Zhang et al. EURASIP Journal on Advances in Signal Processing 2014, **2014**:96
http://asp.eurasipjournals.com/content/2014/1/96

EURASIP Journal on Advances in Signal Processing
a SpringerOpen Journal

RESEARCH Open Access

Worst-case-based robust beamforming for wireless cooperative networks

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Abstract

It is known that distributed beamforming techniques can improve the performance of relay networks by using channel state information (CSI). In practical applications, there exist unavoidably estimation errors of the CSI, which results in outage of quality of service (QoS) or overconsumption of transmit power. In this paper, we propose two worst-case-based distributed beamforming techniques that are robust to the channel estimation errors. In the worst-case-based approaches, the worst case in a set that includes the actual case is optimized. Therefore, the performance of the actual case can be guaranteed. In our first approach, the maximal total relay transmit power in the set is minimized subject to the QoS constraint. This distributed beamforming problem can be approximately solved using second-order cone programming (SOCP). In our second method, the worst QoS in the set is maximized subject to the constraints of individual relay transmit powers. It is shown that the resultant problem can be approximately formulated as a quasi-convex problem and can be solved by using a bisection search method. Simulation results show that the proposed beamforming techniques are robust to the CSI errors and there is no outage of QoS or power in the proposed methods.

Keywords: Distributed beamforming; Worst case; Relay networks; Robust

1 Introduction

Relay networks have recently attracted much interest in the literature as they can not only exploit cooperative diversity of different users in the network but also extend the coverage of wireless communication systems [1-11]. In such networks, users share their communication resources, including bandwidth and transmit power, to help each other in transmitting messages. Different relaying strategies have been proposed to achieve cooperative diversity. Amplify-and-forward (AF), decode-and-forward (DF), and compress-and-forward (CF) relaying protocols have been widely discussed in the literature [3,4,11]. Due to its simplicity and low implementation complexity, the AF protocol has become one of the most popular relaying protocols.

To improve the performance of the receiver, one can exploit distributed beamforming techniques by using channel state information (CSI). Distributed beamforming approaches that assume perfectly known CSI have been well studied in [12-16]. In [12] and [13], relay networks that consist of one source and one destination together with multiple relays are considered and the optimal power allocation at the relays is addressed by using the knowledge of instantaneous CSI. While in [14], the optimal power allocation at the relays is obtained by making use of the second-order statistics of the CSI. The distributed beamforming problems for the relay networks that are comprised of multiple relays and multiple pairs of source and destination are proposed in [15] and [16]. The near-optimal solutions to those beamforming problems are obtained by using second-order cone programming (SOCP) [15] and semi-definite programming (SDP) [16], respectively.

The perfect knowledge of CSI is, however, difficult to obtain due to various reasons such as the error of channel estimation. Without the perfect knowledge of CSI, the conventional distributed beamformers will suffer from severe performance outage in practical applications. Many robust distributed beamforming techniques for relay networks have been proposed in the literature [17-24]. Several Bayesian-based beamforming approaches

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for multiple-input multiple-output (MIMO) relay networks are developed in [17,19] and [20], which consider optimizing an average objective function over channel uncertainties. When the requirement of quality of service (QoS) is considered, worst-case-based beamforming techniques are proposed in [18,21-24]. Relay networks consisting of one source, one relay, and one destination with multiple antennas are considered in [18], while robust beamforming techniques that consider the relay networks consisting of one source, one destination, and multiple relays equipped with single antenna are proposed to relieve the performance degradation [21,23]. In [21], the estimated CSI from the relays to the destination is assumed to have errors, while the CSI from the source to the relays is assumed to be perfectly known. However, both the source-to-relay and relay-to-destination channels are assumed to have estimation errors in [23]. The beamforming approaches in [22] and [24] are proposed for the relay networks consisting of multiple sources, relays and destinations, which consider the case that the channel uncertainty is imposed on the correlation matrices of the channel vectors. It is also assumed that the uncertainties of the correlation matrices are independent to each other. Since the matrices are functions of the same channel vectors, such assumptions may deviate from the practical situations. In addition, the resultant beamforming problems of [22-24] are formulated as SDP problems. Compared with SOCP, SDP has much higher computational complexity.

In this paper, we propose two worst-case-based robust distributed beamforming approaches for AF-based relay networks with one source, one destination, and multiple relays. We assume that both the channel vectors of the source-to-relays and the relays-to-destination have estimation errors with known norm bounds. In both of our beamforming approaches, the worst case of the total/individual relay transmit power and the worst case of the receive QoS in the norm-bound-determined set are found by solving several sub-problems, which are shown to have closed-form solutions. It is shown that the solutions to these sub-problems are consistent to each other, and the common case is referred to the worst case. We then optimize the performance of the worst case to improve the performance of the network. In our first approach, the maximal total relay transmit power in the set is minimized subject to the constraint of the receive QoS. In our second approach, we maximize the worst-case receive QoS under the constraints of individual relay transmit powers. Such a case is of particular interest in mobile communications or *ad hoc* networks when the relays are restricted by their own battery lifetimes.

It is shown in this paper that the first worst-case-based beamforming problem can be approximately solved using SOCP and the second beamforming problem can be approximately formulated as a quasi-convex problem, which can be solved by using a bisection search procedure. The problem of each step in the bisection search procedure is also shown to be an SOCP feasibility problem. Compared to the robust beamforming methods in [22-24] that use SDP, the computational complexity of the proposed method is much lower since the computational complexity of SOCP is $\mathcal{O}(N^{3.5})$ while that of SDP is $\mathcal{O}(N^{6.5})$ [25]. Here, N denotes the number of variables. Numerical simulations show that the proposed beamforming techniques are robust to the CSI estimation errors and can guarantee no outage of either receive QoS or relay transmit power.

The rest of the paper is organized as follows. The signal model is presented in Section 2. The proposed worst-case-based distributed beamforming approaches are developed in Section 3. In Section 4, some simulation results are shown to illustrate the performance of the proposed methods, and the conclusions are given in Section 5.

Notations Throughout this paper, the bold upper- and lowercase letters denote matrices and vectors, respectively. $(\cdot)_{ij}$ denotes the (i,j)th element of a matrix. The ith element of a vector $\mathbf{x}$ is denoted by the corresponding lowercase letter with a subscript, i.e., x_i. $|\cdot|$ and $\|\cdot\|$ denote the norm of a complex number and the Euclidean norm of a vector, respectively. $(\cdot)^*$, $(\cdot)^T$ and $(\cdot)^H$ stand for the conjugate, transpose, and Hermitian transpose, respectively. $\mathrm{E}\{\cdot\}$ is the statistical expectation. $\mathrm{Re}\{\cdot\}$ and $\mathrm{Im}\{\cdot\}$ denote the real and imaginary parts of a complex value, respectively.

2 Signal model

As shown in Figure 1, a half-duplex relay network in flat fading channels with one transmitting source, one destination, and R relays is considered. Each of the node is equipped with one antenna. We assume in this paper that

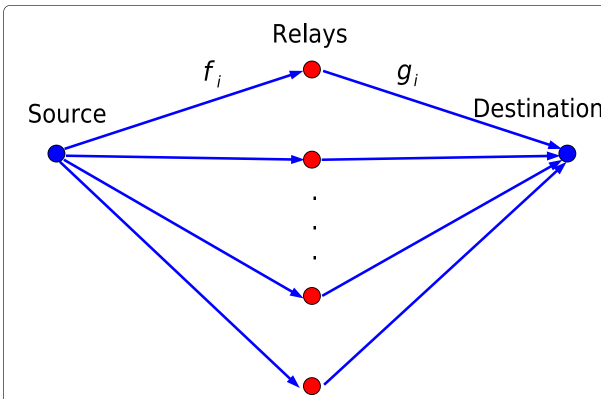

Figure 1 System model.

there is no direct link between the source and the destination due to the poor quality of the channel state. In practical applications, the coefficients of the source-to-relay and relay-to-destination channels are obtained by certain channel estimation methods [26,27]. Therefore, there exist unavoidably some estimation errors. Let $\tilde{f}_i$ and $\tilde{g}_i$, $i = 1, \cdots, R$, denote the actual channel coefficients of the source-to-relay and relay-to-destination channels, respectively, and e_{f_i} and e_{g_i} denote the estimation errors of the channel coefficients $\tilde{f}_i$ and $\tilde{g}_i$, respectively. Assuming that the errors of the channel coefficients are bounded by some known constants $\varepsilon > 0$ and $\beta > 0$ [28]:

$$|e_{f_i}| \leq \varepsilon\,, \qquad |e_{g_i}| \leq \beta\,, \qquad i = 1, \cdots, R\,.$$

Then, the actual channel coefficients belong to the following sets

$$\mathcal{F}_i(\varepsilon) \triangleq \{f_i | f_i = \hat{f}_i + \delta_{f_i}, |\delta_{f_i}| \leq \varepsilon\} \tag{1}$$

$$\mathcal{G}_i(\beta) \triangleq \{g_i | g_i = \hat{g}_i + \delta_{g_i}, |\delta_{g_i}| \leq \beta\} \tag{2}$$

where $\hat{f}_i$ and $\hat{g}_i$ denote the estimates of the channel coefficients $\tilde{f}_i$ and $\tilde{g}_i$, respectively. If $\delta_{f_i} = e_{f_i}$ and $\delta_{g_i} = e_{g_i}$, then $f_i = \tilde{f}_i$ and $g_i = \tilde{g}_i$. Since the actual channel coefficient can be any element of the set, we can optimize the performance of the worst case in the sets to guarantee the performance of the actual case.

Let us consider a two-stage AF relay transmission scheme. In the first stage of each transmission from the source to the destination, the source broadcasts information to the relays. In the second stage, the signal received by each relay is scaled by a complex coefficient and is then retransmitted to the destination. The signal received by the ith relay in the first stage can be written as

$$r_i(n) = f_i s(n) + \eta_i(n), \quad i = 1, \cdots, R \tag{3}$$

where n is the time index, $s(n)$ is the signal transmitted by the source, and $\eta_i(n)$ is the receive noise at the ith relay. In the second stage, the relay transmits a scaled version of its received signal to the destination, which can be written as

$$t_i(n) = w_i^* r_i(n), \quad i = 1, \cdots, R \tag{4}$$

where w_i is the complex weight coefficient of the ith relay. We assume in this paper that the weight coefficients are determined by the destination and are then sent back to the relays via low rate feedback channels. Note that such network beamforming is commonly referred to as 'distributed' because it is assumed that no relay can share its received signals with any other relay. The signal received at the destination is given by

$$y(n) = \sum_{i=1}^{R} g_i t_i(n) + \upsilon(n) \tag{5}$$

where $\upsilon(n)$ denotes the receive noise at the destination. Using (3) and (4), we can rewrite (5) as

$$\begin{aligned} y(n) &= \sum_{i=1}^{R} g_i w_i^* r_i(n) + \upsilon(n) \\ &= \underbrace{\sum_{i=1}^{R} g_i w_i^* f_i s(n)}_{\text{signal}} + \underbrace{\sum_{i=1}^{R} g_i w_i^* \eta_i(n) + \upsilon(n)}_{\text{noise}}\,. \end{aligned} \tag{6}$$

Let $y_s(n) \triangleq \sum_{i=1}^{R} g_i w_i^* f_i s(n)$ and $y_n(n) \triangleq \sum_{i=1}^{R} g_i w_i^* \eta_i(n) + \upsilon(n)$ denote the signal and noise components of the received signal at the destination, respectively. To make the following derivation more concise, we rewrite $y_s(n)$ and $y_n(n)$ as

$$y_s(n) = \mathbf{w}^H(\mathbf{f} \odot \mathbf{g})s(n) = \mathbf{w}^H \mathbf{h} s(n) \tag{7}$$

$$y_n(n) = \mathbf{w}^H \mathbf{G} \boldsymbol{\eta}(n) + \upsilon(n) \tag{8}$$

where $\mathbf{w} \triangleq [w_1, \cdots, w_R]^T$, $\mathbf{f} \triangleq [f_1, \cdots, f_R]^T$, $\mathbf{g} \triangleq [g_1, \cdots, g_R]^T$, $\mathbf{h} \triangleq \mathbf{f} \odot \mathbf{g}$ can be viewed as the vector of the equivalent channel coefficients between the source and the destination, $\boldsymbol{\eta}(n) \triangleq [\eta_1(n), \cdots, \eta_R(n)]^T$, $\odot$ denotes the element-wise Schur-Hamadard product, and $\mathbf{G}$ is a diagonal matrix with g_i, $i = 1, \cdots, R$, on the diagonal, i.e. $\mathbf{G}_{ii} \triangleq g_i$.

3 Robust distributed beamforming

In this section, we propose two worst-case based distributed beamforming approaches that are robust to the channel estimation errors. In our first approach, the maximal total relay transmit power in the predefined set of the channel coefficients is minimized while the condition of the worst-case QoS is satisfied. In our second method, the lowest QoS in the predefined set of the channel coefficients is maximized while the individual relay transmit powers are constrained.

3.1 Minimize the total relay transmit power

In this subsection, let us consider the beamforming problem that minimizes the worst-case total relay transmit power subject to the constraint that the lowest receive signal-to-noise-ratio (SNR) at the destination is satisfied. We use SNR as the measure of QoS in this paper. This worst-case-based beamforming problem can be written as

$$\begin{aligned} &\min_{\mathbf{w}} \max_{\substack{f_i \in \mathcal{F}_i(\varepsilon) \\ i=1,\cdots,R}} P_T(\mathbf{w}, f_i) \\ &\text{s.t.} \min_{\substack{f_i \in \mathcal{F}_i(\varepsilon), g_i \in \mathcal{G}_i(\beta) \\ i=1,\cdots,R}} \text{SNR}(\mathbf{w}, f_i, g_i) \geq \gamma \end{aligned} \tag{9}$$

where P_T is the total relay transmit power and $\gamma > 0$ is the minimal required receive SNR. It should be noticed that the value of γ should not be too large to result in the infeasibility of problem (9). According to (1) and (2), we can rewrite (9) as

$$\begin{aligned} &\min_{\mathbf{w}} \max_{\substack{|\delta_{f_i}|\le\varepsilon \\ i=1,\cdots,R}} P_T(\mathbf{w}, \delta_{f_i}) \\ &\text{s.t.} \min_{\substack{|\delta_{f_i}|\le\varepsilon, |\delta_{g_i}|\le\beta \\ i=1,\cdots,R}} \mathrm{SNR}(\mathbf{w}, \delta_{f_i}, \delta_{g_i}) \ge \gamma \,. \end{aligned} \tag{10}$$

Using (3) and (4), we can write the transmit power of the ith relay as

$$\begin{aligned} P_i &= \mathrm{E}\left\{|t_i(n)|^2\right\} \\ &= \mathrm{E}\left\{|w_i^* f_i s(n) + w_i^* \eta_i(n)|^2\right\} \,. \end{aligned} \tag{11}$$

Assuming that the source signal and the relay noises are statistically independent, we can rewrite (11) as

$$\begin{aligned} P_i &= \mathrm{E}\left\{|w_i^* f_i s(n)|^2\right\} + \mathrm{E}\left\{|w_i^* \eta_i(n)|^2\right\} \\ &= P_0 |w_i|^2 |f_i|^2 + \sigma_\eta^2 |w_i|^2 \end{aligned} \tag{12}$$

where P_0 denotes the power of the source signal $s(n)$ and σ_η^2 is the variance of the relay noise. According to (1), we can rewrite (12) as

$$P_i = |w_i|^2 \left(P_0 |\hat{f}_i + \delta_{f_i}|^2 + {\sigma_\eta}^2\right) . \tag{13}$$

Using (13), the total relay transmit power can be written as

$$P_T = \sum_{i=1}^{R} |w_i|^2 \left(P_0 |\hat{f}_i + \delta_{f_i}|^2 + {\sigma_\eta}^2\right) . \tag{14}$$

Letting the elements on the diagonal matrix $\mathbf{R}$ be $\mathbf{R}_{ii} \triangleq P_0|\hat{f}_i + \delta_{f_i}|^2 + \sigma_\eta^2$, we can rewrite (14) as

$$P_T = \mathbf{w}^H \mathbf{R} \mathbf{w} . \tag{15}$$

The receive SNR at the destination is given by

$$\mathrm{SNR} = \frac{\mathrm{E}\left\{|y_s(n)|^2\right\}}{\mathrm{E}\left\{|y_n(n)|^2\right\}} \,. \tag{16}$$

Using (7), we have

$$\begin{aligned} \mathrm{E}\left\{|y_s(n)|^2\right\} &= \mathrm{E}\left\{|\mathbf{w}^H \mathbf{h} s(n)|^2\right\} \\ &= P_0 \mathbf{w}^H \mathbf{h}\mathbf{h}^H \mathbf{w} \end{aligned} \tag{17}$$

Using (1) and (2) and introducing $\hat{\mathbf{f}} \triangleq [\hat{f}_1, \cdots, \hat{f}_R]^T$, $\hat{\mathbf{g}} \triangleq [\hat{g}_1, \cdots, \hat{g}_R]^T$, $\boldsymbol{\delta}_f \triangleq [\delta_{f_1}, \cdots, \delta_{f_R}]^T$, $\boldsymbol{\delta}_g \triangleq [\delta_{g_1}, \cdots, \delta_{g_R}]^T$, we can rewrite the equivalent channel vector $\mathbf{h}$ as

$$\begin{aligned} \mathbf{h} &= (\hat{\mathbf{f}} + \boldsymbol{\delta}_f) \odot (\hat{\mathbf{g}} + \boldsymbol{\delta}_g) \\ &= \hat{\mathbf{f}} \odot \hat{\mathbf{g}} + \hat{\mathbf{f}} \odot \boldsymbol{\delta}_g + \boldsymbol{\delta}_f \odot \hat{\mathbf{g}} + \boldsymbol{\delta}_f \odot \boldsymbol{\delta}_g \\ &= \hat{\mathbf{h}} + \boldsymbol{\delta}_h \,. \end{aligned} \tag{18}$$

where

$$\begin{aligned} \hat{\mathbf{h}} &\triangleq \hat{\mathbf{f}} \odot \hat{\mathbf{g}} \\ \boldsymbol{\delta}_h &\triangleq \hat{\mathbf{f}} \odot \boldsymbol{\delta}_g + \boldsymbol{\delta}_f \odot \hat{\mathbf{g}} + \boldsymbol{\delta}_f \odot \boldsymbol{\delta}_g \,. \end{aligned} \tag{19}$$

Using (18), we can rewrite (17) as

$$\mathrm{E}\left\{|y_s(n)|^2\right\} = P_0 |\mathbf{w}^H (\hat{\mathbf{h}} + \boldsymbol{\delta}_h)|^2 \,. \tag{20}$$

According to (1) and (2), we have

$$|\delta_{h_i}| \le \beta|\hat{f}_i| + \varepsilon|\hat{g}_i| + \varepsilon\beta \triangleq \xi_i, \quad i = 1, \cdots, R.$$

As a result, the norm of the error vector $\boldsymbol{\delta}_h$ can be written as

$$\|\boldsymbol{\delta}_h\| = \left(\sum_{i=1}^{R} |\delta_{h_i}|^2\right)^{\frac{1}{2}} \le \left(\sum_{i=1}^{R} \xi_i^2\right)^{\frac{1}{2}} \triangleq \zeta \,. \tag{21}$$

Making use of (8), we can write the power of the noise component at the destination as

$$\mathrm{E}\left\{|y_n(n)|^2\right\} = \mathrm{E}\left\{|\mathbf{w}^H \mathbf{G} \boldsymbol{\eta}(n) + \upsilon(n)|^2\right\} \,. \tag{22}$$

Assuming that the relay and the destination noises are statistically independent, we can rewrite (22) as

$$\begin{aligned} \mathrm{E}\left\{|y_n(n)|^2\right\} &= \mathrm{E}\left\{|\mathbf{w}^H \mathbf{G} \boldsymbol{\eta}(n)|^2\right\} + \mathrm{E}\left\{|\upsilon(n)|^2\right\} \\ &= \sigma_\eta^2 \mathbf{w}^H \mathbf{G}\mathbf{G}^H \mathbf{w} + \sigma_\upsilon^2 \\ &= \sigma_\eta^2 \mathbf{w}^H \mathbf{D} \mathbf{w} + \sigma_\upsilon^2 \end{aligned} \tag{23}$$

where σ_υ^2 denotes the variance of the noise at the destination and $\mathbf{D} \triangleq \mathbf{G}\mathbf{G}^H$ is a diagonal matrix with $\mathbf{D}_{ii} = |g_i|^2$. Using (2), the diagonal elements of matrix $\mathbf{D}$ can be rewritten as $\mathbf{D}_{ii} = |\hat{g}_i + \delta_{g_i}|^2$.

Using (15), (20), and (23), the problem in (10) can be rewritten as

$$\min_{\mathbf{w}} \max_{\substack{|\delta_{f_i}|\le\varepsilon \\ i=1,\cdots,R}} \mathbf{w}^H \mathbf{R}(\delta_{f_i}) \mathbf{w} \tag{24}$$

$$\text{s.t.} \min_{\substack{|\delta_{f_i}|\le\varepsilon, |\delta_{g_i}|\le\beta \\ i=1,\cdots,R}} \frac{P_0 |\mathbf{w}^H \left(\hat{\mathbf{h}} + \boldsymbol{\delta}_h(\delta_{f_i}, \delta_{g_i})\right)|^2}{\sigma_\eta^2 \mathbf{w}^H \mathbf{D}(\delta_{g_i}) \mathbf{w} + \sigma_\upsilon^2} \ge \gamma \,.$$

As the problem in (24) is difficult to solve, we solve the following problem instead

$$\min_{\mathbf{w}} \max_{\substack{|\delta_{f_i}|\le\varepsilon \\ i=1,\cdots,R}} \mathbf{w}^H \mathbf{R}(\delta_{f_i}) \mathbf{w} \tag{25}$$

$$\text{s.t.} \min_{\substack{\|\boldsymbol{\delta}_h\|\le\zeta, |\delta_{g_i}|\le\beta \\ i=1,\cdots,R}} \frac{P_0 |\mathbf{w}^H (\hat{\mathbf{h}} + \boldsymbol{\delta}_h)|^2}{\sigma_\eta^2 \mathbf{w}^H \mathbf{D}(\delta_{g_i}) \mathbf{w} + \sigma_\upsilon^2} \ge \gamma \,.$$

The optimal value of problem (25) gives the upper bound for the problem in (24) since the feasibility set of the inner problem in the constraint of (24) is enlarged by replacing $|\delta_{f_i}| \le \varepsilon$, $i = 1, \cdots, R$, with $\|\boldsymbol{\delta}_h\| \le \zeta$.

To simplify the problem in (25), we find first the optimal objective values of the following three sub-problems:

$$\max_{\delta_{f_i}} \ \mathbf{w}^H \mathbf{R}(\delta_{f_i})\mathbf{w} \quad \text{s.t. } |\delta_{f_i}| \le \varepsilon, \ i = 1, \cdots, R \tag{26}$$

$$\min_{\boldsymbol{\delta}_h} \ P_0|\mathbf{w}^H(\hat{\mathbf{h}} + \boldsymbol{\delta}_h)|^2 \quad \text{s.t. } \|\boldsymbol{\delta}_h\| \le \zeta \tag{27}$$

$$\max_{\delta_{g_i}} \ \sigma_\eta^2 \mathbf{w}^H \mathbf{D}(\delta_{g_i})\mathbf{w} + \sigma_\upsilon^2 \quad \text{s.t. } |\delta_{g_i}| \le \beta, \ i = 1, \cdots, R \tag{28}$$

where $\mathbf{w}$ is fixed in these sub-problems. Although we solve the three sub-problems in (26) to (28) independently, we find later that the solutions to these sub-problems are consistent to each other in the worst case.

Using the definition that $\mathbf{R}_{ii} = P_0|\hat{f}_i + \delta_{f_i}|^2 + \sigma_\eta^2$ and the triangle inequality, we have

$$\begin{aligned}
\mathbf{w}^H \mathbf{R}(\delta_{f_i})\mathbf{w} &= \sum_{i=1}^{R} |w_i|^2 \left(P_0|\hat{f}_i + \delta_{f_i}|^2 + \sigma_\eta^2 \right) \\
&\le \sum_{i=1}^{R} |w_i|^2 \left(P_0(|\hat{f}_i| + |\delta_{f_i}|)^2 + \sigma_\eta^2 \right) \\
&= \sum_{i=1}^{R} |w_i|^2 \Big(P_0(|\hat{f}_i|^2 + 2|\hat{f}_i|\,|\delta_{f_i}| + |\delta_{f_i}|^2) \\
&\quad + \sigma_\eta^2 \Big).
\end{aligned} \tag{29}$$

According to the constraints in (26), we can write the maximum of (29) as

$$\begin{aligned}
\mathbf{w}^H \mathbf{R}(\delta_{f_i})\mathbf{w} &\le \sum_{i=1}^{R} |w_i|^2 \left(P_0(|\hat{f}_i|^2 + 2\varepsilon|\hat{f}_i| + \varepsilon^2) + \sigma_\eta^2 \right) \\
&= \mathbf{w}^H \tilde{\mathbf{R}}\mathbf{w}
\end{aligned} \tag{30}$$

where $\tilde{\mathbf{R}}$ is a diagonal matrix with $\tilde{\mathbf{R}}_{ii} \triangleq P_0(|\hat{f}_i|^2 + 2\varepsilon|\hat{f}_i| + \varepsilon^2) + \sigma_\eta^2$. It can be shown that the equalities in (29) and (30) hold true when

$$\delta_{f_i} = \varepsilon \frac{\hat{f}_i}{|\hat{f}_i|}. \tag{31}$$

Making use of (30) and (31), the optimal objective value of sub-problem (26) can be written as

$$\max_{|\delta_{f_i}| \le \varepsilon, \ i=1,\cdots,R} \mathbf{w}^H \mathbf{R}(\delta_{f_i})\mathbf{w} = \mathbf{w}^H \tilde{\mathbf{R}}\mathbf{w}. \tag{32}$$

Using the triangle and Cauchy-Schwarz inequalities, we have

$$\begin{aligned}
|\mathbf{w}^H(\hat{\mathbf{h}} + \boldsymbol{\delta}_h)| &\ge |\mathbf{w}^H\hat{\mathbf{h}}| - |\mathbf{w}^H\boldsymbol{\delta}_h| \\
&\ge |\mathbf{w}^H\hat{\mathbf{h}}| - \|\mathbf{w}\| \cdot \|\boldsymbol{\delta}_h\|
\end{aligned} \tag{33}$$

Considering the constraint in sub-problem (27), the minimum of (33) can be written as

$$|\mathbf{w}^H(\hat{\mathbf{h}} + \boldsymbol{\delta}_h)| \ge |\mathbf{w}^H\hat{\mathbf{h}}| - \zeta\|\mathbf{w}\|. \tag{34}$$

Note that we require the values of ε and β to satisfy $\zeta\|\mathbf{w}\| < |\mathbf{w}^H\hat{\mathbf{h}}|$. Otherwise, the receive SNR may be too low to satisfy the SNR requirement in (25).

It can be verified that the equalities in (33) and (34) hold true when

$$\boldsymbol{\delta}_h = -\zeta \frac{\mathbf{w}}{\|\mathbf{w}\|} e^{j\phi} \tag{35}$$

where $\phi \triangleq \text{angle}(\mathbf{w}^H\hat{\mathbf{h}})$.

Combining (34) and (35), we can obtain the minimum of the objective function in sub-problem (27)

$$\min_{\|\boldsymbol{\delta}_h\| \le \zeta} P_0|\mathbf{w}^H(\hat{\mathbf{h}} + \boldsymbol{\delta}_h)|^2 = P_0(|\mathbf{w}^H\hat{\mathbf{h}}| - \zeta\|\mathbf{w}\|)^2. \tag{36}$$

Using the definition that $\mathbf{D}_{ii} = |\hat{g}_i + \delta_{g_i}|^2$ and the triangle equality, we have

$$\begin{aligned}
\mathbf{w}^H \mathbf{D}(\delta_{g_i})\mathbf{w} &= \sum_{i=1}^{R} |w_i|^2 |\hat{g}_i + \delta_{g_i}|^2 \\
&\le \sum_{i=1}^{R} |w_i|^2 \left(|\hat{g}_i| + |\delta_{g_i}| \right)^2 \\
&= \sum_{i=1}^{R} |w_i|^2 \left(|\hat{g}_i|^2 + 2|\hat{g}_i|\,|\delta_{g_i}| + |\delta_{g_i}|^2 \right).
\end{aligned} \tag{37}$$

Using the constraints in (28), the upper bound of $\mathbf{w}^H\mathbf{D}(\delta_{g_i})\mathbf{w}$ can be written as

$$\begin{aligned}
\mathbf{w}^H \mathbf{D}(\delta_{g_i})\mathbf{w} &\le \sum_{i=1}^{R} |w_i|^2 \left(|\hat{g}_i|^2 + 2\beta|\hat{g}_i| + \beta^2 \right) \\
&= \mathbf{w}^H \tilde{\mathbf{D}}\mathbf{w}
\end{aligned} \tag{38}$$

where $\tilde{\mathbf{D}}$ is a diagonal matrix with $\tilde{\mathbf{D}}_{ii} \triangleq |\hat{g}_i|^2 + 2\beta|\hat{g}_i| + \beta^2$. The equalities in (37) and (38) hold true when

$$\delta_{g_i} = \beta \frac{\hat{g}_i}{|\hat{g}_i|} \quad i = 1, \cdots, R. \tag{39}$$

Using (38) and (39), we can write the optimal value of the objective function in sub-problem (28) as

$$\begin{aligned}
&\max_{|\delta_{g_i}| \le \beta, \ i=1,\cdots,R} \left(\sigma_\eta^2 \mathbf{w}^H \mathbf{D}(\delta_{g_i})\mathbf{w} + \sigma_\upsilon^2 \right) \\
&\quad = \sigma_\eta^2 \mathbf{w}^H \tilde{\mathbf{D}}\mathbf{w} + \sigma_\upsilon^2.
\end{aligned} \tag{40}$$

We can see from (35) and (39) that the sub-problems (27) and (28) achieve their optimal values at the same case if

$$\delta_{f_i} = \frac{-\zeta \frac{w_i}{\|\mathbf{w}\|} e^{j\phi} - \beta \frac{\hat{f}_i \hat{g}_i}{|\hat{g}_i|}}{\hat{g}_i + \beta \frac{\hat{g}_i}{|\hat{g}_i|}}, \ i = 1, \cdots, R. \tag{41}$$

In such case, the optimal value of the inner problem in the constraint of problem (25) can be written as

$$\begin{aligned}\min_{\substack{|\delta_{g_i}|\le\beta,\ \|\boldsymbol{\delta}_h\|\le\zeta\\ i=1,\cdots,R}} &\frac{P_0|\mathbf{w}^H(\hat{\mathbf{h}}+\boldsymbol{\delta}_h)|^2}{\sigma_\eta^2\mathbf{w}^H\mathbf{D}(\delta_{g_i})\mathbf{w}+\sigma_\upsilon^2}\\ &=\frac{P_0(|\mathbf{w}^H\hat{\mathbf{h}}|-\zeta\|\mathbf{w}\|)^2}{\sigma_\eta^2\mathbf{w}^H\tilde{\mathbf{D}}\mathbf{w}+\sigma_\upsilon^2}\,.\end{aligned} \tag{42}$$

We can also see from (30) and (31) that the inner problem in the objective function of problem (25) achieves its optimum if (31) holds true. Therefore, the inner problems in the objective function and the constraint of problem (25) achieves their optima in the case if (31) and (41) hold true at the same time. In such worst case, the channel estimates satisfy the equality

$$-\zeta\frac{w_i}{\|\mathbf{w}\|}e^{j\phi}=\hat{f}_i\frac{\beta\hat{g}_i}{|g_i|}+\hat{g}_i\frac{\varepsilon\hat{f}_i}{|\hat{f}_i|}+\varepsilon\beta\frac{\hat{f}_i\hat{g}_i}{|\hat{f}_i|\,|\hat{g}_i|},\quad i=1,\cdots,R\,. \tag{43}$$

Therefore, we can rewrite problem (25) as following by making use of (32) and (42)

$$\begin{aligned}\min_{\mathbf{w}}\quad &\mathbf{w}^H\tilde{\mathbf{R}}\mathbf{w}\\ \text{s.t.}\quad &\frac{P_0(|\mathbf{w}^H\hat{\mathbf{h}}|-\zeta\|\mathbf{w}\|)^2}{\sigma_\eta^2\mathbf{w}^H\tilde{\mathbf{D}}\mathbf{w}+\sigma_\upsilon^2}\ge\gamma\,.\end{aligned} \tag{44}$$

Noticing that taking the square root of the objective function and both sides of the constraint does not change the solution set, we can rewrite problem (44) as

$$\begin{aligned}\min_{\mathbf{w}}\quad &\|\tilde{\mathbf{R}}^{\frac{1}{2}}\mathbf{w}\|\\ \text{s.t.}\quad &\mathbf{w}^H\hat{\mathbf{h}}\ge\sqrt{\frac{\gamma}{P_0}}\left(\sigma_\eta^2\mathbf{w}^H\tilde{\mathbf{D}}\mathbf{w}+\sigma_\upsilon^2\right)^{\frac{1}{2}}+\zeta\|\mathbf{w}\|\\ &\mathrm{Re}\left\{\mathbf{w}^H\hat{\mathbf{h}}\right\}\ge 0,\ \mathrm{Im}\left\{\mathbf{w}^H\hat{\mathbf{h}}\right\}=0\end{aligned} \tag{45}$$

where we have used the property that $\mathbf{w}$ can be rotated with arbitrary phase without affecting the objective function value. Introducing auxiliary variables τ, τ_1, and τ_2, we can rewrite problem (45) in the form of SOCP as

$$\begin{aligned}\min_{\mathbf{w},\tau,\tau_1,\tau_2}\quad &\tau\\ \text{s.t.}\quad &\tau\ge\|\tilde{\mathbf{R}}^{\frac{1}{2}}\mathbf{w}\|\\ &\mathbf{w}^H\hat{\mathbf{h}}=\tau_1+\tau_2\\ &\tau_1\ge\sqrt{\frac{\gamma}{P_0}}({\sigma_\eta}^2\mathbf{w}^H\tilde{\mathbf{D}}\mathbf{w}+{\sigma_\upsilon}^2)^{\frac{1}{2}}\\ &\tau_2\ge\zeta\|\mathbf{w}\|\\ &\tau,\ \tau_1,\ \tau_2>0\end{aligned} \tag{46}$$

which can be solved in polynomial time using interior point algorithm [29,30]. Compared with the methods of [22-24] that use SDP, the robust beamforming approach (46) has much lower computational complexity. Specifically speaking, the computational complexity of (46) is $\mathcal{O}(R^{3.5})$ and those of the methods in [22-24] are $\mathcal{O}(R^{6.5})$.

3.2 Maximize the receive SNR

In this subsection, we propose another beamforming approach that maximizes the lowest receive SNR in the predefined set subject to the constraints of individual relay transmit powers. This worst-case-based beamforming problem can be formulated as

$$\begin{aligned}\max_{\mathbf{w}}\ \min_{\substack{f_i\in\mathcal{F}_i(\varepsilon),g_i\in\mathcal{G}_i(\beta)\\ i=1,\cdots,R}}\ &\mathrm{SNR}(\mathbf{w},f_i,g_i)\\ \text{s.t.}\ \max_{f_i\in\mathcal{F}_i(\varepsilon)}\ &P_i(\mathbf{w},f_i)\le P_i^{\max}\quad i=1,\cdots,R\end{aligned} \tag{47}$$

where $P_i{}^{\max}$ denotes the maximal allowed individual relay transmit power. Different from the problem in (9), problem (47) considers the constraints of individual relay transmit powers. Such constraints correspond to the applications that the power of the relay is limited. According to (1) and (2), problem (47) can be rewritten as

$$\begin{aligned}\max_{\mathbf{w}}\ \min_{\substack{|\delta_{f_i}|\le\varepsilon,|\delta_{g_i}|\le\beta\\ i=1,\cdots,R}}\ &\mathrm{SNR}(\mathbf{w},\delta_{f_i},\delta_{g_i})\\ \text{s.t.}\ \max_{|\delta_{f_i}|\le\varepsilon}\ &P_i(\mathbf{w},\delta_{f_i})\le P_i^{\max}\quad i=1,\cdots,R\,.\end{aligned} \tag{48}$$

Using (13), (16) to (18), and (23), the problem in (48) can be rewritten as

$$\begin{aligned}\max_{\mathbf{w}}\quad &\min_{\substack{|\delta_{f_i}|\le\varepsilon,\ |\delta_{g_i}|\le\beta\\ i=1,\cdots,R}}\frac{P_0|\mathbf{w}^H\left(\hat{\mathbf{h}}+\boldsymbol{\delta}_h(\delta_{f_i},\delta_{g_i})\right)|^2}{\sigma_\eta^2\mathbf{w}^H\mathbf{D}(\delta_{g_i})\mathbf{w}+\sigma_\upsilon^2}\\ \text{s.t.}\quad &\max_{|\delta_{f_i}|\le\varepsilon}|w_i|^2\left(P_0|\hat{f}_i+\delta_{f_i}|^2+{\sigma_\eta}^2\right)\le P_i^{\max}\\ &i=1,\cdots,R\,.\end{aligned} \tag{49}$$

Instead of solving the problem in (49), we solve the following problem

$$\begin{aligned}\max_{\mathbf{w}}\quad &\min_{\substack{|\delta_{g_i}|\le\beta,\ \|\boldsymbol{\delta}_h\|\le\zeta\\ i=1,\cdots,R}}\frac{P_0|\mathbf{w}^H(\hat{\mathbf{h}}+\boldsymbol{\delta}_h)|^2}{\sigma_\eta^2\mathbf{w}^H\mathbf{D}(\delta_{g_i})\mathbf{w}+\sigma_\upsilon^2}\\ \text{s.t.}\quad &\max_{|\delta_{f_i}|\le\varepsilon}|w_i|^2\left(P_0|\hat{f}_i+\delta_{f_i}|^2+{\sigma_\eta}^2\right)\le P_i^{\max}\\ &i=1,\cdots,R.\end{aligned} \tag{50}$$

It can be seen that the optimal value of (50) is a lower bound of that of problem (49) since the error set of the inner problem of the objective function is enlarged by using $\|\boldsymbol{\delta}_h\|\le\zeta$ instead of $|\delta_{f_i}|\le\varepsilon$, $i=1,\cdots,R$. We consider solving the sub-problem in the constraint of (50) first:

$$\max_{\delta_{f_i}}|w_i|^2(P_0|\hat{f}_i+\delta_{f_i}|^2+\sigma_\eta^2)\quad\text{s.t. }|\delta_{f_i}|\le\varepsilon\,. \tag{51}$$

Applying the triangle inequality to the objective function in (51), we have

$$\begin{aligned} P_i &= |w_i|^2(P_0|\hat{f}_i + \delta_{f_i}|^2 + \sigma_\eta^2) \\ &\leq |w_i|^2(P_0(|\hat{f}_i| + |\delta_{f_i}|)^2 + \sigma_\eta^2) \\ &= |w_i|^2(P_0(|\hat{f}_i|^2 + 2|\hat{f}_i|\,|\delta_{f_i}| + |\delta_{f_i}|^2) + \sigma_\eta^2) \end{aligned} \tag{52}$$

According to the constraint in (51), we can get

$$\begin{aligned} P_i &\leq |w_i|^2(P_0(|\hat{f}_i|^2 + 2\varepsilon|\hat{f}_i| + \varepsilon^2) + \sigma_\eta^2) \\ &= |w_i|^2\alpha_i^2 \end{aligned} \tag{53}$$

where $\alpha_i \triangleq (P_0(|\hat{f}_i|^2 + 2\varepsilon|\hat{f}_i| + \varepsilon^2) + \sigma_\eta^2)^{\frac{1}{2}}$. The equalities in (52) and (53) hold true when

$$\delta_{f_i} = \varepsilon\frac{\hat{f}_i}{|\hat{f}_i|}, \quad i = 1, \cdots, R. \tag{54}$$

Making use of (53) and (54), the maximum of the objective function in (51) can be written as

$$\max_{|\delta_{f_i}| \leq \varepsilon} \quad |w_i|^2(P_0|\hat{f}_i + \delta_{f_i}|^2 + \sigma_\eta^2) = |w_i|^2\alpha_i^2. \tag{55}$$

Similar to the case in Section 3.1, the inner problems in the objective function and the constraints of problem (50) achieve their optima in the same case if (43) holds true, which can be obtained by using (35), (39), and (54). Using (36), (40), and (55), problem (50) can be rewritten as

$$\begin{aligned} \max_{\mathbf{w}} \quad & \frac{P_0(|\mathbf{w}^H\hat{\mathbf{h}}| - \zeta\|\mathbf{w}\|)^2}{\mathbf{w}^H\tilde{\mathbf{D}}\mathbf{w} + \sigma_\upsilon^2} \\ \text{s.t.} \quad & |w_i|^2\alpha_i^2 \leq P_i^{\max},\ i = 1, \cdots, R. \end{aligned} \tag{56}$$

Considering that taking square root of the objective function does not change the solution set and introducing an auxiliary variable $\tau > 0$, we can rewrite (56) as

$$\begin{aligned} \max_{\mathbf{w}, \tau > 0} \quad & \tau \\ \text{s.t.} \quad & \mathbf{w}^H\hat{\mathbf{h}} \geq \frac{\tau}{\sqrt{P_0}}(\mathbf{w}^H\tilde{\mathbf{D}}\mathbf{w} + \sigma_\upsilon^2)^{\frac{1}{2}} + \zeta\|\mathbf{w}\| \\ & |w_i|^2\alpha_i^2 \leq P_i^{\max},\ i = 1, \cdots, R. \\ & \mathrm{Re}\left\{\mathbf{w}^H\hat{\mathbf{h}}\right\} \geq 0,\ \mathrm{Im}\left\{\mathbf{w}^H\hat{\mathbf{h}}\right\} = 0 \end{aligned} \tag{57}$$

where we have used the fact that the weight vector $\mathbf{w}$ can be rotated with arbitrary phase without affecting the objective value.

The problem in (57) is a quasi-convex problem [30], which can be solved by using a bisection search method. In particular, for any given value of τ, we check the feasibility of the following convex problem

$$\begin{aligned} \text{find} \quad & \mathbf{w} \\ \text{s.t.} \quad & \mathbf{w}^H\hat{\mathbf{h}} \geq \frac{\tau}{\sqrt{P_0}}(\mathbf{w}^H\tilde{\mathbf{D}}\mathbf{w} + \sigma_\upsilon^2)^{\frac{1}{2}} + \zeta\|\mathbf{w}\| \\ & |w_i|^2\alpha_i^2 \leq P_i^{\max},\ i = 1, \cdots, R \\ & \mathrm{Re}\left\{\mathbf{w}^H\hat{\mathbf{h}}\right\} \geq 0,\ \mathrm{Im}\left\{\mathbf{w}^H\hat{\mathbf{h}}\right\} = 0. \end{aligned} \tag{58}$$

Let $\tau_{\max}$ denote the optimal objective value of (57). If the feasibility problem in (58) is feasible, then we have $\tau \leq \tau_{\max}$. It can be proven by the contradict method. Assuming that $\tau > \tau_{\max}$ and problem (58) is feasible, we can see that it contradicts the assumption that $\tau_{\max}$ is the optimal value of problem (57). On the contrary, if the feasibility problem (58) is infeasible, then, we can conclude that $\tau > \tau_{\max}$.

As a result, we can apply a bisection search technique by checking the feasibility of the convex problem (58) in each step, which can be summarized as following:

1. $\tau := (\tau_l + \tau_u)/2$.
2. Solve the convex feasibility problem (58). If (58) is feasible, then $\tau_l := \tau$, otherwise $\tau_u := \tau$.
3. If $(\tau_u - \tau_l) < \varepsilon_0$ then stop. Otherwise, go to Step 1.

Here, $[\tau_l, \tau_u]$ determines the interval that contains the optimal value of $\tau_{\max}$ and ε_0 is the tolerance of the error in finding $\tau_{\max}$.

4 Simulation results

In our simulations, we consider a relay network with $R = 20$ relays. For each scenario, 300 simulation runs are used to obtain each simulation point. In all examples, we assume that the channels are of Rayleigh flat-fading with unit variance. In all simulations, the variances of the relay and the destination noises are assumed to be equal to each other and the transmit power of the source is assumed to be 10 dB higher than the noise powers. Except that specified, the norm bounds of the channel estimation errors are assumed to be equal to each other, i.e., $\varepsilon = \beta$. The proposed beamformers are compared with the non-robust distributed beamforming method proposed in [14]. In our simulations, the estimated instant CSI is used in the method of [14] instead of the second-order statistics of the CSI.

4.1 Minimize the total relay transmit power

Figure 2 shows the outage probability of the receive SNR at the destination. An SNR outage occurs when the receive SNR is lower than the minimal required SNR. We assume in this example that $\varepsilon = \beta = 0.1$. We can see from Figure 2 that there is no SNR outage in the proposed robust method. While in the method of [14], there is an outage probability that is larger than 0.55 for all the tested points. In addition, this probability goes higher when the minimal required SNR increases. Figure 3 compares the

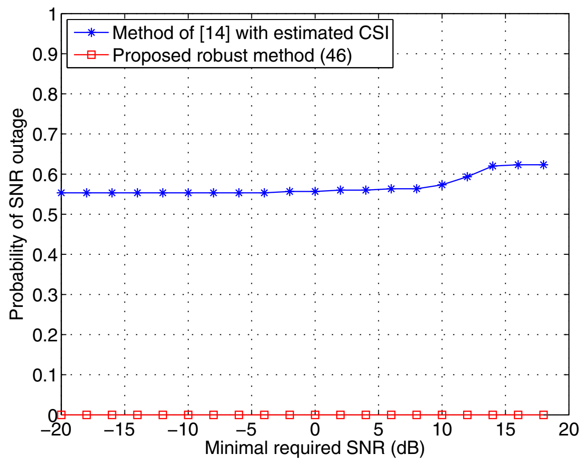

Figure 2 Outage probability of the receive SNR versus minimal required SNR.

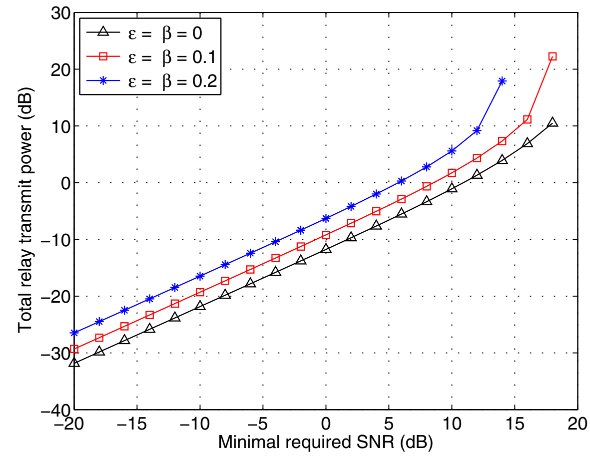

Figure 4 Total relay transmit power versus minimal required SNR for different norm bounds of the channel estimation error.

total relay transmit power versus the minimal required SNR for the case that $\varepsilon = \beta = 0.05$. The performance of the robust method of [23] is also plotted in Figure 3. From the figure, we can see that the minimal relay transmit power of the proposed method is 4 dB lower than that of the method of [23]. We can also see that the relay transmit power of the proposed method is 2 dB higher than the case that the perfect CSI is used, which can be viewed as the price for the robustness.

Figure 4 shows the total relay transmit power versus the minimal required SNR for several different values of ε and β. Figure 5 displays the total relay transmit power versus ε and β for several different values of the minimal required SNR. It can be seen from Figures 4 and 5 that the total relay transmit power increases as the minimal required SNR becomes higher. The beamforming problem is infeasible when the minimal required SNR is higher than certain value. For example, we can see from Figure 4 that the problem is infeasible when $\gamma \geq 18$ dB for the case of $\varepsilon = \beta = 0.1$. In our simulations, it is viewed as infeasible when more than half of the simulation runs are infeasible. It can also be seen from the figures that the relay transmit power increases as the values of ε and β are getting higher. It is reasonable since the values of ε and β determine the sizes of the sets in (1) and (2).

In practical applications, the estimates of the norm bounds of the channel estimation errors may have errors. Let $\hat{\varepsilon}$ and $\hat{\beta}$ denote the estimates of the norm bounds of the channel estimation errors ε and β, respectively. We simulate the case that $\hat{\varepsilon} \neq \varepsilon$ and $\hat{\beta} \neq \beta$. In this example, the channel coefficients are generated according to the values of ε and β, while the problem in (46) is solved

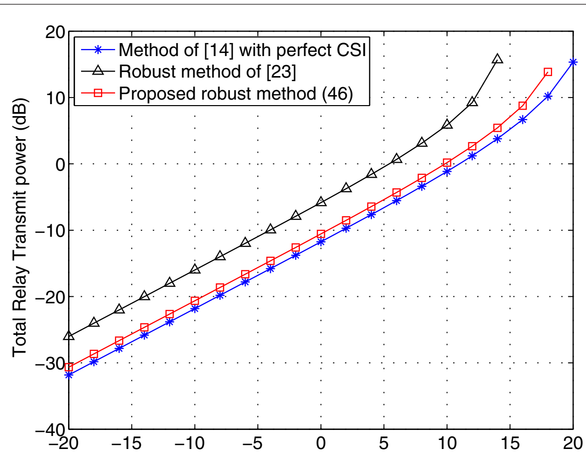

Figure 3 Total relay transmit power versus minimal required SNR.

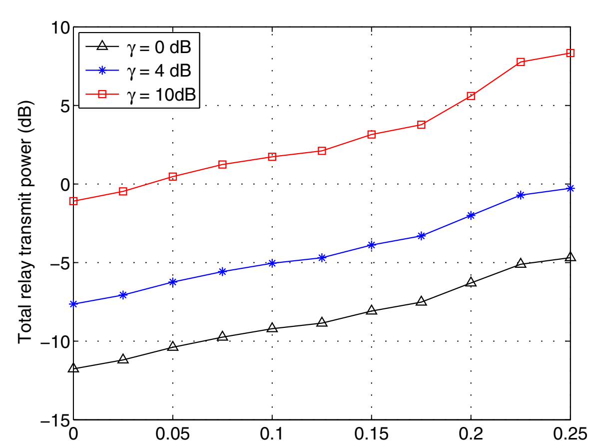

Figure 5 Total relay transmit power versus norm bound of the channel estimation error.

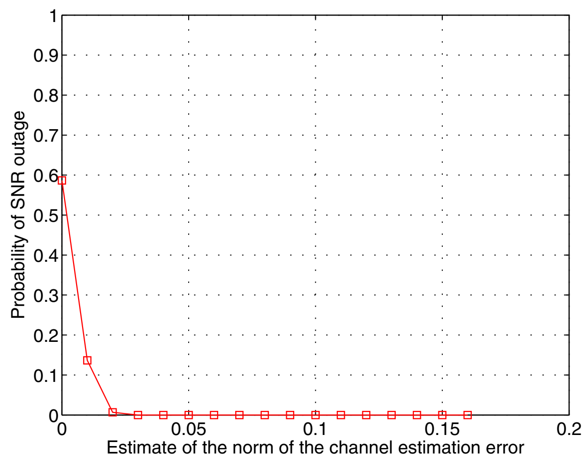

Figure 6 Outage probability of the receive SNR versus estimated value of the norm bound of the channel estimation error.

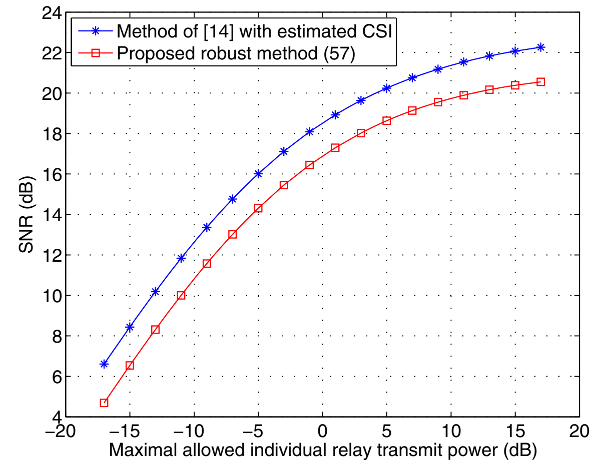

Figure 8 Receive SNR versus individual relay transmit power.

using $\hat{\varepsilon}$ and $\hat{\beta}$. Figure 6 shows the outage probability of the receive SNR versus $\hat{\varepsilon}$ for the case that $\hat{\varepsilon} = \hat{\beta}$ and $\varepsilon = 0.1$. We can see from Figure 6 that the proposed method is robust to the estimation errors of ε and β since the outage probability of the receive SNR is close to 0 when $\hat{\varepsilon} \geq 0.05$. There is no SNR outage when $\hat{\varepsilon} \geq \varepsilon$ since the actual case is included in the sets of (1) and (2).

4.2 Maximize the receive SNR

Figure 7 shows the outage probability of the individual relay transmit power. It is assumed in this example that $\varepsilon = \beta = 0.05$. A power outage occurs when any of the relay transmit power P_i, $i = 1, \cdots, R$, is larger than the maximal allowed relay transmit power $P_i^{\max}$. In the examples of this subsection, it is assumed that all the relays have the same maximal allowed transmit power. We can see from Figure 7 that the proposed robust method guarantees no power outage while the method of [14] with the estimated CSI has power outage in all the simulation points. Figure 8 compares the receive SNR versus individual relay transmit power for both methods. We can see that the maximal SNR of the proposed method is 2 dB lower than the method of [14]. Although the robustness of the proposed method is at the cost of performance degradation, it is necessary in the case that the relays have strict power limitation.

Figure 9 displays the maximal receive SNR versus the individual relay transmit power for several different values of ε and β. Figure 10 shows the maximal receive SNR versus the values of ε and β for several different individual relay transmit power limitations. As can be seen from these figures, the maximal receive SNR increases as

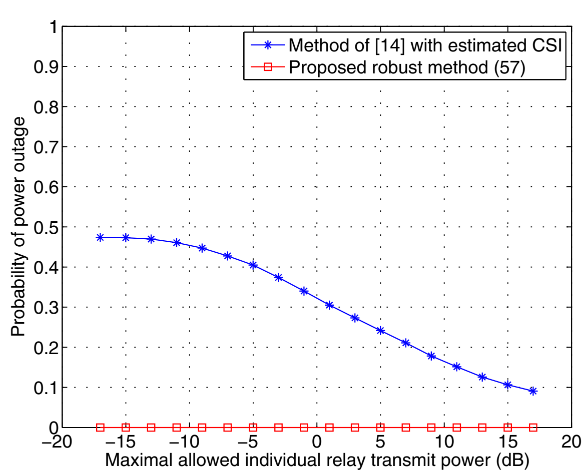

Figure 7 Outage of relay transmit power versus maximal allowed individual relay transmit power.

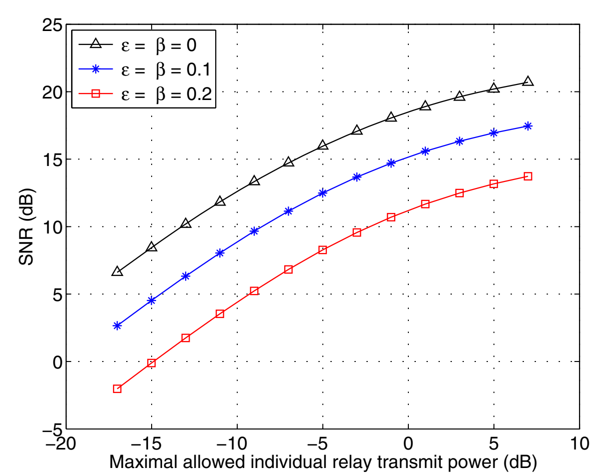

Figure 9 Receive SNR versus individual relay transmit power for different norm bounds of the channel estimation error.

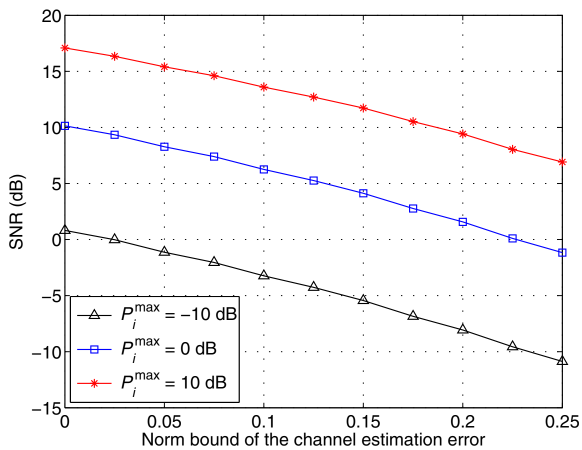

Figure 10 Receive SNR versus norm bound of the channel estimation error.

the transmit power increases. In addition, the maximal receive SNR decreases as the value of ε or β increases. Figure 11 shows the outage probability of the individual relay transmit power versus the estimate of the norm of the channel estimation error ε. In this example, the channel coefficients are generated according to the values of ε and β while problem (57) is solved using $\hat{\varepsilon}$ and $\hat{\beta}$. It is assumed that $\hat{\varepsilon} = \hat{\beta}$ and $\varepsilon = \beta = 0.05$. We can see from this figure that the outage probability of the transmit power is close to 0 when $\hat{\varepsilon}$ deviates from ε.

5 Conclusions

We have developed two worst-case-based robust distributed beamforming approaches for networks that consist of one source, one destination, and multiple relays.

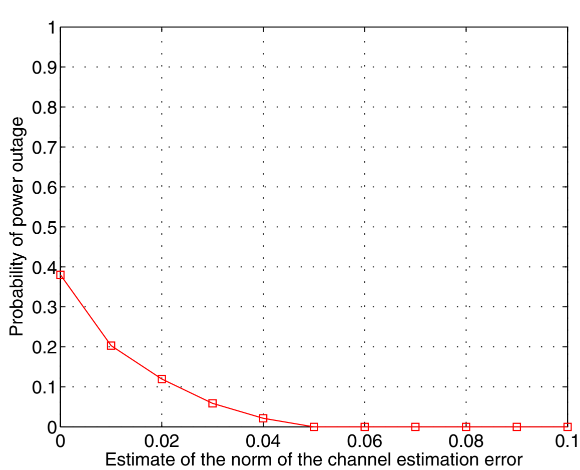

Figure 11 Outage probability of the transmit power versus estimated value of the norm bound of the channel estimation error.

The worst performance in the set that is determined by the norm bounds of the source-to-relay and relay-to-destination channel estimation errors is optimized to guarantee the actual case. In the first distributed beamforming approach, we minimize the maximal total relay transmit power in the set subject to the constraint of receive QoS. In the second distributed beamforming method, the lowest receive QoS in the set is maximized subject to individual relay transmit power constraints. These two distributed beamforming problems are shown to be convex and quasi-convex problems, respectively, which can be efficiently solved by using interior point method. Simulation results demonstrate that the proposed methods are robust to the estimation errors of the source-to-relay and relay-to-destination channels.

Competing interests

The authors declare that they have no competing interests.

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Acknowledgements

This work was supported in part by the Fundamental Research Funds for the Central Universities and by the open research fund of National Mobile Communications Research Laboratory, Southeast University (No. 2011D06). The results of this paper have been presented in part at the International Conference on Acoustics, Speech and Signal Processing (ICASSP), Vancouver, Canada, May 2013, and the 8th International Conference on Communications and Networking in China (Chinacom), Guilin, China, August 2013.

Received: 26 January 2014 Accepted: 7 June 2014
Published: 19 June 2014

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doi:10.1186/1687-6180-2014-96
Cite this article as: Zhang *et al.*: Worst-case-based robust beamforming for wireless cooperative networks. *EURASIP Journal on Advances in Signal Processing* 2014 **2014**:96.