RESEARCH　　　　　　　　　　　　　　　　　　　　　　　　**Open Access**

Optimization of a two-way MIMO amplify-and-forward relay network

Ying Zhang[1*], Yinjiang Chen[1], Chuanyi Pan[1], Huapeng Zhao[2] and Ning Kang[1]

Abstract

In this paper, we consider optimization of a two-way multiple-input multiple-output (MIMO) amplify-and-forward relay network which consists of a pair of transceivers and several relay nodes. Multiple antennas are equipped on the transceivers and relays. Multiple access broadcast scheme which finishes communication in two time slots is considered. In the first time slot, signals received by the relays are scaled by several beamforming matrices. In the second time slot, the relays transmit the scaled signals to the two transceivers. Upon receiving these signals, a MIMO equalizer is implemented at each transceiver to recover the desired signal. In this paper, zero forcing equalizers are used. Joint optimization of the beamforming matrices and the equalizers are realized using the following criteria: 1) the total relay transmission power is minimized subject to the minimal output signal-to-noise ratio (SNR) constraint at each transceiver, 2) the minimal output SNR of the two transceivers is maximized subject to total relay transmission power constraint, and 3) the minimal output SNR of the two transceivers is maximized subject to individual relay transmission power constraint. It is shown that the proposed optimization problems can be formulated as the second-order cone programming problems which can be solved efficiently. The validity of the proposed algorithm is verified by computer simulations.

Keywords: Amplify-and-forward; Beamforming; Multiple-input multiple-output; Relaying; Second-order cone programming; Zero-forcing

Introduction

Relaying technique is capable of extending communication range and coverage by providing link to shadowed users via relay nodes, and it received extensive study in recent years. For collaborative relaying technique, the network with a single pair of users and multiple relay nodes equipped with single antenna has been widely investigated [1-7]. Zheng [1] assumed perfect knowledge of channel-state information (CSI) and proposed to optimize the beamforming vector by maximizing the destination signal-to-noise ratio (SNR) subject to total and local relay transmission power constraints. In [2], similar optimization criterion was used in the case that only the second-order statistics of CSI are available. In [3,4], quantized CSI was considered. Quantizer at each relay and beamforming vectors at destination were optimized to minimize the uncoded bit error rate. In [5],

the minimizing mean square error (MMSE) criterion was adopted to optimize the beamforming coefficients. The advantage of this algorithm lies in its ability to adaptively allocate transmission power of each relay. In recent, a so-called filter-and-forward distributed beamforming technique was proposed in [6]. Different from previous algorithms, this method solved the problem of relay beamforming in frequency selective environments, where a finite impulse response filter is used at each relay. Apart from the above mentioned one-way scheme, a two-way relaying strategy was proposed in [7]. In a two-way relaying scheme, the relays cooperate with each other to establish the connection between two transceivers. The design of the beamformer should simultaneously satisfy the requirements from the two transceivers. In [8], an optimization strategy was proposed to optimize the performance of a two-way single-input single-output relaying network.

Multiple relay nodes create a virtual multiple-input multiple-output (MIMO) environment at the relay layer.

*Correspondence: zh0045ng@e.ntu.edu.sg
[1]College of Electronic Engineering, University of Electronic Science and Technology of China, Xiyuan Avenue, 610066 Chengdu, China
Full list of author information is available at the end of the article

With multiple antennas equipped at transmitting and receiving nodes, user nodes can also employ advantages of MIMO techniques, such as spatial multiplexing, space-time coding, and beamforming. Attracted by these merits, more and more algorithms are proposed to optimize MIMO relay networks [9-14] and virtual MIMO relay networks [15-18]. Most of these algorithms consider one-way communication scheme, where various optimization criteria are adopted, such as maximizing the destination SNR and minimizing the total system/relay transmission power, MMSE, ZF (zero force), etc. For a single pair of users and multiple relays, a unified algorithm which computes the optimal linear transceivers jointly at the source node and the relay nodes for two-way amplify-and-forward (AF) protocols was proposed [14]. The optimization algorithm was designed based on maximization of sum rate and MMSE. In [16], a two-way scheme was considered for a relay network with multiple users and single relay. The network was optimized using MMSE criterion at the destination subject to power constraint on relays.

In this paper, we consider a two-way MIMO relay network with one pair of transceivers and multiple relays, where the two transceivers are each equipped with M antennas, and every relay is equipped with N antennas. We assume that relays receive the mixture of signals from two transceivers in the first time slot. With perfect CSI, relays scale the received signals and then transmit these signals to the transceivers in the second time slot. Finally, a MIMO equalizer is used at each transceiver to recover the desired signal. To achieve power-efficient communication, beamforming coefficient matrices are optimized based on three criteria which are designed based on the minimal output SNR and relay transmission power. Meanwhile, the MIMO equalizer is optimized by imposing the ZF constraint. It is shown that the proposed optimization problems can be formulated as the second-order cone programming (SOCP) problems, which can be efficiently solved using the cvx toolbox [19]. Contributions of this paper are summarized as follows. First, two-way communication scheme of a MIMO relay network consists of multiple relays with multiple antennas is firstly considered. Second, the proposed optimization problem is formulated as an SOCP problem which can be solved efficiently.

The rest of this paper is organized as follows. The 'Problem formulation' section presents the problem formulation. The 'Mathematical approximation' section develops mathematical preparation for optimizing the considered relay network, and 'Optimization of the proposed relay network' section gives a detailed optimization procedure to derive the optimal beamforming and equalization matrices. In the 'Computer simulations' section, computer simulations are conducted to demonstrate validity of the proposed algorithm. Finally, conclusions and discussions are presented in the 'Conclusions' section.

Notation

Bold lower case is used for vectors, while bold capital letters for matrices. $\mathbf{I}$ denotes the unit matrix, $*$ denotes the complex conjugate operation, T denotes the matrix transposition operation, and H denotes the complex conjugate transposition operation. $\mathrm{tr}(\mathbf{A})$ is the trace of matrix $\mathbf{A}$. $\mathrm{vec}(\mathbf{A})$ stacks the columns of $\mathbf{A}$ into a single column vector. $\otimes$ denotes the Kronecker product, and $(\mathbf{A}_0 \oplus \ldots \oplus \mathbf{A}_N)$ yields a block diagonal matrix with block elements given by $\mathbf{A}_i$.

Problem formulation

Figure 1 depicts a two-way MIMO relay scheme, where $\mathbf{W}_i$ ($i = 1, 2, \ldots, L$) is the beamforming matrix for the ith relay and $\mathbf{D}_j$ ($j = 1, 2$) is the equalizer for the jth transceiver. Transceiver 1 and transceiver 2 are both equipped with M antennas. It is supposed that L relays equipped with N antennas are used. Flat fading channels are considered. We assume that no direct link exists between the two transceivers. The channel matrix from transceiver 1 to the ith relay is denoted as $\mathbf{H}_i$ ($i = 1, 2, \ldots, L$), and the one from the ith relay to transceiver 2 is denoted as $\mathbf{G}_i$, where $\mathbf{H}_i \in \mathbb{C}^{N \times M}$ and $\mathbf{G}_i \in \mathbb{C}^{M \times N}$ consist of independent complex Gaussian variables. It is assumed that these channels are reciprocal, i.e., the channel matrix from the ith relay to transceiver 1 is $\mathbf{H}_i^T$, and the one from transceiver 2 to the ith relay is $\mathbf{G}_i^T$. In the first time slot, each transceiver sends messages to the L relays. With the knowledge of $\mathbf{H}_i$ and $\mathbf{G}_i$ (which can be obtained via training), $\mathbf{W}_i$ is computed. In the second time slot, the relays scale the received signals according to $\mathbf{W}_i$ and then transmit these signals to the two transceivers. After receiving signals, a MIMO equalizer denoted as $\mathbf{D}_j$, $j = 1, 2$ is used at each transceiver. The aim of this paper is to optimize the performance of the relay network by designing $\mathbf{W}_i$ and $\mathbf{D}_j$.

The mixture of signals received by the ith relay can be expressed as

$$\mathbf{r}_i = \mathbf{H}_i \mathbf{s}_1 + \mathbf{G}_i^T \mathbf{s}_2 + \mathbf{v}_i, \tag{1}$$

where $\mathbf{s}_1$ and $\mathbf{s}_2$ are transmitted signals with covariance matrices $P_1 \mathbf{I}$ and $P_2 \mathbf{I}$, respectively, and they are independent from each other. $\mathbf{v}_i$ denotes the additive Gaussian noise (AGN) with covariance matrix $\mathbf{R}_{vi}$ at the ith relay. In this paper, it is assumed that AGN is white, and its covariance matrix is identical for all relays, i.e., $\mathbf{R}_{vi} = \sigma_v^2 \mathbf{I}$, $\forall i = 1, \ldots, L$.
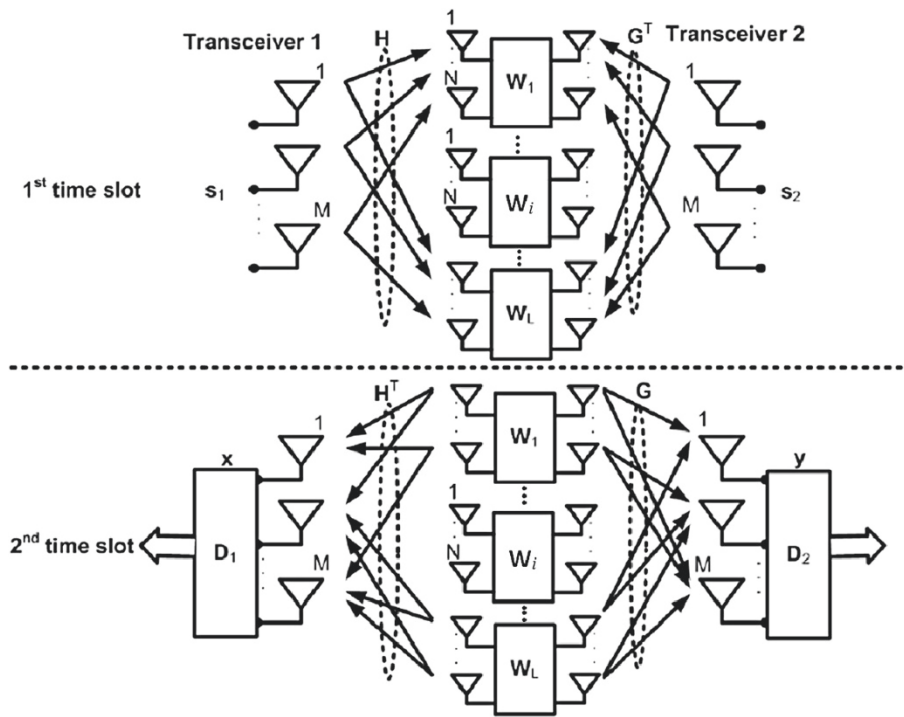
Figure 1 A two-way MIMO relay scheme.

In the second time slot, transceiver 1 and transceiver 2 each receives

$$\mathbf{x} = \sum_{i=1}^{L} \mathbf{H}_i^T \mathbf{W}_i \mathbf{r}_i + \mathbf{v}_x, \tag{2a}$$

$$= \sum_{i=1}^{L} \mathbf{H}_i^T \mathbf{W}_i \mathbf{H}_i \mathbf{s}_1 + \sum_{i=1}^{L} \mathbf{H}_i^T \mathbf{W}_i \mathbf{G}_i^T \mathbf{s}_2$$

$$+ \sum_{i=1}^{L} \mathbf{H}_i^T \mathbf{W}_i \mathbf{v}_i + \mathbf{v}_x,$$

$$\mathbf{y} = \sum_{i=1}^{L} \mathbf{G}_i \mathbf{W}_i \mathbf{r}_i + \mathbf{v}_y \tag{2b}$$

$$= \sum_{i=1}^{L} \mathbf{G}_i \mathbf{W}_i \mathbf{H}_i \mathbf{s}_1 + \sum_{i=1}^{L} \mathbf{G}_i \mathbf{W}_i \mathbf{G}_i^T \mathbf{s}_2$$

$$+ \sum_{i=1}^{L} \mathbf{G}_i \mathbf{W}_i \mathbf{v}_i + \mathbf{v}_y,$$

where $\mathbf{v}_x$ and $\mathbf{v}_y$ denote AGN at transceiver 1 and transceiver 2, and their covariance matrices are assumed to be $\sigma_x^2 \mathbf{I}$ and $\sigma_y^2 \mathbf{I}$, respectively. $\mathbf{s}_1$ and $\mathbf{s}_2$ are known by transceiver 1 and transceiver 2, respectively. If $\mathbf{G}_i$, $\mathbf{H}_i$, and $\mathbf{W}_i$ are available to transceivers, terms containing $\mathbf{s}_1$ and $\mathbf{s}_2$ can be subtracted from Equations 2a and 2b, respectively. Equations 2a and 2b are then rewritten as

$$\bar{\mathbf{x}} = \sum_{i=1}^{L} \mathbf{H}_i^T \mathbf{W}_i \mathbf{G}_i^T \mathbf{s}_2 + \sum_{i=1}^{L} \mathbf{H}_i^T \mathbf{W}_i \mathbf{v}_i + \mathbf{v}_x, \tag{3a}$$

$$\bar{\mathbf{y}} = \sum_{i=1}^{L} \mathbf{G}_i \mathbf{W}_i \mathbf{H}_i \mathbf{s}_1 + \sum_{i=1}^{L} \mathbf{G}_i \mathbf{W}_i \mathbf{v}_i + \mathbf{v}_y. \tag{3b}$$

A MIMO equalizer is used at each transceiver; thereby, the restored signal after equalization is expressed as

$$\widehat{\mathbf{s}}_2 = \mathbf{D}_1 \bar{\mathbf{x}}, \tag{4a}$$

$$\widehat{\mathbf{s}}_1 = \mathbf{D}_2 \bar{\mathbf{y}}. \tag{4b}$$

Based on Equations 1 to 4, the total relay transmission power and terminal SNRs are defined as follows:
1) Total relay transmission power:

$$P_{\mathrm{r}} = \sum_{i=1}^{L} E\left[\|\mathbf{W}_i \mathbf{r}_i\|_2^2\right] \tag{5}$$

$$= \sum_{i=1}^{L} \Big[P_1 \mathrm{tr}\left(\mathbf{W}_i \mathbf{H}_i \mathbf{H}_i^H \mathbf{W}_i^H\right)$$

$$+ P_2 \mathrm{tr}\left(\mathbf{W}_i \mathbf{G}_i^T \mathbf{G}_i^* \mathbf{W}_i^H\right) + \sigma_v^2 \mathrm{tr}\left(\mathbf{W}_i \mathbf{W}_i^H\right) \Big]$$

2) Terminal SNR at transceiver 1:

$$
\begin{aligned}
\text{SNR}_1 &= \frac{E\left[\left\|\mathbf{D}_1 \sum_{i=1}^{L} \mathbf{H}_i^T \mathbf{W}_i \mathbf{G}_i^T \mathbf{s}_2\right\|_2^2\right]}{E\left[\left\|\mathbf{D}_1 \sum_{i=1}^{L} \mathbf{H}_i^T \mathbf{W}_i \mathbf{v}_i + \mathbf{D}_1 \mathbf{v}_x\right\|_2^2\right]} \\
&= \frac{P_2 \text{tr}\left(\mathbf{D}_1 \left(\sum_{i=1}^{L} \mathbf{H}_i^T \mathbf{W}_i \mathbf{G}_i^T\right)\left(\sum_{i=1}^{L} \mathbf{G}_i^* \mathbf{W}_i^H \mathbf{H}_i^*\right)\mathbf{D}_1^H\right)}{\text{tr}\left(\mathbf{D}_1 \left(\sigma_v^2 \sum_{i=1}^{L} \mathbf{H}_i^T \mathbf{W}_i \sum_{i=1}^{L} \mathbf{W}_i^H \mathbf{H}_i^* + \sigma_x^2 \mathbf{I}\right)\mathbf{D}_1^H\right)}
\end{aligned}
\tag{6}
$$

3) Terminal SNR at transceiver 2:

$$
\begin{aligned}
\text{SNR}_2 &= \frac{E\left[\left\|\mathbf{D}_2 \sum_{i=1}^{L} \mathbf{G}_i \mathbf{W}_i \mathbf{H}_i \mathbf{s}_1\right\|_2^2\right]}{E\left[\left\|\mathbf{D}_2 \sum_{i=1}^{L} \mathbf{G}_i \mathbf{W}_i \mathbf{v}_i + \mathbf{D}_2 \mathbf{v}_y\right\|_2^2\right]} \\
&= \frac{P_1 \text{tr}\left(\mathbf{D}_2 \left(\sum_{i=1}^{L} \mathbf{G}_i \mathbf{W}_i \mathbf{H}_i\right)\left(\sum_{i=1}^{L} \mathbf{H}_i^H \mathbf{W}_i^H \mathbf{G}_i^H\right)\mathbf{D}_2^H\right)}{\text{tr}\left(\mathbf{D}_2 \left(\sigma_v^2 \sum_{i=1}^{L} \mathbf{G}_i \mathbf{W}_i \sum_{i=1}^{L} \mathbf{W}_i^H \mathbf{G}_i^H + \sigma_y^2 \mathbf{I}\right)\mathbf{D}_2^H\right)}.
\end{aligned}
\tag{7}
$$

In subsequent sections, P_r, SNR_1, and SNR_2 will be used to optimize beamforming matrices and MIMO equalizers.

Mathematical approximation

From (5) to (7), it seems difficult to directly evaluate $\mathbf{W}_i$ and $\mathbf{D}_j$ because they appear in both signal and noise terms. Therefore, before designing beamformers, three lemmas are derived to make $\mathbf{W}_i$ and $\mathbf{D}_j$ solvable.

Lemma 1. *Total relay transmission power can be expressed as a quadratic function of* $\mathbf{w}$:

$$
P_r = P_1 \mathbf{w}^H \overline{\mathbf{H}} \mathbf{w} + P_2 \mathbf{w}^H \mathbf{G} \mathbf{w} + \sigma_v^2 \mathbf{w}^H \mathbf{w},
\tag{8}
$$

where

$$
\begin{aligned}
\mathbf{w}_i &= vec(\mathbf{W}_i), \ i = 1, \ldots, L, \\
\mathbf{w} &= \left(\mathbf{w}_1^T, \ldots, \mathbf{w}_L^T\right)^T, \\
\overline{\mathbf{H}}_{ik} &= \left(\mathbf{h}_{ik}^T \otimes \mathbf{I}_N\right), \\
\overline{\mathbf{H}} &= \sum_{k=1}^{M} \overline{\mathbf{H}}_{1k}^H \overline{\mathbf{H}}_{1k} \oplus \ldots \oplus \sum_{k=1}^{M} \overline{\mathbf{H}}_{Lk}^H \overline{\mathbf{H}}_{Lk}, \\
\mathbf{G}_{ik} &= \left(\overline{\mathbf{g}}_{ik}^T \otimes \mathbf{I}_N\right), \\
\mathbf{G} &= \sum_{k=1}^{M} \mathbf{G}_{1k}^H \mathbf{G}_{1k} \oplus \ldots \oplus \sum_{k=1}^{M} \mathbf{G}_{Lk}^H \mathbf{G}_{Lk},
\end{aligned}
$$

are defined. The notation of $\mathbf{h}_{ik}$ *and* $\overline{\mathbf{g}}_{ik}$ *is given in the proof.*

Proof. From (5), the kth column of $(\mathbf{W}_i \mathbf{H}_i)$ can be expressed as $\left(\mathbf{h}_{ik}^T \otimes \mathbf{I}_N\right) \mathbf{w}_i$, where $\mathbf{h}_{ik}$ denotes the kth column of matrix $\mathbf{H}_i$. Therefore, the trace of $\left(\mathbf{W}_i \mathbf{H}_i \mathbf{H}_i^H \mathbf{W}_i^H\right)$ is given by the Frobenious norm of $(\mathbf{W}_i \mathbf{H}_i)$, which can be computed as

$$
\text{tr}\left(\mathbf{W}_i \mathbf{H}_i \mathbf{H}_i^H \mathbf{W}_i^H\right) = \mathbf{w}_i^H \sum_{k=1}^{M} \left(\mathbf{h}_{ik}^T \otimes \mathbf{I}_N\right)^H \left(\mathbf{h}_{ik}^T \otimes \mathbf{I}_N\right) \mathbf{w}_i.
\tag{9}
$$

Expressing the trace of $\left(\mathbf{W}_i \mathbf{G}_i^T \mathbf{G}_i^* \mathbf{W}_i^H\right)$ in the similar way of (9) and substituting it and (9) into (5) yield

$$
\begin{aligned}
P_r = {}& P_1 \sum_{i=1}^{L} \mathbf{w}_i^H \sum_{k=1}^{M} \left(\mathbf{h}_{ik}^T \otimes \mathbf{I}_N\right)^H \left(\mathbf{h}_{ik}^T \otimes \mathbf{I}_N\right) \mathbf{w}_i \\
&+ P_2 \sum_{i=1}^{L} \mathbf{w}_i^H \sum_{k=1}^{M} \left(\overline{\mathbf{g}}_{ik}^T \otimes \mathbf{I}_N\right)^H \left(\overline{\mathbf{g}}_{ik}^T \otimes \mathbf{I}_N\right) \mathbf{w}_i \\
&+ \sigma_v^2 \sum_{i=1}^{L} \mathbf{w}_i^H \mathbf{w}_i,
\end{aligned}
$$

where $\overline{\mathbf{g}}_{ik}$ denotes the kth column of matrix $\mathbf{G}_i^T$.

Using the definitions in *Lemma 1*, (8) can be derived. $\square$

SNR constraint is usually used in optimizing a relay network. For our problem, constraints on destination SNRs are expressed as

$$
\text{SNR}_1 \geq \gamma_1,
\tag{10a}
$$

$$
\text{SNR}_2 \geq \gamma_2,
\tag{10b}
$$

where γ_1 and γ_2 are required SNRs at transceiver 1 and transceiver 2, respectively. From (6) and (7), it is seen that (10) is related to $\mathbf{W}_i$ and $\mathbf{D}_j$ in a complicated form. In the rest of this section, two lemmas are derived to transform (10) into a manageable form.

The ZF constraint requires that

$$
\mathbf{D}_1 \sum_{i=1}^{L} \mathbf{H}_i^T \mathbf{W}_i \mathbf{G}_i^T = \mathbf{I},
\tag{11a}
$$

$$
\mathbf{D}_2 \sum_{i=1}^{L} \mathbf{G}_i \mathbf{W}_i \mathbf{H}_i = \mathbf{I},
\tag{11b}
$$

where $\mathbf{D}_1$ and $\mathbf{D}_2$ are defined as the left pseudoinverse of $\sum_{i=1}^{L} \mathbf{H}_i^T \mathbf{W}_i \mathbf{G}_i^T$ and $\sum_{i=1}^{L} \mathbf{G}_i \mathbf{W}_i \mathbf{H}_i$, respectively.

Lemma 2. *From definitions of* $\mathbf{D}_1$ *and* $\mathbf{D}_2$ *given above, if* $\mathbf{D}_1^H\mathbf{D}_1$ *and* $\mathbf{D}_2^H\mathbf{D}_2$ *are diagonal matrices, we have*

$$tr\left(\mathbf{D}_1^H\mathbf{D}_1\right) = \sum_{i=1}^{M} \frac{1}{\mid \phi_{ii}\mid^2 \sigma_i}, \qquad (12a)$$

$$tr\left(\mathbf{D}_2^H\mathbf{D}_2\right) = \sum_{i=1}^{M} \frac{1}{\mid \widetilde{\phi}_{ii}\mid^2 \widetilde{\sigma}_i}, \qquad (12b)$$

where the definitions of ϕ_{ii}, $\widetilde{\phi}_{ii}$, σ_i, *and* $\widetilde{\sigma}_i$ *are given in the proof.*

Proof. It is straightforward to show that

$$\mathbf{D}_1\mathbf{D}_1^H = \left(\sum_{i=1}^{L} \mathbf{G}_i^*\mathbf{W}_i^H\mathbf{H}_i^* \sum_{i=1}^{L} \mathbf{H}_i^T\mathbf{W}_i\mathbf{G}_i^T\right)^{-1}. \qquad (13)$$

If we define $\underline{\mathbf{H}} = \left[\mathbf{H}_1^T, \ldots, \mathbf{H}_L^T\right]^T$, $\underline{\mathbf{W}} = \mathbf{W}_1 \oplus \ldots \mathbf{W}_L$, and $\underline{\mathbf{G}} = [\mathbf{G}_1, \ldots, \mathbf{G}_L]$, (13) can be expressed as

$$\mathbf{D}_1\mathbf{D}_1^H = \left(\underline{\mathbf{G}}^*\underline{\mathbf{W}}^H\underline{\mathbf{H}}^*\underline{\mathbf{H}}^T\underline{\mathbf{W}}\,\underline{\mathbf{G}}^T\right)^{-1}. \qquad (14)$$

Suppose that the eigendecomposition of $\underline{\mathbf{H}}^*\underline{\mathbf{H}}^T$ is given by

$$\underline{\mathbf{H}}^*\underline{\mathbf{H}}^T = \left(\mathbf{U}\ \mathbf{U}_{\parallel}\right) \begin{pmatrix} \mathbf{\Lambda} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix} \begin{pmatrix} \mathbf{U}^H \\ \mathbf{U}_{\parallel}^H \end{pmatrix} \qquad (15)$$
$$= \mathbf{U}\mathbf{\Lambda}\mathbf{U}^H.$$

In (15), the diagonal of $\mathbf{\Lambda}$ consists of M nonzero eigenvalues. The matrix $\mathbf{U}$ consists of all the eigenvectors corresponding to these nonzero eigenvalues. $\mathbf{U}_{\parallel}$ consists of column vectors which are linearly dependent on columns of $\mathbf{U}$. The dependence of eigenvectors is caused by rank deficiency of $\underline{\mathbf{H}}^*\underline{\mathbf{H}}^T$ whose effective rank is M.

We define $\overline{\underline{\mathbf{W}}} = \underline{\mathbf{W}}\,\underline{\mathbf{G}}^T$, and assume that $\overline{\underline{\mathbf{W}}}$ can be represented by the complete orthogonal basis in the NL-dimensional space, where $\mathbf{U}$ is contained in the complete orthogonal basis, i.e.,

$$\overline{\underline{\mathbf{W}}} = \left(\mathbf{U}\ \mathbf{U}_{\perp}\right) \begin{pmatrix} \mathbf{\Phi} \\ \mathbf{\Phi}_{\perp} \end{pmatrix}. \qquad (16)$$

In (16), $\mathbf{U} \in \mathbb{C}^{NL\times M}$, $\mathbf{\Phi} \in \mathbb{C}^{M\times M}$, and $\mathbf{U}_{\perp}$ consist of $N - M$ orthogonal basis of the NL-dimensional space, which can be obtained via Gram-Schmidt procedure based on $\mathbf{U}$.

Substituting (15) and (16) into (14) yields

$$\mathbf{D}_1\mathbf{D}_1^H = \left(\left(\mathbf{\Phi}^H\ \mathbf{\Phi}_{\perp}^H\right) \begin{pmatrix} \mathbf{U}^H \\ \mathbf{U}_{\perp}^H \end{pmatrix} \mathbf{U}\mathbf{\Lambda}\mathbf{U}^H\right. \qquad (17)$$
$$\left.\left(\mathbf{U}\ \mathbf{U}_{\perp}\right) \begin{pmatrix} \mathbf{\Phi} \\ \mathbf{\Phi}_{\perp} \end{pmatrix}\right)^{-1} = \left(\mathbf{\Phi}^H\mathbf{\Lambda}\mathbf{\Phi}\right)^{-1}.$$

From (17), it is seen that

$$\text{tr}\mid\left(\mathbf{D}_1^H\mathbf{D}_1\right) = \text{tr}\left(\mathbf{D}_1\mathbf{D}_1^H\right) \leq \sum_{i=1}^{M} \frac{1}{\mid \phi_{ii}\mid^2 \sigma_i}, \qquad (18)$$

where ϕ_{ii} and σ_i are the ith diagonal element of $\mathbf{\Phi}$ and $\mathbf{\Lambda}$, respectively.

Similarly, for $\mathbf{D}_2$, we have

$$\text{tr}\left(\mathbf{D}_2^H\mathbf{D}_2\right) \leq \sum_{i=1}^{M} \frac{1}{\mid \widetilde{\phi}_{ii}\mid^2 \widetilde{\sigma}_i}, \qquad (19)$$

where $\widetilde{\sigma}_i$ is the ith nonzero eigenvalue of $\underline{\mathbf{G}}^H\underline{\mathbf{G}}$. $\widetilde{\phi}_{ii}$ is the ith diagonal element of $\widetilde{\mathbf{\Phi}}$, where $\underline{\mathbf{W}}\underline{\mathbf{H}} = \left(\widetilde{\mathbf{U}}\ \widetilde{\mathbf{U}}_{\perp}\right) \begin{pmatrix} \widetilde{\mathbf{\Phi}} \\ \widetilde{\mathbf{\Phi}}_{\perp} \end{pmatrix}$, and $\widetilde{\mathbf{U}}$ consists of eigenvectors of $\underline{\mathbf{G}}^H\underline{\mathbf{G}}$ corresponding to its nonzero eigenvalues. □

Lemma 3. *Inequalities (10) can be relaxed as*

$$\frac{\sigma_v^2\mathbf{w}^H\mathbf{H}\mathbf{w} + M\sigma_x^2}{\mathbf{w}^H\mathbf{d}_i\mathbf{d}_i^H\mathbf{w}\sigma_i} \leq \frac{1}{\gamma_1}, \ \forall i = 1,\ldots,M, \qquad (20a)$$

$$\frac{\sigma_v^2\mathbf{w}^H\overline{\mathbf{G}}\mathbf{w} + M\sigma_y^2}{\mathbf{w}^H\widetilde{\mathbf{d}}_i\widetilde{\mathbf{d}}_i^H\mathbf{w}\widetilde{\sigma}_i} \leq \frac{1}{\gamma_2}, \ \forall i = 1,\ldots,M, \qquad (20b)$$

$$\mathbf{w}^H\mathbf{e}_{ij}^* = 0, \ \forall i = 1,\ldots,M, j \neq i, \qquad (20c)$$

$$\mathbf{w}^H\widetilde{\mathbf{e}}_{ij}^* = 0, \ \forall i = 1,\ldots,M, j \neq i, \qquad (20d)$$

where

$$\mathbf{H}_{ik} = \left(\mathbf{I}_N \otimes \mathbf{h}_{ik}\right)^*,$$
$$\mathbf{H} = \sum_{k=1}^{M} \mathbf{H}_{1k}\mathbf{H}_{1k}^H \oplus \ldots \oplus \sum_{k=1}^{M} \mathbf{H}_{Lk}\mathbf{H}_{Lk}^H,$$
$$\overline{\mathbf{G}}_{ik} = \left(\mathbf{I}_N \otimes \overline{\mathbf{g}}_{ik}\right)^*,$$
$$\overline{\mathbf{G}} = \sum_{k=1}^{M} \overline{\mathbf{G}}_{1k}\overline{\mathbf{G}}_{1k}^H \oplus \ldots \oplus \sum_{k=1}^{M} \overline{\mathbf{G}}_{Lk}\overline{\mathbf{G}}_{Lk}^H,$$
$$\mathbf{d}_i = \mathbf{Q}\left(\mathbf{I} \otimes \mathbf{u}_i^*\right)\underline{\overline{\mathbf{g}}}_i,$$
$$\widetilde{\mathbf{d}}_i = \mathbf{Q}\left(\mathbf{I} \otimes \widetilde{\mathbf{u}}_i^*\right)\underline{\mathbf{h}}_i,$$
$$\mathbf{e}_{ij} = \mathbf{Q}\left(\mathbf{I} \otimes \mathbf{u}_i^*\right)\underline{\overline{\mathbf{g}}}_j,$$
$$\widetilde{\mathbf{e}}_{ij} = \mathbf{Q}\left(\mathbf{I} \otimes \widetilde{\mathbf{u}}_i^*\right)\underline{\mathbf{h}}_j,$$

and $\mathbf{u}_i$, $\widetilde{\mathbf{u}}_i$, $\underline{\overline{\mathbf{g}}}_i$, $\underline{\mathbf{h}}_i$ *and* $\mathbf{Q}$ *are defined in the following proof.*

Proof. With the ZF constraint, (6) and (7) can be simplified as

$$\text{SNR}_1 = \frac{P_2M}{\text{tr}\left(\mathbf{D}_1\left(\sigma_v^2\sum_{i=1}^{L}\mathbf{H}_i^T\mathbf{W}_i\sum_{i=1}^{L}\mathbf{W}_i^H\mathbf{H}_i^* + \sigma_x^2\mathbf{I}\right)\mathbf{D}_1^H\right)}$$

$$\text{SNR}_2 = \frac{P_1M}{\text{tr}\left(\mathbf{D}_2\left(\sigma_v^2\sum_{i=1}^{L}\mathbf{G}_i\mathbf{W}_i\sum_{i=1}^{L}\mathbf{W}_i^H\mathbf{G}_i^H + \sigma_y^2\mathbf{I}\right)\mathbf{D}_2^H\right)}.$$

From the property of tr(.), we may relax the inequality of SNR_1 as

$$
\begin{aligned}
SNR_1 &= \frac{P_2 M}{\mathrm{tr}\left(\mathbf{D}_1 \mathbf{B} \mathbf{D}_1^H\right)} \\
&= \frac{P_2 M}{\mathrm{tr}\left(\mathbf{B} \mathbf{D}_1^H \mathbf{D}_1\right)} \\
&\geq \frac{P_2 M}{\mathrm{tr}\left(\mathbf{B}\right)\mathrm{tr}\left(\mathbf{D}_1^H \mathbf{D}_1\right)},
\end{aligned}
\tag{21}
$$

where $\mathbf{B} = \left(\sigma_v^2 \sum_{i=1}^{L} \mathbf{H}_i^T \mathbf{W}_i \sum_{i=1}^{L} \mathbf{W}_i^H \mathbf{H}_i^* + \sigma_x^2 \mathbf{I}\right)$.

Substituting (18) into (21) yields

$$
SNR_1 \geq \frac{P_2 M}{\mathrm{tr}(\mathbf{B}) \sum_{i=1}^{M} \frac{1}{|\phi_{ii}|^2 \sigma_i}}.
\tag{22}
$$

From (16) and the definition of $\overline{\mathbf{W}}$, we have

$$
\begin{aligned}
\mathbf{\Phi} &= \mathbf{U}^H \overline{\mathbf{W}} = \mathbf{U}^H \underline{\mathbf{W}} \underline{\mathbf{G}}^T \\
&= \begin{pmatrix} \mathrm{vec}(\underline{\mathbf{W}})^H \left(\mathbf{I} \otimes \mathbf{u}_1^*\right) \\ \vdots \\ \mathrm{vec}(\underline{\mathbf{W}})^H \left(\mathbf{I} \otimes \mathbf{u}_M^*\right) \end{pmatrix} \underline{\mathbf{G}}^T.
\end{aligned}
\tag{23}
$$

Therefore, the elements of $\mathbf{\Phi}$ can be represented by

$$
\phi_{ij} = \mathrm{vec}(\underline{\mathbf{W}})^H \left(\mathbf{I} \otimes \mathbf{u}_i^*\right) \underline{\overline{\mathbf{g}}}_j,
\tag{24}
$$

where $\mathbf{u}_i$ denotes the ith column of $\mathbf{U}$ and $\underline{\overline{\mathbf{g}}}_j^T$ denotes the jth row of $\underline{\mathbf{G}}$.

Similarly, for SNR_2, we have

$$
SNR_2 \geq \frac{P_1 M}{\mathrm{tr}(\widetilde{\mathbf{B}}) \sum_{i=1}^{M} \frac{1}{|\widetilde{\phi}_{ii}|^2 \widetilde{\sigma}_i}},
\tag{25}
$$

where $\widetilde{\mathbf{B}} = \left(\sigma_v^2 \sum_{i=1}^{L} \mathbf{G}_i \mathbf{W}_i \sum_{i=1}^{L} \mathbf{W}_i^H \mathbf{G}_i^H + \sigma_y^2 \mathbf{I}\right)$, and $\widetilde{\phi}_{ij}$ can be expressed by

$$
\widetilde{\phi}_{ij} = \mathrm{vec}(\underline{\mathbf{W}})^H \left(\mathbf{I} \otimes \widetilde{\mathbf{u}}_i^*\right) \underline{\mathbf{h}}_j,
\tag{26}
$$

where $\widetilde{\mathbf{u}}_i$ denotes the ith column of $\widetilde{\mathbf{U}}$ and $\underline{\mathbf{h}}_j$ denotes the jth column of $\underline{\mathbf{H}}$. It is assumed that the eigendecomposition of $\underline{\mathbf{G}}^H \underline{\mathbf{G}}$ is $\widetilde{\mathbf{U}} \widetilde{\mathbf{\Lambda}} \widetilde{\mathbf{U}}^H$.

Similar to *Lemma 1*, the trace of $\mathbf{B}$ and $\widetilde{\mathbf{B}}$ can be expressed as

$$
\mathrm{tr}(\mathbf{B}) = \sigma_v^2 \mathbf{w}^H \mathbf{H} \mathbf{w} + M\sigma_x^2,
\tag{27a}
$$
$$
\mathrm{tr}(\widetilde{\mathbf{B}}) = \sigma_v^2 \mathbf{w}^H \overline{\mathbf{G}} \mathbf{w} + M\sigma_y^2,
\tag{27b}
$$

where the definitions of $\mathbf{H}$ and $\overline{\mathbf{G}}$ are given in *Lemma 3*.

Substituting (24) and (27a) into (22) and (26), and (27b) into (25) yields

$$
SNR_1 \geq \frac{P_2 M}{\left(\sigma_v^2 \mathbf{w}^H \mathbf{H} \mathbf{w} + M\sigma_x^2\right) \sum_{i=1}^{M} \frac{1}{\left|\mathrm{vec}(\underline{\mathbf{W}})^H \left(\mathbf{I} \otimes \mathbf{u}_i^*\right)\underline{\overline{\mathbf{g}}}_i\right|^2 \sigma_i}},
\tag{28a}
$$

$$
SNR_2 \geq \frac{P_1 M}{\left(\sigma_v^2 \mathbf{w}^H \overline{\mathbf{G}} \mathbf{w} + M\sigma_y^2\right) \sum_{i=1}^{M} \frac{1}{\left|\mathrm{vec}(\underline{\mathbf{W}})^H \left(\mathbf{I} \otimes \widetilde{\mathbf{u}}_i^*\right)\underline{\mathbf{h}}_i\right|^2 \widetilde{\sigma}_i}}.
\tag{28b}
$$

From (28), (10) can be relaxed as

$$
\frac{P_2 M}{\gamma_1} \geq \sum_{i=1}^{M} \frac{\sigma_v^2 \mathbf{w}^H \mathbf{H} \mathbf{w} + M\sigma_x^2}{\left|\mathrm{vec}(\underline{\mathbf{W}})^H \left(\mathbf{I} \otimes \mathbf{u}_i^*\right)\underline{\overline{\mathbf{g}}}_i\right|^2 \sigma_i},
\tag{29a}
$$

$$
\frac{P_1 M}{\gamma_2} \geq \sum_{i=1}^{M} \frac{\sigma_v^2 \mathbf{w}^H \overline{\mathbf{G}} \mathbf{w} + M\sigma_y^2}{\left|\mathrm{vec}(\underline{\mathbf{W}})^H \left(\mathbf{I} \otimes \widetilde{\mathbf{u}}_i^*\right)\underline{\mathbf{h}}_i\right|^2 \widetilde{\sigma}_i}.
\tag{29b}
$$

If every term on the right side of (29a) and (29b) is smaller than $\frac{P_2}{\gamma_1}$ and $\frac{P_1}{\gamma_2}$, respectively, i.e.,

$$
\frac{P_2}{\gamma_1} \geq \frac{\sigma_v^2 \mathbf{w}^H \mathbf{H} \mathbf{w} + M\sigma_x^2}{\left|\mathrm{vec}(\underline{\mathbf{W}})^H \left(\mathbf{I} \otimes \mathbf{u}_i^*\right)\underline{\overline{\mathbf{g}}}_i\right|^2 \sigma_i}, \ \forall i = 1, \ldots, M,
\tag{30a}
$$

$$
\frac{P_1}{\gamma_2} \geq \frac{\sigma_v^2 \mathbf{w}^H \overline{\mathbf{G}} \mathbf{w} + M\sigma_y^2}{\left|\mathrm{vec}(\underline{\mathbf{W}})^H \left(\mathbf{I} \otimes \widetilde{\mathbf{u}}_i^*\right)\underline{\mathbf{h}}_i\right|^2 \widetilde{\sigma}_i}, \ \forall i = 1, \ldots, M,
\tag{30b}
$$

(29) can be satisfied.

Because $\underline{\mathbf{W}}$ is block diagonal matrices, there are many zero elements in $\mathrm{vec}(\underline{\mathbf{W}})$, which do not contribute to the calculation of (30). Suppose $\mathbf{Q}$ is chosen such that

$$
\mathbf{w} = \mathbf{Q}\mathrm{vec}(\underline{\mathbf{W}})
\tag{31}
$$

holds.

To derive (30), we have make assumption that $\mathbf{D}_1^H \mathbf{D}_1$ and $\mathbf{D}_2^H \mathbf{D}_2$ should be diagonal. From (17), we may achieve this by forcing $\mathbf{\Phi}$ to be a diagonal matrix. Therefore, the following equations should be satisfied:

$$
\phi_{ij} = \mathrm{vec}(\underline{\mathbf{W}})^H \left(\mathbf{I} \otimes \mathbf{u}_i^*\right)\underline{\overline{\mathbf{g}}}_j = 0, \ \forall i \neq j,
\tag{32}
$$

$$
\widetilde{\phi}_{ij} = \mathrm{vec}(\underline{\mathbf{W}})^H \left(\mathbf{I} \otimes \widetilde{\mathbf{u}}_i^*\right)\underline{\mathbf{h}}_j = 0, \ \forall i \neq j.
\tag{33}
$$

With (30) to (33) and definitions given in Lemma 3, (20) can be derived. $\square$

Optimization of the proposed relay network

In this section, we introduce optimization of the proposed two-way MIMO relay network using the following three criteria.

Minimizing the total relay transmission power subject to individual minimal output SNR constraint and ZF constraint

Using this criterion, the optimization problem is formulated as

$$\min_{\mathbf{w}} P_r, \tag{34a}$$

subject to $(\text{SNR}_1)_{\text{lower}} \geq \gamma_1,\ (\text{SNR}_2)_{\text{lower}} \geq \gamma_2. \tag{34b}$

where $(\text{SNR}_1)_{\text{lower}}$ and $(\text{SNR}_2)_{\text{lower}}$ denote the minimal output SNR at transceiver 1 and transceiver 2, respectively.

Theorem 1. *(34) can be approximated as an SOCP problem given as*

$$\min_{\mathbf{w}} \|\mathbf{U}_1\mathbf{w}\|_2, \tag{35a}$$

subject to

$$\left\|\mathbf{U}_2\mathbf{w}'\right\|_2 \leq \frac{Real\{\mathbf{w}^H\mathbf{d}_i\}}{\sqrt{\gamma_1}},\ \forall i=1,\ldots,M \tag{35b}$$

$$\left\|\mathbf{U}_3\mathbf{w}'\right\|_2 \leq \frac{Real\{\mathbf{w}^H\widetilde{\mathbf{d}}_i\}}{\sqrt{\gamma_2}},\ \forall i=1,\ldots,M, \tag{35c}$$

$$Imag\{\mathbf{w}^H\mathbf{d}_i\} = 0,\ \forall i=1,\ldots,M, \tag{35d}$$

$$Imag\{\mathbf{w}^H\widetilde{\mathbf{d}}_i\} = 0,\ \forall i=1,\ldots,M, \tag{35e}$$

$$\mathbf{w}^H\mathbf{e}_{ij}^* = 0,\ \forall i=1,\ldots,M, j \neq i, \tag{35f}$$

$$\mathbf{w}^H\widetilde{\mathbf{e}}_{ij}^* = 0,\ \forall i=1,\ldots,M, j \neq i, \tag{35g}$$

$$\mathbf{w}' = \begin{pmatrix} \mathbf{w} \\ 1 \end{pmatrix}. \tag{35h}$$

and $\mathbf{d}_i,\ \widetilde{\mathbf{d}}_i,\ \mathbf{e}_{ij},$ *and* $\widetilde{\mathbf{e}}_{ij}$ *are given in Lemma 3.*

Proof. Define $\mathbf{A}_i = \begin{pmatrix} \overline{\mathbf{H}}_{i1}^H \\ \vdots \\ \overline{\mathbf{H}}_{iM}^H \end{pmatrix}, \mathbf{B}_i = \begin{pmatrix} \mathbf{G}_{i1}^H \\ \vdots \\ \mathbf{G}_{iM}^H \end{pmatrix}$ and

$\mathbf{U}_1 = \begin{pmatrix} \sqrt{P_1}(\mathbf{A}_1 \oplus \ldots \oplus \mathbf{A}_L) \\ \sqrt{P_2}(\mathbf{B}_1 \oplus \ldots \oplus \mathbf{B}_L) \\ \sigma_v\mathbf{I}_{NL} \end{pmatrix}$. From *Lemma 1*, P_r can

represented as $\|\mathbf{U}_1\mathbf{w}\|_2^2$. If we define $\mathbf{C}_i = \begin{pmatrix} \mathbf{H}_{i1}^H \\ \vdots \\ \mathbf{H}_{iM}^H \end{pmatrix}$

and $\mathbf{U}_2 = \begin{pmatrix} \sigma_v(\mathbf{C}_1 \oplus \ldots \mathbf{C}_L) & \mathbf{0} \\ \mathbf{0}^T & \sqrt{M}\sigma_x \end{pmatrix}$, the nominator of the left side of (20a) can be represented as $\|\mathbf{U}_2\mathbf{w}'\|_2^2$. Similarly, we define $\mathbf{E}_i = \begin{pmatrix} \overline{\mathbf{G}}_{i1}^H \\ \vdots \\ \overline{\mathbf{G}}_{iM}^H \end{pmatrix}$ and $\mathbf{U}_3 = \begin{pmatrix} \sigma_v(\mathbf{E}_1 \oplus \ldots \mathbf{E}_L) & \mathbf{0} \\ \mathbf{0}^T & \sqrt{M}\sigma_y \end{pmatrix}$, the nominator of the left side

of (20b) can be represented as $\|\mathbf{U}_3\mathbf{w}'\|_2^2$. Using these definitions, (20a) and (20b) can be expressed as

$$\left\|\mathbf{U}_2\mathbf{w}'\right\|_2^2 \leq \frac{\mathbf{w}^H\mathbf{d}_i\mathbf{d}_i^H\mathbf{w}}{\gamma_1},\ \forall i=1,\ldots,M, \tag{36a}$$

$$\left\|\mathbf{U}_3\mathbf{w}'\right\|_2^2 \leq \frac{\mathbf{w}^H\widetilde{\mathbf{d}}_i\widetilde{\mathbf{d}}_i^H\mathbf{w}}{\gamma_2},\ \forall i=1,\ldots,M. \tag{36b}$$

From the fact that $\text{Real}\{x\} \leq |x|$, (36) can be relaxed by

$$\left\|\mathbf{U}_2\mathbf{w}'\right\|_2 \leq \frac{\text{Real}\{\mathbf{w}^H\mathbf{d}_i\}}{\sqrt{\gamma_1}},\ \forall i=1,\ldots,M, \tag{37a}$$

$$\left\|\mathbf{U}_3\mathbf{w}'\right\|_2^2 \leq \frac{\text{Real}\{\mathbf{w}^H\widetilde{\mathbf{d}}_i\}}{\sqrt{\gamma_2}},\ \forall i=1,\ldots,M, \tag{37b}$$

$$\text{Imag}\{\mathbf{w}^H\mathbf{d}_i\} = 0,\ \forall i=1,\ldots,M, \tag{37c}$$

$$\text{Imag}\{\mathbf{w}^H\widetilde{\mathbf{d}}_i\} = 0,\ \forall i=1,\ldots,M. \tag{37d}$$

Then, with *Lemma 3*, (35) can be derived. $\qquad\square$

Maximizing the minimal output SNR of transceivers subject to total relay transmission power constraint and ZF constraint

Assuming that the minimum SNR required by the two transceivers is t, the optimization problem can be formulated as

$$\max_{\mathbf{w}} t, \tag{38a}$$

subject to $(\text{SNR}_1)_{\text{lower}} \geq t,\ (\text{SNR}_2)_{\text{lower}} \geq t,\ P_r \leq P. \tag{38b}$

where P denotes the maximal total relay transmission power.

Theorem 2. *(38) can be approximated as an SOCP problem:*

$$\max_{\mathbf{w}} t, \tag{39a}$$

subject to $\|\mathbf{U}_1\mathbf{w}\|_2 \leq \sqrt{P}, \tag{39b}$

$$\left\|\mathbf{U}_2\mathbf{w}'\right\|_2 \leq \frac{Real\{\mathbf{w}^H\mathbf{d}_i\}}{\sqrt{t}},\ \forall i=1,\ldots,M, \tag{39c}$$

$$\left\|\mathbf{U}_3\mathbf{w}'\right\|_2 \leq \frac{Real\{\mathbf{w}^H\widetilde{\mathbf{d}}_i\}}{\sqrt{t}},\ \forall i=1,\ldots,M, \tag{39d}$$

$$Imag\{\mathbf{w}^H\mathbf{d}_i\} = 0,\ \forall i=1,\ldots,M, \tag{39e}$$

$$Imag\{\mathbf{w}^H\widetilde{\mathbf{d}}_i\} = 0,\ \forall i=1,\ldots,M, \tag{39f}$$

$$\mathbf{w}^H\mathbf{e}_{ij}^* = 0,\ \forall i=1,\ldots,M, j \neq i, \tag{39g}$$

$$\mathbf{w}^H\widetilde{\mathbf{e}}_{ij}^* = 0,\ \forall i=1,\ldots,M, j \neq i, \tag{39h}$$

$$\mathbf{w}' = \begin{pmatrix} \mathbf{w} \\ 1 \end{pmatrix}. \tag{39i}$$

Proof. It can be easily obtained from *Lemmas 1* to *3* and *Theorem 1.* □

Because (39) is quasi-convex, for any given value of t, it becomes the following SOCP problem:

$$\text{find } \mathbf{w}, \tag{40a}$$

$$\text{subject to (39b) to (39i).} \tag{40b}$$

The bisection search procedure can be applied to solve (40).

Maximizing the minimal output SNR of transceivers subject to individual relay transmission power constraint and ZF constraint

The optimization problem is given as

$$\max_{\mathbf{w}} t, \tag{41a}$$

$$\text{subject to } (\text{SNR}_1)_{\text{lower}} \geq t, \ (\text{SNR}_2)_{\text{lower}} \geq t, \tag{41b}$$

$$P_{ri} \leq P_i, \ i = 1, .., L, \tag{41c}$$

where P_i denotes the maximal transmission power of the ith relay, and $P_{ri} = P_1 \text{tr} \left(\mathbf{W}_i \mathbf{H}_i \mathbf{H}_i^H \mathbf{W}_i^H \right) + P_2 \text{tr} \left(\mathbf{W}_i \mathbf{G}_i^T \mathbf{G}_i^* \mathbf{W}_i^H \right) + \sigma_v^2 \text{tr} \left(\mathbf{W}_i \mathbf{W}_i^H \right).$

Theorem 3. *(41) can be approximated as an SOCP problem:*

$$\max_{\mathbf{w}} t, \tag{42a}$$

$$\text{subject to } \left\| \mathbf{U}_1^i \mathbf{w} \right\|_2 \leq \sqrt{P_i}, \ i = 1, \ldots, L, \tag{42b}$$

$$\left\| \mathbf{U}_2 \mathbf{w}' \right\|_2 \leq \frac{Real\{\mathbf{w}^H \mathbf{d}_i\}}{\sqrt{t}}, \ \forall i = 1, \ldots, M, \tag{42c}$$

$$\left\| \mathbf{U}_3 \mathbf{w}' \right\|_2 \leq \frac{Real\{\mathbf{w}^H \widetilde{\mathbf{d}}_i\}}{\sqrt{t}}, \ \forall i = 1, \ldots, M, \tag{42d}$$

$$Imag\{\mathbf{w}^H \mathbf{d}_i\} = 0, \ \forall i = 1, \ldots, M, \tag{42e}$$

$$Imag\{\mathbf{w}^H \widetilde{\mathbf{d}}_i\} = 0, \ \forall i = 1, \ldots, M, \tag{42f}$$

$$\mathbf{w}^H \mathbf{e}_{ij}^* = 0, \ \forall i = 1, \ldots, M, j \neq i, \tag{42g}$$

$$\mathbf{w}^H \widetilde{\mathbf{e}}_{ij}^* = 0, \ \forall i = 1, \ldots, M, j \neq i, \tag{42h}$$

$$\mathbf{w}' = \begin{pmatrix} \mathbf{w} \\ 1 \end{pmatrix}, \tag{42i}$$

where $\mathbf{U}_1^i$ is defined as $\mathbf{U}_{ri} = \begin{pmatrix} \sqrt{P_1} \mathbf{A}_i \\ \sqrt{P_2} \mathbf{B}_i \\ \sigma_v \mathbf{I}_N \end{pmatrix}.$

Proof. It can be easily obtained from *Lemmas 1* to *3* and *Theorem 1.* □

For any given value of t, (42) reduces to the following SOCP probelm:

$$\text{find } \mathbf{w}, \tag{43a}$$

$$\text{subject to (42b) to (42i).} \tag{43b}$$

Similar to the solution of (40), (43) is solved by the bisection search procedure.

Computer simulations

In order to verify the validity of the proposed algorithm, we devise the following simulation scenario. The number of antennas of transceiver 1, transceiver 2, and relays is assumed to be $M = N = 3$, and the number of relays is $L = 10$. The communication channel coefficients are modeled by complex Gaussian variables with zero mean and variance σ_h^2 and σ_g^2. The two transceivers transmit independent data streams from different antennas with $P_1 = P_2 = 0$ dB. AGN on each antenna is assumed to be complex Gaussian variable with zero mean and unit variance, i.e., $\sigma_x^2 = \sigma_y^2 = \sigma_v^2 = 0$ dB. Sources are generated from a QPSK constellation. The values of SNR are computed from 100 independent trials for each plot. Furthermore, the power consumption to increase the minimal output SNR_2 for 2 dB becomes smaller as the value of γ_1 increases, which means the derived minimal output SNR approaches the output SNR as the value of SNR increases. Therefore, less additional power consumption is needed to increase the same amount of output SNR. This phenomenon can also be demonstrated by Figure 2.

Minimizing the total relay transmission power subject to individual minimal output SNR constraint and ZF constraint

We assume that $\sigma_h^2 = \sigma_g^2 = 0$ dB. Figure 3 depicts the total relay transmission power P_r against the value of γ_1. It is observed that the required transmission power increases as the value of γ_1 increases. Also, for a given γ_1, the total relay transmission power increases with the increase of γ_2.

Figure 2 shows the cumulative distribution function (CDF) of the output SNR at transceiver 2 with different values of γ_2. In Figure 2, the value of γ_2 is assumed to vary from -6 to 6 dB with 2 dB stepsize. From the figure, we see that for a given γ_2, the output SNR at transceiver 2 does not change significantly with the variation of γ_1, and the output SNR_2 is about 2 to 3 dB higher than the value of γ_2 with 90% probability. This is reasonable since the proposed optimization problem uses the minimal output SNR instead of the real output SNR. It can be seen that the difference between the real output SNR_2 and γ_2 decreases as the value of γ_2 increases, which is in accordance with the phenomenon observed in Figure 3.
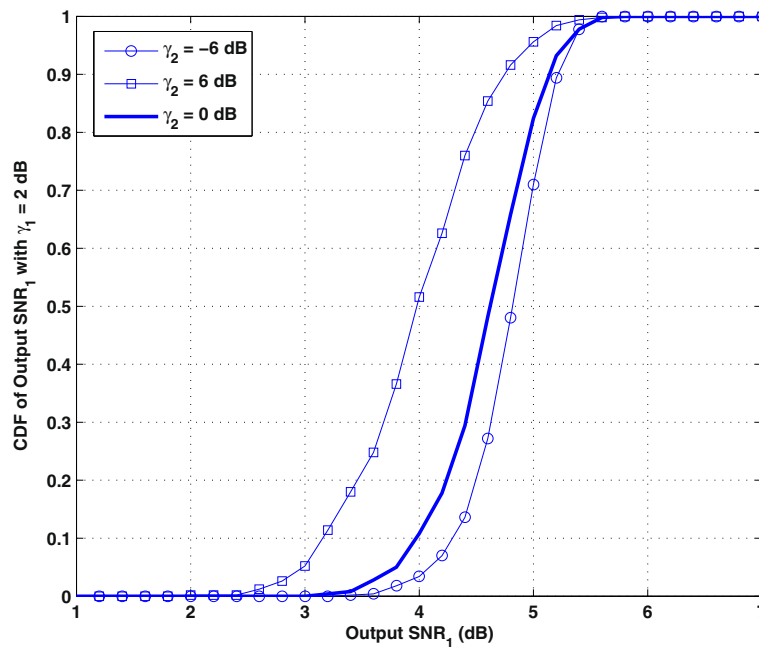
Figure 4 plots the CDF of output SNR_1 with different values of γ_2. It is found that the average output SNR_1
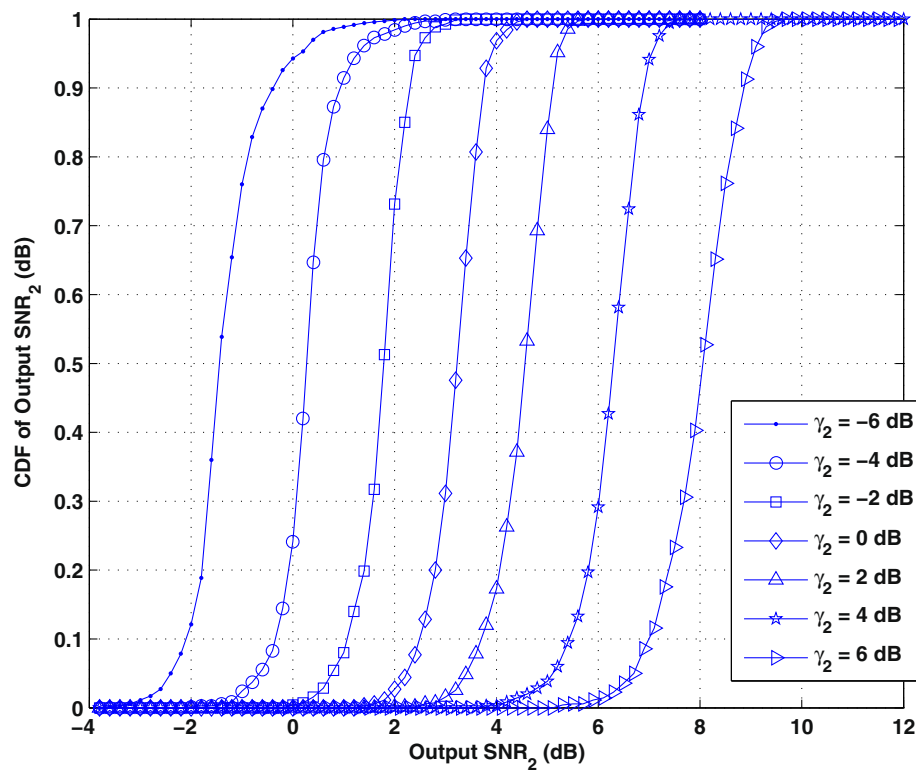
Figure 2 CDF of the output SNR at transceiver 2 with different values of γ_2.
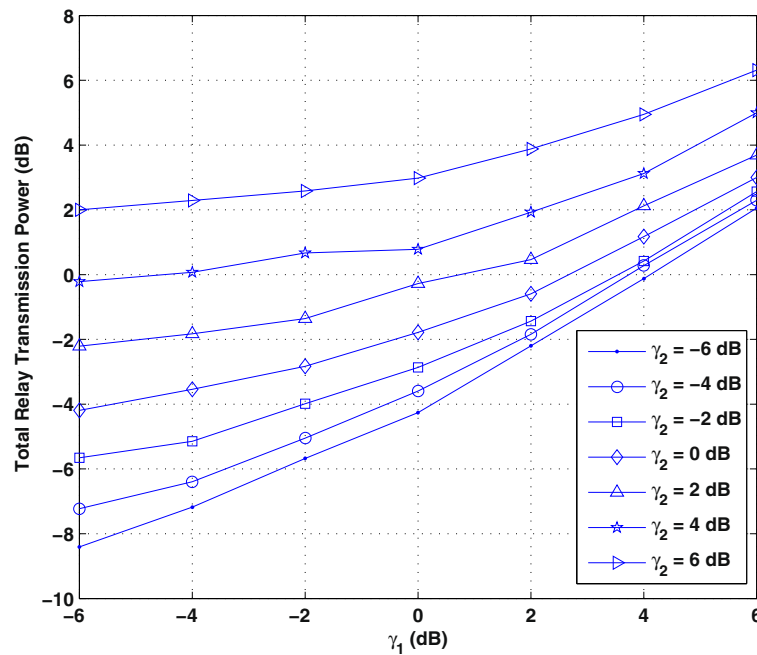
Figure 3 Total relay transmission power P_r versus the value of γ_1.

Figure 4 CDF of the output SNR at transceiver 1 with different values of γ_2.

with $\gamma_2 = 6$ dB is less than that with $\gamma_2 = -6$ dB. It is known that P_r is allocated to the relays such that the two transceivers can simultaneously meet the required SNR. It can be concluded from Figure 4 that P_r allocation tends to emphasize maximizing the output SNR which has higher requirement under the condition that the lower SNR requirement can be satisfied. Therefore, SNR_1 can achieve a higher average value when $\gamma_2 = -6$ dB than when $\gamma_2 = 6$ dB.

Maximizing the minimal output SNR of transceivers subject to total relay transmission power constraint and ZF constraint

Figure 5 depicts the output SNR at transceiver 1 with the value of σ_h^2 changing from 0 to 10 dB. Total relay transmission powers of 0 and 5 dB are considered. It is found that for a given σ_h^2, the output SNR_1 increases with the increase of σ_g^2, while for a given σ_g^2, SNR_1 does not keep increasing with the increase of σ_h^2. This is because as the quality of channels between transceiver 1 and the relays improves, i.e., σ_h^2 increases, the desired transmission power at transceiver 2 to guarantee its output SNR increases [2]. Due to limitation of total relay transmission power, the output SNR at transceiver 1 can not increase consistently. When the quality of channels between transceiver 2 and the relays improves, the output SNR_1 increases with the increase of σ_h^2.

Figure 6 shows the same plot for output SNR_2. It is observed that the output SNR_2 increases with the increase

of σ_h^2, it while does not increase with the increase of σ_g^2 especially when σ_g^2 is high and σ_h^2 is relatively low. The reason is the same as that for SNR_1 versus σ_h^2. Also, as noticed from Figures 5 and 6, the output SNR_1 and SNR_2 increases with the increase of the total relay transmission power.

Maximizing the minimal output SNR of transceivers subject to individual relay transmission power constraint and ZF constraint

In this simulation, we assume that the total relay transmission power is uniformly allocated to the relays. Figures 7 and 8 show the output SNR versus the value of σ_h^2 with individual relay powers of -10 and 0 dB. It is noted that these plots are similar to those with total relay transmission power constraint. With the increase of individual relay transmission power, output SNRs at transceiver 1 and transceiver 2 increase. Compared with Figures 5 and 6, it is found that the output SNR_1 and SNR_2 are slightly lower with individual power constraint than those with total power constraint. This is because individual power constraint is more restrictive than the total power constraint.

Conclusions

In this paper, we focus on the optimization of a two-way MIMO relay network. The proposed optimization criteria yield three SOCP problems which can be solved
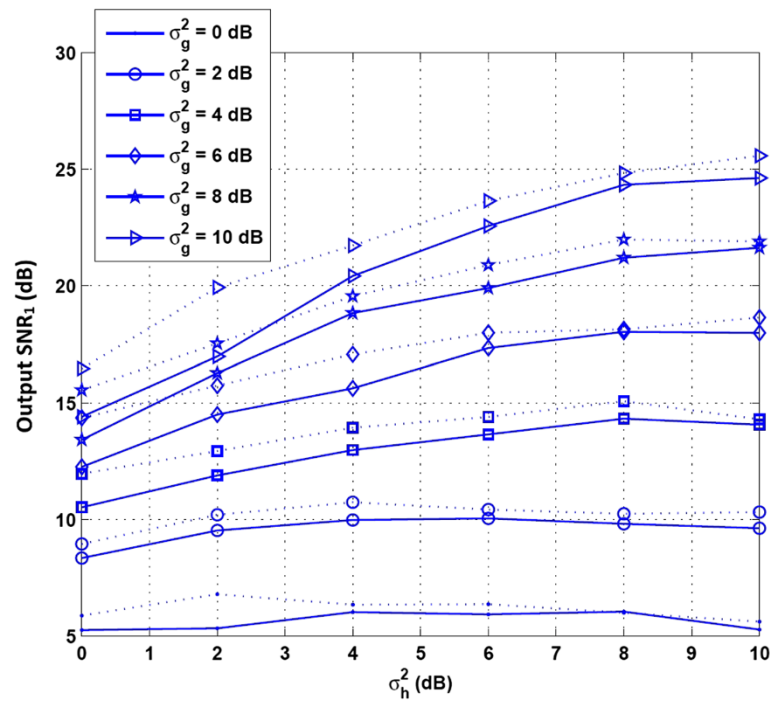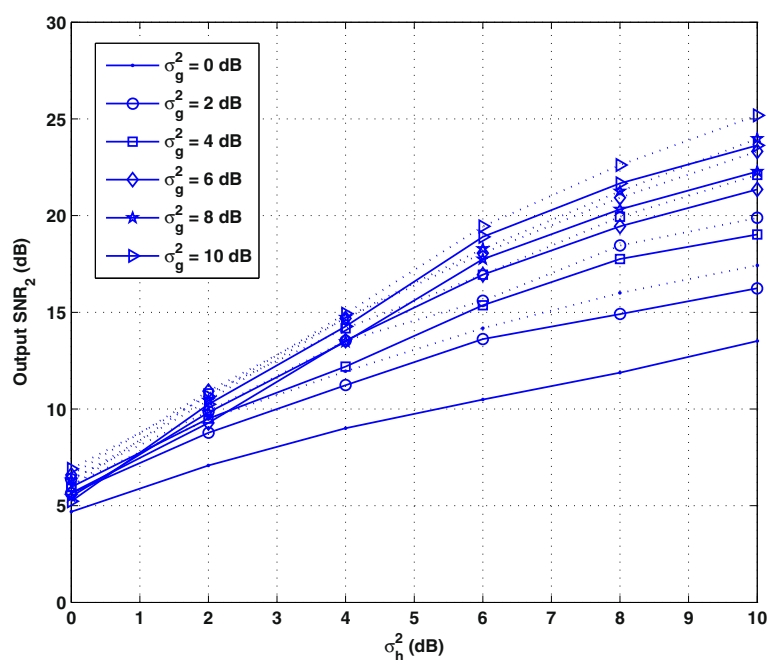
Figure 5 Output SNR$_1$ versus the value of σ_h^2. Solid line: with total relay transmission power of 0 dB, dash line: with total relay transmission power of 5 dB.
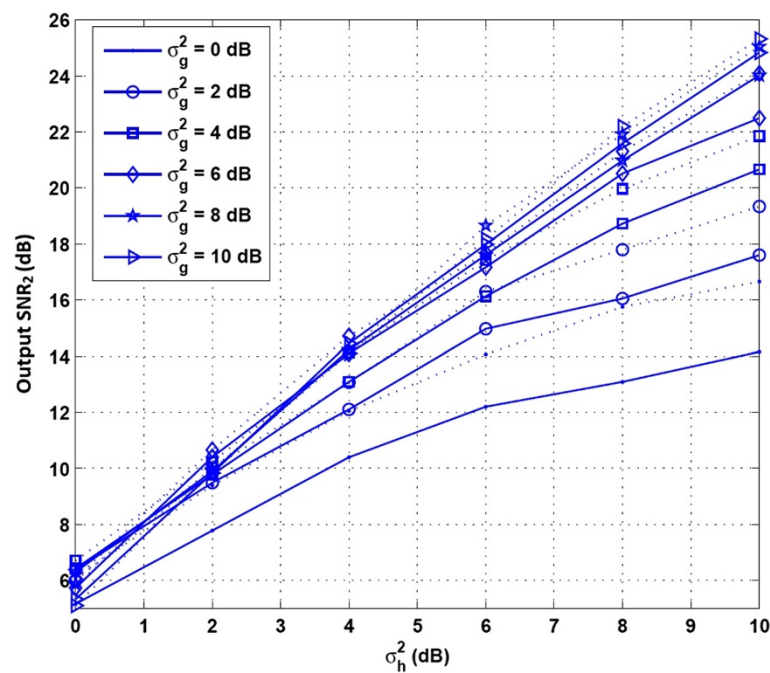
Figure 6 Output SNR$_2$ versus the value of σ_h^2. Solid line: with total relay transmission power of 0 dB, dash line: with total relay transmission power of 5 dB).
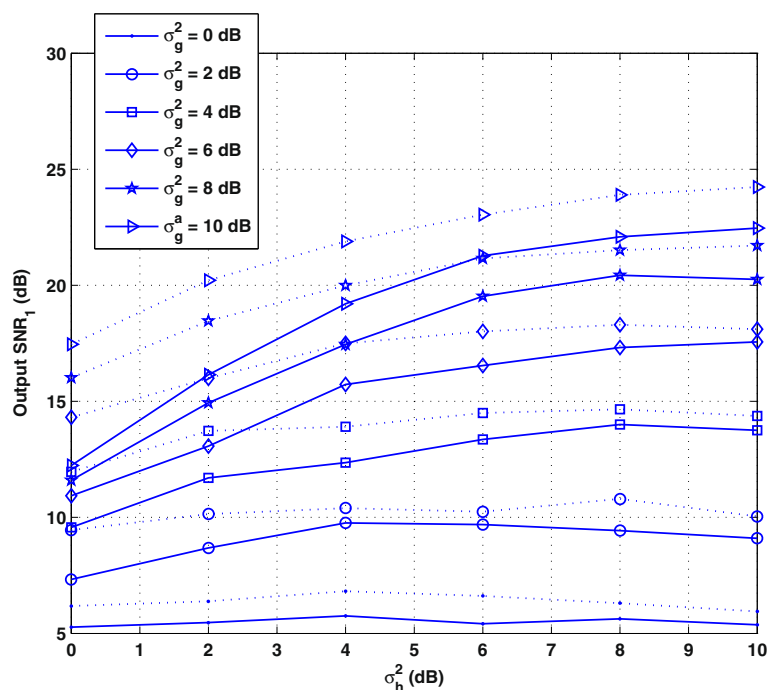
Figure 7 Output SNR$_1$ versus the value of σ_h^2. Solid line: with individual relay transmission power of -10 dB, dash line: with individual relay transmission power of 0 dB.

Figure 8 Output SNR$_2$ versus the value of σ_h^2. Solid line: with individual relay transmission power of -10 dB, dash line: with individual relay transmission power of 0 dB.

efficiently. Computer simulation demonstrates validity of the proposed algorithm. Furthermore, it is straightforward to see that the proposed algorithm can be implemented distributively as long as $\mathbf{U}$ and $\widetilde{\mathbf{U}}$ are broadcasted to all the relays. With $\mathbf{w}$ replaced by $\mathbf{w}_i$, $\mathbf{U}_1$ replaced by $\mathbf{U}_{ri}$, $\mathbf{U}_2$ replaced by $\begin{pmatrix} \sigma_v \mathbf{C}_i & \mathbf{0} \\ \mathbf{0}^T & \sigma_x \end{pmatrix}$, and $\mathbf{U}_3$ replaced by $\begin{pmatrix} \sigma_v \mathbf{D}_i & \mathbf{0} \\ \mathbf{0}^T & \sigma_y \end{pmatrix}$, (35), (39), and (42) can be solved at each relay. The performance of distributed implementation will be analyzed in our future work.

Competing interests

The authors declare that they have no competing interests.

Author details

[1]College of Electronic Engineering, University of Electronic Science and Technology of China, Xiyuan Avenue, 610066 Chengdu, China. [2]Department of Electronics and Photonics, Institute of High Performance Computing, Fusionopolis Way, Singapore 138632, Singapore.

References

1. G Zheng, Collaborative-relay beamforming with perfect CSI: optimum and distributed implementation. IEEE Signal Process Lett. **16**(4), 257 (2009)
2. V Havary-Nassab, S Shahbazpanahi, A Grami, Z-Q Luo, Distributed beamforming for relay networks based on second-order statistics of the channel information. IEEE Trans. Signal Process. **56**(9), 4306 (2008)
3. MM Abdallah, HC Pagadopoulos, Beamforming algorithms for implementation relaying in wireless sensor networks. IEEE Trans. Signal Process. **56**(10), 4772 (2008)
4. E Koyuncu, Y Jing, H Jafarkhani, Distributed beamforming in wireless relay networks with quantized feedback. IEEE J. Selected Areas Commun. **26**(8), 1429 (2008)
5. N Khajehnouri, AH Sayed, Distributed MMSE relay strategies for wireless sensor networks. IEEE Trans. Signal Process. **55**(7), 3336 (2007)
6. H Chen, AB Gershman, S Shahbazpanahi, Filter-and-forward distributed beamforming in relay networks with frequency selective fading. IEEE Trans. Signal Process. **58**(3), 1251 (2010)
7. V Havary-Nassab, S Shahbazpanahi, A Grami, Optimal distributed beamforming for two-way relay networks. IEEE Trans. Signal Process. **58**(3), 1238 (2010)
8. H Chen, Beamforming Optimization for Two-Way Relay Channel (2014). p. 28–31
9. Y Rong, Optimal joint source and relay beamforming for MIMO relays with direct link. IEEE Commun. Lett. **14**(5), 390 (2010)
10. B Koshnevis, W Yu, R Adve, Grassmannian beamforming for MIMO amplify-and-forward relaying. IEEE J. Selected Areas Commun. **26**(8), 1397 (2008)
11. W Guan, H Luo, Joint MMSE transceiver design in non-regenerative MIMO relay systems. IEEE Commun. Lett. J. **12**(7), 517 (2008)
12. Y Rong, F Gao, Optimal beamforming for non-regenerative MIMO relays with direct link. IEEE Commun. Lett. J. **13**(12), 927 (2009)
13. AS Behbahani, R Merched, AM Eltawil, Optimization of a MIMO relay network. IEEE Trans. Signal Process. **56**(10), 5063 (2008)
14. K-J Lee, H Sung, E Park, I Lee, Joint optimization for one and two-way MIMO AF multiple-relay systems. IEEE Trans. Wireless Commun. **9**(12), 3671 (2010)
15. A El-Keyi, B Champagne, Adaptive linearly constrained minimum variance beamforming for multiuser cooperative relaying using the Kalman filter. IEEE Trans. Wireless Commun. **9**(2), 641 (2010)
16. J Joung, AH Sayed, Multiuser two-way amplify-and-forward relay processing and power control methods for beamforming systems. IEEE Trans. Signal Process. **58**(3), 1833 (2010)
17. R Zhang, C Choy Chai, Y-C Liang, Joint beamforming and power control for multiantenna relay broadcase channel with QoS constraints. IEEE Trans. Signal Process. **57**(2), 726 (2009)
18. O Oyman, AJ Paulray, Design and analysis of linear distributed MIMO relaying algorihms. IEEE Proc. Commun. **153**(4), 565 (2006)
19. M Grant, S Boyd, cvx user's guide for cvx version 1.21 (build 790). http://cvxr.com/cvx/. Accessed 16 July 2013