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Adaptive Transmitter Precoding for Time Division Duplex CDMA in Fading Multipath Channels: Strategy and Analysis

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The recently developed blind adaptive techniques for multiuser detection in code division multiple access (CDMA) systems offer an attractive compromise of performance and complexity. However, the desire to further reduce complexity at the mobile unit has led to the investigation of techniques that move signal processing from the mobile unit to the base station. In this paper, we investigate transmitter precoding for downlink time division duplex (TDD) code division multiple access (CDMA) communications. In particular, we develop a linear minimum mean square error precoding strategy using blind channel estimation for fading multipath channels that allows for simple matched filtering at the mobile unit and is easy to make adaptive. We also present a performance analysis using tools developed for the analysis of conventional (receiver-based) linear blind multiuser detection in unknown channels. We compare the analytical and simulation results to traditional receiver-based blind multiuser detection. It is seen that transmitter precoding offers a reasonable alternative for TDD-mode CDMA when minimizing computational complexity at the mobile unit is a priority.

Keywords and phrases: multiuser detection, transmitter precoding, wireless communications, blind channel estimation.

1. INTRODUCTION

The demand for capacity and performance in multiple access wireless systems has spurred the development of sophisticated signal processing techniques for signal reception [1, 2]. However, the goal of maintaining low cost and complexity, especially at the mobile unit, is as important as ever. As a result, researchers have recently begun investigating signal processing techniques that move computational complexity from the mobile unit to the base station, where it can be managed more efficiently. Generally speaking, these techniques involve some kind of transmitter-based multiuser interference cancellation at the base station (precoding) and simple linear processing, for example, matched filtering, at the mobile unit. They are particularly appealing for time division duplex code division multiple access (TDD-CDMA) [3] since the same carrier is used for both uplink and downlink in different time slots. Hence, the downlink channel can be estimated at the base station using the uplink signals [4].

In [5], the authors considered transmitter precoding for synchronous CDMA over additive white Gaussian noise (AWGN) channels. They also present an extension to multipath channels, but a RAKE receiver is required and the channel is assumed perfectly known. A similar technique, pre-RAKE diversity combining, is investigated in [6]. However, RAKE reception is inherently sensitive to channel mismatch and performance is generally inferior to minimum mean square error (MMSE) or decorrelating multiuser interference rejection. The authors in [7] present a comparison of the techniques mentioned above using a typical vehicular channel. In [8], the authors consider transmitter precoding for multipath fading channels but, in contrast to the present work, their prefilter is applied to the output of the spread spectrum encoder, rather than applying the filter, first followed by spreading. It was shown that this approach has inferior average performance unless the spreading codes themselves are allowed to be adaptive.

In this paper, we develop a linear MMSE-based transmitter precoding strategy for the CDMA downlink in synchronous multipath fading channels. No RAKE receiver is required at the mobile unit, only matched filtering. Since we require channel information to construct optimal precoding filters, we implement blind channel estimation at the base-station, where complexity can be managed more efficiently. We also present a performance analysis using tools developed for the analysis of blind multiuser detection with blind channel estimation [9]. Finally, we develop an adaptive implementation that is able to adjust the precoding matrix as users enter and leave the system.

This paper is organized as follows. Section 2 describes the system under consideration. Section 3 develops the transmitter precoding strategy for CDMA over synchronous multipath channels. Section 4 presents a performance analysis. Section 5 discusses an adaptive implementation. Section 6 reports simulation results and Section 7 concludes.

2. SYSTEM DESCRIPTIONS

2.1. *Uplink signal model and blind channel estimation*

We consider a K-user discrete-time synchronous multipath CDMA system with no intersymbol interference (ISI). Such a system is realized either by neglecting the ISI when the multipath delay spread is small compared with the symbol interval, or by inserting guard intervals between symbols when the delay spread is large. The path delays are also assumed to be an integral number of chip periods and are known. We first consider the chip-match filtered uplink signal received at the base station which, during the ith symbol interval, can be written as

$$\mathbf{r}[i] = \sum_{k=1}^{K} b_k[i] \sum_{l=1}^{L} \mathbf{s}_{l,k} f_{l,k} + \mathbf{n}[i], \tag{1}$$

where L is the number of resolvable paths, $b_k[i]$ is the ith symbol for the kth user, $\mathbf{s}_{l,k}$ and $f_{l,k}$ are, respectively, the delayed versions of the spreading waveform (with zero-padding when a guard interval is inserted) and the complex channel fading gain corresponding to the lth path of the kth user, and $\mathbf{n}[i] \sim \mathcal{N}_c(\mathbf{0}, \sigma^2 \mathbf{I}_N)$ is a complex white Gaussian noise vector. Note that $\mathbf{r}[i], \mathbf{n}[i] \in \mathbb{C}^N$ where N is the processing gain. Denote

$$\mathbf{S}_k \triangleq \begin{bmatrix} \mathbf{s}_{1,k} & \mathbf{s}_{2,k} & \cdots & \mathbf{s}_{L,k} \end{bmatrix}, \tag{2}$$

$$\mathbf{f}_k \triangleq \begin{bmatrix} f_{1,k} & f_{2,k} & \cdots & f_{L,k} \end{bmatrix}^T. \tag{3}$$

Then (1) can be written as

$$\mathbf{r}[i] = \sum_{k=1}^{K} \underbrace{\mathbf{S}_k \mathbf{f}_k}_{\mathbf{h}_k} b_k[i] + \mathbf{n}[i] = \mathbf{H}\mathbf{b}[i] + \mathbf{n}[i], \tag{4}$$

where

$$\mathbf{H} \triangleq \begin{bmatrix} \mathbf{h}_1 & \mathbf{h}_2 & \cdots & \mathbf{h}_K \end{bmatrix}, \quad \mathbf{b}[i] \triangleq \begin{bmatrix} b_1[i] & b_2[i] & \cdots & b_K[i] \end{bmatrix}^T. \tag{5}$$

A block diagram of the uplink system appears in Figure 1.

Let the autocorrelation matrix of the received signal $\mathbf{r}[i]$ be

$$\mathbf{C}_r \triangleq E\{\mathbf{r}[i]\mathbf{r}[i]^H\} = \mathbf{H}\mathbf{H}^H + \sigma^2 \mathbf{I}_N \tag{6}$$

$$= \mathbf{U}_s \mathbf{\Lambda}_s \mathbf{U}_s^H + \sigma^2 \mathbf{U}_n \mathbf{U}_n^H, \tag{7}$$

where (7) is the eigendecomposition of $\mathbf{C}_r$. Since the matrix $\mathbf{H}$ has full column rank K, the matrix $\mathbf{H}\mathbf{H}^H$ in (6) has rank K. Therefore, in (7), $\mathbf{\Lambda}_s$ contains the K largest eigenvalues of $\mathbf{C}_r$; $\mathbf{U}_s$ contains the corresponding orthonormal eigenvectors; and $\mathbf{U}_n$ contains the $(N-K)$ orthonormal eigenvectors that correspond to the smallest eigenvalue, σ^2.

Suppose User 1 is the user of interest. Then since $\mathbf{U}_n^H \mathbf{h}_1 = \mathbf{U}_n^H \mathbf{S}_1 \mathbf{f}_1 = 0$, we can estimate $\mathbf{f}_1$ at the base station in the following way [10, 11, 12, 13]:

$$\hat{\mathbf{f}}_1 = \arg\min_{\|\mathbf{f}\|=1} \left\| \mathbf{U}_n^H \mathbf{S}_1 \mathbf{f} \right\|^2 \tag{8}$$

$$= \arg\min_{\|\mathbf{f}\|=1} \mathbf{f}^H \underbrace{\left(\mathbf{S}_1^H \mathbf{U}_n \mathbf{U}_n^H \mathbf{S}_1\right)}_{\mathbf{Q}} \mathbf{f} \tag{9}$$

$$= \text{minimum eigenvector of } \mathbf{Q}. \tag{10}$$

Note that (10) specifies $\mathbf{f}_1$ up to a scale and phase ambiguity and that, in practice, (10) can be implemented blindly in a batch or sequential adaptive manner. In batch mode, we simply replace the noise subspace parameters in (9) with parameters obtained from the eigendecomposition of the sample autocorrelation matrix of the received signal. In sequential adaptive mode, where we update the channel estimates at each time slot, we may employ a suitable subspace tracking algorithm and use the sequential Kalman filtering technique described in Section 5. Note that a necessary condition for our channel estimate to be unique is that $\mathbf{H}$ has rank K, which necessitates this matrix to be tall, that is, $K \leq N$.

2.2. *Downlink signal model, precoding and receiver*

The (downlink) signal transmitted from the base station, during the ith symbol interval, can be written as

$$\mathbf{x}[i] = \mathbf{S}\mathbf{M}\mathbf{b}[i], \tag{11}$$

where

$$\mathbf{S} \triangleq \begin{bmatrix} \mathbf{s}_1 & \mathbf{s}_2 & \cdots & \mathbf{s}_K \end{bmatrix} \tag{12}$$

is the matrix of spreading waveforms and $\mathbf{M} \in \mathbb{C}^{K \times K}$ is a complex precoding filter which we will optimize in the following section. Throughout this paper, we assume that the CDMA system is operating in the TDD mode, so that the downlink and uplink operate using the same carrier frequency in different time slots. We also assume that the time

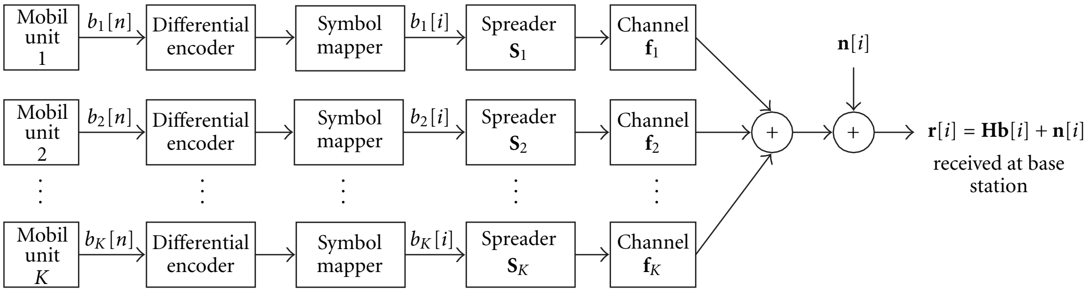

FIGURE 1: The uplink of a K-user CDMA system.

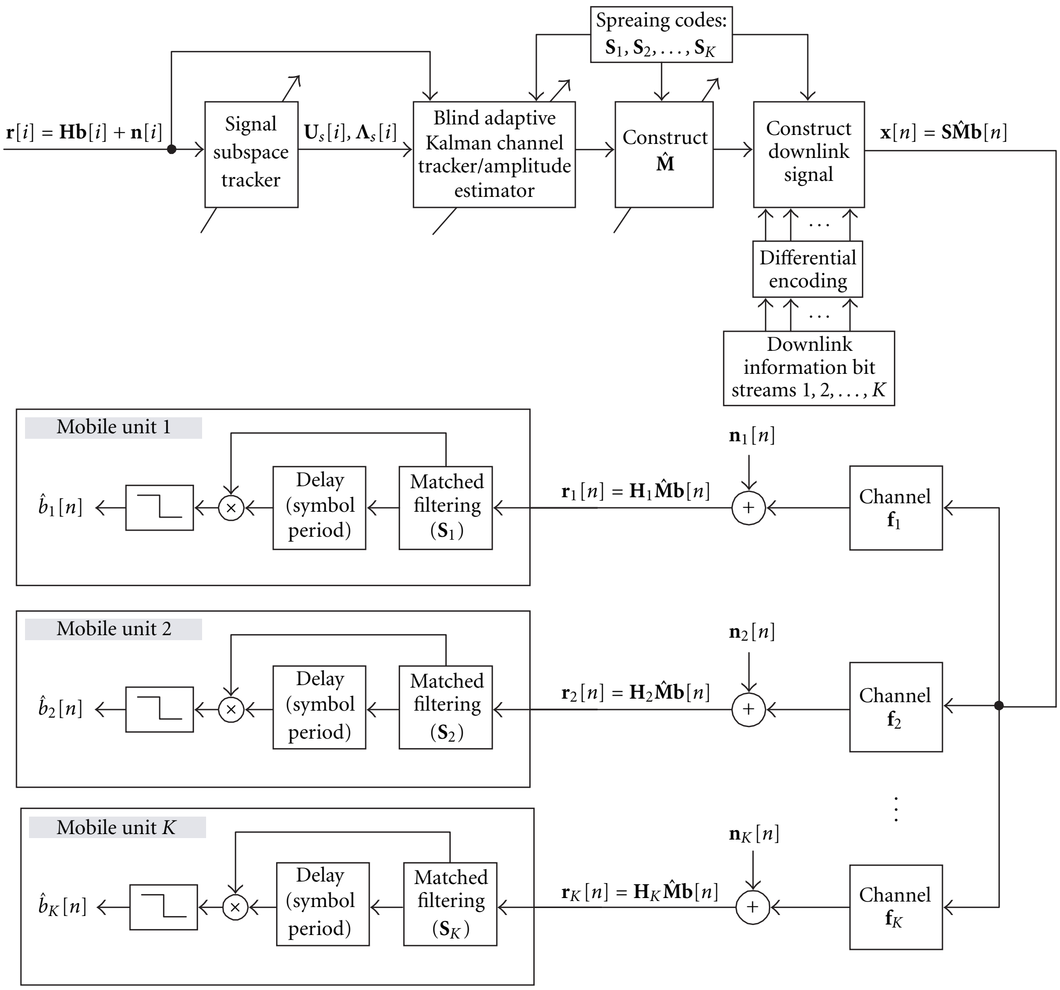

FIGURE 2: Adaptive precoding transmitter structure at the base station for the downlink signal.

elapsing between uplink and downlink transmissions is sufficiently small compared to the coherence time of the channel that the channel impulse response is the same for the uplink and downlink. Then from (2) and (3), the received signal at user 1's mobile unit can be written as

$$\mathbf{r}_1[i] = \underbrace{\begin{bmatrix} \mathbf{S}_1\mathbf{f}_1 & \mathbf{S}_2\mathbf{f}_1 & \cdots & \mathbf{S}_K\mathbf{f}_1 \end{bmatrix}}_{\mathbf{H}_1} \mathbf{Mb}[i] + \mathbf{n}_1[i], \tag{13}$$

where $\mathbf{S}_1, \mathbf{S}_2, \dots, \mathbf{S}_K$ contain shifted versions of their respective signature waveforms as in (2) except that the L shifts are the same for each user's waveform since all spreading codes have been transmitted over user 1's downlink channel. Detection of the downlink information bits is accomplished via matched filtering of the received signal $\mathbf{r}_1[i]$ with user 1's signature waveform, $\mathbf{s}_1$. Figure 2 contains a block diagram of the signal processing that takes place at the base station when an adaptive implementation is employed. We will see more details in Section 5.

3. TRANSMITTER PRECODING FOR SYNCHRONOUS MULTIPATH CDMA

We seek to choose the precoding matrix $\mathbf{M}$ so as to provide the best downlink performance possible when the mobile units are constrained to the use of a matched filter receiver. We choose the MMSE criterion, so $\mathbf{M}$ is chosen to minimize

$$J = E\left\{\left\|\begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_K \end{bmatrix} - \begin{bmatrix} \mathbf{s}_1^H\mathbf{r}_1 \\ \mathbf{s}_2^H\mathbf{r}_2 \\ \vdots \\ \mathbf{s}_K^H\mathbf{r}_K \end{bmatrix}\right\|^2\right\}, \tag{14}$$

where we have dropped the time index for clarity. It is easy to see that

$$\begin{bmatrix} \mathbf{s}_1^H\mathbf{r}_1 \\ \mathbf{s}_2^H\mathbf{r}_2 \\ \vdots \\ \mathbf{s}_K^H\mathbf{r}_K \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{s}_1^H\mathbf{H}_1 \\ \mathbf{s}_2^H\mathbf{H}_2 \\ \vdots \\ \mathbf{s}_K^H\mathbf{H}_K \end{bmatrix}}_{\mathcal{H}} \mathbf{M}\mathbf{b} + \underbrace{\begin{bmatrix} \mathbf{s}_1^H\mathbf{n}_1 \\ \mathbf{s}_2^H\mathbf{n}_2 \\ \vdots \\ \mathbf{s}_K^H\mathbf{n}_K \end{bmatrix}}_{\mathbf{v}}. \tag{15}$$

Then

$$J = E\{\|\mathbf{b} - \mathcal{H}\mathbf{M}\mathbf{b} - \mathbf{v}\|^2\}. \tag{16}$$

The following proposition gives the optimal precoding matrix.

Proposition 1. *The choice of* $\mathbf{M}$ *that minimizes* J *is* $\mathbf{M} = \mathcal{H}^{-1}$.

Proof. Although cumbersome, a direct proof can be constructed via a complex analog to the proof in [5, Appendix A]. We offer the following simple alternative proof by contradiction. Suppose there exists a choice of $\mathbf{M}$, say $\mathbf{M} = \mathbf{M}_0$, that results in a smaller J than $\mathbf{M} = \mathcal{H}^{-1}$. Then

$$E\{\|\mathbf{b} - \mathcal{H}\mathbf{M}_0\mathbf{b} - \mathbf{v}\|^2\} < E\{\|\mathbf{v}\|^2\}. \tag{17}$$

We may evaluate the left-hand side of (17) as

$$\begin{aligned} E\{\|\mathbf{b} - \mathcal{H}\mathbf{M}_0\mathbf{b} - \mathbf{v}\|^2\} = K &- 2E\{\mathrm{Re}[\mathbf{b}^H\mathcal{H}\mathbf{M}_0\mathbf{b}]\} \\ &+ E\{\mathbf{b}^H\mathbf{M}_0^H\mathcal{H}^H\mathcal{H}\mathbf{M}_0\mathbf{b}\} \\ &+ E\{\mathbf{v}^H\mathbf{v}\}. \end{aligned} \tag{18}$$

Then (17) implies

$$K - 2E\{\mathrm{Re}[\mathbf{b}^H\mathcal{H}\mathbf{M}_0\mathbf{b}]\} + E\{\mathbf{b}^H\mathbf{M}_0^H\mathcal{H}^H\mathcal{H}\mathbf{M}_0\mathbf{b}\} < 0. \tag{19}$$

However, the left-hand side of (19) is equal to $E\{\|\mathbf{b} - \mathcal{H}\mathbf{M}_0\mathbf{b}\|^2\}$ which can never be less than zero. Hence, we have a contradiction. □

Denote by $\hat{\mathbf{H}}_i$ $(1 \le i \le K)$ the matrix $\mathbf{H}_i$ where the channel $\mathbf{f}_i$ has been replaced with the blind estimate $\hat{\mathbf{f}}_i$ obtained from (10). Then we may form an initial blind estimate of $\mathbf{M}$ at the base station as

$$\hat{\mathbf{M}} = \begin{bmatrix} \mathbf{s}_1^H\hat{\mathbf{H}}_1 \\ \mathbf{s}_2^H\hat{\mathbf{H}}_2 \\ \vdots \\ \mathbf{s}_K^H\hat{\mathbf{H}}_K \end{bmatrix}^{-1}. \tag{20}$$

There remain amplitude and phase ambiguities in $\hat{\mathbf{M}}$ that are addressed in the following sections.

Remarks

We should expect the performance of transmitter-based multiuser detection to be somewhat inferior to traditional receiver-based approaches since the transmitter precoding filter must satisfy more requirements than the detector in a receiver-based approach. More specifically, notice that the objective function in (14) requires that the choice of $\mathbf{M}$ maximize performance for all users *simultaneously*. Choosing $\mathbf{M}$ to minimize an objective of the form

$$\begin{aligned} J' &= E\{|b_1 - \mathbf{s}_1^H\mathbf{r}_1|^2\} \\ &= E\{|b_1 - \mathbf{s}_1^H(\mathbf{H}_1\mathbf{M}\mathbf{b}[i] + \mathbf{n}_1[i])|^2\} \end{aligned} \tag{21}$$

could provide better downlink performance for User 1 than J of (14) but at the expense of the other users. Hence J' is not an acceptable cost function. In this sense, the simultaneity requirement means that transmitter precoding has fewer degrees of freedom for combatting interference than does MMSE or decorrelating receiver-based multiuser detection.

Notice that choosing the optimal precoding matrix by minimizing J places no explicit constraint on average transmit power. In fact, it was found in a related work on MMSE precoding [5] that unconstrained optimization with simple power scaling provides superior performance at high signal-to-noise ratio (SNR) to constrained optimization. As a result, we will focus on the former. We will also suppress the power scale factor for simplicity.

4. PERFORMANCE ANALYSIS

In [9, 14, 15, 16, 17], the authors developed analytical tools to investigate the performance of blind and group-blind linear MMSE multiuser detection. In this section, we adapt these tools to the analysis of transmitter precoding with blind channel estimation. In particular, we will derive signal-to-interference-plus-noise ratio (SINR) and bit error rate (BER) expressions that take residual multiple-access interference and channel estimation error into account. In Section 6, we will compare these expressions to simulation results.

Notice that the estimate $\hat{\mathbf{M}}$ given by (20) is not a consistent estimate of $\mathbf{M}$ because of the unknown phase and scaling factors. However, there is a diagonal matrix $\mathbf{\Phi}$ so that $\hat{\mathbf{M}}\mathbf{\Phi}^{-1}$

is a consistent estimate. The matrix $\mathbf{\Phi}$ is of the form

$$\mathbf{\Phi} \triangleq \text{diag}(||\mathbf{f}_1||e^{J\phi_1}, ||\mathbf{f}_2||e^{J\phi_2}, \ldots, ||\mathbf{f}_K||e^{J\phi_K}), \tag{22}$$

where ϕ_k, $k = 1, \ldots, K$, are phase factors that depend on how the estimation is implemented. With this in mind, we state the following result which is proved in the appendix.

Theorem 1. *Let $\hat{\mathbf{M}}$ be given by* (20) *and let* $\mathbf{b}$ *be i.i.d. QPSK symbols independent of* $\hat{\mathbf{M}}$. *Then*

$$\sqrt{M}([\hat{\mathbf{M}}\mathbf{\Phi}^{-1} - \mathbf{M}]\mathbf{b}) \longrightarrow \mathcal{N}_c(\mathbf{0}, \mathbf{C}_m) \tag{23}$$

in distribution as $M \longrightarrow \infty$ with

$$\mathbf{C}_m = \mathcal{H}^{-1}\mathbf{D}\mathcal{H}^{-H}, \tag{24}$$

where the diagonal elements of $\mathbf{D}$ *are given by*

$$\begin{gathered}[\mathbf{D}]_{i,i} = \beta_i \sum_{k=1}^{K}\sum_{l=1}^{K} [\mathcal{H}^{-1}\mathcal{H}^{-H}]_{k,l}\mathbf{s}_i^H\mathbf{S}_k\mathbf{Q}_i^{\dagger}\mathbf{S}_l^H\mathbf{s}_i, \\ \beta_i \triangleq \sigma^2\mathbf{h}_i^H\mathbf{U}_s\mathbf{\Lambda}_s(\mathbf{\Lambda}_s - \eta\mathbf{I}_K)^{-2}\mathbf{U}_s^H\mathbf{h}_i,\end{gathered} \tag{25}$$

while the off-diagonal elements can be ignored with good accuracy. Here, $\mathbf{Q}_i^{\dagger}$ denotes the Moore-Penrose generalized inverse [18] of the matrix $\mathbf{Q}_i \triangleq \mathbf{S}_i^H\mathbf{U}_n\mathbf{U}_n^H\mathbf{S}_i$.

The SINR at the output of the matched filter for User 1 is given by [9, 14]

$$\text{SINR} \triangleq \frac{|E\{\mathbf{s}_1^H\mathbf{r}_1[i] \mid b_1[i]\}|^2}{E\{\text{Var}(\mathbf{s}_1^H\mathbf{r}_1[i] \mid b_1[i])\}}. \tag{26}$$

Now suppose that the phase and amplitude factors in $\mathbf{\Phi}$ have been determined. Write the estimated matrix $\hat{\mathbf{M}}$ as $\hat{\mathbf{M}}\mathbf{\Phi}^{-1} = \mathbf{M} + \Delta\mathbf{M}$, where $\Delta\mathbf{M}$ is the estimation error. Dropping the time index for clarity, the received signal can then be written as

$$\begin{aligned} r_1 &= \mathbf{s}_1^H\mathbf{r}_1 && (27)\\ &= (\mathbf{s}_1^H\mathbf{H}_1)\hat{\mathbf{M}}\mathbf{\Phi}^{-1}\mathbf{b} + \mathbf{s}_1^H\mathbf{n}_1 && (28)\\ &= (\mathbf{s}_1^H\mathbf{H}_1)\mathbf{M}\mathbf{b} + (\mathbf{s}_1^H\mathbf{H}_1)\Delta\mathbf{M}\mathbf{b} + \mathbf{s}_1^H\mathbf{n}_1 && (29)\\ &= (\mathbf{s}_1^H\mathbf{H}_1)[\mathbf{M}]_{:,1}b_1 + (\mathbf{s}_1^H\mathbf{H}_1)[\mathbf{M}]_{:,2:K}[\mathbf{b}]_{2:K} \\ &\quad + (\mathbf{s}_1^H\mathbf{H}_1)\Delta\mathbf{M}\mathbf{b} + \mathbf{s}_1^H\mathbf{n}_1, && (30)\end{aligned}$$

where the notation $[\mathbf{M}]_{:,2:K}$ indicates the matrix composed of columns 2 through K of the matrix $\mathbf{M}$. According to Theorem 1, for large M, the third term in (30) is also Gaussian distributed (independent of the other terms) with variance

$$v_1^2 = \frac{1}{M}(\mathbf{s}_1^H\mathbf{H}_1)\mathbf{C}_m\mathbf{H}_1^H\mathbf{s}_1. \tag{31}$$

Since $\mathbf{M}$ represents an MMSE detector, we can also make the approximate assumption that the residual multiple access interference is Gaussian distributed [19]. We can therefore calculate the BER via a single Q-function as

$$P_b(e) \cong Q(\sqrt{\text{SINR}}) \tag{32}$$

with

$$\text{SINR} = \frac{[(\mathbf{s}_1^H\mathbf{H}_1)[\mathbf{M}]_{:,1}]^2}{\sum_{k=2}^{K}|(\mathbf{s}_1^H\mathbf{H}_1)[\mathbf{M}]_{:,k}|^2 + \sigma^2||\mathbf{s}_1||^2 + (1/M)(\mathbf{s}_1^H\mathbf{H}_1)\mathbf{C}_m\mathbf{H}_1^H\mathbf{s}_1}. \tag{33}$$

Notice that the first term in the denominator of the SINR expression is due to residual multiple-access interference. The second term is the ambient noise, and the third term is due to the channel estimation error.

5. ADAPTIVE IMPLEMENTATION

In this section, we present an adaptive implementation of the transmitter precoding strategy discussed in Section 3 that updates the precoding matrix $\mathbf{M}$ at each time slot. This sequential updating allows the implementation to adapt as the channel changes and as users enter and leave the system. A block diagram of the signal processing at the base station appears in Figure 2. Note that we have suppressed the signal processing necessary for detection of the uplink bits. The uplink signal received at the base station is used in a signal subspace tracker, along with the known spreading codes of all users, to construct channel estimates by solving (39). Recall that since we are assuming TDD mode, the uplink channel estimates also serve as downlink channel estimates that can be used to construct $\mathbf{M}$. As previously mentioned, these channel estimates have amplitude and phase ambiguities. Since nearly all cellular CDMA systems employ power control, it is likely that the base station has some knowledge of each users' transmit power. This information, coupled with estimates of the received power, can be used to estimate the channel amplitude for each user. More specifically, let the diagonal matrices $\mathbf{A} = \text{diag}(\alpha_1, \alpha_2, \ldots, \alpha_K)$ and $\mathbf{P} = \text{diag}(\sqrt{p_1}, \sqrt{p_2}, \ldots, \sqrt{p_K})$ contain the unknown channel amplitudes and the known uplink transmit powers, respectively. Also define $\bar{\mathbf{H}}$ such that $\mathbf{H} = \bar{\mathbf{H}}\mathbf{A}\mathbf{P}$ so that the columns of $\bar{\mathbf{H}}$ have unit norm.

We propose an estimator based on the following fact.

Proposition 2. *The amplitude matrix,* $\mathbf{A}$, *may be expressed as*

$$\mathbf{A} = \left[\bar{\mathbf{H}}^H\mathbf{U}_s(\mathbf{\Lambda}_s - \sigma^2\mathbf{I}_K)^{-1}\mathbf{U}_s^H\bar{\mathbf{H}}\mathbf{P}^2\right]^{-1/2}, \tag{34}$$

where $\mathbf{U}_s$ and $\mathbf{\Lambda}_s$ are signal subspace components derived from an eigendecomposition of the autocorrelation matrix $\mathbf{C}_r$, of the received signal, as in (6) *and* (7).

Proof. Since

$$\begin{aligned}\bar{\mathbf{H}}\mathbf{A}^2\mathbf{P}^2\bar{\mathbf{H}}^H + \sigma^2\mathbf{I}_N &= \mathbf{U}_s\mathbf{\Lambda}_s\mathbf{U}_s^H + \sigma^2\mathbf{U}_n\mathbf{U}_n^H \\ &= \mathbf{H}\mathbf{H}^H + \sigma^2\mathbf{I}_N = \mathbf{C}_r,\end{aligned} \tag{35}$$

it is easy to see that $\bar{\mathbf{H}}\mathbf{A}^2\mathbf{P}^2\bar{\mathbf{H}}^H = \mathbf{U}_s(\mathbf{\Lambda}_s - \sigma^2\mathbf{I}_N)\mathbf{U}_s^H$. It can also be verified using the definition of the Moore-Penrose generalized matrix inverse that $(\bar{\mathbf{H}}\mathbf{A}^2\mathbf{P}^2\bar{\mathbf{H}}^H)^\dagger = (\bar{\mathbf{H}}^H)^\dagger\mathbf{A}^{-2}\mathbf{P}^{-2}\bar{\mathbf{H}}^\dagger$. Then

$$(\bar{\mathbf{H}}^H)^\dagger\mathbf{A}^{-2}\mathbf{P}^{-2}\bar{\mathbf{H}}^\dagger = \mathbf{U}_s(\mathbf{\Lambda}_s - \sigma^2\mathbf{I}_K)^{-1}\mathbf{U}_s^H \tag{36}$$

and, solving for $\mathbf{A}^{-2}$, we have

$$\mathbf{A}^{-2} = \bar{\mathbf{H}}^H\mathbf{U}_s(\mathbf{\Lambda}_s - \sigma^2\mathbf{I}_K)^{-1}\mathbf{U}_s^H\bar{\mathbf{H}}\mathbf{P}^2 \tag{37}$$

and the proposition follows. □

We may obtain an estimate $\hat{\mathbf{A}}$ of $\mathbf{A}$ by replacing $\bar{\mathbf{H}}$, $\mathbf{U}_s$, $\mathbf{\Lambda}_s$, and σ^2 of (34) with their respective estimates obtained from subspace tracking and the solution to (39). The subspace tracker we have chosen to use for this adaptive implementation is NAHJ-FST (noise averaged Hermitian Jacobi fast subspace tracking) which has complexity $O(NK)$ floating operations per user per bit and which performs close to the lower bound for all subspace trackers that are based on the singular value decomposition [20]. The application of NAHJ-FST to the current tracking problem is a straightforward modification of [20] and will not be discussed in detail. The channel phase ambiguity can be circumvented by the use of differential encoding and decoding of the data. After channel and amplitude estimation, the amplitude corrected channel information is then used, along with the known spreading codes, to construct the precoding matrix $\hat{\mathbf{M}}$. Finally, the downlink information bits are differentially encoded and filtered with $\hat{\mathbf{M}}$ before spreading and transmission. At the mobile unit, matched filtering and differential detection are performed to obtain estimates of the downlink information bits.

5.1. Blind sequential adaptive channel estimation

Here, we describe the adaptive channel estimator used in Figure 2. Consider User 1 the user of interest and denote by $\mathbf{z}[i]$ the projection of received uplink signal $\mathbf{r}[i]$ onto the noise subspace, that is,

$$\mathbf{z}[i] = \mathbf{r}[i] - \mathbf{U}_s\mathbf{U}_s^H\mathbf{r}[i] = \mathbf{U}_n\mathbf{U}_n^H\mathbf{r}[i]. \tag{38}$$

Since $\mathbf{z}[i]$ lies in the noise subspace, it is orthogonal to any signal in the signal subspace and, in particular, it is orthogonal to $\mathbf{S}_1\mathbf{f}_1$. Hence, $\mathbf{f}_1$ is the solution to the following constrained optimization problem:

$$\begin{gathered}\min_{\mathbf{f}_1\in\mathbb{C}^L} E\{||\mathbf{z}[i]^H\mathbf{S}_1\mathbf{f}_1||^2\} = \min_{\mathbf{f}_1\in\mathbb{C}^L} E\Big\{||(\mathbf{S}_1^H\mathbf{z}[i])^H\mathbf{f}_1||^2\Big\} \\ \text{s.t. } ||\mathbf{f}_1|| = 1.\end{gathered} \tag{39}$$

In order to obtain a sequential algorithm to solve the above optimization problem, we write it in the following (trivial) state space form:

$$\begin{aligned}\mathbf{f}_1[i+1] &= \mathbf{f}_1[i], \quad \text{state equation,} \\ 0 &= [\mathbf{S}_1^H\mathbf{z}[i]]^H\mathbf{f}_1[i], \quad \text{observation equation.}\end{aligned} \tag{40}$$

Denote $\mathbf{x}[i] \triangleq \mathbf{S}_1^H\mathbf{z}[i]$. Then the standard Kalman filter can be applied to the above system as

$$\begin{aligned}\mathbf{k}[i] &= \mathbf{\Sigma}[i-1]\mathbf{x}[i](\mathbf{x}[i]^H\mathbf{\Sigma}[i-1]\mathbf{x}[i])^{-1}, \\ \mathbf{f}_1[i] &= \frac{\mathbf{f}_1[i-1] - \mathbf{k}[i](\mathbf{x}[i]^H\mathbf{f}_1[i-1])}{||\mathbf{f}_1[i-1] - \mathbf{k}[i](\mathbf{x}[i]^H\mathbf{f}_1[i-1])||}, \\ \mathbf{\Sigma}[i] &= \mathbf{\Sigma}[i-1] - \mathbf{k}[i]\mathbf{x}[i]^H\mathbf{\Sigma}[i-1].\end{aligned} \tag{41}$$

with the initial condition $\mathbf{\Sigma}[0] = \mathbf{I}_L$.

5.2. Algorithm summary

We may summarize the adaptive implementation at the base station as follows.

Algorithm 1 (Sequential adaptive transmitter precoding for synchronous multipath CDMA).

(1) *Using a suitable signal subspace tracking algorithm, for example, NAHJ-FST, update the signal subspace components* $\mathbf{U}_s[i]$, $\mathbf{\Lambda}_s[i]$, *and* $\sigma^2[i]$ *at each time slot i using the uplink signals.*

(2) *Track the channels* $\{\mathbf{f}_k\}_{k=1}^K$ *as follows:*

$$\begin{aligned}\mathbf{z}[i] &= \mathbf{r}[i] - \mathbf{U}_s[i]\mathbf{U}_s[i]^H\mathbf{r}[i], \\ \mathbf{x}[i] &= \mathbf{S}_k^H\mathbf{z}[i], \\ \mathbf{k}[i] &= \mathbf{\Sigma}[i-1]\,\mathbf{x}[i]\,(\mathbf{x}[i]^H\mathbf{\Sigma}[i-1]\mathbf{x}[i])^{-1}, \\ \mathbf{f}_k[i] &= \frac{\mathbf{f}_k[i-1] - \mathbf{k}[i](\mathbf{x}[i]^H\mathbf{f}_k[i-1])}{||\mathbf{f}_k[i-1] - \mathbf{k}[i](\mathbf{x}[i]^H\mathbf{f}_k[i-1])||}, \\ \mathbf{\Sigma}[i] &= \mathbf{\Sigma}[i-1] - \mathbf{k}[i]\mathbf{x}[i]^H\mathbf{\Sigma}[i-1].\end{aligned} \tag{42}$$

(3) *Calculate the channel amplitudes via* (34) *using the channel estimates, the signal subspace parameters, the known spreading codes, and the known transmit powers.*

(4) *Using* (15) *and the information from steps (1), (2), and (3), calculate* $\mathcal{H}$ *and set* $\mathbf{M} = \mathcal{H}^{-1}$.

(5) *Differentially encode the downlink bit streams for each user to form* $\mathbf{b}[i]$.

(6) *Transmit the precoded downlink signal* $\mathbf{x}[i] = \mathbf{SMb}[i]$.

(7) *Perform matched filtering and differential detection at the mobile units.*

6. SIMULATION RESULTS

6.1. Analytical performance versus simulated performance

Here, we compare the BER expression in (33) to simulation results. We will also compare the analytical and simulated performance to conventional receiver-based subspace blind MMSE multiuser detection. The simulated system is a QPSK modulated synchronous multipath CDMA system as described in Section 2, that is,

$$b_k[i] \in \left\{\frac{1+j}{\sqrt{2}}, \frac{1-j}{\sqrt{2}}, \frac{-1+j}{\sqrt{2}}, \frac{-1-j}{\sqrt{2}}\right\}. \tag{43}$$

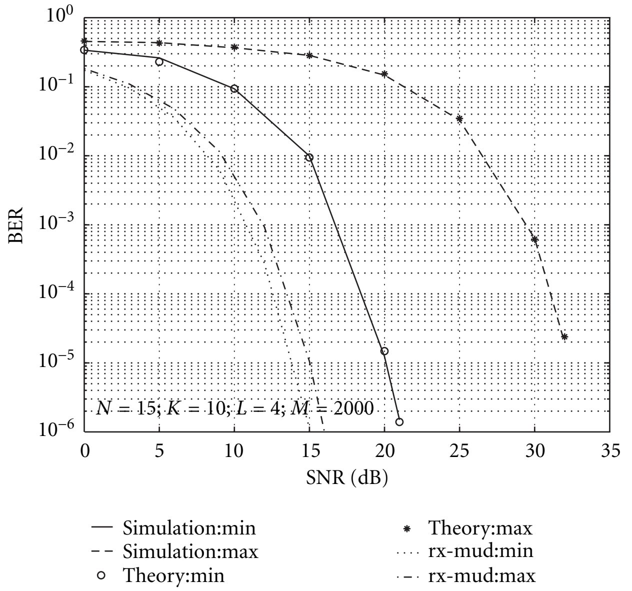

Figure 3: Comparison of analytical and simulated performance results for transmitter precoding. Also presented for comparison, is the analytical performance of (receiver-based) subspace blind MMSE multiuser detection. The number of signal samples used to construct the precoders and detectors is 2000.

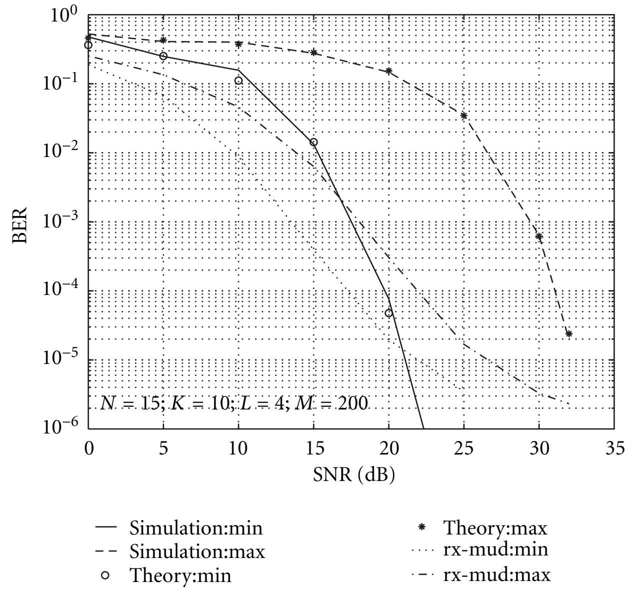

Figure 4: Comparison of analytical and simulated performance results for transmitter precoding. Also presented for comparison, is the analytical performance of (receiver-based) subspace blind MMSE multiuser detection. The number of signal samples used to construct the precoders and detectors is 200.

The spreading codes for each user are m-sequences of length 15 and their shifted versions. The number of users in the system is 10. The number of paths, induced by each user's channel, is $L = 4$. Each path delay is an integral number of chip periods T_c, chosen with equal probability from the set $\{0, T_c, 2T_c, \ldots, (N-1)T_c\}$. The fading gain for each users' channel is generated from a complex Gaussian distribution. The channel parameters are fixed for all simulations. Blind channel estimation is performed on the uplink signals using (10), where the exact noise subspace is replaced by an estimated noise subspace obtained from an eigendecomposition of the received signal, as in [9]. The frame length used for estimating the channel (and the precoding filter) is either $M = 200$ or $M = 2000$, as noted on the figures. As mentioned previously, there is an amplitude and phase ambiguity inherent in (10). For now, they are assumed known. These ambiguities are resolved in the adaptive implementation discussed next.

Figure 3 illustrates the best and worst BER performance among the 10 users using (32) for the analytical performance of transmitter precoding, and [9, equation (98)] for the analytical performance of receiver-based subspace blind MMSE multiuser detection (denoted rx-mud:min and rx-mud:max in the figure). The SNR is defined as $E_b/(2\sigma^2)$. Notice that there is a very good match between the simulated and analytical performance results for transmitter precoding. The previously mentioned performance penalty of transmitter precoding relative to receiver-based multiuser detection is also evident. The number of signal samples used to estimate the channels and, hence, **M** is 2000.

The simulation parameters for Figure 4 are identical to those for Figure 3 except that the number of signal samples used to estimate **M** and the receiver-based multiuser detector has been reduced from 2000 to 200. Although the performance of subspace blind MMSE multiuser detection is still superior, the error floor that appears, its performance does not appear in the performance of transmitter precoding. It is clear that an error floor must exist for both techniques since the detector and precoder are estimated from noisy received signals, but this result suggests that the performance of transmitter precoding may degrade more gracefully than that of subspace blind MMSE multiuser detection when the number of signal samples available diminishes.

Figure 5 contains plots of the analytical and simulated SINR versus the number of signal samples used to construct the precoder matrix for the best performing user. The analytical values are obtained from the SINR expression in (33). The simulated values are taken from

$$\mathrm{SINR}_k = \frac{|E\{z_k[i] \mid b_k[i]\}|^2}{E\{\mathrm{Var}[z_k[i] \mid b_k[i]]\}}, \tag{44}$$

where the expectations are replaced with time averaging and where $z_k[i]$ is the decision statistics for user k at time slot i. This figure also contains an indication of the theoretical SINR when perfect channel information is available, that is, for $M = \infty$. As was the case with the BER plots, there is good agreement between the simulated and theoretical results.

6.2. *Performance of adaptation implementation*

Here, we examine the simulated performance of the adaptive implementation of transmitter precoding discussed in Section 5. We consider the same K-user synchronous

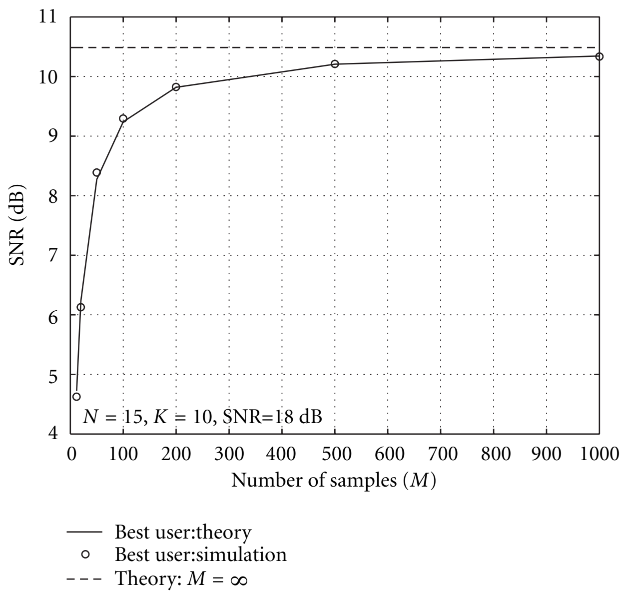

Figure 5: Analytical and simulated SINR results for transmitter precoding. The SNR is fixed at 18 dB.

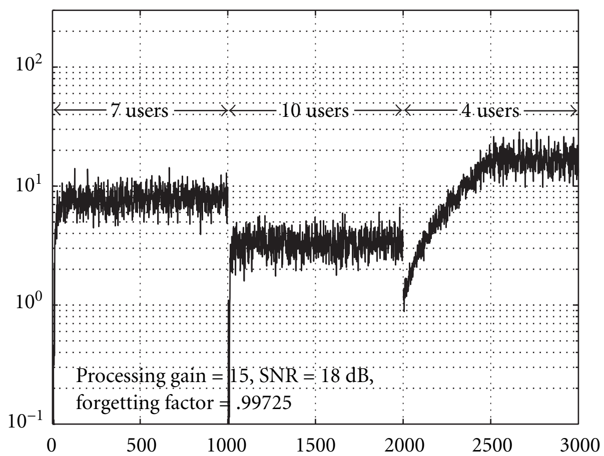

Figure 6: Adaptation performance of adaptive transmitter precoding.

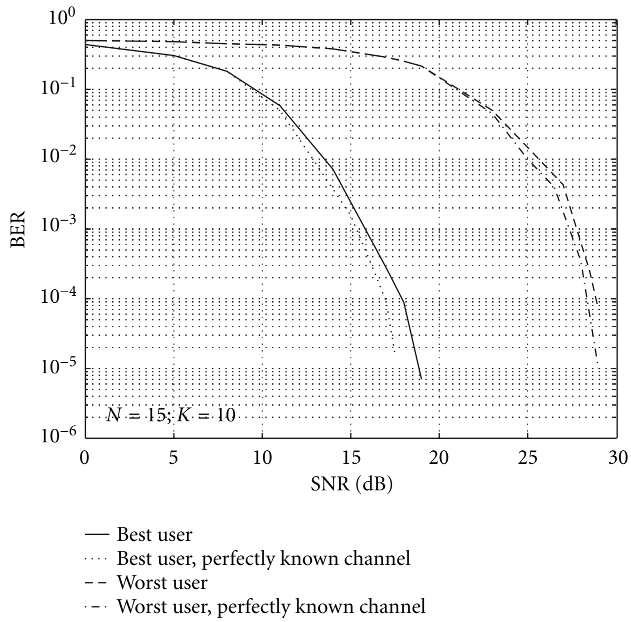

Figure 7: Steady-state performance of adaptive transmitter precoding.

multipath CDMA system used in the previous simulation results, except that the modulation used is BPSK instead of QPSK. The channel is kept constant during all simulations and the SNR is fixed at 18 dB.

Figure 6 contains a plot of the average SINR for the first four users versus the time index. Since we are using differential detection, the decision statistic for detecting bit i of user k is

$$z_k[i] = \mathrm{Re}\Big\{\big(\mathbf{s}_k^H \mathbf{r}_k[i-1]\big)^H \big(\mathbf{s}_k^H \mathbf{r}_k[i]\big)\Big\} \tag{45}$$

and the SINR is defined here as

$$\mathrm{SINR} = \frac{\big|E\{z_k[i] \mid d_k^*[i-1]d_k[i]\}\big|^2}{E\{\mathrm{Var}[z_k[i] \mid d_k^*[i-1]d_k[i]]\}}, \tag{46}$$

where the $d_k[i]$ represents the ith differentially coded bit for user k. For these simulations, the expectation is replaced with time averaging. During the first 1000 time slots, there are 7 users in the system. At time slot 1001, 3 users are added to the system and at iteration 2001 and 6 users are removed from the system. We see that the adaptive system rapidly adjusts (within 500 time slots) when users leave and enter the system.

Figure 7 contains plots of the BER versus SNR. The system has been allowed 500 received signal samples to reach steady state before errors are accumulated. For comparison, this figure also contains plots generated with perfect knowledge of the channel. It is tempting to compare this figure to Figures 3 and 4, but it must be noted that the blind adaptive implementation uses BPSK instead of QPSK and differential encoding/decoding rather than coherent detection.

7. CONCLUSIONS

In this paper, we have developed a transmitter precoding strategy for the downlink of a CDMA system in fading multipath channels. The technique presented precodes the downlink bits before spreading and transmission and is optimal in the mean square error sense. We have presented a performance analysis using tools developed for the analysis of (receiver-based) blind multiuser detection. Also we have developed an adaptive implementation of the precoding strategy that is able to adjust the precoding matrix as users enter and leave the system. Simulation results indicate a very close match between simulated and analytical performance. We also see through simulation that the adaptive implementation is able to quickly and successfully adapt as users enter and leave the system. Future work will address asynchronous multipath channels and cases where the uplink and downlink channels are not identical, but only statistically highly correlated.

APPENDIX

In this appendix, we prove Theorem 1. We begin with the following lemma.

Lemma A.1 (see [9, Corollary 1]). *Let* $\mathbf{f}_1$ *be the true channel of user 1 and let* $\hat{\mathbf{f}}_1$ *be the channel estimate given by* (10). *Then there exists a phase factor* $e^{j\phi}$ *such that*

$$\sqrt{M}\left(\hat{\mathbf{f}}_1 - \|\mathbf{f}_1\|^{-1}e^{j\phi}\mathbf{f}_1\right) \longrightarrow \mathcal{N}_c\left(\mathbf{0}, \beta_1\|\mathbf{f}_1\|^{-2}\mathbf{Q}_1^{\dagger}, \mathbf{0}\right) \quad \text{(A.1)}$$

in distribution as $M \to \infty$ *with*

$$\beta_1 \triangleq \sigma^2 \mathbf{h}_1^H \mathbf{U}_s \mathbf{\Lambda}_s \left(\mathbf{\Lambda}_s - \eta \mathbf{I}_K\right)^{-2} \mathbf{U}_s^H \mathbf{h}_1, \qquad \mathbf{Q}_1 \triangleq \mathbf{S}_1^H \mathbf{U}_n \mathbf{U}_n^H \mathbf{S}_1 \quad \text{(A.2)}$$

and where the notation $\mathcal{N}_c(\boldsymbol{\mu}, \mathbf{C}, \bar{\mathbf{C}})$ *indicates a complex Gaussian distribution with mean* $\boldsymbol{\mu}$, *Hermitian covariance matrix* $\mathbf{C}$, *and symmetric covariance matrix* $\bar{\mathbf{C}}$.

Proof of Theorem 1. Let $\Delta\mathbf{M}$ denote the differential of the function $\hat{\mathbf{M}}$ [14, 21]. Then

$$\Delta\mathbf{M} = -\mathcal{H}^{-1}\Delta\mathcal{H}\mathcal{H}^{-1},$$

$$\Delta\mathcal{H} = \begin{bmatrix} \mathbf{s}_1^H \Delta\mathbf{H}_1 \\ \mathbf{s}_2^H \Delta\mathbf{H}_2 \\ \vdots \\ \mathbf{s}_K^H \Delta\mathbf{H}_K \end{bmatrix}, \quad \text{(A.3)}$$

$$\Delta\mathbf{H}_k = \left[\mathbf{S}_1\Delta\mathbf{f}_k \;\; \mathbf{S}_2\Delta\mathbf{f}_k \;\; \cdots \;\; \mathbf{S}_K\Delta\mathbf{f}_k\right].$$

Now each $\Delta\mathbf{f}_k$ is asymptotically circularly Gaussian distributed by Lemma A.1. It follows that the same holds for $\Delta\mathbf{M}$, and since $\mathbf{b}$ is independent of $\hat{\mathbf{M}}$, this is also true for $\Delta\mathbf{M}\mathbf{b}$, and the theorem follows. It remains to calculate $\mathbf{C}_m$. To this end notice that

$$\Delta\mathcal{H} = \begin{bmatrix} \mathbf{s}_1^H\mathbf{S}_1\Delta\mathbf{f}_1 & \mathbf{s}_1^H\mathbf{S}_2\Delta\mathbf{f}_1 & \cdots & \mathbf{s}_1^H\mathbf{S}_K\Delta\mathbf{f}_1 \\ \mathbf{s}_2^H\mathbf{S}_1\Delta\mathbf{f}_2 & \mathbf{s}_2^H\mathbf{S}_2\Delta\mathbf{f}_2 & \cdots & \mathbf{s}_2^H\mathbf{S}_K\Delta\mathbf{f}_2 \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{s}_K^H\mathbf{S}_1\Delta\mathbf{f}_K & \mathbf{s}_K^H\mathbf{S}_2\Delta\mathbf{f}_K & \cdots & \mathbf{s}_K^H\mathbf{S}_K\Delta\mathbf{f}_K \end{bmatrix}. \quad \text{(A.4)}$$

Then

$$ME\{\Delta\mathbf{M}\mathbf{b}\mathbf{b}^H\Delta\mathbf{M}^H\} = ME\{\Delta\mathbf{M}\Delta\mathbf{M}^H\} \quad \text{(A.5)}$$

$$= \mathcal{H}^{-1}M\underbrace{E\{\Delta\mathcal{H}\mathcal{H}^{-1}\mathcal{H}^{-H}\Delta\mathcal{H}^H\}}_{\mathbf{D}}\mathcal{H}^{-H}. \quad \text{(A.6)}$$

Here, we have

$$\begin{aligned} M[\mathbf{D}]_{i,j} &= ME\left\{\left[\mathbf{s}_i^H\mathbf{S}_1\Delta\mathbf{f}_i \;\; \mathbf{s}_i^H\mathbf{S}_2\Delta\mathbf{f}_i \;\; \cdots \;\; \mathbf{s}_i^H\mathbf{S}_K\Delta\mathbf{f}_i\right] \times \mathcal{H}^{-1}\mathcal{H}^{-H}\begin{bmatrix} \mathbf{s}_j^H\mathbf{S}_1\Delta\mathbf{f}_j^* \\ \mathbf{s}_j^H\mathbf{S}_2\Delta\mathbf{f}_j^* \\ \vdots \\ \mathbf{s}_j^H\mathbf{S}_K\Delta\mathbf{f}_j^* \end{bmatrix}\right\} \quad \text{(A.7)} \\ &= \sum_{k=1}^{K}\sum_{l=1}^{K}\left[\mathcal{H}^{-1}\mathcal{H}^{-H}\right]_{k,l}\mathbf{s}_i^H\mathbf{S}_k ME\{\Delta\mathbf{f}_i\Delta\mathbf{f}_j^H\}\mathbf{S}_l^H\mathbf{s}_j. \quad \text{(A.8)} \end{aligned}$$

We can disregard cross-correlation between different channel estimates and, therefore, we have $[\mathbf{X}]_{i,j} \approx 0$ for $i \neq j$, and for $i = j$, we have, by Lemma A.1,

$$ME\{\Delta\mathbf{f}_i\Delta\mathbf{f}_i^H\} = \beta_i\|\mathbf{f}_i\|^{-2}\mathbf{Q}_i^{\dagger}. \quad \text{(A.9)}$$

If we substitute (A.9) into (A.8) and compensate for the amplitude and phase factors, then (A.6) is equal to (24) and the proof is complete. □

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REFERENCES

[1] S. Verdú, *Multiuser Detection*, Cambridge University Press, Cambridge, UK, 1998.

[2] M. L. Honig and H. V. Poor, "Adaptive interference suppression," in *Wireless Communications: Signal Processing Perspectives*, H. V. Poor and G. W. Wornell, Eds., pp. 64–128, Prentice-Hall, Upper Saddle River, NJ, USA, 1998.

[3] G. J. R. Povey and M. Nakagawa, "A review of time division duplex-CDMA techniques," in *Proc. IEEE 5th International Symposium on Spread Spectrum Techniques and Applications (ISSSTA '98)*, pp. 630–633, Sun City, South Africa, September 1998.

[4] M. Meurer, P. W. Baier, T. Weber, Y. Lu, and A. Papathanassiou, "Joint transmission: advantageous downlink concept for CDMA mobile radio systems using time division duplexing," *Electronics Letters*, vol. 36, no. 10, pp. 900–901, 2000.

[5] B. R. Vojčić and W. M. Jang, "Transmitter precoding in synchronous multiuser communications," *IEEE Trans. Communications*, vol. 46, no. 10, pp. 1346–1355, 1998.

[6] R. Esmailzadeh, E. Sourour, and M. Nakagawa, "Prerake diversity combining in time-division duplex CDMA mobile communications," *IEEE Trans. Vehicular Technology*, vol. 48, no. 3, pp. 795–801, 1999.

[7] S. Georgoulis and D. G. M. Cruickshank, "Pre-equalization, transmitter precoding and joint transmission techniques for

time division duplex CDMA," in *Proc. 2001 Conf. on 3G Mobile Communication Technologies*, pp. 257–261, London, UK, March 2001.
[8] M. Brandt-Pearce and A. Dharap, "Transmitter-based multiuser interference rejection for the down-link of a wireless CDMA system in a multipath environment," *IEEE Journal on Selected Areas in Communications*, vol. 18, no. 3, pp. 407–417, 2000.
[9] A. Høst-Madsen and X. Wang, "Performance of blind multiuser detectors with blind channel estimation," preprint, http://ee.tamu.edu/~wangx/perf.html.
[10] S. E. Bensley and B. Aazhang, "Subspace-based channel estimation for code division multiple access communication systems," *IEEE Trans. Communications*, vol. 44, no. 8, pp. 1009–1020, 1996.
[11] H. Liu and G. Xu, "A subspace method for signal waveform estimation in synchronous CDMA systems," *IEEE Trans. Communications*, vol. 44, no. 10, pp. 1346–1354, 1996.
[12] M. Torlak and G. Xu, "Blind multiuser channel estimation in asynchronous CDMA systems," *IEEE Trans. Signal Processing*, vol. 45, no. 1, pp. 137–147, 1997.
[13] X. Wang and H. V. Poor, "Blind equalization and multiuser detection in dispersive CDMA channels," *IEEE Trans. Communications*, vol. 46, no. 1, pp. 91–103, 1998.
[14] A. Høst-Madsen and X. Wang, "Performance of blind and group-blind multiuser detection," *IEEE Transactions on Information Theory*, vol. 48, no. 7, pp. 1849–1872, 2002.
[15] A. Høst-Madsen and X. Wang, "Performance of blind and group-blind multiuser detectors," in *Proc. 38th Annual Allerton Conference on Communications, Computing and Control*, pp. 4–6, Monticello, Ill, USA, October 2000.
[16] A. Høst-Madsen and X. Wang, "Performance analysis of subspace-based blind and group-blind multiuser detection," in *Proc. 10th International Symposium on Information Theory and Its Applications (ISITA '00)*, Honolulu, Hawaii, USA, November 2000.
[17] A. Høst-Madsen and X. Wang, "Performance of subspace-based multiuser detection," in *Proc. 2001 IEEE International Symposium on Information Theory (ISIT '01)*, Washington, DC, USA, June 2001.
[18] R. A. Horn and C. R. Johnson, *Matrix Analysis*, Cambridge University Press, Cambridge, UK, 1985.
[19] H. V. Poor and S. Verdú, "Probability of error in MMSE multiuser detection," *IEEE Transactions on Information Theory*, vol. 43, no. 3, pp. 858–871, 1997.
[20] D. Reynolds and X. Wang, "Adaptive group-blind multiuser detection based on a new subspace tracking algorithm," *IEEE Trans. Communications*, vol. 49, no. 7, pp. 1135–1141, 2001.
[21] G. S. Rogers, *Matrix Derivatives*, vol. 2 of *Lecture Notes in Statistics*, Marcel Dekker, New York, USA, 1980.

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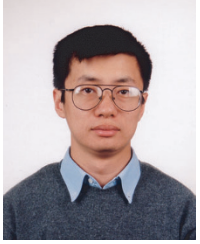

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