

Research Article

Decentralized Detection in Censoring Sensor Networks under Correlated Observations

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The majority of optimal rules derived for different decentralized detection application scenarios are based on an assumption that the sensors' observations are statistically independent. Deriving the optimal decision rule in the canonical decentralized setting with correlated observations was shown to be complicated even for the simple case of two sensors. We introduce an alternative suboptimal rule to deal with correlated observations in decentralized detection with censoring sensors using a modified generalized likelihood ratio test (mGLRT). In the censoring scheme, sensors either send or do not send their complete observations to the fusion center. Using ML estimation to estimate the censored values, the decentralized problem is converted to a centralized problem. Our simulation results indicate that, when sensor observations are correlated, the mGLRT gives considerably better performance in terms of probability of detection than does the optimal decision rule derived for uncorrelated observations.

1. Introduction

The theory of signal detection and estimation is used in a wide variety of target-detection applications. Different formulations for the target detection problem have been suggested depending on the cost of communication. The classical detection theory considered the canonical detection problem in which a decision is made based on observations present in one central location, that is, centralized detection. The classical detection theory suits applications in which the complete observations are available at one location for decision making, for example, detection of the presence of a target using a radar.

Decentralized detection addresses the issue of detection in sensor networks. A typical decentralized detection system with fusion is shown in Figure 1, in which a fusion-center produces an estimate of the state of nature based on the data sent by geographically dispersed sensor nodes. Some preliminary processing of data is carried out at each sensor node. For example, in the canonical decentralized detection problem, each sensor node quantizes the likelihood ratio of its observation before sending it [1]. The performance of decentralized detection systems is suboptimal compared to

the centralized systems because the decision maker (fusion-center) does not receive a sufficient statistic. Nonetheless, factors such as communication bandwidth and limited energy motivate the use of decentralized detection systems. Moreover, in systems with a large number of sensor nodes, uncompressed information could overwhelm the fusion-center.

In battery-powered sensor networks, the most valuable resource is the limited energy available to each sensor. In a scenario where the absence of the target (null hypothesis) is much more likely than its presence (target hypothesis), an alternative formulation of the decentralized detection problem would allow the sensors to not communicate all the time to the fusion center, and thereby conserve energy. In such a case, the sensors are said to *censor* their observations by not sending observations that fall within a certain criterion. Rago et al. [2] considered a censoring scheme in which sensors either send or do not send some real-valued function of their observation to a fusion center based on a communication-rate constraint. The work in [2] shows that in the censoring scenario with independent sensors' observations, it is optimal (in both the Bayesian and the Neyman-Pearson (NP) sense) for the sensors to not send

their likelihood ratios if they fall in a particular interval and that the optimal censoring region is a single interval of the likelihood ratio.

In the canonical decentralized detection problem introduced in [1], an optimal decision rule should come up with the optimal quantizer at each sensor and the optimal decision rule at the fusion center. To obtain the censoring thresholds and the optimum decision rule in the censoring formulation, joint optimization is required across the sensors and the fusion center. Appadwedula et al. [3, 4] provided a useful simplification of the censoring interval R_i^c . It was proven in [3, 4] that setting the lower threshold of the censoring interval to 0 for any false-alarm constraints less than or equal to $\alpha^* = 1 - \prod_{i=1}^N (1 - \kappa_i)$ is optimal in the NP sense, where κ_i is the communication rate constraint for sensor i . This reduces the complexity of finding the censoring region to dealing with one threshold per sensor instead of two thresholds per sensor.

Most of the work done in the area of deriving optimal rules was based on an assumption that the observations at each sensor node are conditionally independent. Although such an assumption reduces the complexity of analysis noticeably, many wireless sensor network applications experience correlated observations, for example, in target-detection problems in which the sensors are close to each other and are prone to the same noise sources.

The effect of correlation on the performance of a decentralized detection system has been explored in the literature. Some of this work was done in [5–9]. The results obtained are often not easy to implement because the observation space cannot be divided into two contiguous portions. Willett et al. [8], for example, derive the optimal sensor rule for a two-sensor system with correlated noise. Their findings show that as the correlation between the sensors' observations increases, the optimal detection scheme cannot use single-interval decision regions at both sensors. In fact, the optimal scheme, if present, would be that either one sensor uses a single-interval decision region and the other sensor's observation is ignored, or both sensors use non-single-interval decision regions and neither sensor is ignored. Furthermore, it was shown in [9] that finding the optimal decision rule at the fusion center requires complete knowledge of the observation statistics.

In Section 2, we introduce the problem of detection under correlated observations. We propose a modified generalized likelihood ratio test (mGLRT) as a test implemented at a fusion-center receiving correlated observations. We then show how the mGLRT could be applied in censoring sensor networks. In Section 3, we evaluate the performance of the mGLRT through simulations. We consider two examples, a two-sensor network, and an eight-sensor network. We then find when the mGLRT has a performance similar to that of the optimal centralized test. In Section 4 we consider an example with real data.

2. Correlated Observations and the mGLRT

In wireless sensor networks used for detection applications, sensors communicate to detect a certain phenomenon. As

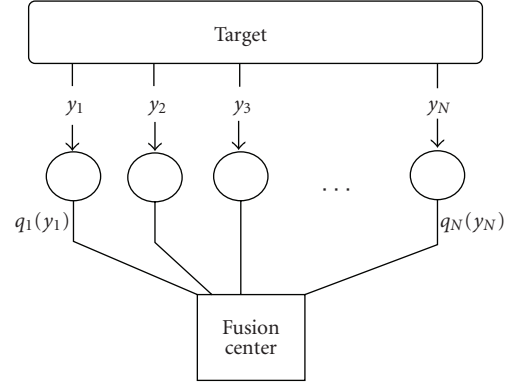


FIGURE 1: Decentralized detection system with fusion center.

mentioned earlier, the sensor observations are sent to a fusion center for decision making. The observations received at the fusion center may have a certain degree of correlation depending on the nature of the communication links between the sensors and the fusion center and the sources of signal and noise. Correlated noise is encountered in different communication scenarios such as wireless communication in fading channels. In the following discussion, we consider the problem of detection using censoring sensors under correlated noise. We introduce the problem by considering a simple sensor network consisting of two sensors communicating to a fusion center.

Consider a decentralized detection system consisting of two sensors sending their censored observations to a fusion center in which the final decision is made. The optimal fusion rule in the NP sense would be one that solves the following optimization problem:

$$\begin{aligned} \max_{\tau} \quad & \{P_D = P_{01|H_1}P_{D|01} + P_{10|H_1}P_{D|10} + P_{11|H_1}P_{D|11}\} \\ \text{subject to} \quad & P_F = P_{01|H_0}P_{F|01} + P_{10|H_0}P_{F|10} + P_{11|H_0}P_{F|11} \leq \alpha, \end{aligned} \quad (1)$$

where P_D is the probability of detection, P_F is the probability of false alarm, $P_{\cdot|H_i}$ is the conditional probability under H_i , $i = 0, 1$, P_{01} is P (first sensor censors, second sensor sends), P_{10} is P (first sensor sends, second sensor censors), P_{11} is P (first sensor sends, second sensor sends), and τ is the fusion center's decision-rule threshold.

Finding the optimal solution requires joint optimization for the censoring and the fusion center decision thresholds even with the assumption of independence across sensor observations. With the independence assumption removed, finding the optimal decision rule becomes intractable for the original decentralized detection problem [1, 6–8]. For a two-sensor system, censoring converts the simple binary hypothesis problem to a composite hypothesis problem in which we get four different sensor-output combinations under each hypothesis. These combinations are

$$\begin{aligned} H_0 : \quad & \{(0, 0), (0, \phi), (\theta, 0), (\theta, \phi)\}, \\ H_1 : \quad & \{(0, 0), (0, \phi), (\theta, 0), (\theta, \phi)\}, \end{aligned} \quad (2)$$

where $\theta, \phi \in (-\infty, \infty)$ is the received data at the fusion center from sensor 1 and sensor 2, respectively. When the sensor censors its observation, the received data at the fusion-center will be zero. H_0 and H_1 are the target absent and the target present hypotheses, respectively.

The optimal NP test in such a case is a uniformly most powerful (UMP) test. However, the UMP test does not exist in this case because the likelihood ratio between the pair of observations depends on which one of the four different sensor-output combinations was received at the fusion center.

When the UMP test fails, a common practice in solving composite hypothesis problems is to apply a generalized likelihood ratio test (GLRT) which is known to give a performance close to the optimal rule in a wide range of detection problems when the optimal rule is hard to find [10]. The GLRT uses the following likelihood ratio test (LRT) for the case of two sensors:

$$\frac{\max_{\theta \in \Lambda_1} p_{\theta}(y_1, y_2)}{\max_{\theta \in \Lambda_0} p_{\theta}(y_1, y_2)}, \quad (3)$$

where $\Lambda_i, i \in \{0, 1\}$, is the set of all possible parameters (θ) under hypothesis i . The GLRT uses the probability distribution that corresponds to the parameter (θ) with the highest posterior probability based on the observations in the LRT. A modified generalized likelihood ratio test (mGLRT) proposed here uses the following likelihood ratio for decision:

$$\frac{\max_{y_i \in \mathcal{R}_i^c} p_1(y_1, y_2)}{\max_{y_i \in \mathcal{R}_i^c} p_0(y_1, y_2)}. \quad (4)$$

If none of the sensors' observations are censored, then the optimal NP test in the centralized case is applied. However, if observations get censored, then the ML estimate of each censored observation under both hypotheses is used in the LRT.

For a composite hypothesis-testing problem, the generalized likelihood ratio test uses the probability density function (pdf) of the state of nature with the highest posterior probability based on the observations under the two hypotheses in the final likelihood ratio. We use the mGLRT to estimate the values of the censored observations under both hypotheses. The use of mGLRT in the censoring sensors scenario will be illustrated for a two-sensor network. The generalization to an N -sensor network will then be considered. The following assumptions are made throughout the discussion.

- (i) The sensors censor their observations; that is, they either send their complete observations to a fusion center or send nothing.
- (ii) The sensors communicate with the fusion center through an ideal noise-free channel.
- (iii) The noise accompanied with the sensors' observations is Gaussian. Therefore, the notions of statistical correlation and dependence will be used interchangeably [11].

- (iv) The fusion center knows the statistics of the sensors' observations.
- (v) The covariance matrix of the sensors' observations is positive definite.
- (vi) If all sensors' observations are censored, then decide H_0 .

2.1. A Two-Sensor Network. To demonstrate how the mGLRT can be applied to a censoring sensor network, we begin with a simple two-sensor network. Consider the problem of detecting a mean-shift in Gaussian noise using a censoring two-sensor network. In detection-theory terminology, the problem can be expressed as $H_0 : Y \sim \mathcal{N}(0, \Sigma_N)$ versus $H_1 : Y \sim \mathcal{N}(\mu, \Sigma_N)$, where the covariance matrix

$$\Sigma_N = \begin{bmatrix} \sigma_1^2 & \sigma_{12}^2 \\ \sigma_{21}^2 & \sigma_2^2 \end{bmatrix} \quad (5)$$

is assumed to be positive definite and $\sigma_{12}^2 = \sigma_{21}^2$. The joint probability density function of the sensor observations is

$$p_i(y_1, y_2) = \frac{1}{2\pi\sqrt{|\Sigma_N|}} \exp\left(-\frac{(\underline{y} - \underline{\mu})\Sigma_N^{-1}(\underline{y} - \underline{\mu})^T}{2}\right), \quad (6)$$

where $\underline{\mu} = [\mu \ \mu]$, $\underline{y} = [y_1 \ y_2]$, and y_1, y_2 are sensor observations.

To limit our attention to finding the fusion-center decision rule, the communication-rate constraints are assumed to be identical across the sensors, and so $\kappa_1 = \kappa_2 = \kappa/2$ for each sensor. We found experimentally that assigning an equal communication-rate constraint among the sensors performs best. This agrees with the finding in [12] for the statistically independent observations case. When one of the sensors sends its observation to the fusion center and the other does not, the censored observation could be estimated based on the correlation information. For example, if sensor 1 censors while sensor 2 sends, the fusion center could estimate the value of y_1 using maximum likelihood (ML) estimation based on y_2 under each hypothesis:

$$\begin{aligned} H_1 : \hat{y} &= \arg \max_{y_1} p_1(y_1 | y_2, y_2), \\ H_0 : \tilde{y} &= \arg \max_{y_1} p_0(y_1 | y_2, y_2), \end{aligned} \quad (7)$$

where $p_i(\cdot)$ is the joint pdf of the observations under H_i .
Let

$$\Sigma_N^{-1} = \begin{bmatrix} \delta_1 & \delta_{12} \\ \delta_{21} & \delta_2 \end{bmatrix}, \quad (8)$$

with $\delta_{12} = \delta_{21}$. Applying (7) results in

$$\begin{aligned} H_1 : \hat{y}_1 &= y_1 | y_2 = \frac{-\delta_{12}(y_2 - \mu) + \delta_1 \cdot \mu}{\delta_1}, \\ H_0 : \tilde{y}_1 &= y_1 | y_2 = \frac{-\delta_{12} \cdot y_2}{\delta_1}. \end{aligned} \quad (9)$$

The likelihood ratio of the censored and uncensored observations at the fusion center is

$$l_{\text{FC}}(\hat{y}_1, \tilde{y}_1, y_2) = \frac{p_1(\hat{y}_1, y_2)}{p_0(\tilde{y}_1, y_2)} \quad (10)$$

which can be expressed as a function of y_2 alone

$$l_{\text{FC}}(y_2) = \frac{(1/2\pi\sqrt{|\Sigma_N|}) \exp((1/2)(y_2 - \mu)^2(\delta_{12}^2/\delta_1 - \delta_2))}{(1/2\pi\sqrt{|\Sigma_N|}) \exp((1/2)y_2^2(\delta_{12}^2/\delta_1 - \delta_2))}, \quad (11)$$

which can easily be simplified to

$$l_{\text{FC}}(y_2) = \exp\left(\left(\mu y_2 - \frac{\mu^2}{2}\right)\left(\delta_2 - \frac{\delta_{12}^2}{\delta_1}\right)\right). \quad (12)$$

So, by calculating the inverse of the covariance matrix and estimating the mean of the observations, y_1 can be estimated based on the observation y_2 .

2.2. Applying mGLRT to an N -Sensor System. In the previous discussion, we considered applying the mGLRT to a system consisting of two sensors communicating with a fusion center. The mGLRT can be easily generalized to N sensors using the same mathematical formulation. Consider a system with N sensors that detect the presence of a target under correlated Gaussian noise. Let

- (i) R^c be the set of indices of the sensors that censored their observations,
- (ii) R^s be the set of indices of the sensors that sent their complete observations to the fusion center,
- (iii) y_i be the observation of the i th sensor,
- (iv) \underline{y}_s be the vector containing the uncensored observations.

The fusion center will have $|R^s|$ observations and $R^s \cap R^c = \emptyset$. Using ML estimation, the fusion center can estimate the censored observations based on the received observations and the knowledge of the sensors' observation statistics (covariance matrix, Σ_N , and mean of the observations, $\underline{\mu}$). Since the observations are assumed to be Gaussian, we need to estimate the censored observations such that they minimize the exponent, or equivalently maximize the negative of the exponent, of the joint probability density of the sensors' observations under each hypothesis. This is done by solving the following two optimization problems.

Problem 1 (under H_1). We have

$$\{y_i : i \in R^c\} = \hat{\underline{y}}_c = \max_{\{y_i, i \in R^c\} | \{y_j, j \in R^s\}} [\underline{y} - \underline{\mu}] \Sigma_N^{-1} [\underline{y} - \underline{\mu}]^T, \quad (13)$$

such that $y_i \leq t_i$, $i \in R^c$.

Problem 2 (under H_0). We have

$$\{y_i : i \in R^c\} = \tilde{\underline{y}}_c = \max_{\{y_i, i \in R^c\} | \{y_j, j \in R^s\}} [\underline{y}] \Sigma_N^{-1} [\underline{y}]^T, \quad (14)$$

such that $y_i \leq t_i$, $i \in R^c$, where t_i is the censoring threshold for the i th sensor.

The censoring interval of the i th sensor is assumed to be (t_{1i}, t_{2i}) . Again, the communication-rate constraints are chosen to be identical across all sensors. The results in [2] state that choosing the censoring interval to be a single interval does not reduce the probability of detection. The choice of t_{1i} and t_{2i} depends on the nature of correlation between the observations. For observations with positive correlations and a positive mean, the negative observations are considered *uninformative*, and therefore t_{1i} is chosen to be $-\infty$. t_{2i} is then found based on the communication rate constraint imposed on each sensor. When the observations are negatively correlated, the negative observations may be *informative*; therefore, the censoring intervals should be chosen differently [13].

The optimization problems under consideration are convex because the equations to be maximized are quadratic. Also, the set over which the optimization is carried out is convex because the censoring intervals are continuous in \mathbb{R} . The solution of those problems gives us an estimate of the censored observations. The objective function of Problem 1 can be rewritten as

$$[\underline{y} - \underline{\mu}] \Sigma_N^{-1} [\underline{y} - \underline{\mu}]^T = \sum_{i=1}^N \delta_i z_i^2 + \sum_{i,j=1, i \neq j}^N \delta_{ij} z_i z_j, \quad (15)$$

where $z_k = (y_k - \mu_k)$, and for Problem 2

$$[\underline{y}] \Sigma_N^{-1} [\underline{y}]^T = \sum_{i=1}^N \delta_i y_i^2 + \sum_{i,j=1, i \neq j}^N \delta_{ij} y_i y_j. \quad (16)$$

The solution to Problem 1 is

$$\hat{\underline{y}}_c = \Sigma_c^{-1} \hat{\underline{x}} + \underline{\mu}, \quad (17)$$

and to Problem 2 is

$$\tilde{\underline{y}}_c = \Sigma_c^{-1} \tilde{\underline{x}}, \quad (18)$$

where

- (i) $\hat{\underline{x}}, \tilde{\underline{x}}$ are vectors with elements

$$\hat{x}_i = \sum_{j \in R^s} \delta_{ij} (y_j - \mu_j), \quad i \in R^c, \quad (19)$$

$$\tilde{x}_i = \sum_{j \in R^s} \delta_{ij} y_j, \quad i \in R^c,$$

and δ_{ij} is the element in the i th row and j th column of Σ_N^{-1} (inverse of the covariance matrix).

- (ii) Σ_c^{-1} is an $|R_c| \times |R_c|$ matrix obtained by eliminating the elements in Σ_N^{-1} that are related to the uncensored observations.

After estimating the censored observations under the two hypotheses, the values of the received observations (\underline{y}_s) and the estimated observations under both hypotheses ($\hat{\underline{y}}_c, \tilde{\underline{y}}_c$) are plugged into a regular LRT at the fusion center, so

$$l(\hat{\underline{y}}_c, \tilde{\underline{y}}_c, \underline{y}_s) = \frac{p_1(\underline{y}_s, \hat{\underline{y}}_c)}{p_0(\underline{y}_s, \tilde{\underline{y}}_c)}. \quad (20)$$

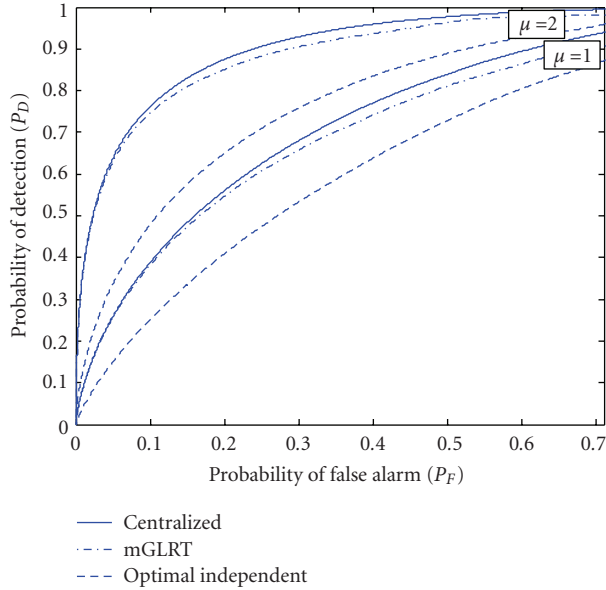


FIGURE 2: ROC curve of the mGLRT and the optimal independent test for the two-sensor Gaussian example with $\sigma_1^2 = 5$, $\sigma_2^2 = 2$, $\sigma_{21}^2 = \sigma_{12}^2 = 3$, $\kappa_i = 0.75$, and different signal amplitudes ($\mu = 1, 2$).

TABLE 1: Simulation details for each figure.

Figure	Points per trial	Trials	P_F tolerance
2, 6	5000	100	N/A
3, 4, 7	5000	100	10%

3. Performance Evaluation of the mGLRT through Simulations

Analytically comparing mGLRT to the optimal independent test is difficult. Therefore, in this section, we compare the proposed mGLRT to the optimal independent test through simulations. Two sensor-network scenarios will be considered, a two-sensor network and a network consisting of eight sensors. The results are obtained using Monte Carlo (MC) simulations. Table 1 shows the required number of trials for each figure. The second column shows the number of points per MC trial. At each point, a value is obtained on the graph.

Decentralized detection has a suboptimal performance when compared to centralized detection, because in centralized detection all the data are available at one location for decision making; therefore, the performance of the centralized detector is an upper bound on the decentralized detection performance. Consequently, when evaluating a decentralized detection scheme, our comparison criteria will be based on the difference between the centralized detection performance and the performance of the scheme under consideration. We assume the following about the fusion center in the centralized detection:

- (i) complete observations are available at the fusion center with no censoring or quantization.

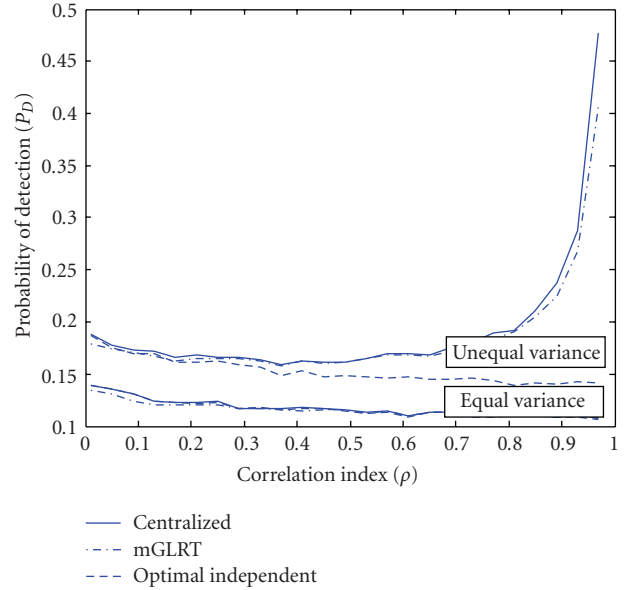


FIGURE 3: Performance of the mGLRT compared to the optimal independent test as the correlation index changes at $P_F = 0.05$ with: $\sigma_1^2 = 5$, $\sigma_2^2 = 2$, $\kappa_i = 0.5$ (unequal variance), $\sigma_1^2 = 5$, $\sigma_2^2 = 5$, $\kappa_i = 0.5$ (equal variance).

- (ii) the fusion center has complete knowledge of the correlation structure between the observations.

3.1. Two-Sensor Network. In the following simulations, the performance of the mGLRT is compared to the optimal independent test for a two-sensor network that communicates to a fusion center. The sensors receive Gaussian observations that have a certain correlation structure.

As discussed earlier, the mGLRT test exploits the correlation information with a slight increase in computations compared to the optimal independent test derived in [3, 4]. Therefore, we expect the mGLRT to have a better performance compared to the optimal independent test when the observations are significantly correlated. The results of our simulations match the expectation. Figure 2 shows the performance of the mGLRT compared to the optimal independent test in the case of correlated observations for different values of signal amplitudes. The figure shows that there is a slight degradation in performance when using the mGLRT in a censoring environment compared to our performance benchmark. This degradation in performance is sacrificed to save energy in the sensors via censoring. It is worth noting that for low probabilities of false alarm, there is essentially no loss in performance due to using censoring with the mGLRT.

The performance of the mGLRT compared to the optimal independent test should vary depending on the degree of correlation between the sensor observations. The reason is that the fusion center using the mGLRT uses the correlation information and thus gets better performance, unlike the optimal independent test which assumes that the sensor observations are independent. To examine the effect

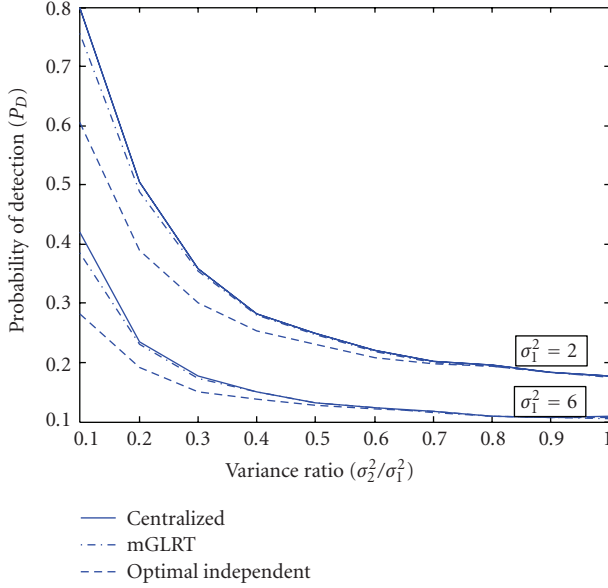


FIGURE 4: Performance of the mGLRT compared to the optimal independent test and the centralized detection with complete observations at $P_F = 0.05$ as the variances of the sensors become closer to each other for $\kappa_i = 0.5$, $\rho = 0.5$, and $\sigma_1^2 = 2, 6$.

of correlation, we define a correlation index ρ that captures the degree of correlation:

$$\rho = \frac{\sigma_{12}^2 \sigma_{21}^2}{\sigma_1^2 \sigma_2^2}. \quad (21)$$

Figure 3 shows that as the sensor observations become more correlated with unequal variance (i.e., the diagonal elements in the covariance matrix are not equal), the mGLRT performs considerably better than the optimal independent test and does not differ much from the ideal centralized detector. The optimal independent test has a better performance than the mGLRT when ρ is in the vicinity of 0 (i.e., closer to the independent case), since it is optimal. Also, in the case of equal variance, the performance of the optimal independent test is close to that of the ideal centralized detector.

As seen in Figure 3, the structure of the correlation matrix plays a role in the performance of the mGLRT compared to the optimal independent test. Figure 4 shows that as the diagonal elements of the correlation matrix (variance of each sensor's observation) get close to each other, the performance of the mGLRT degrades until it matches the optimal independent test when the variance of the sensor observations is equal. However, as the diagonal elements of the covariance matrix stray from each other, the mGLRT performs better.

Based on the observations from Figures 3 and 4, we conclude that when the sensor observations are highly correlated with unequal variances the mGLRT performs much better than the optimal independent test. When the variances are equal and the sensor observations are highly correlated, either both sensors get the same reading, which is equivalent to a one-sensor observation, or both sensors

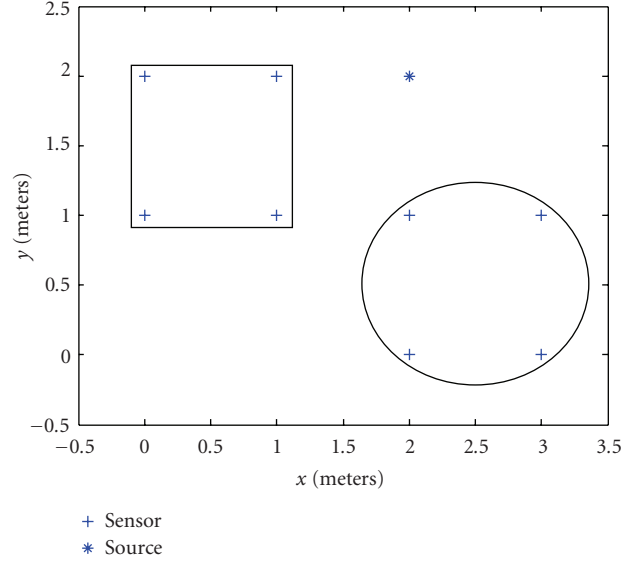


FIGURE 5: Locations of the eight sensors and the source.

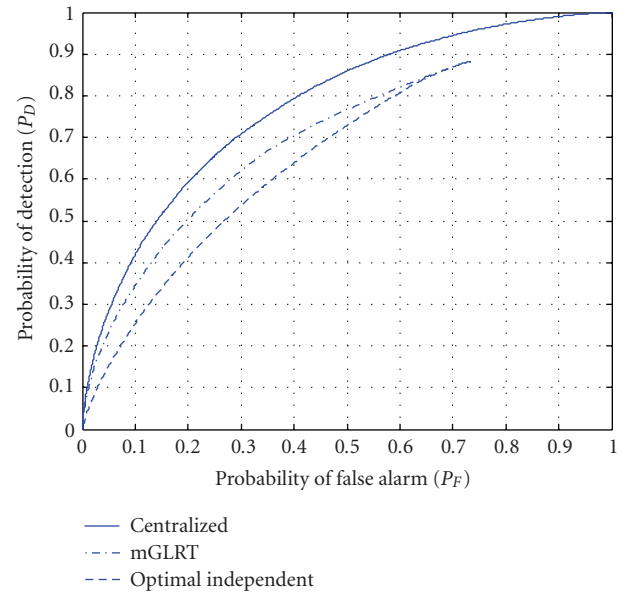


FIGURE 6: ROC curve of the mGLRT, and optimal independent test for the eight-sensor example with $\sigma_{ii}^2 = 5$, $\sigma_{ij}^2 = 4$ for sensors in the same set, $\sigma_{ij}^2 = 2$ for sensors in different sets. The communication rate constraint for each sensor is $\kappa_i = 0.375$.

do not get any. Therefore, the performance of the mGLRT degrades since the fusion-center receives less information. Also, the optimal independent test appears to be optimal for correlated observations when the correlation matrix is symmetric. Since the optimal independent rule partitions the observation space into two continuous parts via the censoring threshold, the result obtained here agrees with Corollary 1 in [14] which states that for a two-sensor system with $\sigma_1^2 = \sigma_2^2$, $\mu_1 = \mu_2$, and nonnegative correlation, an optimal solution exists in which both quantizers use the same contiguous partition of the observation space.

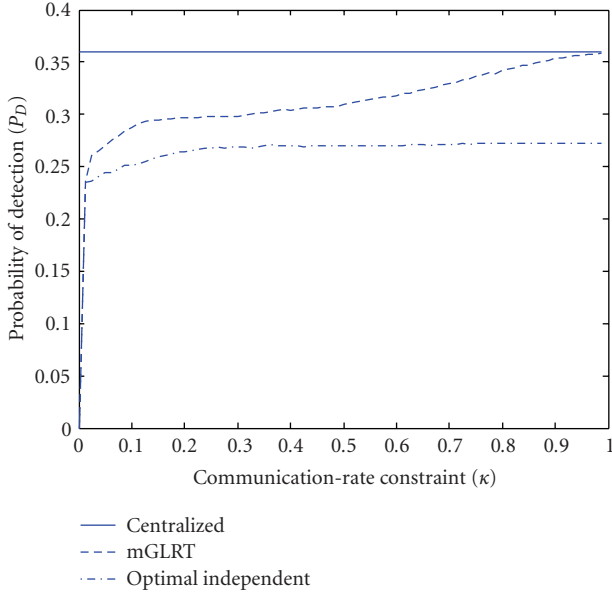


FIGURE 7: Performance of the mGLRT compared to the optimal independent test as the communication-rate constraint varies with $P_F = 0.05$ for the eight-sensor example with $\sigma_{ii}^2 = 5$, $\sigma_{ij}^2 = 3$ for sensors in the same set, $\sigma_{ij}^2 = 0.6$ for sensors in different sets.

The performance loss due to using the optimal independent test is higher compared to the mGLRT as the correlation increases. However, when the sensors’ observations are of equal variance and all observations are highly correlated, the performance of the optimal independent test matches that of the mGLRT. The reader is referred to [13] for a more thorough comparison between the mGLRT and the optimal independent test.

3.2. Eight-Sensor Network. In the following simulations, the performance of the mGLRT is compared to the optimal independent test for an eight-sensor network that communicates to a fusion center with the following assumptions.

- (i) The sensors are placed as shown in Figure 5.
- (ii) The amplitude of the signal emitted by the source to be detected decreases as $1/r$, where r is the distance from the source to the sensor.
- (iii) The observations of the sensors contained in the circle and the square are highly correlated among their peers in the same group; whereas the sensors in the circle are weakly correlated with the sensors in the square.

Figure 6 shows the performance of the mGLRT compared to the optimal independent test and our performance benchmark for the system described above. The figure shows that the mGLRT performs better even when the sensors’ observations are of equal variance. Moreover, the ideal centralized detection performs slightly better than the mGLRT at low false-alarm probabilities. Because of censoring, the mGLRT and the optimal independent test

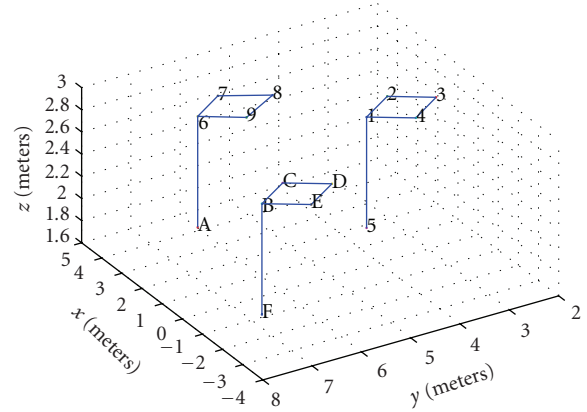


FIGURE 8: Microphones’ locations.

could not get to the point where $P_F = P_D = 1$. This agrees with the results in [3].

The performance of the mGLRT compared to the optimal independent and centralized tests depends on the communication-rate constraint. As the communication-rate constraint increases, the mGLRT should get closer in performance to the centralized test because they both have similar structure. Figure 7 shows a comparison between the mGLRT and the optimal independent test as the communication-rate constraint changes. The optimal independent test does not take advantage from increasing the communication rate constraint beyond 1/8.

4. A Real-Life Application

The performance analysis carried out for the detection tests so far was based on pure simulations. We will now consider a real-world detection problem from the work in [15], where the presence of a frog is to be detected using sound signals collected from an array of microphones placed as shown in Figure 8. For our censoring setup, the sensors (microphone and a transmitter) are assumed to have a communication-rate constraint of 0.33; that is, the sensors will communicate at most one-third of the time when the frog is not active. The signals collected from the microphones are shown in Figure 9. The spikes represent the call of a frog.

To apply the mGLRT, we assume that the noise is Gaussian, $\mathcal{N}(0, \Sigma_{15})$, where Σ_{15} is known and obtained by calculating the correlation between the different microphone signals during an interval when the frog is not calling. For the signals shown in Figure 9, Σ_{15} has both negative and positive entries. Figure 10 shows the performance of the mGLRT compared to the optimal centralized test. The performance of the mGLRT is almost similar to that of the optimal centralized test while saving energy through censoring. On the other hand, using the optimal independent test results in a degradation in performance especially for low false alarm probabilities.

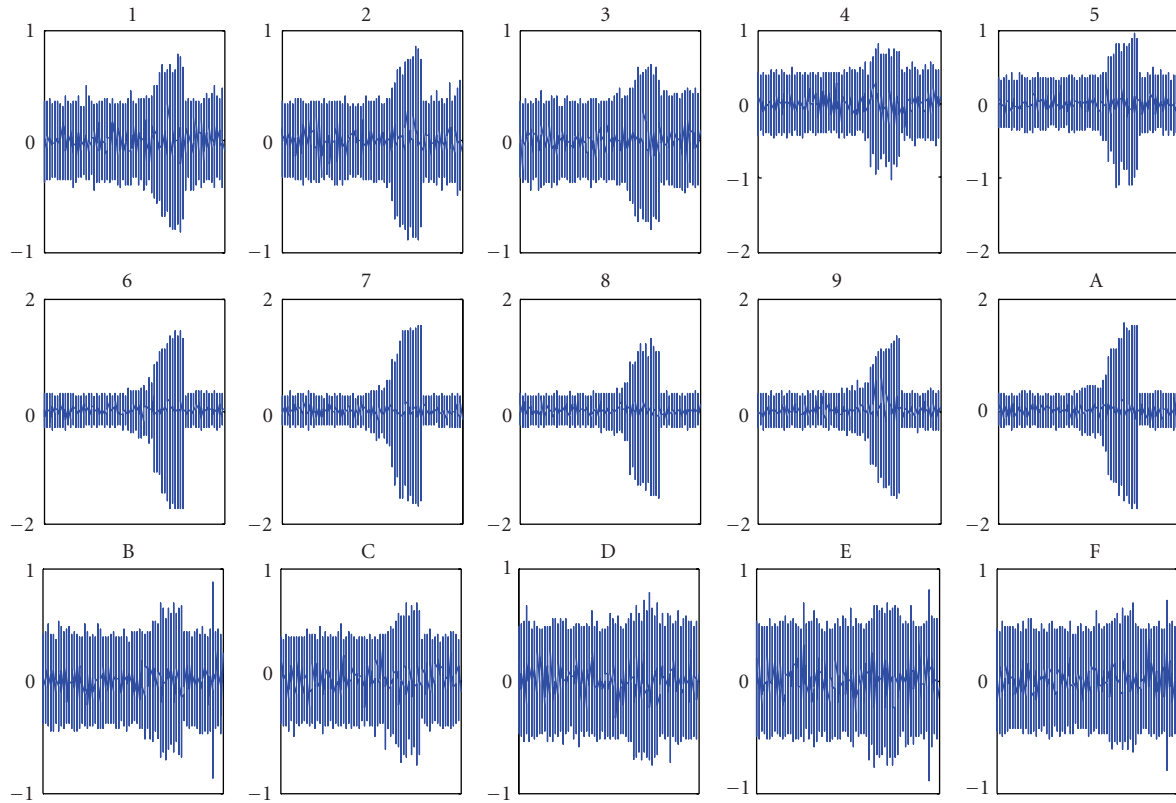


FIGURE 9: Signals (amplitude versus time) collected from the 15 microphones with a duration of 30 seconds for each signal.

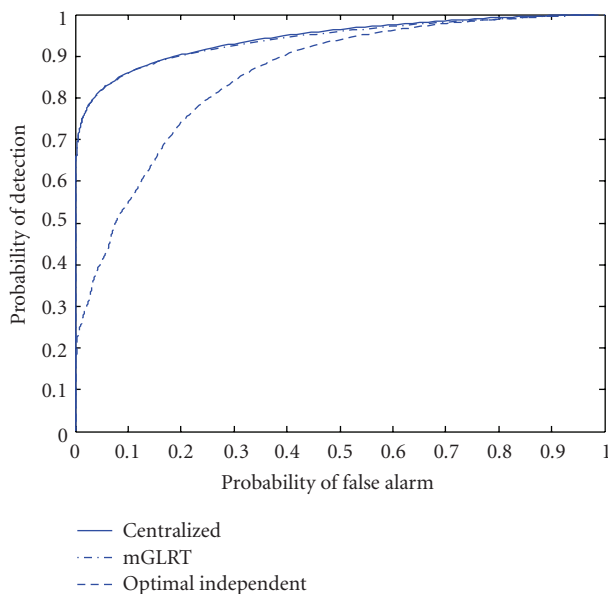


FIGURE 10: ROC curve of the mGLRT with a 2/3 censoring probability and the optimal independent test for the frog detection problem.

5. Conclusion

A modified GLRT provides a simple and well-performing method for decentralized detection with censoring sensors in

correlated noise. Our proposed mGLRT generally performs better than the optimal independent test when the sensor observations are significantly correlated. In our analysis, we showed where the mGLRT could be used while preserving performance and saving energy.

The mGLRT uses the knowledge of correlation between sensor observations to estimate the censored values at the fusion center. Depending on the degree of correlation between the sensors' observations and the variance of the noise affecting the sensors, the performance of the mGLRT varies when compared to the optimal independent test. The mGLRT out-performs the optimal independent test the most when the sensors' observations are highly correlated and the variances of the sensors' observations are not equal. Interestingly, when the sensors' observations are of equal variance and all observations are highly correlated, the performance of the optimal independent test matches that of the mGLRT.

Allowing sensors to censor in the case when the null hypothesis is more likely to occur prolongs the lifetime of battery-powered sensors by saving energy while sacrificing some of the system's performance. Our simulations showed that for a wide range of values for the correlation indices and variance ratios, the degradation in performance when using the mGLRT with censoring is very low. Moreover, as the variance of the observations increases, the mGLRT gets closer in performance to the optimal centralized test.

In a scenario where the fusion center could estimate the covariance matrix of the sensors' observations, the fusion center could choose between either using the optimal independent test or resorting to the mGLRT. If the observations are statistically independent, the fusion center could save on computations by using the optimal independent test. However, if the mGLRT is used all the time, a slight degradation in performance will be experienced in the following two cases.

- (i) The sensors' observations are independent.
- (ii) The sensors' observations have equal variance and the correlation among them is equal.

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