Hindawi Publishing Corporation
EURASIP Journal on Applied Signal Processing
Volume 2006, Article ID 91567, Pages 1–10
DOI 10.1155/ASP/2006/91567

Adaptive DOA Estimation Using a Database of PARCOR Coefficients

Eiji Mochida and Youji Iiguni

Department of Systems Innovation, Graduate School of Engineering Science, Osaka University, 1–3 Machikaneyama Toyonaka, Osaka 560-8531, Japan

Received 6 July 2005; Revised 8 March 2006; Accepted 23 March 2006

Recommended for Publication by Benoit Champagne

An adaptive direction-of-arrival (DOA) tracking method based upon a linear predictive model is developed. This method estimates the DOA by using a database that stores PARCOR coefficients as key attributes and the corresponding DOAs as non-key attributes. The k-dimensional digital search tree is used as the data structure to allow efficient multidimensional searching. The nearest neighbour to the current PARCOR coefficient is retrieved from the database, and the corresponding DOA is regarded as the estimate. The processing speed is very fast since the DOA estimation is obtained by the multidimensional searching. Simulations are performed to show the effectiveness of the proposed method.

1. INTRODUCTION

Estimation of the direction-of-arrival (DOA) for multiple sources plays an important role in the fields of radar, sonar, high-resolution spectral analysis, and communication systems. A lot of high-resolution DOA estimation methods using a linear array antenna [1–3] or using two identical subarrays [4] have been developed. The linear prediction (LP) method [5] is one of the well-known methods. The LP method characterises the bearing spectrum by the LP coefficients, and provides a high-resolution spectrum even with a small number of antenna elements. However, the LP method requires to find local maxima (peak) of the bearing spectrum. The peak searching is computationally heavy, and thus the LP method is unsuitable for DOA tracking when DOAs change with time. Recently, Markov chain, Monte Carlo (MCMC) [6, 7] method, and Gershman's optimisation method [8, 9] have been studied. MCMC method has high-resolution and Gershman's method can be used for estimation of moving sources. These methods achieve a high estimation accuracy, however their computational complexities are very large since optimisation problems need to be solved.

An adaptive DOA estimation method using a database has been proposed by one of the authors [10, 11]. This method uses autocorrelation coefficients as key attributes, and DOAs as non-key attributes. The nearest neighbour to the autocorrelation coefficients estimated from observation signals is retrieved from the database, and the corresponding DOA is regarded as the estimate. This method estimates the DOA by only a database retrieval method, and thus the processing speed is fast. However, the dimension of the key vector increases in proportion to the number of antenna elements. Therefore, as the number of antenna elements increases, the database size becomes larger and thus the processing speed is slower.

There is a one-to-one correspondence between the LP coefficients and the partial autocorrelation (PARCOR) coefficients, and therefore the PARCOR coefficients also characterise the bearing spectrum. The PARCOR coefficient is more suitable as a key vector than the LP coefficient, because the PARCOR coefficient is robust against rounding errors and the absolute value is assured to be less than or equal to unity.

We propose an adaptive DOA tracking method using a database of PARCOR coefficients. We put the PARCOR coefficients as key attributes and the DOAs as non-key attributes. In the database construction process, we quantise DOAs and signal powers, and compute a set of true auto-correlation matrices for various combinations of the quantised DOAs and signal powers. We further compute a set of PARCOR coefficients from the set of true auto-correlation matrices by using the modified Levinson-Durbin algorithm, and then store pairs of PARCOR coefficients and the corresponding DOAs into a database. In the estimation process, we estimate the PARCOR coefficients from observation signals by

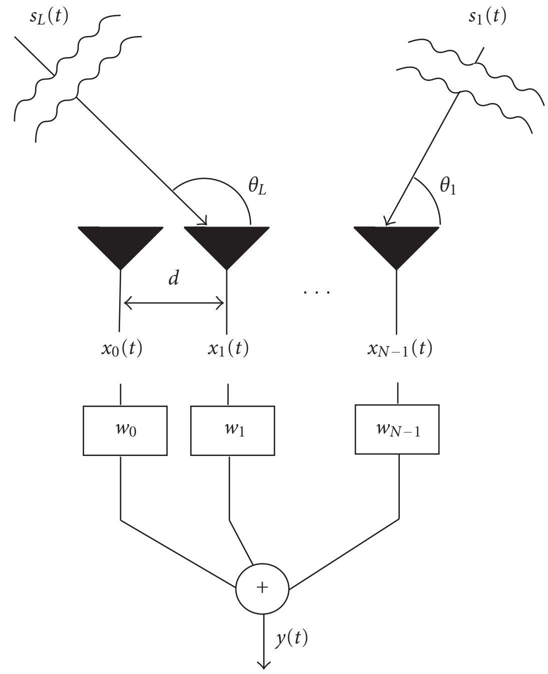


FIGURE 1: Distant wave source and linear array antenna.

using the Levinson-Durbin algorithm, retrieve a record with a key value nearest to the current key from the database, and use the corresponding DOA as the estimate. We then use the k-d trie (k-dimensional digital search tree) [12] as the data structure to allow efficient multidimensional searching. The proposed method does not require exhaustive peak searching, and provides the estimation by only the database retrieval method. Using this, we can reduce the dimension of the key vector to the number of signal sources even if the number of antenna elements is larger than the number of signal sources. The size reduction of the key vector is extremely useful in decreasing search time.

2. DOA ESTIMATION PROBLEM AND LINEAR PREDICTION

2.1. DOA estimation problem

Consider L mutually uncorrelated signals with center frequency f_c (wavelength λ_c) arriving at a linear array antenna of N ($N > L$) inter-elements with distance d. We assume that the signals are narrow banded and the signal sources are far apart from the array. Let the ith arriving signal at time t, the DOA, and the signal power be $s_i(t)$, θ_i, and σ_i^2, respectively. Let the signal received by the jth element, the noise input on the jth antenna element, and the output of the array antenna be $x_j(t)$, $n_j(t)$, and $y(t)$, respectively. The relation between the signal sources and the linear array antenna is illustrated in Figure 1. The output vector from the array antenna is expressed as

$$\mathbf{x}(t) = \left(x_0(t), \ldots, x_{N-1}(t)\right)^{\mathrm{T}} = \sum_{i=1}^{L} \mathbf{a}(\theta_i) s_i(t) + \mathbf{n}(t). \tag{1}$$

Here $\mathbf{n}(t) = (n_0(t), n_1(t), \ldots, n_{N-1}(t))^{\mathrm{T}}$ is an N-dimensional complex white noise vector, and $(\cdot)^{\mathrm{T}}$ denotes the transpose. We assume that noises $\{n_j(t)\}_{j=0}^{N-1}$ and signals $\{s_i(t)\}_{i=1}^{L}$ are mutually uncorrelated.

In the case of the omnidirectional element, the response vector $\mathbf{a}(\theta_i)$ is given by

$$\mathbf{a}(\theta_i) = \left(1, e^{j\varphi_i}, \ldots, e^{j\varphi_i(N-1)}\right)^{\mathrm{T}} \tag{2}$$

with $\varphi_i = 2\pi d \cos\theta_i / \lambda_c$. We define the weight coefficient on the jth array output as w_j ($j = 0, \ldots, N-1$) and the weight coefficient vector as $w = (w_0, w_1, \ldots, w_{N-1})^{\mathrm{T}}$. The array output is then expressed as

$$y(t) = \sum_{j=0}^{N-1} \bar{w}_j x_j(t) = \mathbf{w}^{\mathrm{H}} \mathbf{x}(t), \tag{3}$$

where $\bar{(\cdot)}$ denotes the conjugation and $(\cdot)^{\mathrm{H}}$ denotes the Hermitian transpose. We define the auto-correlation matrix of the output signal $\mathbf{x}(t)$ by

$$\begin{aligned}\mathbf{R} &= \mathrm{E}\left[\mathbf{x}(t)\mathbf{x}^{\mathrm{H}}(t)\right] = \sum_{i=1}^{L} \sigma_i^2 \mathbf{a}(\theta_i)\mathbf{a}(\theta_i)^{\mathrm{H}} + \sigma^2 \mathbf{I} \\ &= \begin{pmatrix} r_0 & r_1 & r_2 & \cdots & r_{N-1} \\ \bar{r}_1 & r_0 & r_1 & & r_{N-2} \\ \bar{r}_2 & \bar{r}_1 & r_0 & & \vdots \\ \vdots & & & \ddots & r_1 \\ \bar{r}_{N-1} & \cdots & & \bar{r}_1 & r_0 \end{pmatrix},\end{aligned} \tag{4}$$

where $\mathbf{I}$ denotes the identity matrix of size N, $\mathrm{E}[\cdot]$ denotes the expectation operator, and σ^2 denotes the noise power. The first term of the right-hand side of (4) is the signal term, of which rank is always L if $\theta_i \neq \theta_j$ ($i \neq j$), and the second term is the noise term. The inclusion of the noise term guarantees $\mathbf{R}$ to be full-rank of N. Using the auto-correlation matrix, the output power is represented by

$$\mathrm{E}\left[\,|y(t)|^2\right] = \mathrm{E}\left[\,|\mathbf{w}^{\mathrm{H}}\mathbf{x}(t)|^2\right] = \mathbf{w}^{\mathrm{H}}\mathbf{R}\mathbf{w}. \tag{5}$$

2.2. Linear prediction

When we set $w_0 = 1$ in (3), we can have

$$x_0(t) = -\sum_{j=1}^{N-1} \bar{w}_j x_j(t) + y(t). \tag{6}$$

When we predict $x_0(t)$ with a weighted linear combination of the output signals $\{x_j(t)\}_{j=1}^{N-1}$, we can regard $y(t)$ as the prediction error. We will determine the weight coefficients $\{w_j\}_{j=1}^{N-1}$ so that the mean-square error is minimised. This is formulated as

$$\min_{\mathbf{w}} \mathbf{w}^{\mathrm{H}}\mathbf{R}\mathbf{w} \quad \text{subject to} \quad \mathbf{c}^{\mathrm{H}}\mathbf{w} = 1, \tag{7}$$

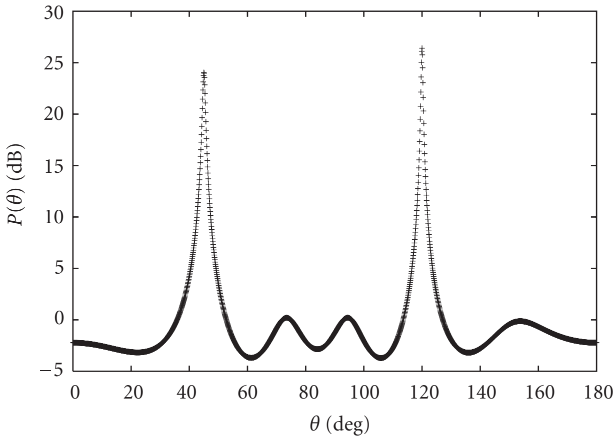


FIGURE 2: DOA estimation using the LP method for the case of $(\theta_1, \theta_2) = (45^\circ, 120^\circ)$.

where $\mathbf{c} = (1, \underbrace{0, \ldots, 0}_{N-1})^{\mathrm{T}}$. The constrained optimisation problem is easily solved by using the Lagrange multiplier method. The solution is given by

$$\mathbf{w}^* = (1, w_1^*, \ldots, w_{N-1}^*)^{\mathrm{T}} = \frac{1}{\mathbf{c}^{\mathrm{H}}\mathbf{R}^{-1}\mathbf{c}}\mathbf{R}^{-1}\mathbf{c}. \quad (8)$$

Here the weight coefficients $\{w_j^*\}_{j=1}^{N-1}$ are referred to as the "LP coefficients." It is here noted that the Capon spectrum is obtained by replacing $\mathbf{c}$ by $\mathbf{a}(\theta)$ in (7).

The conventional LP method estimates the DOAs by locally maximising the following bearing spectrum:

$$P(\theta) = \frac{1}{|\mathbf{a}^{\mathrm{H}}(\theta)\mathbf{w}^*|^2}. \quad (9)$$

Figure 2 shows an example of the bearing spectrum obtained by the LP method for the case of $(\theta_1, \theta_2) = (45^\circ, 120^\circ)$. The extremely large peaks correspond with the DOAs, and the other small peaks are spurious. We have to perform the computationally expensive peak searching to find the two large peaks. The peak searching requires $O(NK)$ computation steps, where K is the number of bins. When the DOAs change with time, the peak searching has to be performed at each time. The iterative use of the peak searching requires a large amount of processing time. Thus the conventional LP method is unsuitable for adaptive DOA estimation.

3. DOA ESTIMATION USING A DATABASE RETRIEVAL SYSTEM

We have explained in Section 2 that the peaks of the bearing spectrum are uniquely characterised by the LP coefficients. We can thus estimate the DOAs by searching the nearest neighbour to the current LP coefficients in the database which stores pairs of the LP coefficients and the DOAs. This method can estimate the DOAs by only a database retrieval method. The processing speed is very fast, since exhaustive peak searching is not required. We first explain how to construct the database, and then how to estimate the DOAs by database searching.

3.1. Database construction

3.1.1. Selection of model coefficients

We construct a database, which stores model coefficients as key attributes and DOAs as non-key attributes. The LP coefficients $\{w_j^*\}_{j=1}^{N-1}$ seem to be good candidates for the model coefficients. However, the LP coefficients are unsuitable as keys, because they take values in the range $(-\infty, \infty)$. Instead of the LP coefficients, we use the PARCOR coefficients $\{\rho^j\}_{j=1}^{N-1}$ which have a one-to-one correspondence to the LP coefficients, as the keys.

We define the jth LP coefficient of order i as $w_j^{(i)*}$. When the PARCOR coefficients $\{\rho^j\}_{j=1}^{N-1}$ are given, the corresponding LP coefficients $\{w_j^{(N-1)*}\}_{j=1}^{N-1}$ are computed by using the recursion

$$w_j^{(i)*} = w_j^{(i-1)*} + \rho^i \bar{w}_{i-j}^{(i-1)*} \quad (j = 1, 2, \ldots, i). \quad (10)$$

Here the recursion is initiated with $i = 2$ and stopped when i reaches the final value $N - 1$. On the other hand, when $\{w_j^{(N-1)*}\}_{j=1}^{N-1}$ are given, the corresponding PARCOR coefficients $\{\rho^j\}_{j=1}^{N-1}$ are computed by using the recursion

$$w_j^{(i-1)*} = \frac{w_j^{(i)*} - \rho^i \bar{w}_{i-j}^{(i)*}}{1 - |\rho^i|^2} \quad (j = 1, 2, \ldots, i-1) \quad (11)$$

and the fact that $w_{i-1}^{(i-1)*} = \rho^{i-1}$. Here the recursion is initiated with $i = N - 1$ and stopped when i reaches 2. Equations (10) and (11) show that there is a one-to-one relationship between the LP coefficients and the PARCOR coefficients. The PARCOR coefficients are more suitable as keys than the LP coefficients, because the PARCOR coefficients are robust against rounding errors and the absolute values are assured to be less than or equal to unity [13].

We see from (8) that the LP coefficients $\{w_j^{(N-1)*}\}_{j=1}^{N-1}$ are uniquely computed from the auto-correlation matrix $\mathbf{R}$. Consequently, the PARCOR coefficients $\{\rho^j\}_{j=1}^{N-1}$ are also uniquely computed from $\mathbf{R}$. We also see from (4) that $\mathbf{R}$ is expressed as functions of θ_i, σ_i^2, and σ^2. As a result, $\{\rho^j\}_{j=1}^{N-1}$ is expressed as functions of θ_i, σ_i^2, and σ^2. We define the noise-free auto-correlation matrix by

$$\widetilde{\mathbf{R}} = \mathbf{R} - \sigma^2\mathbf{I} = \sum_{i=1}^{L} \sigma_i^2 \mathbf{a}(\theta_i)\mathbf{a}(\theta_i)^{\mathrm{H}}, \quad (12)$$

and then define the jth noise-free PARCOR coefficient computed from $\widetilde{\mathbf{R}}$ by $\tilde{\rho}^j$. Since $\tilde{\rho}^j$ does not depend on the noise power σ^2, it is a function of only (θ_i, σ_i^2).

Let the rank of $\widetilde{\mathbf{R}}$ be p. When L DOAs are different from each other, we have $p = L$. Otherwise, we have $p < L$. Therefore, p is always less than N, and the $(N \times N)$ auto-correlation matrix $\widetilde{\mathbf{R}}$ is not invertible. Consequently, we cannot compute the noise-free LP coefficients from $\widetilde{\mathbf{R}}$ by the standard

$$
\begin{aligned}
&\varepsilon_0^2 = r_0\\
&j = 1, 2, \dots, N-1\\
&\Delta^j = \bar{r}_j + \sum_{i=1}^{j} w_i^{(j-1)*} \bar{r}_{j-i}\\
&\widetilde{\rho}^j = w_j^{(j)*} = -\frac{\Delta^j}{\varepsilon_{j-1}^2} \cdots \qquad \text{(A)}\\
&\text{if } |\widetilde{\rho}^j|^2 > \alpha, \quad \text{then stop}\\
&\varepsilon_j^2 = \varepsilon_{j-1}^2 (1 - |\widetilde{\rho}^j|^2)\\
&i = 1, 2, \dots, j-1\\
&w_i^{(j)*} = w_i^{(j-1)*} + \widetilde{\rho}^j \bar{w}_{j-i}^{(j-1)*}
\end{aligned}
$$

Algorithm 1: Modified Levinson-Durbin algorithm.

Levinson-Durbin algorithm. To solve this problem, we develop a modified Levinson-Durbin (L-D) algorithm which recursively computes the LP and the PARCOR coefficients from the auto-correlation matrix by utilising the Toeplitz structure of $\widetilde{\mathbf{R}}$. Using this algorithm, we can determine the noise-free LP coefficients and the noise-free PARCOR coefficients of order p from $\widetilde{\mathbf{R}}$.

Algorithm 1 summarises the modified L-D algorithm. When applying the standard L-D algorithm to the noise-free auto-correlation matrix $\widetilde{\mathbf{R}}$ of order p, the value of $|\widetilde{\rho}^p|$ becomes unity during order update, and then ε_p^2 becomes zero. We cannot compute the succeeding PARCOR coefficients $\{\widetilde{\rho}^j\}_{j=p+1}^{N-1}$, because division by zero occurs in (A). For the solution, when $|\widetilde{\rho}^p|$ is larger than a threshold $\alpha(\simeq 1)$, we regard $|\widetilde{\rho}^p|$ as unity, terminate the update, and set the succeeding noise-free PARCOR coefficients as zeros, that is, $\widetilde{\rho}^{p+1} = \cdots = \widetilde{\rho}^{N-1} = 0$. The reason for using this procedure is that the value of $|\widetilde{\rho}^p|$ does not become exactly equal to unity due to estimation errors. Using the modified L-D algorithm, we can obtain $N-1$ noise-free PARCOR coefficients $(\widetilde{\rho}^1, \widetilde{\rho}^2, \dots, \widetilde{\rho}^p, \underbrace{0, 0, \dots, 0}_{N-1-p})$. Since $p \le L$, we always have $\widetilde{\rho}^j = 0$ for $j = L+1, L+2, \dots, N-1$. Zero coefficients do not depend on the DOAs. Thus we use the L noise-free PARCOR coefficients $(\widetilde{\rho}^1, \widetilde{\rho}^2, \dots, \widetilde{\rho}^L)$ as the database key.

3.1.2. Quantisation of data

We quantise the DOAs θ_i into $\theta_i(u)$ $(u = 1, 2, \dots, U)$ and the signal powers σ_i^2 into $\sigma_i^2(v)$ $(v = 1, 2, \dots, V)$, where U and V are the numbers of the DOA and signal power bins, respectively. Denoting the total number of the quantised data as M, we have

$$M = U^L \times V^L. \tag{13}$$

We put the quantisation step sizes of θ_i and σ_i^2 as $\delta\theta_i$ and $\delta\sigma_i^2$, respectively. As $\delta\theta_i$ and $\delta\sigma_i^2$ are smaller, the estimation accuracy is higher while the database size is larger. We therefore have to determine the values of $\delta\theta_i$ and $\delta\sigma_i^2$ so that a good tradeoff between the estimation accuracy and the database size is achieved. While θ_i takes values in the range $[0, \pi)$, σ_i^2 may take a very large value. The straightforward quantisation of σ_i^2 significantly increases the size of V. We have thus normalised the signal power σ_i^2 with respect to $\sum_i \sigma_i^2$ so that the normalised signal power is restricted to the range $(0, 1)$.

We define the noise-free auto-correlation matrices as $\{\widetilde{\mathbf{R}}(m)\}_{m=1}^{M}$, and the noise-free PARCOR coefficients corresponding to each of the M quantised data as $\{\widetilde{\rho}^j(m)\}_{m=1}^{M}$. We compute $\widetilde{\mathbf{R}}(m)$ by using (12), and then compute $\widetilde{\rho}^j(m)$ from $\widetilde{\mathbf{R}}(m)$ by using the modified L-D algorithm. We further quantise the real and imaginary parts of $\widetilde{\rho}^j(m)$ to the integer values $z_{2j-1}(m)$ and $z_{2j}(m)$ with b bits. Then we can have

$$
\begin{aligned}
&(z_1(m), z_2(m), \dots, z_{2L}(m))\\
&\quad = \big(Q(\operatorname{Re}[\widetilde{\rho}^1(m)]), Q(\operatorname{Im}[\widetilde{\rho}^1(m)]),\\
&\qquad Q(\operatorname{Re}[\widetilde{\rho}^2(m)]), Q(\operatorname{Im}[\widetilde{\rho}^2(m)]), \dots,\\
&\qquad Q(\operatorname{Re}[\widetilde{\rho}^L(m)]), Q(\operatorname{Im}[\widetilde{\rho}^L(m)])\big),
\end{aligned}
\tag{14}
$$

where Q is the output of the quantiser, and $\operatorname{Re}[x]$ and $\operatorname{Im}[x]$ denote the real and imaginary parts of x, respectively. Note that $z_j(m)$ takes value in the range $[0, 2^b - 1]$.

3.1.3. Database storage

We define the PARCOR vector corresponding to the mth quantised data as

$$\boldsymbol{\rho}(m) = (z_1(m), z_2(m), \dots, z_{2L}(m)) \quad (m = 1, 2, \dots, M) \tag{15}$$

and the DOA vector corresponding to $\boldsymbol{\rho}(m)$ as

$$\boldsymbol{\theta}(m) = (\theta_1(m), \theta_2(m), \dots, \theta_L(m)) \quad (m = 1, 2, \dots, M). \tag{16}$$

We successively store the pairs of $\{(\boldsymbol{\rho}(m), \boldsymbol{\theta}(m))\}_{m=1}^{M}$ into the database. If the database has already stored the same PARCOR vector as the current one, we delete it. We denote the number of data sets which are actually stored in the database as C. Then C is much smaller than M due to the deletion of data sets.

3.2. DOA estimation

3.2.1. Estimation of PARCOR coefficients

We will present a method of estimating the auto-correlation matrix $\mathbf{R}$ from observation signals $x_j(t)$ $(j = 0, 1, \dots, N-1)$. When the DOAs change with time, we recursively estimate it by

$$\begin{aligned}\hat{\mathbf{R}}_t &= \frac{\mathbf{x}_t\mathbf{x}_t^{\mathrm{H}} + \lambda\mathbf{x}_{t-1}\mathbf{x}_{t-1}^{\mathrm{H}} + \lambda^2\mathbf{x}_{t-2}\mathbf{x}_{t-2}^{\mathrm{H}} + \cdots}{1+\lambda+\lambda^2+\cdots} \\ &= \lambda\frac{\mathbf{x}_{t-1}\mathbf{x}_{t-1}^{\mathrm{H}} + \lambda\mathbf{x}_{t-2}\mathbf{x}_{t-2}^{\mathrm{H}} + \lambda^2\mathbf{x}_{t-3}\mathbf{x}_{t-3}^{\mathrm{H}} + \cdots}{1+\lambda+\lambda^2+\cdots} \\ &\quad + \frac{1}{1+\lambda+\lambda^2+\cdots}\mathbf{x}_t\mathbf{x}_t^{\mathrm{H}} \\ &= \lambda\hat{\mathbf{R}}_{t-1} + (1-\lambda)\mathbf{x}_t\mathbf{x}_t^{\mathrm{H}}.\end{aligned} \tag{17}$$

Here, λ (usually $0.95 \le \lambda \le 0.995$) is a forgetting factor that controls the influence of the previous estimations, and $\hat{\mathbf{R}}_t$ is the estimation of the auto-correlation matrix at time t. Unfortunately, the recursive estimation using (17) does not preserve the Toeplitz structure of $\mathbf{R}$. We thus average the diagonal elements of $\hat{\mathbf{R}}_t$ to obtain the estimation of r_j as follows:

$$\hat{r}_j = \frac{\sum_{l=1}^{N-j}(\hat{\mathbf{R}}_t)_{l,l+j}}{N-j} \quad (j = 0, 1, \ldots, N-1), \tag{18}$$

where $(\hat{\mathbf{R}}_t)_{i,j}$ denotes the ijth element of $\hat{\mathbf{R}}_t$. We next subtract the noise power σ^2 from the diagonal elements of $\hat{\mathbf{R}}_t$ to estimate the noise-free auto-correlation matrix $\tilde{\mathbf{R}}$ as follows:

$$\hat{\tilde{\mathbf{R}}}_t = \hat{\mathbf{R}}_t - \sigma^2\mathbf{I}. \tag{19}$$

Here the noise power σ^2 is assumed to be known. It needs to be estimated a priori in the absence of source signals or needs to be estimated by using the eigenvalue decomposition of auto-correlation matrix $\mathbf{R}$. We denote the estimation of $\tilde{\rho}^j$ as $\hat{\tilde{\rho}}^j$. We recursively calculate $\{\hat{\tilde{\rho}}^j\}_{j=1}^{N-1}$ from $\hat{\tilde{\mathbf{R}}}_t$ by using the modified L-D algorithm. In the same way as in the database construction, when $|\hat{\tilde{\rho}}^j| > \alpha$, we put $\hat{\tilde{\rho}}^{j+1} = \cdots = \hat{\tilde{\rho}}^{N-1} = 0$, and take the estimation of the PARCOR vector as

$$\begin{aligned}\hat{\boldsymbol{\rho}} &= \Big(Q\big(\mathrm{Re}\big[\hat{\tilde{\rho}}^1\big]\big), Q\big(\mathrm{Im}\big[\hat{\tilde{\rho}}^1\big]\big), Q\big(\mathrm{Re}\big[\hat{\tilde{\rho}}^2\big]\big), \\ &\qquad Q\big(\mathrm{Im}\big[\hat{\tilde{\rho}}^2\big]\big), \ldots, Q\big(\mathrm{Re}\big[\hat{\tilde{\rho}}^L\big]\big), Q\big(\mathrm{Im}\big[\hat{\tilde{\rho}}^L\big]\big)\Big) \\ &\equiv (\hat{z}_1, \hat{z}_2, \ldots, \hat{z}_{2L}).\end{aligned} \tag{20}$$

3.2.2. Database retrieval

Mutidimensional searching is performed to retrieve the PARCOR vector nearest to $\hat{\boldsymbol{\rho}}$ from the database. More concretely, the PARCOR vectors lying in the hypercube $\{(z_1, z_2, \ldots, z_{2L}) \mid |\hat{z}_j - z_j| \le D,\ j = 1, 2, \ldots, 2L\}$ are retrieved from the database. Here D denotes the searching range which is a positive integer number such that $0 \le D \le 2^b - 1$. We take the DOA vector corresponding to the retrieved PARCOR vector as the DOA estimate, and denote the DOA estimation at time t as $\hat{\boldsymbol{\theta}}_t$. When more than one PARCOR vector is retrieved during the multidimensional searching, we select the PARCOR vector which minimises the Euclidean distance $\sum_{j=1}^{2L}\sqrt{(\hat{z}_j - z_j)^2}$ out of the retrieved ones. If no data are retrieved, we take the previous estimation $\hat{\boldsymbol{\theta}}_{t-1}$ as the current estimation $\hat{\boldsymbol{\theta}}_t$.

4. PERFORMANCE EVALUATION

We performed simulations for the cases of $L = 2$ and $L = 3$ to evaluate the estimation performance of the proposed method.

4.1. DOA estimation for two signals

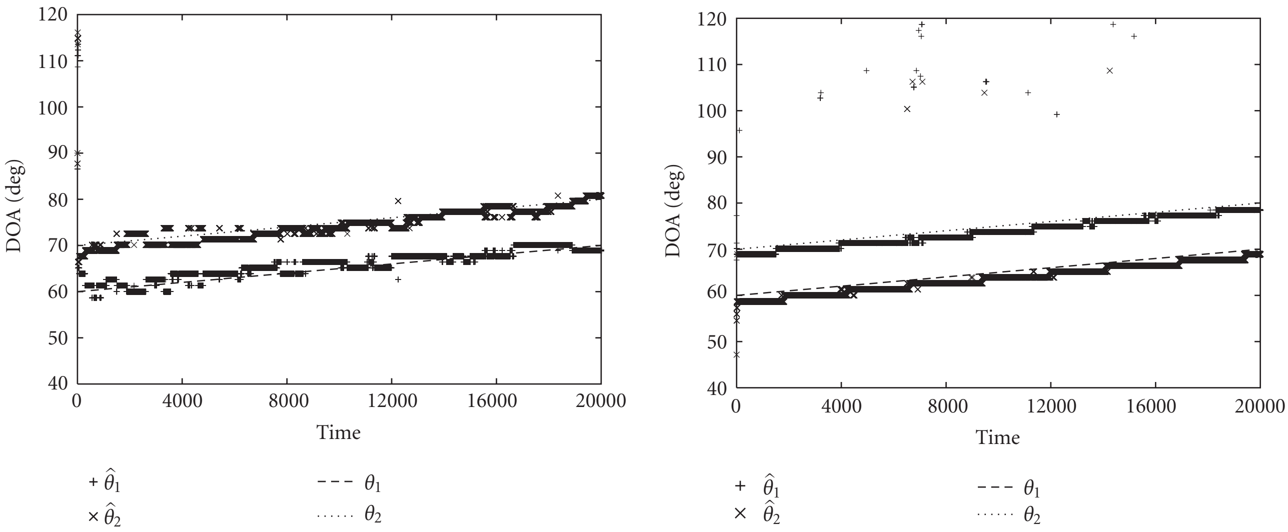
We constructed the database of $L = 2$, and estimated the DOAs of two moving sources.

4.1.1. Database construction

We consider the case where two signals arrive on the linear array antenna of $N = 6$ and $d = \lambda_c/2$. We quantise the DOA by sampling $\cos\theta$ with constant sampling interval 0.02, and quantise the normalised power with the constant sampling interval 0.25. Then we have $U = 99$ and $V = 4$, and therefore $M = U^L \times V^L = 156816$. We put $b = 8$ and $\alpha = 1 - 2/2^b = 0.992$ so that better estimation accuracy was obtained. We successively entered the data set $\{(\boldsymbol{\rho}(m), \boldsymbol{\theta}(m))\}_{m=1}^{M}$ into the database. Then $C = 22229$ $(= 0.14 \times M)$, and the size of the database was about 776 (KB).

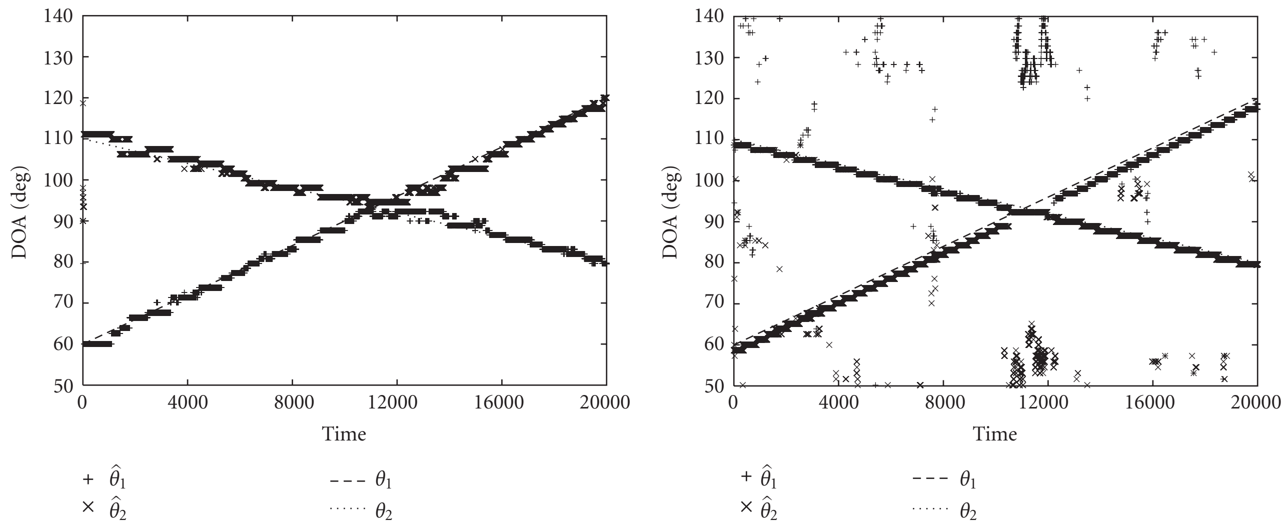
4.1.2. DOA estimation

We estimated the DOAs of two moving signals, where we put $\sigma_1^2 = 40$, $\sigma_2^2 = 50$, and $\sigma^2 = 1$. Then we have $\mathrm{SNR}_1 = 16$ dB and $\mathrm{SNR}_2 = 17$ dB. We have recursively estimated $\hat{\mathbf{R}}_t$ by (17) with $\lambda = 0.995$. As λ is smaller, tracking capability is improved while stability of the estimations is lost. Therefore we have to make a tradeoff between tracking capability and stability in the choice of λ (usually $0.95 \le \lambda \le 0.995$). Since the nonstationarity is weak in this case, we put $\lambda = 0.995$. We put the searching range $D = 10$. Figure 3 shows the results for the case where θ_1 and θ_2 change by 1° per 4000 snapshots starting from 60° and 70°, respectively. For example, when the sampling frequency f_s is 1.0 (MHz), the time interval τ is $\tau = 1/f_s = 1.0$ (μs). Then the duration of 4000 snapshots is 4.0 (ms). Figure 4 shows the results for the case where θ_1 changes by 1° per 333 snapshots starting from 60° and θ_2 changes by −1° per 666 snapshots starting from 110°. Figures 3(a) and 4(a) show the results of the proposed method. Figures 3(b) and 4(b) show the results of the conventional LP method, where the peaks of $P(\theta)$ were obtained by sampling $\cos\theta$ with constant sampling interval 0.02. We see that the proposed method well tracks the DOA changes. The erratic results of the proposed method are due to the quantisation errors of PARCOR coefficients. The MSEs of the proposed method and the LP method are 22.81 and 7.22, respectively, and the estimation accuracy of the LP method is better than that of the proposed method. However, the estimation of the LP method sometimes fails due to the existence of the spurious of the bearing spectrum. Moreover the proposed method is much faster than the the LP method as shown later.



(a) (b)

Figure 3: Estimation results for two moving signals: (a) proposed method (b) LP method.



(a) (b)

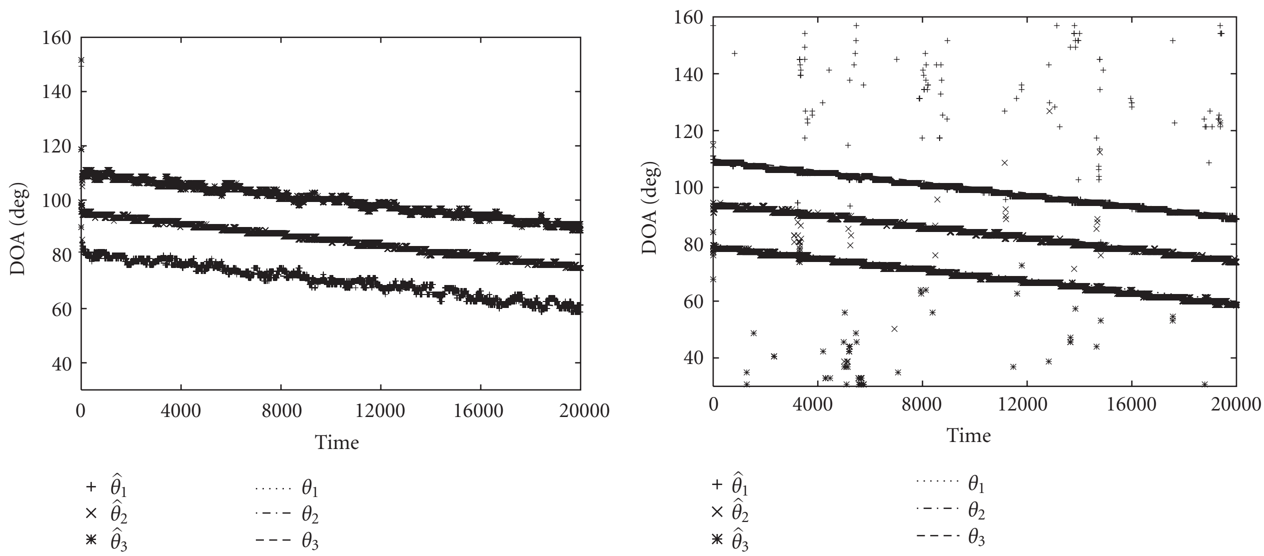
Figure 4: Estimation results for two moving signals: (a) proposed method (b) LP method.

4.2. *DOA estimation for three signals*

We constructed the database of $L = 3$, and estimated the DOAs of three moving signals. We used the same quantisation step sizes as the previous ones. Then we had $M = 62099136$ and $C = 3821007$ $(= 0.06 \times M)$. The database size was about 64 (MB).

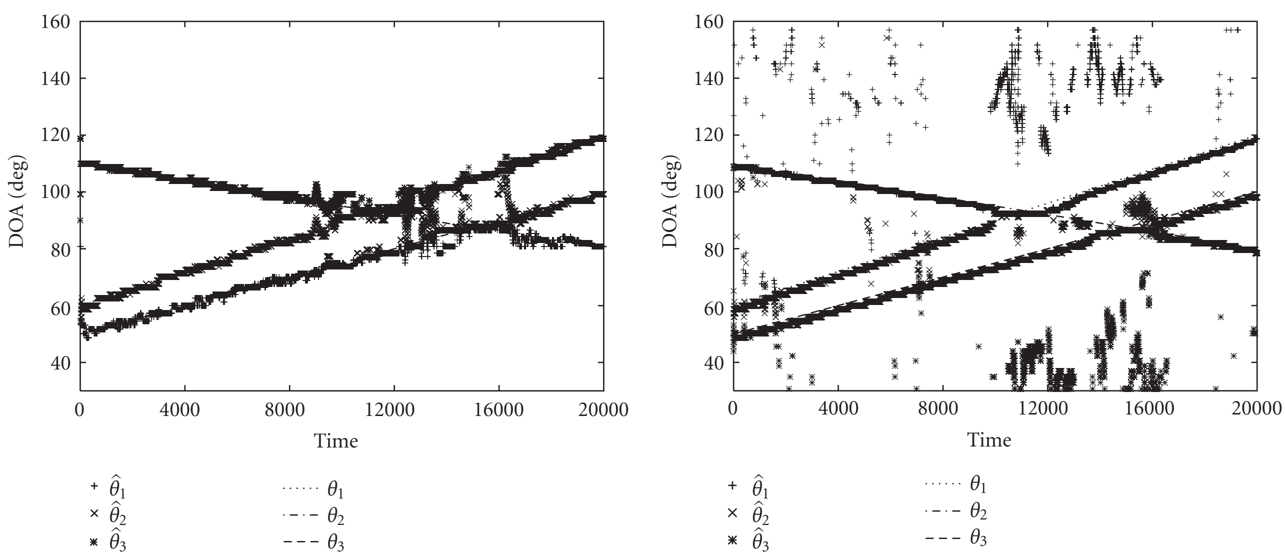
4.2.1. *DOA estimation*

We put $\lambda = 0.995$ and $D = 10$ in the same way as in the previous case. We estimated the DOAs of three moving signals ($\text{SNR}_1 = 16$ dB, $\text{SNR}_2 = 17$ dB, $\text{SNR}_3 = 17$ dB). Figure 5 shows the results for the case where θ_1, θ_2, and θ_3 change by -1° per 1000 snapshots starting from 80°, 95°, and 110°, respectively. Figure 6 shows the results for the case where θ_1 changes by 1° per 333 snapshots starting from 60°, θ_2 changes by -1° per 666 snapshots starting from 110°, and θ_3 changes by 1° per 400 snapshots starting from 50°. Figures 5(a) and 6(a) show the results of the proposed method. Figures 5(b) and 6(b) show the results of the conventional LP method. We see that the proposed method well tracks the DOA changes. Similarly, the estimation accuracy of the LP method is better than that of the proposed method, however the estimation of the LP



(a) (b)

Figure 5: Estimation results for three moving signals: (a) proposed method (b) LP method.



(a) (b)

Figure 6: Estimation results for three moving signals: (a) proposed method (b) LP method.

method sometimes fails due to the existence of the spurious peaks of the bearing spectrum, and the proposed method is much faster than the the LP method as shown later.

The proposed method requires a priori knowledge of the number of signals L, because the database contents depend on the value of L. Consequently, L needs to be estimated by using the model selection method such as Akaike information criteria (AIC) [14, 15]. Fortunately, the proposed method can well estimate the DOAs of L' signals using the database designed for $L(>L')$ signals, although it fails when $L<L'$. The reason is that estimation of L' signals is equivalent to the estimation of L signals where $L-L'$ signals arrive at the same angle.

We will denote a database designed for the L signals as $DB(L)$. Figure 7 shows the results of estimating the DOAs of two signals with $DB(3)$. We see that the proposed method using $DB(3)$ correctly estimates the DOAs of two signals. Figure 8 shows the results of estimating the DOAs of three signals with $DB(2)$. We see that the proposed method fails to estimate the DOAs.

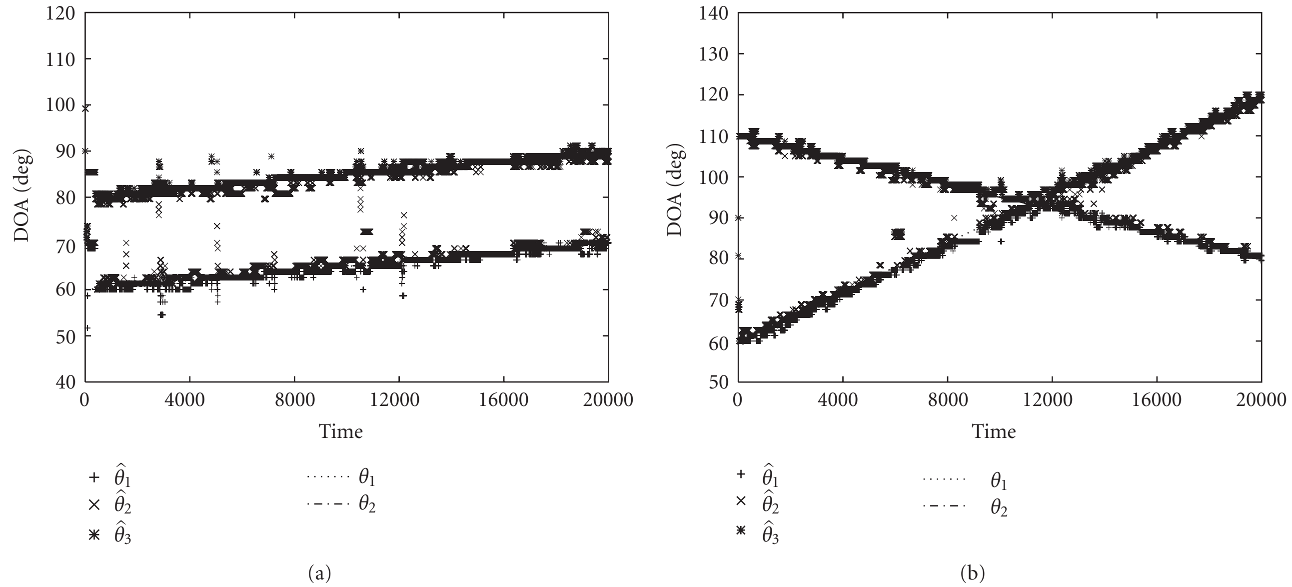


Figure 7: Estimation results for two moving signals using $DB(3)$.

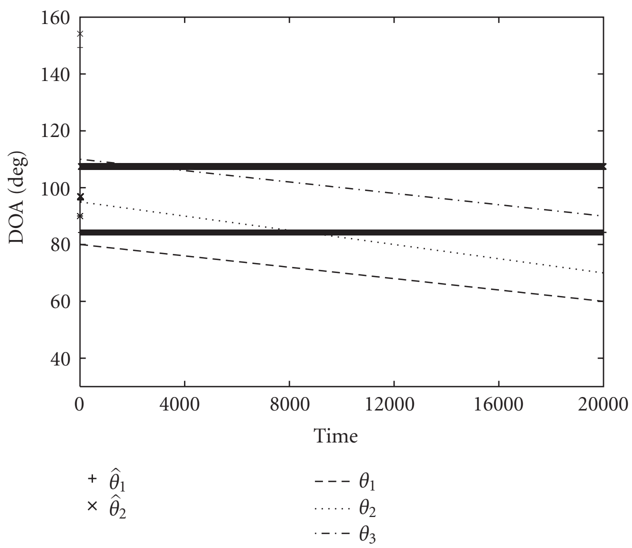


Figure 8: Estimation results for three moving signals using $DB(2)$.

4.3. Processing time

Table 1 summarises the computation times of the proposed method and the LP method. In the proposed method, the values in the columns "$\hat{\mathbf{R}}_t$," "$\tilde{\rho}^j$," "k-d trie," and "total" are the time requirements of computing $\hat{\mathbf{R}}_t$ by (17), estimating $\{\hat{\tilde{\rho}}^j\}_{j=1}^{L}$ by using the modified L-D algorithm, multidimensional searching, and the total processing time, respectively. In the LP method, the values in the columns "$\hat{w}_j^*$" and "peak searching" are the time requirements of estimating $\{\hat{w}_j^*\}_{j=1}^{N-1}$ by using the L-D algorithm and peak searching, respectively. In the proposed method, the database has been constructed a priori, and it has been fixed during the estimation. Therefore, we do not need to include the time requirement of database construction in the processing time. All computations were done on an IBM PC/AT compatible computer with an Intel Pentium IV 2.4 GHz. The time of computing $\hat{\mathbf{R}}_t$ is the same in both methods, that is, about 10.5 μs per snapshot. When comparing the computation times excluding it, the proposed method with $L = 2(L = 3)$ is about 50(30) times faster than the LP method. As the number of signal sources L increases, the database size gets larger and the processing time increases.

4.4. Determination of searching range

We have measured the estimation accuracy and the processing time for different values of the searching range D. We have evaluated the estimation accuracy by

$$J = \frac{1}{T}\sum_{t=1}^{T}\sum_{i=1}^{L}\left(\hat{\theta}_i^t - \theta_i^t\right)^2, \tag{21}$$

where θ_i^t denotes the ith DOA at time t, and T denotes the total snapshot.

Figure 9 shows the estimation accuracy for different values of D. We examined six cases of $(\theta_1, \theta_2) = (a)(30^\circ, 135^\circ)$, $(b)(30^\circ, 90^\circ)$, $(c)(45^\circ, 100^\circ)$, $(d)(45^\circ, 90^\circ)$, $(e)(60^\circ, 100^\circ)$, $(f)(60^\circ, 135^\circ)$. We set $T = 10000$ and $(\mathrm{SNR}_1, \mathrm{SNR}_2) = (10\,\mathrm{dB}, 11\,\mathrm{dB})$. We see that the estimation accuracy is improved as the value of D is larger, and that the estimation accuracy is fixed at some value for D larger than 10. The reason is that, when choosing $D = 10$, we can retrieve the nearest neighbour to the current key by multidimensional searching in almost all cases. Figure 10 shows the processing time per snapshot for different values of D. We see that the processing time

Table 1: Comparisons of processing time (per snapshot).

Simulation	Proposed method (μs)				LP method (μs)			
	$\hat{\mathbf{R}}_t$	$\hat{\tilde{\rho}}^j$	k-d trie	Total	$\hat{\mathbf{R}}_t$	$\hat{w}_j^*$	Peak searching	Total
$L = 2$	10.5	7.0	2.3	19.8	10.5	6.6	441.9	459.0
$L = 3$	10.5	8.0	7.8	26.3	10.5	6.6	441.9	459.0

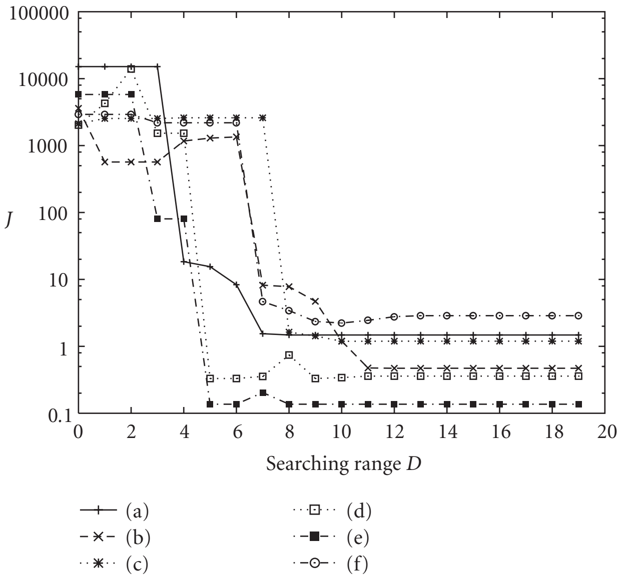


Figure 9: Estimation accuracy for different values of D.

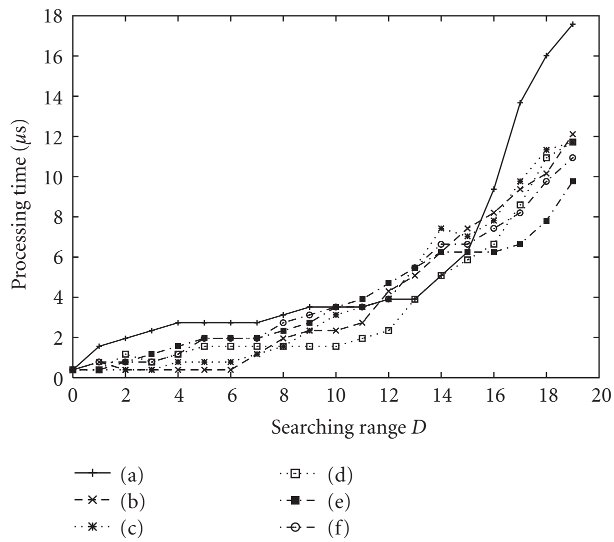


Figure 10: Processing time for different values of D.

increases as the value of D is larger. There is a tradeoff between the estimation accuracy and the processing time in determining D. We thus judged from Figures 9 and 10 that the appropriate value is 10, and put $D = 10$ in the previous simulations.

5. CONCLUSION

We proposed the adaptive DOA estimation method using the database of PARCOR coefficients. In this method, the dimension of key vector is equal to the number of signal sources and does not depend on the number of antenna elements. Thus the database size becomes relatively small and the processing speed is very fast. Although we found from simulation results that some erratic behaviours were observed due to quantisations of PARCOR coefficients, the proposed method is much faster than the LP method and is robust against the spurious of the bearing spectrum.

REFERENCES

[1] J. Capon, "High-resolution frequency-wavenumber spectrum analysis," *Proceedings of the IEEE*, vol. 57, no. 8, pp. 1408–1418, 1969.

[2] R. O. Schmidt, "Multiple emitter location and signal parameter estimation," *IEEE Transactions on Antennas and Propagation*, vol. 34, no. 3, pp. 276–280, 1986.

[3] B. D. Rao and K. V. S. Hari, "Performance analysis of Root-Music," *IEEE Transactions on Acoustics, Speech, and Signal Processing*, vol. 37, no. 12, pp. 1939–1949, 1989.

[4] R. Roy and T. Kailath, "ESPRIT - estimation of signal parameters via rotational invariance techniques," *IEEE Transactions on Acoustics, Speech, and Signal Processing*, vol. 37, no. 7, pp. 984–995, 1989.

[5] D. H. Johnson, "The application of spectral estimation methods to bearing estimation problems," *Proceedings of the IEEE*, vol. 70, no. 9, pp. 1018–1028, 1982.

[6] C. Andrieu, P. M. Djurić, and A. Doucet, "Model selection by MCMC computation," *Signal Processing*, vol. 81, no. 1, pp. 19–37, 2001.

[7] C. Andrieu and A. Doucet, "Joint Bayesian model selection and estimation of noisy sinusoids via reversible jump MCMC," *IEEE Transactions on Signal Processing*, vol. 47, no. 10, pp. 2667–2676, 1999.

[8] V. Katkovnik and A. B. Gershman, "Performance study of the local polynomial approximation based beamforming in the presence of moving sources," *IEEE Transactions on Antennas and Propagation*, vol. 50, no. 8, pp. 1151–1157, 2002.

[9] V. Katkovnik and A. B. Gershman, "A local polynomial approximation based beamforming for source localization and tracking in nonstationary environments," *IEEE Signal Processing Letters*, vol. 7, no. 1, pp. 3–5, 2000.

[10] I. Setiawan, Y. Iiguni, and H. Maeda, "New approach to adaptive DOA estimation based upon a database retrieval technique," *IEICE Transactions on Communications*, vol. E83-B, no. 12, pp. 2694–2701, 2000.

[11] I. Setiawan and Y. Iiguni, "DOA estimation based on the database retrieval technique with nonuniform quantization

and clustering," *Digital Signal Processing*, vol. 14, no. 6, pp. 590–613, 2004.

[12] J. A. Orenstein, "Multidimensional tries used for associative searching," *Information Processing Letters*, vol. 14, no. 4, pp. 150–157, 1982.

[13] J. Makhoul, "Linear prediction: a tutorial review," *Proceedings of the IEEE*, vol. 63, no. 4, pp. 561–580, 1975.

[14] M. Wax and T. Kailath, "Detection of signals by information theoretic criteria," *IEEE Transactions on Acoustics, Speech, and Signal Processing*, vol. 33, no. 2, pp. 387–392, 1985.

[15] L. C. Godara, "Application of antenna arrays to mobile communications, part II: beam-forming and direction-of-arrival considerations," *Proceedings of the IEEE*, vol. 85, no. 8, pp. 1195–1245, 1997.

Eiji Mochida received the B.E. and M.E. degrees in communications engineering from Osaka University, Osaka, Japan, in 2001 and 2003, respectively, and the D.E. degree from Osaka University in 2006. He is now working on hardware development for communication systems at the Pixela Corporation, Osaka, Japan.

Youji Iiguni received the B.E. and M.E. degrees in applied mathematics and physics from Kyoto University, Kyoto, Japan, in 1982 and 1984, respectively, and the D.E. degree from Kyoto University in 1990. He was an Assistant Professor at Kyoto University from 1984 to 1995, and an Associate Professor at Osaka University from 1995 to 2003. Since 2003, he has been a Professor at Osaka University. His research interests include signal processing and image processing.