

MIMO-OFDM Channel Estimation in the Presence of Carrier Frequency Offset

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A multiple-input multiple-output (MIMO) wireless communication system with orthogonal frequency division multiplexing (OFDM) is expected to be a promising scheme. However, the estimation of the carrier frequency offset (CFO) and the channel parameters is a great challenging task. In this paper, a maximum-likelihood- (ML-) based algorithm is proposed to jointly estimate the frequency-selective channels and the CFO in MIMO-OFDM by using a block-type pilot. The proposed algorithm is capable of dealing with the CFO range nearly $\pm 1/2$ useful OFDM signal bandwidth. Furthermore, the cases with timing error and unknown channel order are discussed. The Cramér-Rao bound (CRB) for the problem is developed to evaluate the performance of the algorithm. Computer simulations show that the proposed algorithm can exploit the gain from multiantenna to improve effectively the estimation performance and achieve the CRB in high signal-to-noise ratio (SNR).

Keywords and phrases: MIMO-OFDM, CFO estimation, channel estimation, CRB, ML.

1. INTRODUCTION

Multiple-input multiple-output system with orthogonal frequency division multiplexing (MIMO-OFDM), with its ability to improve the capacity and simplify equalization in frequency-selective channels, is expected to play an important role in future wireless communications. Recent laboratory test and field trial results show the “encouraging” performance of this system [1]. However, OFDM techniques are more sensitive to carrier frequency offset (CFO) than single-carrier (SC) techniques. The CFO gives rise to intercarrier interference (ICI), which dramatically degrades the performance. In addition, with the influence of the channel fading, the theoretical benefits of a MIMO-OFDM system may not be fully achieved. The estimation of the CFO and the channel parameters is a crucial task.

The channel estimation problem for MIMO-OFDM was first studied by Li et al. [2], and a corresponding simplified algorithm was presented in [3]. Roman et al. [4] proposed a channel tracking and equalization method in MIMO-OFDM

stemming from Kalman filtering. In the literatures, most channel estimation methods assume perfect CFO knowledge. The conventional CFO estimators for OFDM proposed by van de Beek et al. [5] and Schmidl and Cox [6] have low complexity and work well even for MIMO-OFDM systems. However, using one OFDM symbol, they are only capable of dealing with the CFO range within one- or two-subcarrier bandwidth. Furthermore, they can not exploit the gain from multiantennas to improve estimation performance. Mody and Stuber [7] presented a time and frequency synchronization technique for a MIMO-OFDM system, and Honan and Tureli [8] also proposed a blind algorithm for CFO estimation in a MIMO-OFDM system. However, the algorithms aforementioned did not consider the joint channel and CFO estimation. Although Besson and Stoica [9] addressed the joint CFO and channel gains estimation problem for MIMO using a training sequence, their method is only applied to flat fading channels. In addition, Morelli and Mengali [10] proposed a method for joint frequency and channel estimation in SC single-input single-output (SISO) systems.

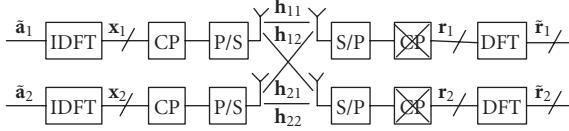


FIGURE 1: MIMO-OFDM transmission.

In this paper, the problem of joint frequency-selective channels and CFO estimation is considered, and a new estimator is proposed to resolve it, which can effectively exploit the gain from multiantennas to improve the estimation performance. Our approach can be viewed as an extension of Morelli and Mengali's algorithm [10].

The organization of this paper is as follows. Section 2 presents the system model and the algorithm for joint channel and CFO estimation is developed in Section 3. The Cramér-Rao bound (CRB) is given in Section 4. Computer simulations are conducted in Section 5 to demonstrate the performance of a proposed algorithm in various scenarios. Finally, Section 6 concludes the paper.

Notation

Upper (lower) boldface letters denote matrices (column vectors), and frequency domain components are indicated by a tilde. $(\cdot)^T$ and $(\cdot)^H$ represent transpose and conjugate transpose, respectively. $\|\cdot\|$ represents the Frobenius norm, and $\mathbf{I}_{N \times N}$ denotes the $N \times N$ identity matrix. $(\cdot) \bmod N$ denotes integer (\cdot) modulo N . $\text{Re}(\cdot)$ and $\text{Im}(\cdot)$ denote the real part and imaginary part of complex number (\cdot) . $\text{diag}(\cdot)$ denotes a vector constructed by the diagonal elements of the matrix (\cdot) .

2. SYSTEM MODEL

The MIMO-OFDM transmission model used in this paper is illustrated in Figure 1 [4]. Although a 2-transmit/2-receive (2T/2R) antenna configuration is considered, it can be extended to any transmit/receive antenna case.

In the following mathematic model, we assume perfect timing synchronization and negligible sampling frequency offset. Let $\{h_{i,j,l}\}_{l=0,1,\dots,L-1}$ be the channel impulse response (including the transmitting and receiving filters) from the i th transmitter to the j th receiver that is supposed to be uncorrelated with each other, where L is the channel order. Consider the OFDM block with N subcarriers, equispaced at a separation of $1/T$, so the useful OFDM signal bandwidth is N/T . The k th OFDM modulation block of the i th transmit antenna is expressed as $\mathbf{x}_i(k) = \mathbf{F}_N \tilde{\mathbf{a}}_i(k)$, where \mathbf{F}_N is the $N \times N$ inverse discrete Fourier transform (IDFT) matrix, and $\tilde{\mathbf{a}}_i(k)$ is the $N \times 1$ complex symbol vector sent from antenna i . Each OFDM modulation block is preceded by a cyclic prefix (CP) of size L_{CP} which is not longer than the channel impulse response. The CFO between the transmitters and receivers is normalized by the useful OFDM signal bandwidth, and the normalized CFO is denoted by f . At the receivers, after the CP removal, the received signal with CFO

can be expressed as

$$\begin{bmatrix} \mathbf{r}_1(k) \\ \mathbf{r}_2(k) \end{bmatrix} = \begin{bmatrix} \mathbf{C}_0(f) & \mathbf{0} \\ \mathbf{0} & \mathbf{C}_0(f) \end{bmatrix} \begin{bmatrix} \mathbf{H}_{11}(k) & \mathbf{H}_{21}(k) \\ \mathbf{H}_{12}(k) & \mathbf{H}_{22}(k) \end{bmatrix} \begin{bmatrix} \mathbf{x}_1(k) \\ \mathbf{x}_2(k) \end{bmatrix} \xi + \begin{bmatrix} \mathbf{w}_1(k) \\ \mathbf{w}_2(k) \end{bmatrix}, \quad (1)$$

or, in a more compact form,

$$\mathbf{r}(k) = \mathbf{C}(f)\mathbf{H}(k)\mathbf{x}(k)\xi + \mathbf{w}(k), \quad (2)$$

where $\mathbf{r}_j(k)$ is the k th received block of size $N \times 1$ at antenna j . $\mathbf{C}_0(f) = \text{diag}[1, e^{j2\pi f}, \dots, e^{j2\pi f(N-1)}]$, $\xi = e^{j2\pi f((k-1)N+kG)}$, and \mathbf{H}_{ij} is the circulant matrix constructed by channel taps $\{h_{i,j,l}\}_{l=0,1,\dots,L-1}$ with the (r, l) th entry given by $h_{i,j,(r-l) \bmod N}$. $\mathbf{w}(k)$ is assumed to be a circular white Gaussian noise of size $2N \times 1$.

After having performed the discrete Fourier transform (DFT) of $\mathbf{r}_j(k)$, we obtain

$$\begin{aligned} \tilde{\mathbf{r}}_j(k) &= \mathbf{F}_N^H \mathbf{C}_0(f) \mathbf{H}_{1j}(k) \mathbf{F}_N \tilde{\mathbf{a}}_1(k) \xi \\ &\quad + \mathbf{F}_N^H \mathbf{C}_0(f) \mathbf{H}_{2j}(k) \mathbf{F}_N \tilde{\mathbf{a}}_2(k) \xi + \mathbf{F}_N^H \mathbf{w}_j(k) \\ &= \tilde{\mathbf{D}}_{1j}(k) \tilde{\mathbf{a}}_1(k) \xi + \tilde{\mathbf{D}}_{2j}(k) \tilde{\mathbf{a}}_2(k) \xi + \tilde{\mathbf{n}}_j(k), \end{aligned} \quad (3)$$

where $j = 1, 2$, $\tilde{\mathbf{n}}_j = \mathbf{F}_N^H \mathbf{w}_j(k)$, \mathbf{F}_N^H is a DFT matrix, and $\tilde{\mathbf{D}}_{ij}(k) = \mathbf{F}_N^H \mathbf{C}_0(f) \mathbf{H}_{ij}(k) \mathbf{F}_N$. Without CFO, $\mathbf{C}_0(f)$ is an identity matrix, so $\tilde{\mathbf{D}}_{ij}(k)$ is a diagonal matrix. However, if $\mathbf{C}_0(f)$ is not an identity matrix when CFO exists, then $\tilde{\mathbf{D}}_{ij}(k)$ is no longer a diagonal matrix and ICI is introduced. Once f is estimated, $\mathbf{C}(f)$ can be canceled to diagonalize $\tilde{\mathbf{D}}_{ij}(k)$. In virtue of the special structure of $\tilde{\mathbf{D}}_{ij}(k)$ and channel information, equalization can be realized in lower complexity.

3. ESTIMATION OF CFO AND CHANNELS

From Section 2, we can find that the estimation of CFO and channel parameters is a crucial task for MIMO-OFDM systems. In this section, maximum-likelihood (ML) estimation methods are applied to the derivation of joint CFO and channel estimation algorithm. The estimation in the case of an unknown channel order (UCO) is also discussed.

3.1. CFO and channel estimation algorithm

We assume that channel parameters and CFO are invariant during several pairs of MIMO-OFDM modulation blocks, and use a pair of blocks as a pilot. Since estimation can be achieved in one block, k is dropped in (1) first, and then (1) can be rewritten as

$$\begin{aligned} \mathbf{r} &= \mathbf{C}(f) \begin{bmatrix} \mathbf{X}_1 & \mathbf{X}_2 & \mathbf{0}_{N \times L} & \mathbf{0}_{N \times L} \\ \mathbf{0}_{N \times L} & \mathbf{0}_{N \times L} & \mathbf{X}_1 & \mathbf{X}_2 \end{bmatrix} \begin{bmatrix} \mathbf{h}_{11} \\ \mathbf{h}_{21} \\ \mathbf{h}_{12} \\ \mathbf{h}_{22} \end{bmatrix} + \mathbf{w} \\ &= \mathbf{C}(f)\mathbf{X}\mathbf{h} + \mathbf{w}, \end{aligned} \quad (4)$$

where $\mathbf{h} = [\mathbf{h}_{11}^T \mathbf{h}_{21}^T \mathbf{h}_{12}^T \mathbf{h}_{22}^T]^T$, $\mathbf{h}_{ij} = [h_{i,j,0} h_{i,j,1} \dots h_{i,j,L-1}]^T$, and \mathbf{X}_i denotes the $N \times L$ circulant matrix stacked by modulate block \mathbf{x}_i at the i th transmitter with the (r, l) th entry given by $x_{i,(r-l) \bmod N}$.

Since all parameters except noise are determinant, the log-likelihood function of received data is given by

$$\ln(L) = \text{const} - 2N \ln(\sigma^2) - \frac{1}{\sigma^2} \|\mathbf{r} - \mathbf{C}(f)\mathbf{X}\mathbf{h}\|^2. \quad (5)$$

The estimation of f and \mathbf{h} is the solution of the following joint optimization problem:

$$\min_{\hat{f}, \hat{\mathbf{h}}} \|\mathbf{r} - \mathbf{C}(f)\mathbf{X}\mathbf{h}\|^2. \quad (6)$$

Let f be given, we can obtain

$$\hat{\mathbf{h}} = (\mathbf{X}^H \mathbf{X})^{-1} \mathbf{X}^H \mathbf{C}^H(f) \mathbf{r}, \quad (7)$$

with $\mathbf{C}(f)\mathbf{C}^H(f) = \mathbf{I}_{2N \times 2N}$ and f can be obtained by the following cost function:

$$J(f) = \mathbf{r}^H \mathbf{C}(f) \mathbf{P} \mathbf{C}^H(f) \mathbf{r}, \quad (8)$$

where $\mathbf{P} = \mathbf{X}(\mathbf{X}^H \mathbf{X})^{-1} \mathbf{X}^H$.

To simplify the algorithm, Newton method is applied to achieve fine search of CFO after coarse search.

The resulting algorithm is summarized in the following steps.

Step 1 (the coarse search). In this step, we search the maximum of $J(f_k)$ at frequency grid f_k , where $f_k = k/MT$, $k = 0, 1, \dots, M-1$, and M is commonly selected as $2N$ or $4N$ [11]. In practice, the grid space can be determined flexibly according to search range.

Step 2 (the fine search). With the value f_k being the initial value, some optimization methods such as Newton method [12] can be used to obtain the more accurate estimation of \hat{f} .

Step 3 (channel estimation). Substituting \hat{f} into (7), we can obtain the channel parameters.

The iterations of Newton method are listed as

$$f_{k+1} = f_k - [\nabla^2 J(f_k)]^{-1} \nabla J(f_k), \quad (9)$$

where

$$\begin{aligned} \nabla J(f_k) &= j2\pi \mathbf{r}^H \mathbf{C}(f_k) (\mathbf{B}\mathbf{P} - \mathbf{P}\mathbf{B}) \mathbf{C}^H(f_k) \mathbf{r}, \\ \nabla^2 J(f_k) &= 4\pi^2 \mathbf{r}^H \mathbf{C}(f_k) (2\mathbf{B}\mathbf{P}\mathbf{B} - \mathbf{P}\mathbf{B}^2 - \mathbf{B}^2\mathbf{P}) \mathbf{C}^H(f_k) \mathbf{r}. \end{aligned} \quad (10)$$

Here, the subscript k denotes the k th iteration, and $\mathbf{B} = \text{diag}\{0, 1, 2, \dots, N-1, 0, 1, 2, \dots, N-1\}$.

Remarks

(1) Computer simulations show that the convergent point can be achieved using Newton method after 2 or 3 iterations.

(2) Since the matrices \mathbf{X} and \mathbf{P} are only related to the pilot block and \mathbf{B} is determinate, the terms consisting of them in (7), (8), and (10) can be computed offline, which largely reduces the burden of online computation.

(3) It is shown that the estimated value of CFO is independent of the estimated channels, but the estimation accuracy of f influences the final estimation accuracy of channel parameter \mathbf{h} .

(4) From the expression of $\mathbf{C}(f)$, we can see that the proposed cost function (8) is periodic. In order to avoid the ambiguity caused by the periodicity of the cost function, the estimation range of normalized CFO of the estimator is from -0.5 to 0.5 (within $\pm N/2T$).

3.2. UCO case

In order to construct the circulant matrix \mathbf{X}_i , the channel order L should be known in advance. So the additional algorithm for the channel order estimation is needed. Furthermore, because channel order is variant in practice, the matrices \mathbf{X} and \mathbf{P} have to be reconstructed according to different L . However, we find that the estimator is robust to the overestimated channel order. So the channel order L can be simply replaced by L_{CP} under the condition $L_{CP} \geq L$ which is generally satisfied in MIMO-OFDM systems. Therefore, we do not need to estimate L and reconstruct \mathbf{X} and \mathbf{P} . Simulations show that the replacement only causes a slight loss in estimation performance (see Section 5).

4. THE CRB

The deterministic CRB is given here to judge the performance of the proposed algorithm. We now construct the Fisher information matrix (FIM) by calculating the derivative of (5) with respect to $\eta = [f \text{Re}(\mathbf{h})^T \text{Im}(\mathbf{h})^T]$; the expression for FIM is shown as

$$\text{FIM} = \frac{2}{\sigma^2} \begin{bmatrix} \text{Re}(\mathbf{X}^H \mathbf{X}) & -\text{Im}(\mathbf{X}^H \mathbf{X}) & -2\pi \text{Im}(\mathbf{X}^H \mathbf{B} \mathbf{X} \mathbf{h}) \\ \text{Im}(\mathbf{X}^H \mathbf{X}) & \text{Re}(\mathbf{X}^H \mathbf{X}) & 2\pi \text{Re}(\mathbf{X}^H \mathbf{B} \mathbf{X} \mathbf{h}) \\ 2\pi \text{Im}(\mathbf{h}^H \mathbf{X}^H \mathbf{B} \mathbf{X}) & 2\pi \text{Re}(\mathbf{h}^H \mathbf{X}^H \mathbf{B} \mathbf{X}) & 4\pi^2 \text{Re}(\mathbf{h}^H \mathbf{X}^H \mathbf{B}^2 \mathbf{X} \mathbf{h}) \end{bmatrix}. \quad (11)$$

The derivation of FIM, which is similar to that in [13], is given in the appendix.

We can compute the variance of an individual estimated parameter by inverting the FIM, namely, $\text{CRB}(\eta) = \text{diag}\{\text{FIM}^{-1}\}$. The CRB value of CFO parameter is the last element of $\text{CRB}(\eta)$. The rest of the elements of $\text{CRB}(\eta)$ are the CRB values of channel parameters.

Since there are too many independent channel parameters, the performance evaluation of channel estimation is complicated. For example, in a 2T/2R system, there are $4L$ CRB values after the combination of their real part and imaginary part. Because all of the channel parameters are independent, we can evaluate the estimation performance using the average of all channel variances. This is used in the simulation of the next section.

5. PERFORMANCE EVALUATION THROUGH SIMULATIONS

In this section, some simulations are conducted to assess the effectiveness of the proposed algorithm by comparison with Schmidl and Cox algorithm (SCA) [6] and CRB. For comparison reason, a pair of MIMO-OFDM modulation blocks with the same structure in [6] is used as pilot tones at 2 transmitters separately, and QPSK symbol modulation is employed. The additive channel noise is white Gaussian with zeros mean. Channel parameters corresponding to different transmit or receive antennas are independent and identically distributed (i.i.d.), and delay-power-spectrum function is exponential. The channel order and the length of CP are $L = 6$ and $L_{\text{CP}} = 8$, respectively, in Simulations 2, 3, and 4. For each simulation, 500 Monte Carlo trials are run. The estimation performance is evaluated by mean squared error (MSE). Corresponding to the CRB in Section 4, the MSE of a channel is defined as

$$\text{MSE}_{\text{ch}} = \frac{1}{4L} \sum_{i=1}^2 \sum_{j=1}^2 \sum_{l=1}^L E\{|h_{i,j,l} - \hat{h}_{i,j,l}|^2\}. \quad (12)$$

The MSE of CFO is defined as

$$\text{MSE}_{\hat{f}} = E\{|f - \hat{f}|^2\}. \quad (13)$$

5.1. Simulation 1: the influence of the symbol timing error

The proposed scheme assumes perfect timing synchronization. In the presence of timing error, the matrix X_i is not circulant anymore and the interference from the adjacent OFDM symbol is introduced. This will influence the performance of CFO and channel estimation. To obtain more insight of it, we can rewrite the cost function (8) as

$$J(f, \tau_0) = \mathbf{r}(\tau_0)^H \mathbf{C}(f) \mathbf{P} \mathbf{C}^H(f) \mathbf{r}(\tau_0), \quad (14)$$

where τ_0 is the symbol timing offset (TO) relative to perfect timing position in the received data. The cost function surface is plotted in Figure 2. It is shown that a sharp peak

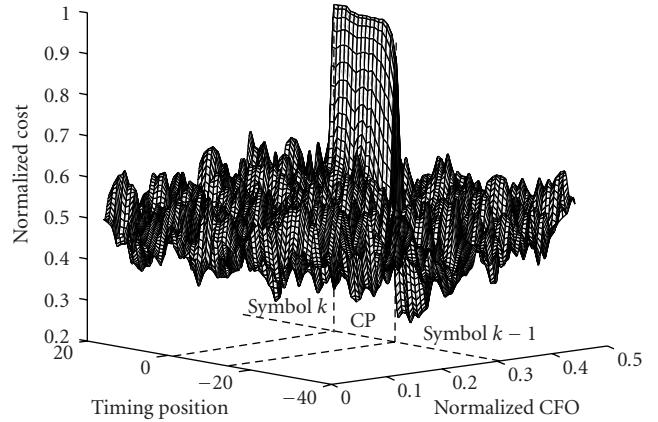


FIGURE 2: Cost function surface for CFO estimation ($N = 64$, $L_{\text{CP}} = 16$, SNR = 15 dB, $f = 0.3$, $L = 8$).

appears at the true CFO point when the timing position is within the CP. If the timing position lies beyond the CP, it is difficult to find a unique tall peak in the range of CFO. So the frequency estimation is robust when the timing position is within the CP, but the performance of CFO estimation degrades severely when the timing position is beyond the CP. Subsequent simulations will show the influence of timing error on the MSE of CFO estimation when the timing position lies within the CP.

5.2. Simulation 2: the influence of SNR

The influence of SNR is studied for $N = 64$. For comparison reason, the normalized CFO is selected as 0.01 (within a subcarrier). We illustrate the MSE obtained by simulation of 2T/2R antenna systems using (a) the proposed algorithm with perfect symbol timing and known channel order; (b) the proposed algorithm with perfect symbol timing and UCO; (c) the proposed algorithm with UCO and symbol TO (the timing position swing randomly inside the CP); and (d) the SCA. The results are shown in Figure 3. $\text{CRB}_{f_{22}}$ and $\text{CRB}_{h_{22}}$ denote the CRB of CFO and channel using 2T/2R antennas, and $\text{CRB}_{f_{21}}$ is the CRB of CFO using 2T/1R antennas. It can be observed from Figure 3a that the performance of CFO estimation of the proposed algorithm can achieve the $\text{CRB}_{f_{22}}$ at certain SNR, while SCA only achieves $\text{CRB}_{f_{21}}$. The result shows that the proposed algorithm can exploit gain obtained from multiantennas to improve effectively the estimation performance.

The MSE of both algorithms is apart from the CRB when SNR is low, which is caused by the threshold effects [11]. We can observe that the proposed algorithm is more sensitive to threshold effects. Because of the influence of threshold effects of the CFO estimation, the performance of channel estimation in Figure 3b degrades at the corresponding SNR.

It is also shown in Figure 3 that there is only slight loss in the performance of CFO and channel estimation when the channel order is unknown. In the presence of timing error, the MSE of CFO approaches $\text{CRB}_{f_{22}}$ at certain SNR.

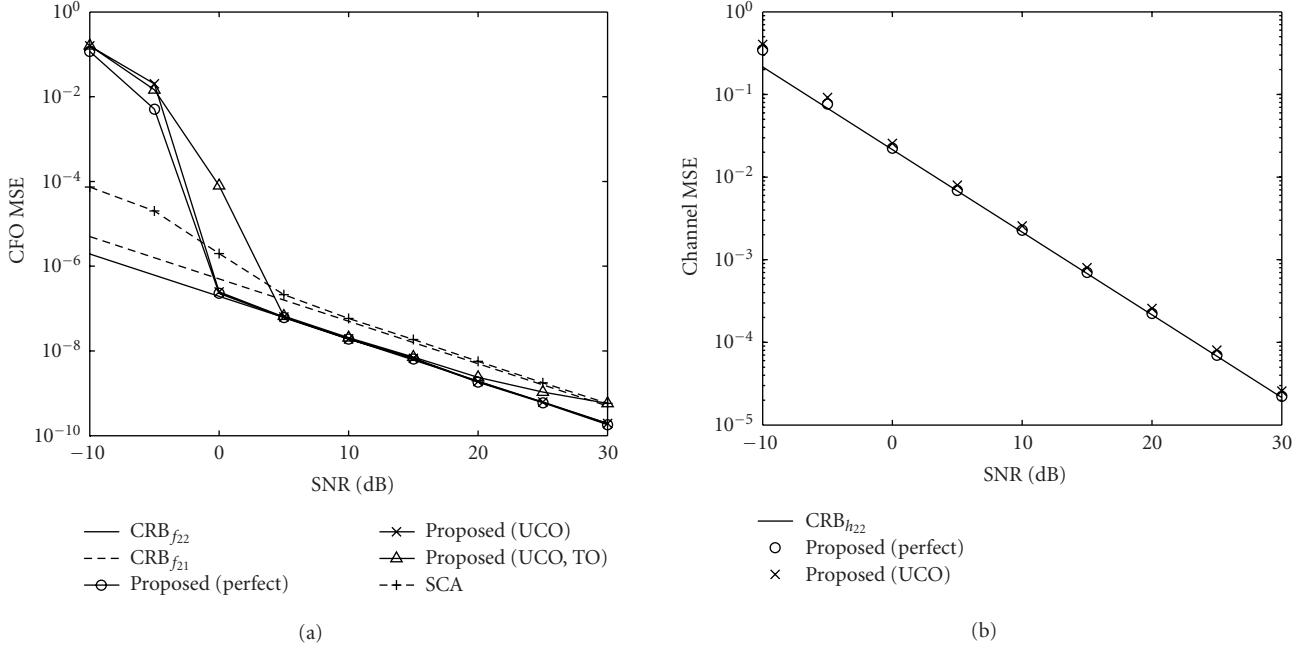


FIGURE 3: The influence of SNR ($N = 64$). (a) CFO estimation performance. (b) Channel estimation performance.

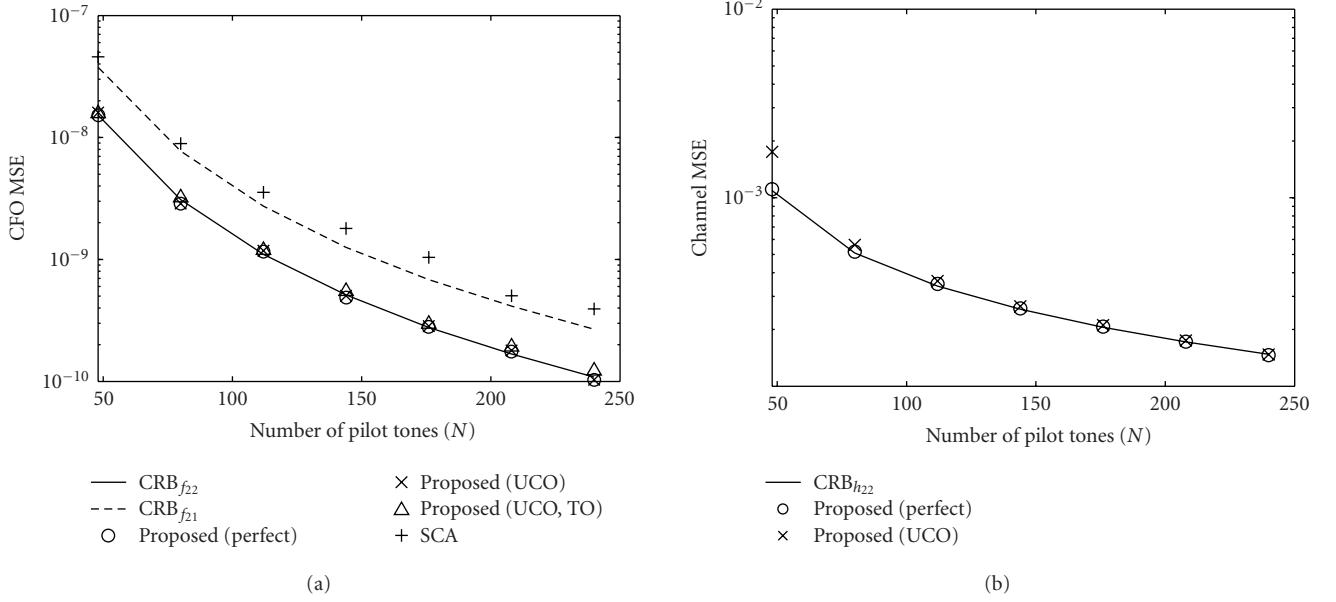


FIGURE 4: The influence of the number of pilot tones N (SNR = 15 dB). (a) CFO estimation performance. (b) Channel estimation performance.

5.3. Simulation 3: the influence of the number of pilot tones

The SNR is fixed at 15 dB. For comparison reason, the normalized CFO is selected as $0.6/N$ (within a subcarrier). In Figure 4, we plot the estimation performance of

the channel and frequency offset with different numbers of pilot tones, respectively. From Figure 4, we can observe that

- (a) the proposed algorithm is very close to CRB, even in the case of UCO and in the presence of symbol TO;

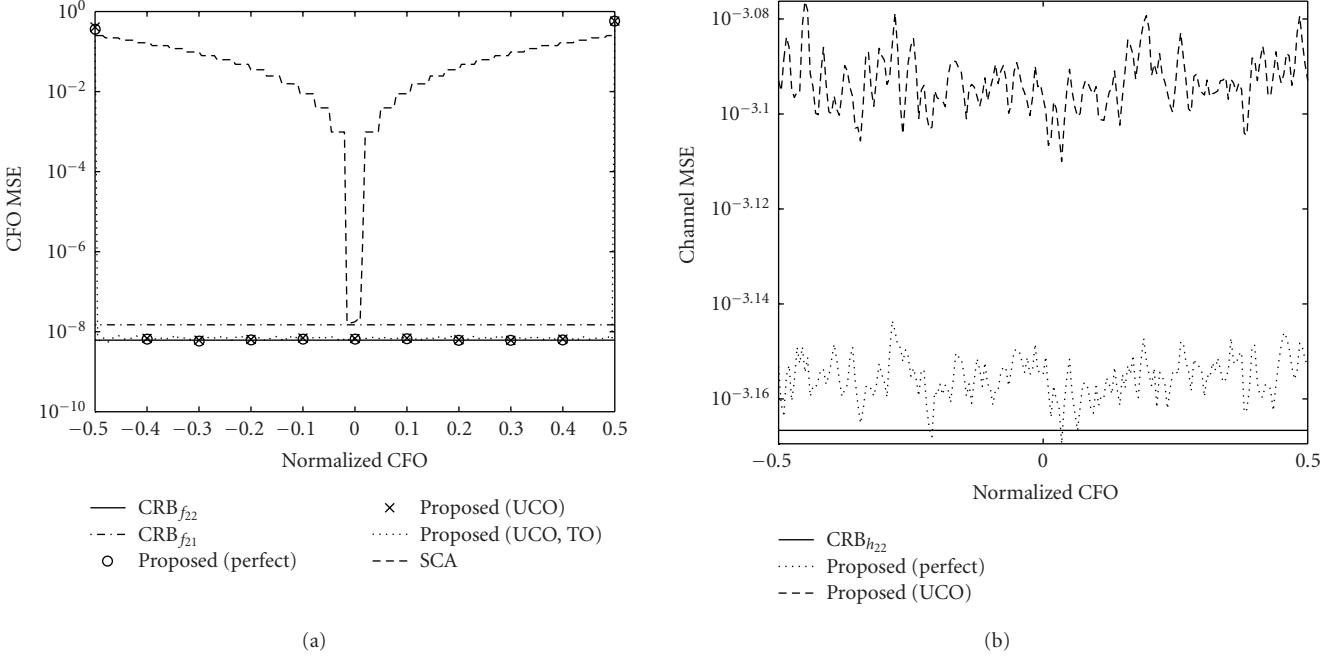


FIGURE 5: The influence of CFO. (a) CFO estimation performance. (b) Channel estimation performance.

- (b) with more pilot tones, the estimation accuracy is improved, but it is not improved linearly with the number of pilot tones;
- (c) the CFO estimation performances of the proposed algorithm is better than that of SCA;
- (d) in fact, CRB is related to pilot data, so the CRB plotted in simulation is one corresponding to certain pilot tones. A lower MSE of estimation can be achieved by selecting optimum pilot tones [13].

5.4. Simulation 4: the influence of CFO

The dependence of the proposed estimator and SCA on the normalized CFO is highlighted in Figure 5a and the performance of channel estimation is plotted in Figure 5b. Here, SNR = 15 dB and $N = 64$. It is shown that the CFO performance of the proposed estimator is invariant in the normalized CFO range from -0.5 to 0.5 (CFO within $\pm N/2T$), while the channel estimation performance fluctuates slightly above the CRB. Compared with the proposed estimator, the SCA [6] can only estimate the CFO range within $\pm 1/T$ (two subcarrier distances) when using a pair of modulation blocks in $2T/2R$ antenna communication systems.

6. CONCLUSIONS

A joint channel and CFO estimator using block-type pilot in a MIMO-OFDM system is proposed and the CRB for the problem is also developed. Because the estimator is robust to overestimated channel order, the channel order can be replaced by the length of CP, which makes the estimation in the case of UCO possible. The CFO estimation is also robust to

the TO when the timing position is inside the CP. Finally, the computer simulation results show that the proposed estimator can exploit the gain from multiantennas effectively and the MSE of estimation is close to the corresponding CRB at certain SNR.

APPENDIX

In this appendix, we calculate each element of FIM in (11).

Before calculating the FIM, we need the following assumption:

$$\begin{aligned} E\{\mathbf{w}(k)\} &= \mathbf{0}, & E\{\mathbf{w}(k)\mathbf{w}^H(k)\} &= \sigma^2 \mathbf{I}, \\ E\{\mathbf{w}(k)\mathbf{w}^T(k)\} &= \mathbf{0} \end{aligned} \quad (\text{A.1})$$

and we can obtain the following result from (4):

$$\mathbf{w} = \mathbf{r} - \mathbf{C}(f)\mathbf{X}\mathbf{h}. \quad (\text{A.2})$$

Based on the assumption and result above, the derivative of (5) with respect to the parameters to be estimated can be listed as

$$\begin{aligned} \frac{\partial \ln L}{\partial \operatorname{Re}(\mathbf{h})} &= \frac{2}{\sigma^2} \operatorname{Re}(\mathbf{X}^H \mathbf{C}(f)^H \mathbf{w}), \\ \frac{\partial \ln L}{\partial \operatorname{Im}(\mathbf{h})} &= \frac{2}{\sigma^2} \operatorname{Im}(\mathbf{X}^H \mathbf{C}(f)^H \mathbf{w}), \\ \frac{\partial \ln L}{\partial \bar{f}} &= \frac{4\pi}{\sigma^2} \operatorname{Im}(\mathbf{h}^H \mathbf{X}^H \mathbf{B} \mathbf{C}(f) \mathbf{w}). \end{aligned} \quad (\text{A.3})$$

The elements of FIM are given by

$$\begin{aligned}
 \text{FIM}_{1,1} &= E\left[\frac{\partial \ln L}{\partial \text{Re}(\mathbf{h})}\left(\frac{\partial \ln L}{\partial \text{Re}(\mathbf{h})}\right)^T\right] = \frac{2}{\sigma^2} \text{Re}(\mathbf{X}^H \mathbf{X}), \\
 \text{FIM}_{1,2} &= E\left[\frac{\partial \ln L}{\partial \text{Re}(\mathbf{h})}\left(\frac{\partial \ln L}{\partial \text{Im}(\mathbf{h})}\right)^T\right] = -\frac{2}{\sigma^2} \text{Im}(\mathbf{X}^H \mathbf{X}), \\
 \text{FIM}_{1,3} &= E\left[\frac{\partial \ln L}{\partial \text{Re}(\mathbf{h})}\left(\frac{\partial \ln L}{\partial f}\right)^T\right] = -\frac{4\pi}{\sigma^2} \text{Im}(\mathbf{X}^H \mathbf{B} \mathbf{X} \mathbf{h}), \\
 \text{FIM}_{2,1} &= E\left[\frac{\partial \ln L}{\partial \text{Im}(\mathbf{h})}\left(\frac{\partial \ln L}{\partial \text{Re}(\mathbf{h})}\right)^T\right] = \frac{2}{\sigma^2} \text{Im}(\mathbf{X}^H \mathbf{X}), \\
 \text{FIM}_{2,2} &= E\left[\frac{\partial \ln L}{\partial \text{Im}(\mathbf{h})}\left(\frac{\partial \ln L}{\partial \text{Im}(\mathbf{h})}\right)^T\right] = \frac{2}{\sigma^2} \text{Re}(\mathbf{X}^H \mathbf{X}), \\
 \text{FIM}_{2,3} &= E\left[\frac{\partial \ln L}{\partial \text{Im}(\mathbf{h})}\left(\frac{\partial \ln L}{\partial f}\right)^T\right] = \frac{4\pi}{\sigma^2} \text{Re}(\mathbf{X}^H \mathbf{B} \mathbf{X} \mathbf{h}), \\
 \text{FIM}_{3,3} &= E\left[\frac{\partial \ln L}{\partial f}\left(\frac{\partial \ln L}{\partial f}\right)^T\right] = \frac{8\pi^2}{\sigma^2} \text{Re}(\mathbf{h}^H \mathbf{X}^H \mathbf{B}^2 \mathbf{X} \mathbf{h}).
 \end{aligned} \tag{A.4}$$

As FIM is a symmetry matrix, the FIM in (11) can be constructed by (A.4).

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