillator system to test expected signal, which consumes 0.125×10^6 times iteration. In each iteration, 76 real additions and 102 real multiplications are needed. Therefore 9.51×10^6 real additions and 12.8×10^6 real multiplications are needed totally. Computation burden is reduced to one fiftieth of conventional method.

7. Conclusions

In this paper chaotic oscillator is used to acquire weak GPS signal. The suggested acquisition method belongs to application of nonlinear dynamics which is different from conventional method. LE is employed to determine states of chaotic oscillator system. A procedure flow is given to illustrate the acquisition method. The simulation results demonstrate that chaotic oscillator acquisition preponderates conventional ones. Chaotic oscillator can detect signal at 16 dBHz CNR at detection ratio 0.9, with false alarm ratio 10^{-6}. Furthermore, oscillator acquisition algorithm consumes much less acquisition time, which is important for tacking signal and decoding navigation message. The advantage derives from properties of nonlinear dynamics.

Some other details in engineering have not been discussed in this paper. For example, in order to cover the possible Doppler shift range 20 KHz [2], oscillator acquisition system can be implemented by parallel structure, with multiple oscillator acquisition channels. Also, relationship among threshold value of LE, undetected rate, and Doppler resolution is another topic needed to be studied further.

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