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An Approximate Algorithm for Robust Adaptive Beamforming

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This paper presents an adaptive weight computation algorithm for a robust array antenna based on the sample matrix inversion technique. The adaptive array minimizes the mean output power under the constraint that the mean square deviation between the desired and actual responses satisfies a certain magnitude bound. The Lagrange multiplier method is used to solve the constrained minimization problem. An efficient and accurate approximation is then used to derive the fast and recursive computation algorithm. Several simulation results are presented to support the effectiveness of the proposed adaptive computation algorithm.

Keywords and phrases: robust array antenna, Lagrange multiplier method, Taylor series approximation, direction of arrival.

1. INTRODUCTION

The directionally constrained minimization of power (DCMP) adaptive array adjusts the array weights to minimize the mean output power while keeping the antenna response to the direction of arrival (DOA) of the desired signal [1, 2]. When the true DOA is known a priori, the DCMP array achieves a good performance. More precisely, the array provides spatial filtering that maximizes the radar's sensitivity in the desired direction while suppressing interference signals coming from other directions and measurement noises. However, if there is a mismatch between the prescribed and actual DOAs, the desired signal is viewed as an interference and then suppressed [3]. Even a small mismatch may cause a significant performance degradation.

For the solution, a number of robust array antennas that impose the directional derivative constraints [4, 5, 6, 7, 8, 9], the inequality directional constraints [10, 11, 12, 13], and the mean-square deviation constraints [14, 15, 16] have been developed. These methods succeed in achieving flat main beam magnitude responses and decreasing the array sensitivity to look-direction errors. However, the adaptive weight computation algorithm to solve the constrained minimization problem at each time step is not provided, which is required to follow changing interference environment. Although some adaptive algorithms were presented in [6, 7, 10], they were derived based on the steepest descent technique and therefore exhibit slower convergence than the sample matrix inversion (SMI) technique [17, 18].

We here consider the robust array antenna with the inequality directional constraints [10, 11, 12, 13]. The robust array antenna is designed so that the mean output power is minimized under the constraint that the mean square deviation between the desired and actual responses satisfies a certain magnitude bound. The constrained minimization problem can be solved by using the Lagrange multiplier method. However, when the interference environment changes with time, we have to find a root of a nonlinear equation at each time step, which is computationally expensive. We thus apply second-order Taylor series approximations to the nonlinear equation to obtain the closed-form solution, and then derive an adaptive weight computation algorithm based on the SMI technique. The derived adaptive algorithm recursively compute the weight vector in $O(N^2)$ computation time at each time step, where N is the number of array elements. Several simulation results are performed to show the effectiveness of the proposed adaptive computation algorithm.

2. DCMP ARRAY ANTENNA

Consider a narrowband adaptive array antenna of N sensors. We define the kth array input at a discrete time t as $x_{k,t}$ and the kth weight as w_k. We further define the array input vector and the weight vector as $\mathbf{x}_t = (x_{1,t}, x_{2,t}, \ldots, x_{N,t})^{\mathrm{T}}$ and

$\mathbf{w} = (w_1, w_2, \dots, w_N)^{\mathrm{T}}$, respectively, where "T" denotes the transpose operator. The array output is then given by

$$y_t = \mathbf{w}^{\mathrm{H}}\mathbf{x}_t, \tag{1}$$

where "H" denotes the complex conjugate transpose. Consider a desired sinusoidal signal with a DOA θ_d. Putting the phase shift at the kth input as $\Phi_k(\theta_d)$, the constraint of the DCMP array is formulated as

$$\mathbf{c}^{\mathrm{H}}\mathbf{w} = h, \tag{2}$$

where $\mathbf{c}$ is the constraint vector defined by $\mathbf{c}^{\mathrm{H}} = (e^{-j\Phi_1(\theta_d)}, e^{-j\Phi_2(\theta_d)}, \dots, e^{-j\Phi_N(\theta_d)})$ and h is the desired response. Although we here treat a single constraint, the extension to multiple (L) direction constraints is possible by replacing $\mathbf{c}$ by the $L \times N$ matrix $(\mathbf{c}_1^{\mathrm{T}}, \mathbf{c}_2^{\mathrm{T}}, \dots, \mathbf{c}_L^{\mathrm{T}})^{\mathrm{T}}$, where L is the number of constraints.

When the DOA θ_d is given, the DCMP array determines the weight vector $\mathbf{w}$ so that the mean output power $E[(y_t)^2]$ is minimized subject to the constraint (2), where $E[\cdot]$ denotes the expectation operator. Using the Lagrange multiplier method, the solution to the linearly constrained minimization problem is obtained by [1, 2]

$$\mathbf{w} = \mathbf{R}^{-1}\mathbf{c}\left(\mathbf{c}^{\mathrm{H}}\mathbf{R}^{-1}\mathbf{c}\right)^{-1}h, \tag{3}$$

where $\mathbf{R}$ is the covariance matrix of $\mathbf{x}_t$, defined by $\mathbf{R} = E[\mathbf{x}_t\mathbf{x}_t^{\mathrm{H}}]$. Adaptive weight estimation algorithms to follow changing interference environment have been derived based on the SD and SMI techniques [1, 17].

3. ADAPTIVE ALGORITHM FOR ROBUST ARRAY ANTENNA

3.1. Constrained minimization problem

The use of the equality constraint (2) causes performance degradation in the presence of look-direction errors. For the solution, a robust array antenna, which minimizes the mean output power under the constraint that the mean square deviation between the desired and actual responses satisfies a certain magnitude bound, has been proposed [14, 15, 16]. This is formulated as

$$\min_{\mathbf{w}} \mathbf{w}^{\mathrm{H}}\mathbf{R}\mathbf{w} \tag{4}$$

$$\text{subject to } \frac{1}{2\Delta}\int_{\theta_d-\Delta}^{\theta_d+\Delta} \left|\mathbf{c}^{\mathrm{T}}(\theta)\mathbf{w} - h\right|^2 d\theta \le \varepsilon^2, \tag{5}$$

where ε and Δ are small positive constants representing the severity of the constraint and the angle width considered in the constraint, respectively. While the equality constraint (2) restricts the output response to h only at the angle θ_d, the inequality constraint (5) makes the response close (in a least squares sense) to h in the angle range $[\theta_d - \Delta, \theta_d + \Delta]$. The resulting array therefore has robustness against look-direction errors.

The inequality constraint must be an active equality constraint. If the constraint is not active, the solution to the optimization problem becomes $\mathbf{w} = \mathbf{0}$, which does not make sense. Hence we replace (5) by the equality constraint so that the Lagrange multiplier method is immediately applied. The Lagrangian function is then given by

$$H(\mathbf{w}) = \mathbf{w}^{\mathrm{H}}\mathbf{R}\mathbf{w} + \lambda\left(\frac{1}{2\Delta}\int_{\theta_d-\Delta}^{\theta_d+\Delta} \left|\mathbf{c}^{\mathrm{H}}(\theta)\mathbf{w} - h\right|^2 d\theta - \varepsilon^2\right), \tag{6}$$

where λ is the Lagrange multiplier. The solution to the constrained minimization problem must satisfy the following relations:

$$\frac{\partial H(\mathbf{w})}{\partial \mathbf{w}} = 0, \tag{7}$$

$$\frac{1}{2\Delta}\int_{\theta_d-\Delta}^{\theta_d+\Delta} \left|\mathbf{c}^{\mathrm{H}}(\theta)\mathbf{w} - h\right|^2 d\theta = \varepsilon^2. \tag{8}$$

We put

$$\begin{aligned}\mathbf{S} &= \mathbf{R} + \frac{\lambda}{\Delta}\int_{\theta_d-\Delta}^{\theta_d+\Delta} \mathbf{c}(\theta)\mathbf{c}^{\mathrm{H}}(\theta)\,d\theta, \\ \mathbf{u} &= \frac{\lambda}{\Delta}\int_{\theta_d-\Delta}^{\theta_d+\Delta} \mathbf{c}(\theta)\,d\theta\end{aligned} \tag{9}$$

to have

$$\begin{aligned} H(\mathbf{w}) &= \frac{1}{2}\mathbf{w}^{\mathrm{H}}\mathbf{S}\mathbf{w} - \frac{h}{2}\mathbf{w}^{\mathrm{H}}\mathbf{u} - \frac{h^*}{2}\mathbf{u}^{\mathrm{H}}\mathbf{w} + \lambda\left(|h|^2 - \varepsilon^2\right) \\ &= \frac{1}{2}\left(\mathbf{w} - h\mathbf{S}^{-1}\mathbf{u}\right)^{\mathrm{H}}\mathbf{S}\left(\mathbf{w} - h\mathbf{S}^{-1}\mathbf{u}\right) \\ &\quad - \frac{|h|^2}{2}\mathbf{u}^{\mathrm{H}}\mathbf{S}^{-1}\mathbf{u} + \lambda\left(|h|^2 - \varepsilon^2\right). \end{aligned} \tag{10}$$

Since $\mathbf{S}$ is positive definite and Hermitian, $H(\mathbf{w})$ is minimized by putting

$$\mathbf{w} = \frac{\lambda h}{\Delta}\left(\mathbf{R} + \frac{\lambda}{\Delta}\int_{\theta_d-\Delta}^{\theta_d+\Delta} \mathbf{c}(\theta)\mathbf{c}^{\mathrm{H}}(\theta)\,d\theta\right)^{-1}\int_{\theta_d-\Delta}^{\theta_d+\Delta} \mathbf{c}(\theta)\,d\theta. \tag{11}$$

The constraint (8) is rewritten as

$$\begin{aligned} 0 = \mathbf{w}^{\mathrm{H}}&\left(\int_{\theta_d-\Delta}^{\theta_d+\Delta} \mathbf{c}(\theta)\mathbf{c}(\theta)^{\mathrm{H}}\,d\theta\right)\mathbf{w} - \mathbf{w}^{\mathrm{H}}\left(\int_{\theta_d-\Delta}^{\theta_d+\Delta} \mathbf{c}(\theta)\,d\theta\right)h \\ &- \left(\int_{\theta_d-\Delta}^{\theta_d+\Delta} \mathbf{c}(\theta)^{\mathrm{H}}\,d\theta\right)\mathbf{w}h^* + 2\Delta\left(|h|^2 - \varepsilon^2\right), \end{aligned} \tag{12}$$

where "$*$" denotes the complex conjugate. The Lagrange multiplier λ can be determined by substituting (11) into (12) and then solving it for λ. However, the closed-form solution is difficult to obtain due to its nonlinearity.

When the generalized singular value decomposition of $\mathbf{R}$ is obtained, the value of λ can be determined by finding a root of a nonlinear equation, referred to as "secular equation" [19, 20]. A standard root-finding technique such as Newton's method is applicable to the solution of the nonlinear equation. Both root-finding algorithms and singular value decomposition algorithms use iterative methods, in which an iterative scheme is continued until convergence is obtained, that is, until the new value is very close to the previous value. When $\mathbf{R}$ changes with time as often happens, root-finding and singular value decomposition need to be performed at each time step. The iterative methods require $O(N^2)$ computation time per iteration. The computational complexity increases with an increase in the number of iterations. Moreover, the use of the iterative methods at each time step is not suited for adaptive array processing where the maximum processing time is crucial. We thus derive the adaptive computation algorithm by applying second-order Taylor series approximations to the nonlinear equation. We here consider a single constraint to derive the adaptive algorithm, as shown in (5). When there are multiple (L) direction constraints, we can use a similar technique to derive the adaptive algorithm by replacing $\mathbf{c}$ and $\mathbf{c}\mathbf{c}^{\mathrm{H}}$ by $\mathbf{c}_1 + \cdots + \mathbf{c}_L$ and $\mathbf{c}_1\mathbf{c}_1^{\mathrm{H}} + \cdots + \mathbf{c}_L\mathbf{c}_L^{\mathrm{H}}$, respectively, in (9), (10), (11), and (12).

3.2. Computation of weight vector

We define the N-dimensional vectors $\mathbf{p}$, $\mathbf{q}$, and $\mathbf{r}$ as

$$\mathbf{p} = \mathbf{c}(\theta_d), \qquad \mathbf{q} = \left.\frac{d\mathbf{c}(\theta)}{d\theta}\right|_{\theta=\theta_d}, \qquad \mathbf{r} = \left.\frac{d^2\mathbf{c}(\theta)}{d\theta^2}\right|_{\theta=\theta_d}, \tag{13}$$

and the $(N \times N)$ matrices $\mathbf{G}$, $\mathbf{V}^{-1}$, and $\mathbf{Q}_3$ as

$$\mathbf{G} = \mathbf{r}\mathbf{p}^{\mathrm{H}} + 2\mathbf{q}\mathbf{q}^{\mathrm{H}} + \mathbf{p}\mathbf{r}^{\mathrm{H}}, \tag{14}$$

$$\mathbf{V}^{-1} = \left(\mathbf{R} + 2\lambda\mathbf{p}\mathbf{p}^{\mathrm{H}}\right)^{-1} = \mathbf{R}^{-1} - \frac{2\lambda\mathbf{R}^{-1}\mathbf{p}\mathbf{p}^{\mathrm{H}}\mathbf{R}^{-1}}{1 + 2\lambda\mathbf{p}^{\mathrm{H}}\mathbf{R}^{-1}\mathbf{p}}, \tag{15}$$

$$\mathbf{Q}_3 = \left(\mathbf{I} + \frac{\Delta^2\lambda}{3}\mathbf{V}^{-1}\mathbf{G}\right)^{-1}. \tag{16}$$

Using the second-order Taylor series expansion, we approximately have

$$\begin{aligned}
&\int_{\theta_d-\Delta}^{\theta_d+\Delta} \mathbf{c}(\theta)\mathbf{c}^{\mathrm{H}}(\theta)d\theta \\
&\quad = 2\Delta\mathbf{c}(\theta_d)\mathbf{c}^{\mathrm{H}}(\theta_d) + \frac{\Delta^3}{3}\frac{d^2}{d\theta^2}\mathbf{c}(\theta)\mathbf{c}^{\mathrm{H}}(\theta)\Big|_{\theta=\theta_d} + \cdots \\
&\quad \approx 2\Delta\mathbf{p}\mathbf{p}^{\mathrm{H}} + \frac{\Delta^3}{3}\mathbf{G}, \\
&\int_{\theta_d-\Delta}^{\theta_d+\Delta} \mathbf{c}(\theta)d\theta \simeq 2\Delta\mathbf{p} + \frac{\Delta^3}{3}\mathbf{r}.
\end{aligned} \tag{17}$$

Substituting (17) into (11) yields

$$\begin{aligned}
\mathbf{w} &\simeq \frac{\lambda h}{\Delta}\left\{\mathbf{R} + \frac{\lambda}{\Delta}\left(2\Delta\mathbf{p}\mathbf{p}^{\mathrm{H}} + \frac{\Delta^3}{3}\mathbf{G}\right)^{-1}\right\}\left(2\Delta\mathbf{p} + \frac{\Delta^3}{3}\mathbf{r}\right) \\
&= \lambda h\left(\mathbf{R} + 2\lambda\mathbf{p}\mathbf{p}^{\mathrm{H}} + \frac{\Delta^2\lambda}{3}\mathbf{G}\right)^{-1}\left(2\mathbf{p} + \frac{\Delta^2}{3}\mathbf{r}\right) \\
&= \lambda h\left(\mathbf{I} + \frac{\Delta^2\lambda}{3}\mathbf{V}^{-1}\mathbf{G}\right)^{-1}\mathbf{V}^{-1}\left(2\mathbf{p} + \frac{\Delta^2}{3}\mathbf{r}\right) \\
&= \lambda h\mathbf{Q}_3\mathbf{V}^{-1}\left(2\mathbf{p} + \frac{\Delta^2}{3}\mathbf{r}\right).
\end{aligned} \tag{18}$$

Putting the N-dimensional vectors $\mathbf{v}_r$, $\mathbf{v}_q$, and $\mathbf{v}_p$ as

$$\mathbf{v}_p = \frac{\Delta^2\lambda}{3}\mathbf{V}^{-1}\mathbf{p}, \qquad \mathbf{v}_q = \frac{2\Delta^2\lambda}{3}\mathbf{V}^{-1}\mathbf{q}, \qquad \mathbf{v}_r = \frac{\Delta^2\lambda}{3}\mathbf{V}^{-1}\mathbf{r}, \tag{19}$$

the matrix $\mathbf{Q}_3$ in (18) is rewritten as

$$\mathbf{Q}_3 = \left(\mathbf{I} + \mathbf{v}_r\mathbf{p}^{\mathrm{H}} + \mathbf{v}_q\mathbf{q}^{\mathrm{H}} + \mathbf{v}_p\mathbf{r}^{\mathrm{H}}\right)^{-1}. \tag{20}$$

Therefore, we can compute $\mathbf{Q}_3$ in $O(N^2)$ computation time by recursive use of the matrix inversion lemma:

$$\begin{aligned}
\mathbf{Q}_1 &= \mathbf{I} - \frac{\mathbf{v}_r\mathbf{p}^{\mathrm{H}}}{1 + \mathbf{p}^{\mathrm{H}}\mathbf{v}_r}, \\
\mathbf{Q}_2 &= \mathbf{Q}_1 - \frac{\mathbf{Q}_1\mathbf{v}_q\mathbf{q}^{\mathrm{H}}\mathbf{Q}_1}{1 + \mathbf{q}^{\mathrm{H}}\mathbf{Q}_1\mathbf{v}_q}, \\
\mathbf{Q}_3 &= \mathbf{Q}_2 - \frac{\mathbf{Q}_2\mathbf{v}_p\mathbf{r}^{\mathrm{H}}\mathbf{Q}_2}{1 + \mathbf{r}^{\mathrm{H}}\mathbf{Q}_2\mathbf{v}_p}.
\end{aligned} \tag{21}$$

3.3. Computation of Lagrange multiplier

We define several real values as

$$\begin{aligned}
&\alpha = \mathbf{p}^{\mathrm{H}}\mathbf{R}^{-1}\mathbf{p}, \qquad \beta = \mathbf{p}^{\mathrm{H}}\mathbf{R}^{-1}\mathbf{q}, \qquad \gamma = \mathbf{p}^{\mathrm{H}}\mathbf{R}^{-1}\mathbf{r}, \\
&\xi = \alpha(\gamma + \gamma^*) + 2|\beta|^2, \qquad \varphi = \frac{|h|}{\varepsilon}, \qquad \nu = 1 + 2\lambda\alpha.
\end{aligned} \tag{22}$$

Then we have

$$\begin{aligned}
\mathbf{p}^{\mathrm{H}}\mathbf{V}^{-1}\mathbf{p} &= \frac{\alpha}{\nu}, \\
\mathbf{p}^{\mathrm{H}}\mathbf{V}^{-1}\mathbf{q} &= \frac{\beta}{\nu}, \\
\mathbf{p}^{\mathrm{H}}\mathbf{V}^{-1}\mathbf{r} &= \frac{\gamma}{\nu}, \\
\mathbf{p}^{h}\mathbf{V}^{-1}\mathbf{G}\mathbf{V}^{-1}\mathbf{p} &= \frac{\xi}{\nu^2}.
\end{aligned} \tag{23}$$

Neglecting small quantities of order Δ^4 in (16), we approximately have

$$\mathbf{Q}_3 = \left(\mathbf{I} + \frac{\Delta^2\lambda}{3}\mathbf{V}^{-1}\mathbf{G}\right)^{-1} \simeq \mathbf{I} - \frac{\Delta^2\lambda}{3}\mathbf{V}^{-1}\mathbf{G}. \tag{24}$$

Substituting (24) into (18) yields

$$\begin{aligned}\mathbf{w} &\simeq \lambda h\left(\mathbf{I}-\frac{\Delta^2\lambda}{3}\mathbf{V}^{-1}\mathbf{G}\right)\mathbf{V}^{-1}\left(2\mathbf{p}+\frac{\Delta^2}{3}\mathbf{r}\right)\\ &\simeq \lambda h\mathbf{V}^{-1}\left(2\mathbf{p}-\frac{2\lambda\Delta^2}{3}\mathbf{G}\mathbf{V}^{-1}\mathbf{p}+\frac{\Delta^2}{3}\mathbf{r}\right).\end{aligned} \tag{25}$$

We now obtain two different ways of computing $\mathbf{w}$, that is, (18) and (25). The weight vector computed by (18) is more accurate than the one by (25), because (18) is derived using only approximations (17). We thus use (18) in the computation of $\mathbf{w}$ and (25) in the computation of λ.

Using (17), (23), and (25), we can approximately have

$$\begin{aligned}&\mathbf{w}^{\mathrm{H}}\left(\int_{\theta_d-\Delta}^{\theta_d+\Delta}\mathbf{c}(\theta)\mathbf{c}(\theta)^{\mathrm{H}}d\theta\right)\mathbf{w}\\ &\quad=\Delta\lambda^2|h|^2\left(\frac{8\alpha^2}{\nu^2}+\frac{8(\xi-\nu|\beta|^2)}{3\nu^3}\Delta^2\right),\\ &\left(\int_{\theta_d-\Delta}^{\theta_d+\Delta}\mathbf{c}(\theta)d\theta\right)\mathbf{w}\\ &\quad=\Delta\lambda h\left(\frac{4\alpha}{\nu}+\frac{2(\xi-2\nu|\beta|^2)}{3\alpha\nu^2}\Delta^2\right).\end{aligned} \tag{26}$$

Substituting (26) into (12) yields

$$\begin{aligned}&\lambda^2|h|^2\left(\frac{4\alpha^2}{\nu^2}+\frac{4(\xi-\nu|\beta|^2)}{3\nu^3}\Delta^2\right)\\ &\qquad-\lambda|h|^2\left(\frac{4\alpha}{\nu}+\frac{2(\xi-2\nu|\beta|^2)}{3\alpha\nu^2}\Delta^2\right)+|h|^2\\ &\quad=\varepsilon^2.\end{aligned} \tag{27}$$

After some manipulation, (27) is reduced to

$$\left(1-\frac{\nu^2}{\varphi^2}\right)+\Delta^2\frac{(\nu-1)}{3\alpha^2\nu}\left(|\beta|^2(\nu+1)\nu-\xi\right)=0. \tag{28}$$

Solving (28) for ν yields

$$\nu=\varphi+\frac{\varphi-1}{6\alpha^2}\{\varphi(\varphi+1)|\beta|^2-\xi\}\Delta^2. \tag{29}$$

Thus we have

$$\lambda=\frac{\nu-1}{2\alpha}=\frac{\varphi-1}{2\alpha}+\Delta^2\frac{(\varphi-1)}{12\alpha^3}\{\varphi(\varphi+1)|\beta|^2-\xi\}. \tag{30}$$

We see that the Lagrange multiplier is expressed independently of the weight vector $\mathbf{w}$. We can now obtain the closed-form solution to the constrained minimization problem (4), (5).

3.4. Summary of the proposed adaptive algorithm

To follow changing interference environment, we recursively estimate $\mathbf{R}^{-1}$ by

$$\mathbf{R}_t^{-1}=\frac{1}{1-\mu}\left(\mathbf{R}_{t-1}^{-1}-\frac{\mu\mathbf{R}_{t-1}^{-1}\mathbf{x}_t\mathbf{x}_t^{\mathrm{H}}\mathbf{R}_{t-1}^{-1}}{(1-\mu)+\mu\mathbf{x}_t^{\mathrm{H}}\mathbf{R}_{t-1}^{-1}\mathbf{x}_t}\right), \tag{31}$$

$t=1,2,\ldots$

$$\mathbf{R}_t^{-1}=\frac{1}{1-\mu}\left(\mathbf{R}_{t-1}^{-1}-\frac{\mu\mathbf{R}_{t-1}^{-1}\mathbf{x}_t\mathbf{x}_t^{\mathrm{H}}\mathbf{R}_{t-1}^{-1}}{(1-\mu)+\mu\mathbf{x}_t^{\mathrm{H}}\mathbf{R}_{t-1}^{-1}\mathbf{x}_t}\right)$$

$$\alpha=\mathbf{p}^{\mathrm{H}}\mathbf{R}_t^{-1}\mathbf{p}$$

$$\beta=\mathbf{p}^{\mathrm{H}}\mathbf{R}_t^{-1}\mathbf{q}$$

$$\gamma=\mathbf{p}^{\mathrm{H}}\mathbf{R}_t^{-1}\mathbf{r}$$

$$\xi=\alpha(\gamma+\gamma^*)+2|\beta|^2$$

$$\lambda=\frac{\varphi-1}{2\alpha}+\Delta^2\frac{(\varphi-1)}{12\alpha^3}\{\varphi(\varphi+1)|\beta|^2-\xi\}$$

$$\mathbf{V}^{-1}=\mathbf{R}_t^{-1}-\frac{2\lambda\mathbf{R}_t^{-1}\mathbf{p}\mathbf{p}^{\mathrm{H}}\mathbf{R}_t^{-1}}{1+2\lambda\mathbf{p}^{\mathrm{H}}\mathbf{R}_t^{-1}\mathbf{p}}$$

$$\mathbf{v}_p=\frac{\Delta^2\lambda}{3}\mathbf{V}^{-1}\mathbf{p}$$

$$\mathbf{v}_q=\frac{2\Delta^2\lambda}{3}\mathbf{V}^{-1}\mathbf{q}$$

$$\mathbf{v}_r=\frac{\Delta^2\lambda}{3}\mathbf{V}^{-1}\mathbf{r}$$

$$\mathbf{Q}_1=\mathbf{I}-\frac{\mathbf{v}_r\mathbf{p}^{\mathrm{H}}}{1+\mathbf{p}^{\mathrm{H}}\mathbf{v}_r}$$

$$\mathbf{Q}_2=\mathbf{Q}_1-\frac{\mathbf{Q}_1\mathbf{v}_q\mathbf{q}^{\mathrm{H}}\mathbf{Q}_1}{1+\mathbf{q}^{\mathrm{H}}\mathbf{Q}_1\mathbf{v}_q}$$

$$\mathbf{Q}_3=\mathbf{Q}_2-\frac{\mathbf{Q}_2\mathbf{v}_p\mathbf{r}^{\mathrm{H}}\mathbf{Q}_2}{1+\mathbf{r}^{\mathrm{H}}\mathbf{Q}_2\mathbf{v}_p}$$

$$\mathbf{w}_t=\lambda h\mathbf{Q}_3\mathbf{V}^{-1}\left(2\mathbf{p}+\frac{\Delta^2\mathbf{r}}{3}\right)$$

Algorithm 1: Proposed adaptive algorithm.

where $\mathbf{R}_t$ is the estimates of $\mathbf{R}$ at time t and μ is a forgetting factor such that $\mu \ll 1$. The computational complexity per sample is of order N^2. The direct computation of (31) causes the problem of numerical stability when using a short word-length processor. The use of the numerically stable updating scheme based on the UD or square-root decomposition may be helpful. But we avoided the problem by using floating-point double precision arithmetics in the following simulation.

Algorithm 1 summarizes the proposed algorithm that recursively computes the weight vector $\mathbf{w}_t$ from the array input $\mathbf{x}_t$ in $O(N^2)$ computation time. It is here noted that $\mathbf{p}$, $\mathbf{q}$, $\mathbf{r}$, and φ can be computed a priori. We can consider that the true and approximated solutions are very close to each other because (18) and (30) are derived using second-order Taylor series approximations. This will be verified through computer simulations below.

4. COMPUTER SIMULATION

We consider a desired signal with a frequency 100 MHz, a power 1, and a DOA $\theta_d = 90^\circ$, and an interference with a frequency 100 MHz, a power 10, and a DOA $\theta_i = 150^\circ$. We set $h=1$, $N=4$, $\Delta=0.5^\circ$, $\varepsilon=0.02$, $T=2$ nanoseconds. We chose the element spacing equal to one-half wavelength, and added a white noise with mean 0 and variance $0.01(=\sigma_n^2)$ to the array input.

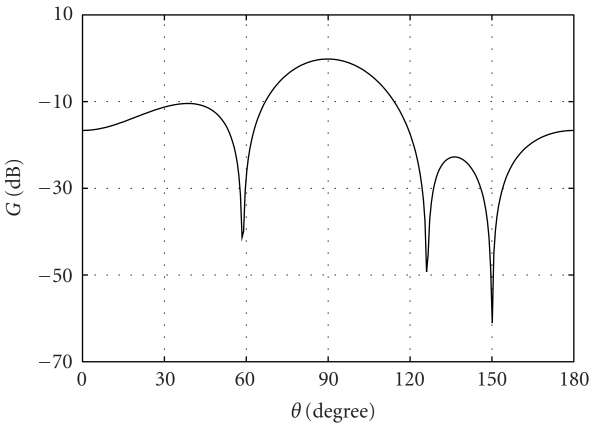

Figure 1: Array pattern.

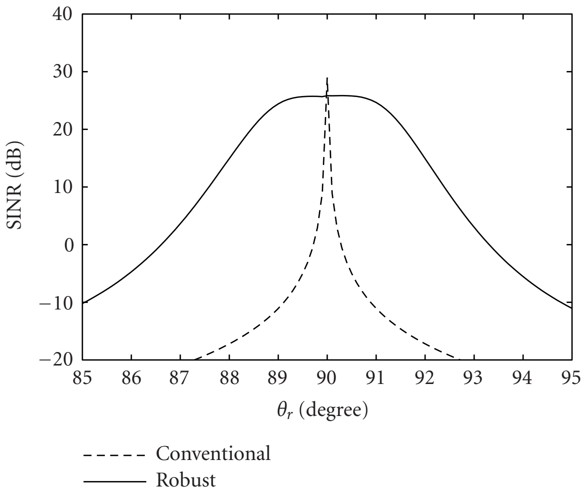

Figure 2: Comparison of SINRs.

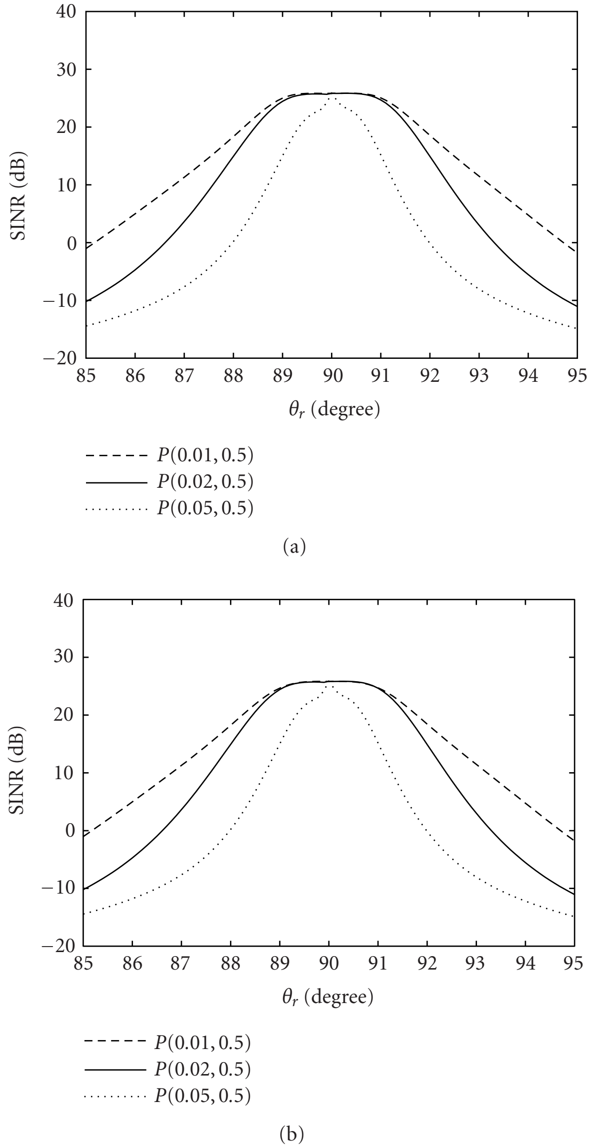

Figure 3: SINR for various values of ε. (a) True solution. (b) Approximated solution.

When the desired signal s_t is coming from a direction θ, the covariance matrix of the array input is represented by

$$\mathbf{R}(\theta) = E[\mathbf{x}_t\mathbf{x}_t^{\mathrm{H}}] = E\Big[\,|\,s_t\,|^2\Big]\mathbf{c}(\theta)\mathbf{c}(\theta)^{\mathrm{H}}. \tag{32}$$

Let the optimal weight vector computed off-line be $\mathbf{w}_o$. The array pattern with respect to θ is then represented by

$$G(\theta) = E\Big[\,|\,y_t\,|^2\Big] = \mathbf{w}_o^{\mathrm{H}}\mathbf{R}(\theta)\mathbf{w}_o = E\Big[\,|\,s_t\,|^2\Big]\,|\,\mathbf{w}_o^{\mathrm{H}}\mathbf{c}(\theta)\,|^2. \tag{33}$$

Figure 1 shows the array pattern of the robust array. We see that the array antenna places a null in the direction of the interference, 150°, while keeping a large antenna response to the desired direction, 90°.

The array input $\mathbf{x}_t$ is decomposed into the sum of the desired signal component $\mathbf{d}_t$, the interference component $\mathbf{i}_t$, and the observation noise component $\mathbf{e}_t$. The powers of $\mathbf{d}_t$, $\mathbf{i}_t$, and $\mathbf{e}_t$ are expressed as

$$P_d = \mathbf{w}^{\mathrm{H}}E[\mathbf{d}_t\mathbf{d}_t^{\mathrm{T}}]\mathbf{w}, \qquad P_i = \mathbf{w}^{\mathrm{H}}E[\mathbf{i}_t\mathbf{i}_t^{\mathrm{T}}]\mathbf{w}, \\ P_e = \mathbf{w}^{\mathrm{H}}E[\mathbf{e}_t\mathbf{e}_t^{\mathrm{T}}]\mathbf{w}, \tag{34}$$

respectively. The signal-to-interference-plus-noise ratio (SINR) is then defined by

$$\mathrm{SINR} = \frac{P_d}{P_i + P_e}. \tag{35}$$

Let the actual and prescribed DOAs of the desired signal be θ_r and θ_d, respectively. We put $\theta_d = 90°$ to design the constraint vector $\mathbf{c}$, and computed the weight vector $\mathbf{w}$ for various values of θ_r. Figure 2 plots the SINR as the function of θ_r. The result for the conventional array computed by (3) is also shown for comparison purposes. It is found that the robust array offers a flat SINR in the look direction, although there is a tradeoff in the noise rejection capability of the processor in look directions which are far away from the desired signal.

Figure 3 shows the SINRs for $\varepsilon = 0.01$, 0.02, and 0.05 with $\Delta = 0.5°$, where Figures 3a and 3b are the results of the

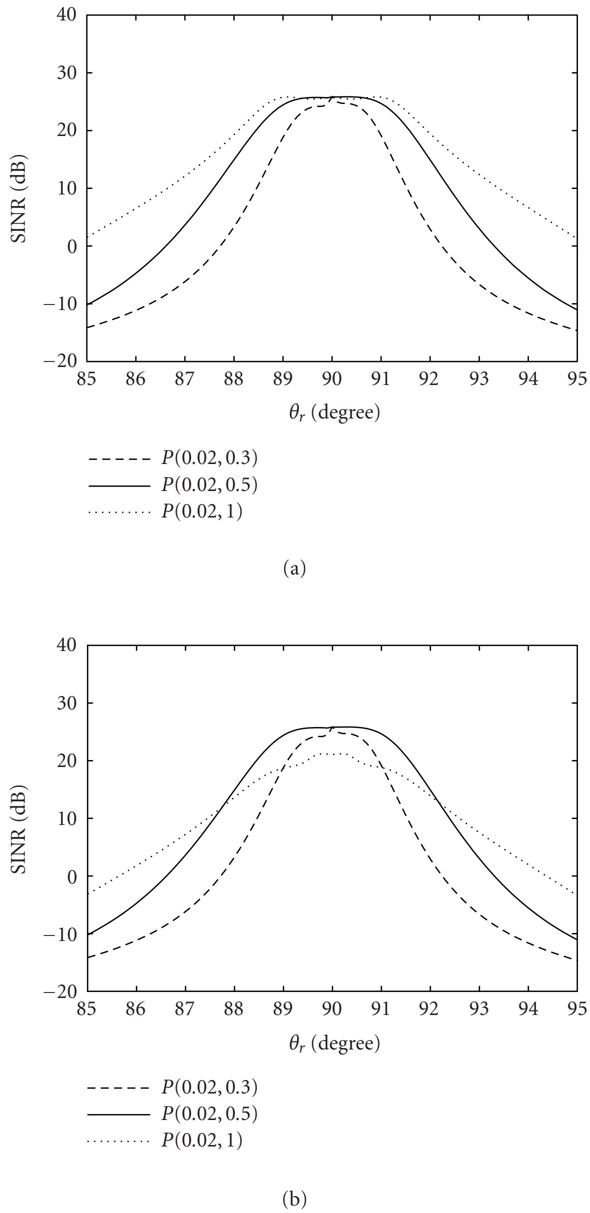

Figure 4: SINR for various values of Δ. (a) True solution. (b) Approximated solution.

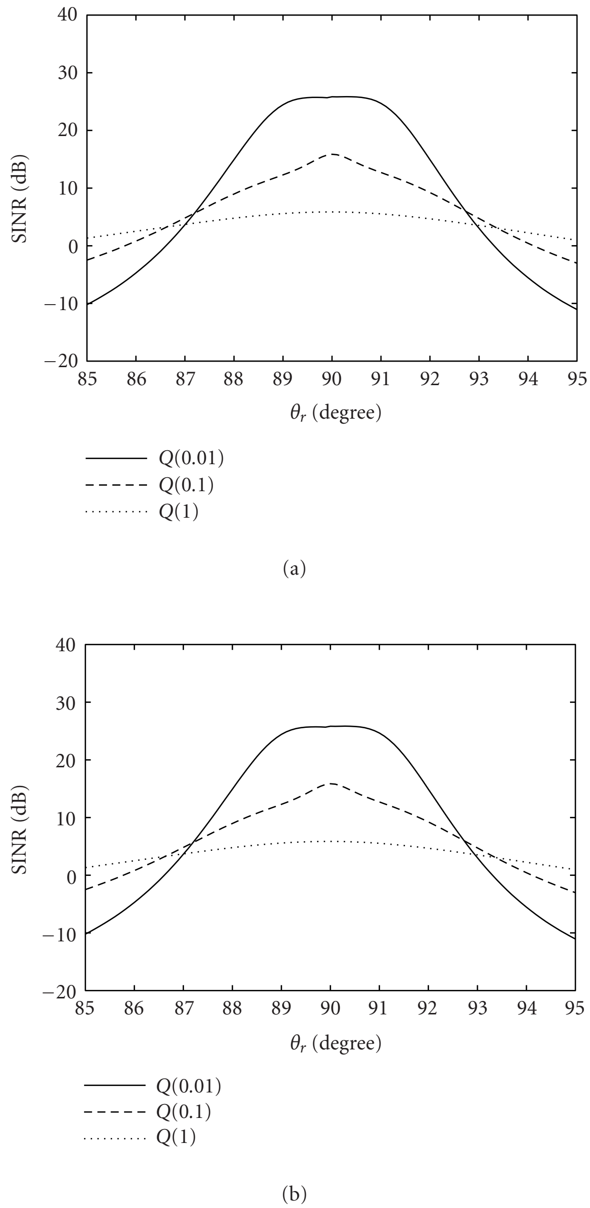

Figure 5: SINR for various values of SNR. (a) True solution. (b) Approximated solution.

exact and approximated solutions, respectively, and $P(a, b)$ denotes the result for $\varepsilon = a$ and $\Delta = b$. The exact solution was obtained by (11) and (12), and the approximated solution was obtained by (18) and (30). We see that robustness against look-direction errors is increased as ε is smaller, while resolution capability of the desired and interference signals is decreased. Therefore, we have to make a tradeoff between robustness and resolution capability in determining the value of ε. We also see that the exact and approximated solutions are very close to each other.

Figure 4 shows the SINRs for $\Delta = 0.3^\circ, 0.5^\circ$, and 1.0° with $\varepsilon = 0.02$. We see that robustness against look-direction errors is increased as Δ is larger, while resolution capability is decreased. Figure 5 shows the SINRs for $\sigma_n^2 = 0.01, 0.1$, and 1 with $\varepsilon = 0.02$ and $\Delta = 0.5^\circ$, where $Q(c)$ denotes the result for $\sigma_n^2 = c$. Figure 6 shows the SINRs for $N = 4, 6$, and 8 with $\varepsilon = 0.02$, $\Delta = 0.5^\circ$, $\sigma_n^2 = 0.01$, where $R(d)$ denotes the result for $N = d$. We see that robustness is decreased as σ_n^2 is larger or N is larger. We also see that the exact and approximated solutions are very close to each other except for the case of $N = 8$.

We quantitatively evaluated the approximation errors of the Lagrange multiplier and the weight vector computed by the proposed algorithm. Table 1 summarizes the true and

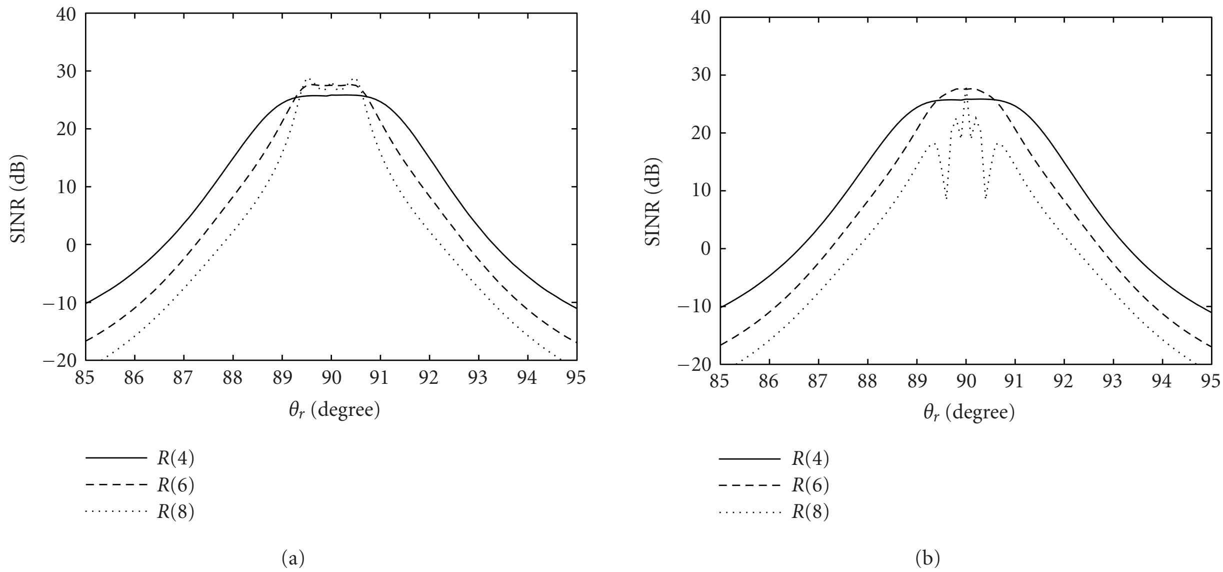

Figure 6: SINR for various numbers of array elements. (a) True solution. (b) Approximated solution.

Table 1: Approximation accuracies.

N	σ_n^2	ε	Δ	$\hat{\lambda}$	λ	$\lvert\mathbf{w}-\hat{\mathbf{w}}\rvert^2$	$\lvert\mathbf{w}-\hat{\mathbf{w}}\rvert^2/\lvert\mathbf{w}\rvert^2$
4	0.01	0.02	0.5	24.6107	24.5686	7.44582*e*-08	2.97194*e*-07
4	0.01	0.01	0.5	49.963	49.6534	3.14965*e*-07	1.23124*e*-06
4	0.01	0.03	0.5	16.2252	16.2136	3.14146*e*-08	1.28040*e*-07
4	0.01	0.05	0.5	9.52998	9.52965	1.02032*e*-08	4.33819*e*-08
4	0.01	0.02	0.3	24.5805	24.5686	1.33783*e*-09	5.35673*e*-09
4	0.01	0.02	1	24.7523	24.5986	1.51836*e*-05	5.86991*e*-05
4	0.1	0.02	0.5	25.1836	25.1605	9.75074*e*-10	3.91061*e*-09
4	1	0.02	0.5	30.9070	30.9052	1.31363*e*-11	5.27997*e*-11
6	0.01	0.02	0.5	24.5654	24.5569	3.25641*e*-06	1.92091*e*-05
8	0.01	0.02	0.5	24.5626	24.5561	2.76087*e*-05	0.000189067

approximated Lagrange multipliers, the squared error between the true and approximated weights, and the normalized error. The approximation is found to be very accurate. Figure 7 plots the normalized error between the true and approximated weights as the function of the angle width Δ, where Figure 7a is the result for $\varepsilon = 0.01, 0.02, 0.05$, Figure 7b is the result for $\sigma_n^2 = 0.01, 0.1, 1$, and Figure 7c is the result for $N = 4, 6, 8$. It is evident that the normalized error increases with an increase of Δ.

Finally, we compared the robust array trained by the proposed algorithm to the conventional array trained by the SMI algorithm in convergence performance. Figure 8 depicts the convergence trajectories of the SINR, where Figures 8a and 8b are the results for $\theta_r = 90^\circ$ and $\theta_r = 91^\circ$, respectively. We used the same parameters as in Figure 2. We see from Figure 8a that both methods show almost the same performance in the absence of look-direction errors. We see from Figure 8b that the conventional method fails when there is a mismatch between the prescribed and actual DOAs, while the proposed method exhibits almost the same convergence performance due to its robustness against look-direction errors.

5. CONCLUSION

We have derived the adaptive weight computation algorithm for the robust array antenna based on the SMI technique by using second-order Taylor series approximations. The adaptive algorithm can recursively compute the weight vector in only $O(N^2)$ computation time. Simulation results have shown that we have to tune parameters Δ and ε so that a good tradeoff between robustness and resolution capability is achieved, and that robustness depends upon the array size and the SNR.

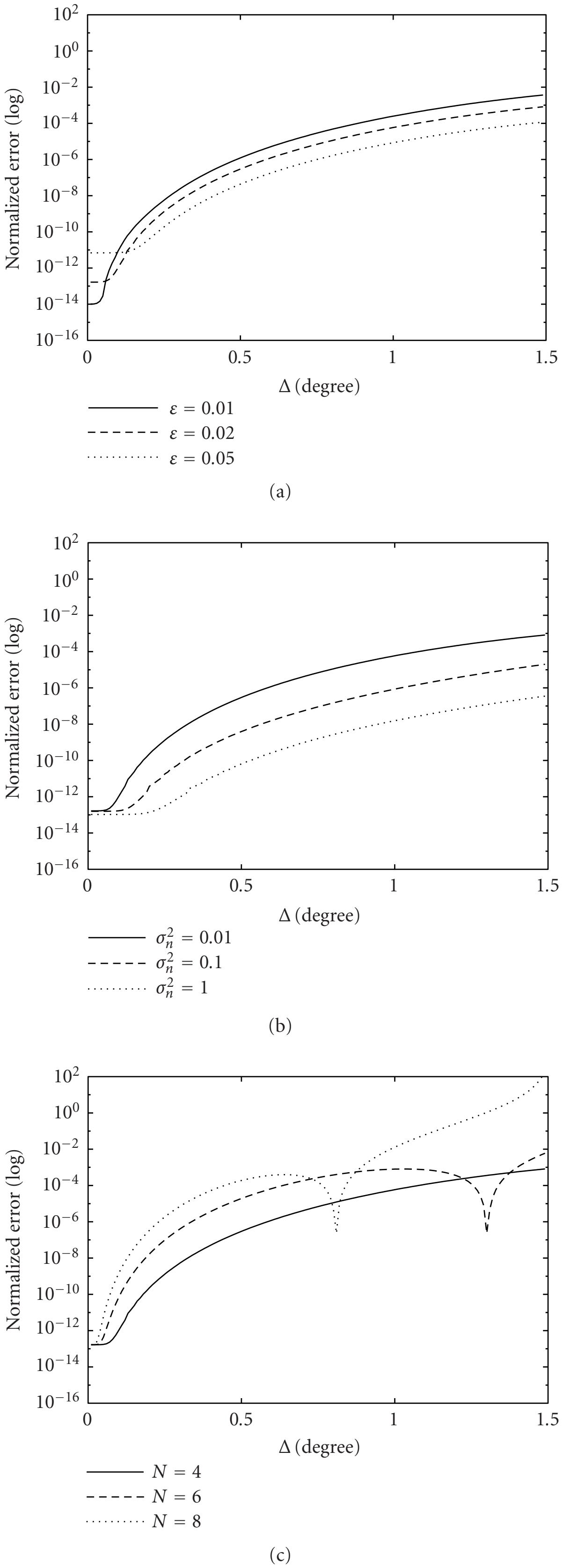

Figure 7: Approximation accuracies: (a) Case I ($\varepsilon = 0.01, 0.02, 0.05$). (b) Case II ($\sigma_n^2 = 0.01, 0.1, 1$). (c) Case III ($N = 4, 6, 8$).

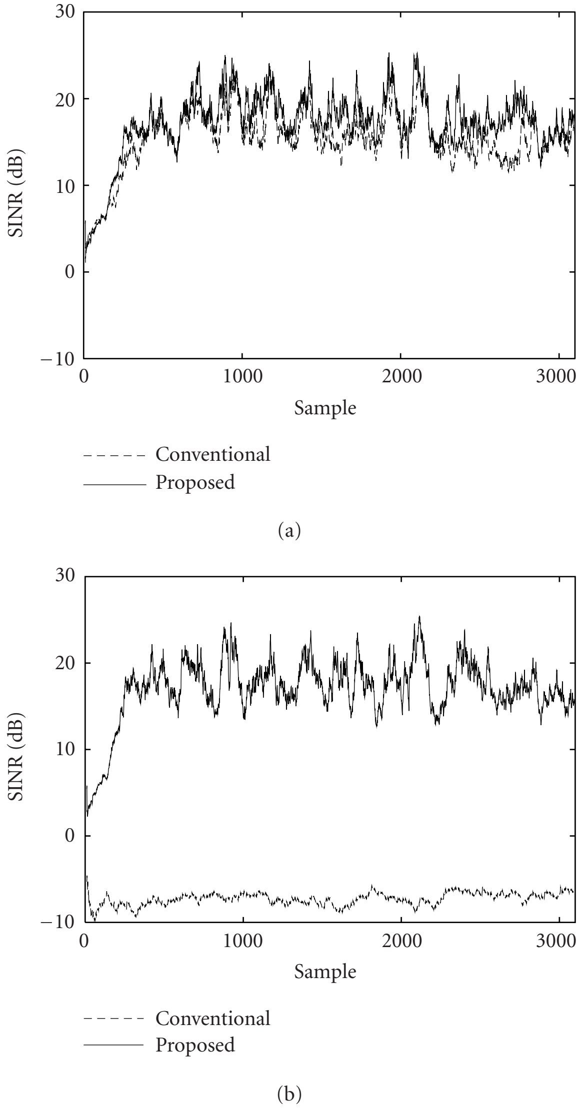

Figure 8: Convergence comparisons. (a) $\theta_r = 90^\circ$. (b) $\theta_r = 91^\circ$.

The inequality constraint for the case of broadband sources was considered in [14, 16]. Using the same approximation method, the result for a narrowband source will be extended to broadband sources.

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