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RESEARCH

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Reduced-dimension space-time adaptive processing based on angle-Doppler correlation coefficient

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Abstract

Traditional space-time adaptive processing (STAP) is a strategy for clutter suppression in airborne radar, which requires a large number of computational complexity and secondary data. In order to address the problem, reduced-dimension (RD) STAP is generally used. We propose a novel RD STAP through searching the best channels as the auxiliary channels to cancel the interference. Based on the estimation of the clutter Fourier basis vectors offline, a parameter named angle-Doppler correlation coefficient (ADC^2) is constructed to evaluate the capability of each auxiliary channel in clutter suppression, and the best sets of RD channels can be selected. The proposed algorithm can achieve the best detection performance with the fixed number of auxiliary channel. When the degrees of freedom (DOF) are restricted to a small value, only one auxiliary channel is needed to guarantee the SINR loss less than 3 dB. Therefore, the requirement of the training sample can be reduced, which makes the proposed approach more suitable for the heterogeneous clutter environments.

Keywords: Airborne radar, Space-time adaptive processing, Reduced-dimension, Clutter covariance matrix

1 Introduction

Space-time adaptive processing (STAP) plays an important role in the areas of airborne radar and sonar systems, which collect signals linearly from an array to detect weak targets within severe clutter and jamming environments [1, 2]. The clutter-plus-noise covariance matrix (CCM) is employed to calculate the filter weights for clutter suppression. It has been long known that increasing the number of degrees of freedom (DOF) enables excellent detection performance, but since the computational complexity and the number of samples for estimation CCM are limited, it is difficult to be implemented in practical work [3]. In recent years, a large amount of productive works have been studied aiming at STAP with few DOF and secondary data and provide a better detection performance in heterogeneous clutter and strong jammer environment, including the knowledge-aided radar, the multiple-input multiple-output radar, and the jamming suppression in complex environment [4–6].

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The foremost theory of STAP is to adjust the space-time filter weights to maximize output signal-to-interference-plus-noise ratio (SINR) adaptively with DOF as less as possible. Therefore, the dimensionality reduction and rank reduction techniques are explored extensively and addressed in the literature. When it comes to reduced-dimension (RD) STAP, some typical suboptimal approaches like the factor approach [7], the joint domain localized (JDL) [8], and the space-time multiple-beam (STMB) [9] have been proposed, which employ a fixed dimension reducing transformation prior to the processing. Lately, a modified RD STAP was proposed by selecting auxiliary channels near the clutter ridge [10]. However, none of the traditional RD STAP can select auxiliary channels adaptively. A multistage multiple-beam (MSMB) technique is proposed in [11, 12], based on the principle of selecting auxiliary channels to cancel the interference components in the main channel clearly. But it is not appropriate for engineering applications because a large amount of calculation is needed.

On the other hand, reduced-rank (RR) STAP approaches make use of data-dependent transformations, such as the principal components (PC) inverse [13], the cross-spectral metric (CSM) [14], and multistage Wiener filter (MSWF) [15, 16]. However, the performance of the RR STAP algorithm aforementioned drops rapidly when the processing DOFs decrease less than the clutter rank. For the purpose of reducing the DOF during interference processing, a reconfigurable array beamforming approach is proposed in [17], and [18] extends this approach to an antenna-pulse selection strategy. According to the authors described in [19], a thinned STAP method is proposed and generates even better SINR loss performance for slow-moving target compared to the full configuration STAP, but the SINR will never exceed the full one because of the decreased amount of antenna and pulse.

In this article, a novel RD STAP approach consisting of an auxiliary channel selection strategy based on a parameter referred to as ADC^2 is proposed. To construct the mode of CCM offline, the clutter Fourier basis can be achieved by the geometry of airborne array, and the amplitude of each basis can be calculated by estimating the ratio of clutter to noise utilizing a least squares method. Furthermore, ADC^2 will be formulated after a serious transformation of CCM, which is in direct proportion to detection performance. The best channels will be found by selecting the maximum ADC^2 in both angular and Doppler domains, and the general transformation matrix will be derived. We process the radar data in the localized region after RD processing, so that a sub-optimal detection performance and low calculated amount can be achieved.

The paper is organized as follows. In Section 2, we formulate the signal mode and review the STAP approach in the transform domain. We transform the clairvoyant CCM and derive the ADC^2 parameter in Section 3. The principle of selecting auxiliary channels is presented in Section 4. The contrastive simulations and performance analysis are provided in Section 5. Finally, we conclude this paper in Section 6.

2 Signal mode and reduced-dimension processor

2.1 Detection performance of airborne radar

In airborne radar system travelling at velocity v_p, let us suppose the antenna array has a side-looking uniform linear arrays, consisting of N-spaced antennas and transmits M pulses in one coherent processing interval. Under the signal absence hypothesis H_0 and signal presence hypothesis H_1, the $NM \times 1$ received signal $\mathbf{x}$ can be expressed as

$$\begin{aligned} H_0 &: \mathbf{x} = \mathbf{c} + \mathbf{n} \\ H_1 &: \mathbf{x} = \alpha\mathbf{s} + \mathbf{c} + \mathbf{n}, \end{aligned} \tag{1}$$

where $\mathbf{c}+\mathbf{n}$ denotes the interference matrix which is composed of clutter and noise components, respectively, and has a complex Gaussian distribution with zero mean and covariance matrix $\mathbf{R}$. $\mathbf{s}$ is the target space-time steering vector with an unknown complex amplitude α

$$\mathbf{s} = \mathbf{s}_d \otimes \mathbf{s}_s, \tag{2}$$

where $\otimes$ denotes the Kronecker product and $\mathbf{s}_d$ and $\mathbf{s}_s$ are the temporal steering vector and spatial steering vector, respectively. The adaptive matched filter (AMF) [20] detector under the assumption is given by

$$\frac{\left|\mathbf{s}^H\mathbf{R}^{-1}\mathbf{x}\right|^2}{\mathbf{s}^H\mathbf{R}^{-1}\mathbf{s}} \underset{H_0}{\overset{H_1}{\gtrless}} \tau, \tag{3}$$

where τ is the threshold with a constant false alarm probability. According to [21], the probability distribution of detection statistic is chi-square distribution

$$\begin{aligned} T_s \left|H_0\right. &\sim \tfrac{1}{2}\chi_2^2 \\ T_s \left|H_1\right. &\sim \tfrac{1}{2}\chi_2^2\left(2|\alpha|^2\mathbf{s}^H\mathbf{R}^{-1}\mathbf{s}\right), \end{aligned} \tag{4}$$

where χ_2^2 denotes the chi-square distribution whose DOF is 2. In case of target present, the non-centrality parameter is $2|\alpha|^2\mathbf{s}^H\mathbf{R}^{-1}\mathbf{s}$. Note that the detection probability is a monotone-increasing function of the non-centrality parameter. In case of target present, a method for comparing detection performance of radar systems is proposed by evaluating its non-centrality parameter. As a consequence, we can draw a conclusion that maximizing the parameter $|\alpha|^2\mathbf{s}^H\mathbf{R}^{-1}\mathbf{s}$ leads to better detection performance.

2.2 RD STAP processor

According to the RD STAP algorithm, the received signal after matched filtering should be transformed to the angle-Doppler domain firstly; the transform vector of the nth angular channel and the mth Doppler channel is

$$\begin{aligned} \mathbf{T}(m,n) = \mathbf{s}_d\left(f_{dm}\right) \otimes \mathbf{s}_s\left(f_{sn}\right), n \in \{0, 1, \cdots, \\ \times N-1\}, m \in \{0, 1, \cdots, M-1\}, \end{aligned} \tag{5}$$

where $\mathbf{T}(m,n) \in \mathbb{C}^{NM\times 1}$ represents the $mN+n$th column of the transform matrix $\mathbf{T}$. We define f_{dt} and f_{st} as the spatial frequency and Doppler frequency of the target. $\mathbf{s}_d\left(f_{dm}\right)$ is the temporal steering vector with the Doppler frequency $f_{dm} = f_{dt} + m/M$, and $\mathbf{s}_s\left(f_{sn}\right)$ is the spatial steering vector with the spatial frequency $f_{sn} = f_{st} + n/N$ respectively.

$$\begin{aligned} \mathbf{s}_d\left(f_{dm}\right) &= \left[1, e^{j2\pi f_{dm}}, \cdots, e^{j2\pi(M-1)f_{dm}}\right]^T \\ \mathbf{s}_s\left(f_{sn}\right) &= \left[1, e^{j2\pi f_{sn}}, \cdots, e^{j2\pi(N-1)f_{sn}}\right]^T, \end{aligned} \tag{6}$$

so that each angular-Doppler channel which corresponds to the steering vector above will be used to form each column of the transform matrix $\mathbf{T}$, and the first column of $\mathbf{T}$ corresponds to the detected channel which can be written as follows:

$$\mathbf{T}(0,0) = \mathbf{s}_d\left(f_{d0}\right) \otimes \mathbf{s}_s\left(f_{s0}\right) \tag{7}$$

and is set as the main channel, while all other angular-Doppler channels are used to construct other columns of $\mathbf{T}$. In the practical application, the main channel is used for detecting target signals embedded in the interference, and the auxiliary channels receive the interference and are correlated to those in the main channel. Assume that the dimension of receiving data after dimension reduced is D; thus, the transform matrix can be formed by arranging the main channel followed by $D-1$ auxiliary channels.

Consequently, many RD approaches are provided to select only few channels to achieve a good detection performance. The difference among RD STAP algorithms is that different column sets of $\mathbf{T}$ are selected. For the JDL approaches [8], the auxiliary channels are selected around the detection channel as a localized processing region, which has different sizes such as 3×3, 5×3, or other else. One kind of the STMB approach [9] selects auxiliary channels near the detected one in four different directions. These two modes of eight auxiliary channels are illustrated in Fig. 1. It is easy to see that the transform matrix of the two approach have the same dimension. However, a fixed sets of RD channels is not optimal in practice. Hence, we have to seek a new approach which not only selects auxiliary channels adaptively but also achieves better performance with dimension as lower as possible.

3 Angle-Doppler correlation coefficient based on CCM estimation in the transform domain

3.1 A prior mode of CCM

Suppose that all components of received signal $\mathbf{x}$ are mutually uncorrelated. The CCM of one range cell is the sum of the clutter and noise covariance matrices

$$\mathbf{R} = E\left[(\mathbf{c}+\mathbf{n})(\mathbf{c}+\mathbf{n})^H\right] = \mathbf{R}_c + \sigma_n^2 \mathbf{I}_{NM}, \tag{8}$$

where σ_n^2 is the noise power, $\mathbf{R}_c \in \mathbb{C}^{NM\times NM}$ represents the clutter covariance matrix whose clutter rank is estimated to be [22]

$$N_r = \lceil N + \gamma (M-1) \rceil . \tag{9}$$

Note that $\lceil\rceil$ rounds up to the next integer. The clutter energy is concentrated about a line on the normalized space-time plane with the slope $\gamma = 2v_p T_r/d$, where d is the inner space of antennas and T_r is the pulse repetition interval. Through eigen decomposition, the clutter covariance matrix can be expressed in terms of its eigenvalues and eigenvectors as follows:

$$\mathbf{R}_c = \sum_{j=1}^{N_r} \lambda_j \mathbf{e}_j \mathbf{e}_j^H, \tag{10}$$

where λ_j and $\mathbf{e}_j$ are the jth eigenvalue and the corresponding eigenvector of $\mathbf{R}_c$. However, it is not applicable in practice for the reason that the estimation of clutter eigenvectors requires training data to make eigen decomposition and is computationally complex. For the clutter intensity heavy enough, the Fourier basis is also defined in [23] to span the clutter subspace and can be used to construct CCM offline, whose number is approximate to the clutter rank

$$\mathbf{R}_c = \sum_{i=1}^{N_r} \varepsilon_i^2 \mathbf{v}_i \mathbf{v}_i^H = \mathbf{V}_I \Sigma_I \mathbf{V}_I^H, \tag{11}$$

where $\mathbf{v}_i$ is called the Fourier basis with voltage coefficients ε_i, composes the Fourier matrix $\mathbf{V}_I = \left[\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_{N_r}\right] \in \mathbb{C}^{NM\times N_r}$, and arranges each power coefficient into a diagonal matrix $\Sigma_I = \mathrm{diag}\left(\varepsilon_1^2, \varepsilon_2^2, \cdots, \varepsilon_{N_r}^2\right)$. Therefore, the clutter subspace can be spanned by either clutter basis vectors or eigenvectors; each eigenvector can be obtained by a linear combination of Fourier basis, i.e., $\mathbf{e}_j = \sum_{i=1}^{N_r} \eta_i^j \mathbf{v}_i$.

In order to construct the CCM offline, we can estimate the Fourier basis instead of eigen decomposition. Based on the known angle of flight and antenna, the trajectory of the clutter spectrum can be deduced. For a side-looking

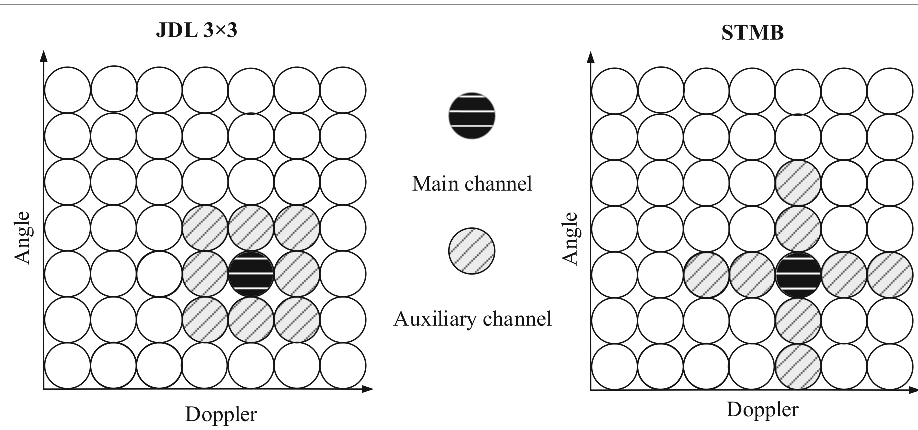

Fig. 1 Mode of nine angular-Doppler channels

array, the clutter ridge is a straight line which concentrates most of the clutter energy, and the Brennans rule [24] implies that there are at most N_r clutter eigenvalues larger than the noise floor. An approximate set of Fourier basis of clutter subspace is given as follows:

$$\mathbf{v}_i = e^{j2\pi \mathbf{m} f_d^i} \otimes e^{j2\pi \mathbf{n} f_s^i}, i = 1, 2, \cdots, N_r, \tag{12}$$

where the Doppler frequency and spatial frequency of each Fourier basis are $f_d^i = \frac{4v_p T_r}{\lambda} \cdot \frac{i}{N_r}, f_s^i = \frac{2d}{\lambda} \cdot \frac{i}{N_r}$ by dividing the clutter spectrum into N_r basis averagely.

Assume that the received data in the cell under test is $\mathbf{x}$ with target absent, which can be written using the Fourier basis as

$$\mathbf{x} = \sum_{i=1}^{N_r} \varepsilon_i \mathbf{v}_i + \mathbf{n} = \mathbf{V}_I \varepsilon_I + \mathbf{n}. \tag{13}$$

Given the collection of space-time Fourier basis, then the major objective is to estimate the voltage of each Fourier basis. A least squares estimate of ε_I satisfies [25]

$$\min_{\varepsilon_I} \|\mathbf{x} - \mathbf{V}_I \varepsilon_I\|_2^2, \tag{14}$$

where $\varepsilon_I = \left[\varepsilon_1, \varepsilon_2, \cdots, \varepsilon_{N_r}\right]^T$ is a vector composed by complex scalars. As a consequence, the set of the clutter Fourier basis is achieved and receiving data is required. In the sample deficient case, the proposed method provides a more effective approach for CCM estimating, using the a priori knowledge of the clutter basis. In some practical scenarios, $\mathbf{V}_I$ should be represented by eigenvectors through decomposition, because the characterization of clutter subspace and clutter rank may be inaccurate. Therefore, the proposed representation of CCM is applicative especially under the condition of few samples, which leads to a more accurate selection of RD channels.

3.2 Angle-Doppler correlation coefficient

The RD processing which can transform the radar received data $\mathbf{x}$ by a subset of the transform matrix defined in Eq. 5 yields a vector $\mathbf{x}_T = \mathbf{T}_D^H \mathbf{x} \in \mathbb{C}^{D\times 1}$ in the transform domain where $\mathbf{T}_D \in \mathbb{C}^{NM\times D}$ denotes the matrix composed by D selected columns of $\mathbf{T}$. Similarly, the CCM in the transform domain can be expressed as follows:

$$\mathbf{R}_T = \mathbf{T}_D^H \mathbf{R} \mathbf{T}_D. \tag{15}$$

Substituting Eqs. 8 and 11 into Eq. 15, it has

$$\mathbf{R}_T = \mathbf{T}_D^H \left(\sum_{i=1}^{N_r} \varepsilon_i^2 \mathbf{v}_i \mathbf{v}_i^H + \sigma_n^2 \mathbf{I} \right) \mathbf{T}_D = \left(\mathbf{T}_D^H \mathbf{V}_I\right) \Sigma_I \left(\mathbf{T}_D^H \mathbf{V}_I\right)^H + \sigma_n^2 \mathbf{T}_D^H \mathbf{T}_D. \tag{16}$$

The columns of $\mathbf{T}_D$ are orthogonal to each other and have the effect of concentrating energy. Assume that the RD matrix consists of D columns, by replacing $\mathbf{T}_D^H \mathbf{T}_D$ in (16) with $NM\mathbf{I}_D$, we can derive

$$\mathbf{R}_T = \mathbf{V}_T \Sigma_I \mathbf{V}_T^H + NM\sigma_n^2 \mathbf{I}_D = \mathbf{V}_T \Sigma_I \mathbf{V}_T^H + \sigma_T^2 \mathbf{I}_D \in \mathbb{C}^{D\times D}, \tag{17}$$

where $\mathbf{V}_T = \mathbf{T}_D^H \mathbf{V}_I \in \mathbb{C}^{D\times N_r}$ denotes the clutter Fourier basis matrix in the transform domain and $\sigma_T^2 = NM\sigma_n^2$ is the noise power with a gain of NM by RD processing. The matrix inversion lemma yields

$$\begin{aligned} \mathbf{R}_T^{-1} &= \frac{1}{\sigma_T^2} \left(\mathbf{I}_D - \mathbf{V}_T \left(\sigma_T^2 \Sigma_I^{-1} + \mathbf{V}_T^H \mathbf{V}_T \right)^{-1} \mathbf{V}_T^H \right) \\ &= \frac{1}{\sigma_T^2} \left(\mathbf{I}_D - \mathbf{V}_T \left(\Phi_I + \mathbf{V}_T^H \mathbf{V}_T\right)^{-1} \mathbf{V}_T^H \right), \end{aligned} \tag{18}$$

where $\Phi_I = \sigma_T^2 \Sigma_I^{-1} = NM\sigma_n^2 \mathrm{diag}\left(1/\varepsilon_1^2, 1/\varepsilon_2^2, \cdots, 1/\varepsilon_{N_r}^2\right)$ arranged by the ratio of concentrated noise power to each power of Fourier basis.

As discussed in Section 2, the detection performance is directly related to the parameter $|\alpha|^2 \mathbf{s}^H \mathbf{R}^{-1} \mathbf{s}$. In the domain of RD transformation, the desired signal and CCM have been dimension reduced, which has the following form:

$$|\alpha|^2 \mathbf{s}_T^H \mathbf{R}_T^{-1} \mathbf{s}_T = \frac{|\alpha|^2}{\sigma_T^2} \mathbf{s}_T^H \left(\mathbf{I}_D - \mathbf{V}_T \left(\Phi_I + \mathbf{V}_T^H \mathbf{V}_T\right)^{-1} \mathbf{V}_T^H \right) \mathbf{s}_T, \tag{19}$$

where $\mathbf{s}_T = \mathbf{T}_D^H \mathbf{s} \in \mathbb{C}^{D\times 1}$ is the desired signal steering vector in the transform domain and $\mathrm{SNR} = |\alpha|^2/\sigma_n^2$. It can be inferred from Eq. 19 that the detection performance depends on two factors: the signal-to-noise ratio and the following term:

$$a = \mathbf{s}_T^H \left(\mathbf{I}_D - \mathbf{V}_T (\Phi_I + \mathbf{V}_T^H \mathbf{V}_T)^{-1} \mathbf{V}_T^H \right) \mathbf{s}_T \tag{20}$$

Since the number of auxiliary channels is fixed, the optimal performance can be achieved by changing the form of $\mathbf{s}_T$ and $\mathbf{V}_T$ to maximize the ADC^2, i.e., selecting the appropriate auxiliary channels.

However, the expression given in Eq. 20 is not in a convenient form for auxiliary channel selection; a concise formula in terms of matrix determinants is derived as follows:

The middle term in Eq. 20 is denoted with $\mathbf{G}_T$ and is expressed as:

$$\mathbf{G}_T = \mathbf{V}_T^H \mathbf{V}_T + \Phi_I \tag{21}$$

By embedding the desired signal in the clutter-plus-noise cross-correlation matrix, $\mathbf{G}_T \in \mathbb{C}^{N_r \times N_r}$ in the transform domain yields

$$\mathbf{G}_{sT} = \begin{bmatrix} r_{ss} & \mathbf{s}_T^H \mathbf{V}_T \\ \mathbf{V}_T^H \mathbf{s}_T & \mathbf{G}_T \end{bmatrix} \tag{22}$$

where $r_{ss} = \mathbf{s}_T^H \mathbf{s}_T = (NM)^2$ denotes the self-correlation of the desired signal in the transform domain. Furthermore, due to the fact that $\mathbf{T}\mathbf{T}^H = NM\mathbf{I}_{NM}$, utilizing the determinant of block matrix results in

$$|\mathbf{G}_{sT}| = |\mathbf{G}_T| \left(r_{ss} - \mathbf{s}_T^H \mathbf{V}_T \mathbf{G}_T^{-1} \mathbf{V}_T^H \mathbf{s}_T \right) = r_{ss} \left| \mathbf{G}_T - \frac{1}{r_{ss}} \mathbf{V}_T^H \mathbf{s}_T \mathbf{s}_T^H \mathbf{V}_T \right|. \tag{23}$$

It is evident that

$$\mathbf{s}_T^H \mathbf{V}_T \left(\Phi_I + \mathbf{V}_T^H \mathbf{V}_T \right)^{-1} \mathbf{V}_T^H \mathbf{s}_T = \mathbf{s}_T^H \mathbf{V}_T \mathbf{G}_T^{-1} \mathbf{V}_T^H \mathbf{s}_T = r_{ss} - \frac{|\mathbf{G}_{sT}|}{|\mathbf{G}_T|}. \tag{24}$$

Substituting Eq. 24 into Eq. 20, the ADC^2 can be simplified as

$$a = \frac{|\mathbf{G}_{sT}|}{|\mathbf{G}_T|} \tag{25}$$

It will be shown that the more DOF exists, the more computing times is necessary for calculating the ADC^2.

From the analysis above, the detection performance can be expressed as a function of RD matrix which is decided by the selected auxiliary channels. In the next section, an approach for finding the most appropriate auxiliary channels is provided, utilizing the ADC^2.

4 The best channels in the transform domain

4.1 The flow of selecting the best channels

As discussed already, different RD matrix corresponds to a different output of ADC^2, the main goal of maximizing the ADC^2 is to achieve the optimum RD matrix. However, we cannot select the best angular-Doppler channels at one time, for the reason that there are so many combinations of auxiliary channels which are used to compose the RD matrix $\mathbf{T}$.

Therefore, we select the best channels step by step. First of all, only the desired signal is already known to set as the main channel, that is set as $\mathbf{s}$. Assume that the current number of selected optimal channels is d, naturally, there are $d-1$ auxiliary channels. We denote $a_d(m,n)$ as the dth ADC^2 corresponding to each angular-Doppler channel to evaluate the effect of clutter cancellation and can be represented as

$$a_d(m,n) = (NM)^2 \left| \mathbf{G}_T^{(d)} - \left(\frac{1}{NM} \right)^2 \left(\mathbf{V}_T^{(d)} \right)^H \mathbf{s}_T^{(d)} \left(\mathbf{s}_T^{(d)} \right)^H \mathbf{V}_T^{(d)} \right| / \left| \mathbf{G}_T^{(d)} \right| \tag{26}$$

which could be inferred from Section 3 with the fact that

$$\mathbf{V}_T^{(d)}(m,n) = \mathbf{T}_d^H(m,n)\mathbf{V}_I, \mathbf{s}_T^{(d)}(m,n) = \mathbf{T}_d^H(m,n)\,\mathbf{s}$$
$$\mathbf{G}_T^{(d)}(m,n) = \left(\mathbf{V}_T^{(d)}(m,n) \right)^H \mathbf{V}_T^{(d)}(m,n) + \Phi_I, \tag{27}$$

where $\mathbf{T}_d(m,n)$ is the transform matrix of dth step composed by the matrix $\mathbf{T}_{d-1}$ derived from the former $d-1$ steps. Before the selection of the next auxiliary channel, the influence of the current RD channels should be considered through updating the RD matrix, and then selecting the next optimal channel by evaluating each transform vector $\mathbf{s}_d(f_{dm}) \otimes \mathbf{s}_s(f_{sn})$ that corresponds to each angular and Doppler frequency, which has the form

$$\mathbf{T}_d(m,n) = \left[\mathbf{T}_{d-1}, \mathbf{s}_d(f_{dm}) \otimes \mathbf{s}_s(f_{sn}) \right] \in \mathbb{C}^{NM \times d}. \tag{28}$$

Note that the first transform vector corresponds to the desired signal, i.e., $\mathbf{T}_1 = \mathbf{s}$. Then, we select the channel with maximum $a_d(m,n)$ as the current optimal channel, so that the transform matrix of dth step is achieved and denoted by $\mathbf{T}_d$. For adding the effect of the former selected channels, the ADC^2 should be updated until the optimal channel selection progress is finished.

After the RD matrix $\mathbf{T}_d$ with D auxiliary channels has been achieved, the transformation of the received signal $\mathbf{x}$ by the operation $\mathbf{T}_d$ defined in Eq. 5 yields a lower dimensional vector $\mathbf{x}_T$. The CCM in the transform domain can be obtained by 15; therefore, the weight vector in the transform domain is given by

$$\mathbf{w}_T = \mu \mathbf{R}_T^{-1} \mathbf{s}_T, \tag{29}$$

where $\mu = \left(\mathbf{s}_T \mathbf{R}_T^{-1} \mathbf{s}_T \right)^{-1}$ is a non-zero constant that does not affect the output SINR.

Comparing the proposed methods with MSMB presented in [14], both of them select auxiliary channels adaptively in the RD STAP. We summarise the computational complexities (Fig. 2) of the step that selects the dth channel: for MSMB, the computational complexity is about

$$O\left(J^4 + J^3 + (d+1)\left(J^2 + Jd + d^2 \right) \right), \tag{30}$$

where $J = NM$ is the dimension of received data, and there exists a time-consuming eigen decomposition of MSMB at the beginning of this algorithm. While the computational complexity of the proposed method is

$$O\left(J\left(N_r^3 + N_r d^2 + N_r^2 d \right) + J^2 (N_r + 1) \right). \tag{31}$$

We can see that the proposed method is calculated much easier than MSMB. However, because of the increased step of channel selection, the computational complexity of the proposed method is higher than the RD STAP of fixed channels, but better performance is achieved.

4.2 Prediction for the possible location of auxiliary channels

According to the generalized sidelobe canceler, the principle of RD STAP can be treated as a process that cancels the interference components in the main channel by a

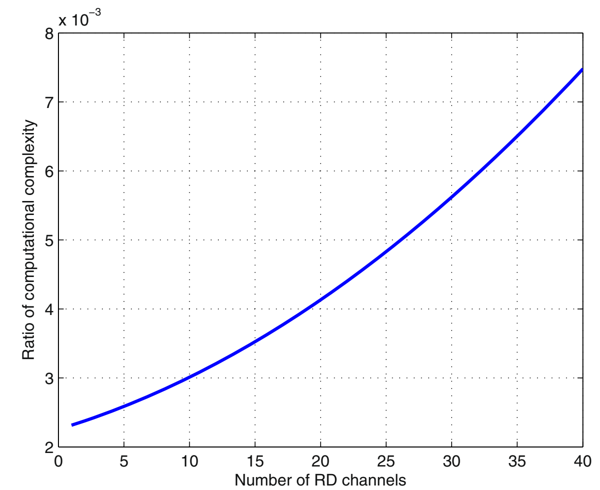

Fig. 2 Comparison of computational complexity between two adaptive RD methods

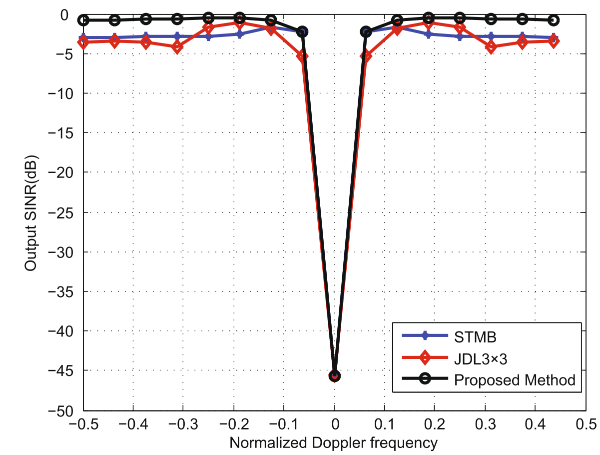

Fig. 3 Output SINR loss of each RD method

weighted summation of the interference components in auxiliary channels, which can be expressed as

$$\mathbf{x}_T = \mathbf{T}_D^H \mathbf{x} = [\mathbf{s}, \mathbf{T}_{D-1}]^H \mathbf{x} \in \mathbb{C}^{D\times NM} \tag{32}$$

All the columns of $\mathbf{T}_{D-1}$ correspond to the orthogonal channels which are orthogonal to the desired signal, i.e., $\mathbf{T}_{D-1}^H \mathbf{s} = \mathbf{0}$. In order to facilitate the analysis, assume that only one clutter Fourier basis with voltage is taken into consideration, the vectors of the signal and clutter components in the transform domain, respectively, are

$$\mathbf{s}_T = \begin{bmatrix} \mathbf{s}^H \\ \mathbf{T}_{D-1}^H \end{bmatrix} \mathbf{s} = \begin{pmatrix} \alpha_0 \\ \alpha \end{pmatrix}, \mathbf{v}_T = \begin{bmatrix} \mathbf{s}^H \\ \mathbf{T}_{D-1}^H \end{bmatrix} \mathbf{v} = \begin{pmatrix} \beta_0 \\ \beta \end{pmatrix}, \tag{33}$$

where $\alpha_0 = NM$, $\alpha = \mathbf{T}_{D-1}^H \mathbf{s} = \mathbf{0}$, $\beta_0 = \mathbf{s}^H \mathbf{v}$ is already known due to the prior estimation of clutter basis, and $\beta = \mathbf{T}_{D-1}^H \mathbf{v} \in \mathbb{C}^{(D-1)\times N_r}$ depends on the closeness of clutter basis and the selected channels. Substituting Eq. 32 into Eq. 20, the expression of ADC^2 can be written as

$$\begin{aligned} a &= \left(\alpha_0^* \; \alpha^H\right) \left[\mathbf{I}_D - \begin{pmatrix} \beta_0 \\ \beta \end{pmatrix} \left(\Phi + \left(\beta_0^* \; \beta^H\right) \begin{pmatrix} \beta_0 \\ \beta \end{pmatrix} \right)^{-1} \left(\beta_0^* \; \beta^H\right) \right] \begin{pmatrix} \alpha_0 \\ \alpha \end{pmatrix} \\ &= \left(\alpha_0^* \; \alpha^H\right) \left[\mathbf{I}_D - \frac{1}{\Phi + \beta_0^2 + \beta^H \beta} \begin{pmatrix} \beta_0^2 & \beta_0 \beta^H \\ \beta \beta_0^* & \beta \beta^H \end{pmatrix} \right] \begin{pmatrix} \alpha_0 \\ \alpha \end{pmatrix} \\ &= \alpha_0^2 - \frac{\alpha_0^2 \beta_0^2}{\Phi + \beta_0^2 + \beta^H \beta} = \frac{\alpha_0^2 \left(\Phi + \beta^H \beta\right)}{\Phi + \beta_0^2 + \beta^H \beta}, \end{aligned} \tag{34}$$

where $\Phi = NM\sigma_n^2/\varepsilon^2$ is scalar. From the last row of Eq. 34, we can infer that the ADC^2 is directly proportional to $\beta^H \beta$ and inversely proportional to β_0, which means that the inner product of β plays a driving role to the detection performance. Furthermore, we can draw a conclusion that

Table 1 Basic radar system parameters

Symbol	Description	Value
N	Number of antenna elements	16
M	Number of pulses	16
f_c	Carrier frequency	1200 MHz
f_r	Pulse repetition frequency	2400 Hz
d	Space between antenna elements	0.5 m
v_p	Radar platform velocity	150 m/s
H	Platform altitude	5000 m
R	Detection range	100 km
CNR	Clutter-to-noise ratio	50 dB

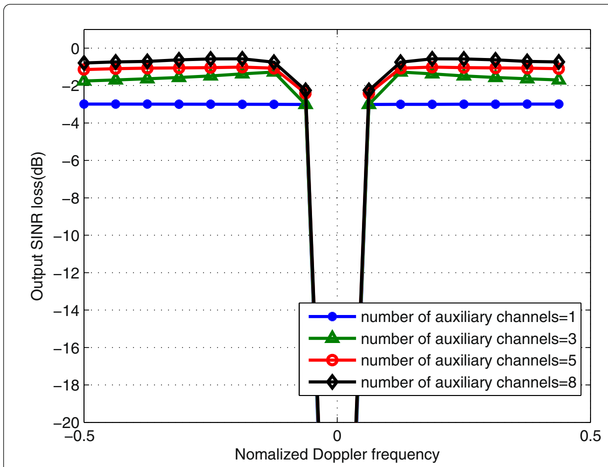

Fig. 4 Output SINR loss of the proposed method with different numbers of auxiliary channels

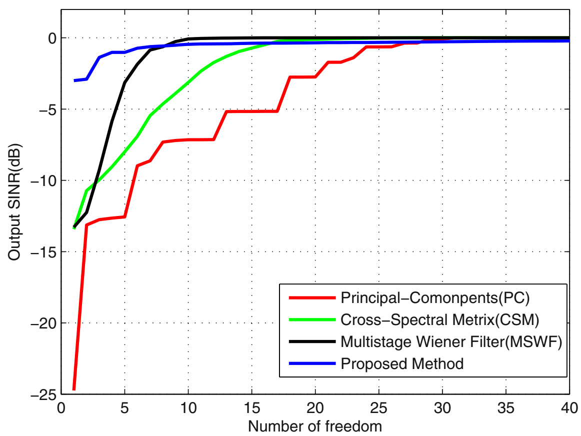

Fig. 5 Output SINR loss of RD processors as a function of number of freedom for testing Doppler channel 4

the more transform vectors similar to clutter basis are, and the better detection performance is achieved. This further demonstrates the conclusion in [10], and a more reasonable explanation and flexible method of channel selection is proposed.

In addition, the power of β_0 plays a blocking effect on the detection performance, which is easy to understand by slow-moving target detection: when the radial velocity of target is low, the clutter power becomes bigger because the target gets closer to the clutter ridge and then leads to a larger power of β_0. So that the correctness of ADC^2 can be further proved.

5 Simulations

In this section, we illustrate the validity of the effectiveness of the proposed method by presenting the simulation results. The basic parameters used in the following simulations are listed in Table 1. It is assumed that the receiving noise after matched filtering is Gaussian white noise whose power is calculated according to the thermal noise. In all simulations, we assume that the target is in broadside, that is, it has an azimuth of 0°.

Firstly, we describe the calculation ratio of the proposed method to MSMB [11, 12] as a function of the number of RD channels, using the formula provided in Eqs. 30 and 31. It is obvious that the proposed method has less complexity, especially in the scenario that RD channels is less and further confirms that the proposed method is more applicable in the sample deficient cases.

Figure 3 shows the SINR performance of the three RD STAP methods, the JDL3 × 3 approach, STMB approach, and the proposed approach, respectively. The normalized Doppler frequency sweeps over the range $[-0.5, 0.5]$, and the best channels are calculated at each frequency point. It is clear to see that the proposed RD approach shows better capability of suppressing clutter than the other two, even if they have the same dimension of RD matrix. In detail, they exhibit similar performance when the velocity of target is low, but the proposed approach generates better SINR output compared to the two stationary RD approaches when the target is away from the clutter ridge. Under the condition that there is no target ambiguity in Doppler, we can infer that a larger performance gap will be generated when the velocity of target becomes larger.

In the second experiment, we evaluate the performance of the proposed approach for different numbers of auxiliary channels, which is a partial enlarged picture exhibited in Fig. 4. We can conclude that the proposed method has a slight output SINR improvement as the number of auxiliary channels increases. By detailed observation, the proposed approach with only one auxiliary channel

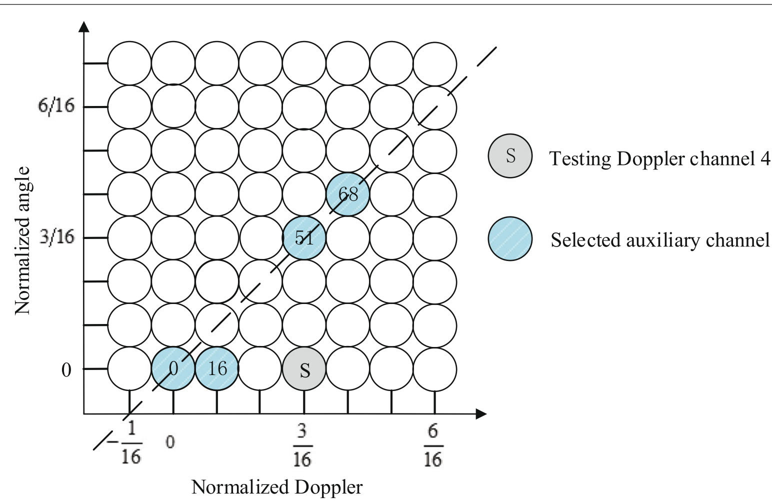

Fig. 6 The four best channels for testing Doppler channel 4

can achieve a SINR loss about less than 3 dB, and a little performance improvement is achieved for eight auxiliary channels compared with three auxiliary channels. As a result, an excellent performance can be achieved by using only few channels to cancel the interference, then computational amount and the secondary data used to estimate the CCM can be decreased.

We illustrate the relationship between the output SINR loss and the number of freedom with a fixed testing Doppler channel 4, i.e., the normalized Doppler frequency is 3/16. The results, shown in Fig. 5, compare the proposed method with the RR approach such as PC inverse algorithm [9], the cross-spectral metric algorithm (CSM) [10], and the multistage Wiener filter algorithm (MSWF) [11], as a function of Wiener filter order (adaptive DOF). One can see that the PC and CSM approaches have poorer convergence property to the optimal SINR and lower SINR while the MSWF has better convergence performance, and the proposed approach has the fastest convergence rate. On the other hand, in the case that the DOF is low, that is, only few auxiliary channels are selected, the output SINR of the proposed approach is less than 3 dB, while the other approaches cannot achieve the same performance and result in poorer clutter suppression. Only when the DOF of the MSWF, CSM, and PC reaches 11, 20, and 31, respectively, can they achieve the same performance as the proposed approach, while the proposed approach exhibits good performance with no restriction on DOF. Therefore, it is certain that the proposed approach produces a desirable detection performance and less amount of calculation.

Last but not the least, the location of the four best channels are shown in Fig. 6, where the solid circles correspond to the RD channels and the diagonal denote the clutter ridge. The serial number of auxiliary channels is labeled according to the rules as follows: $N(m-1)+n-1$ denotes the mth Doppler channel and the nth angular channel, and the smaller number of n and m means the channel is closer to the frequency of 0, both in the angular and Doppler domains. It can be seen that the auxiliary channels are mainly distributed in the area around the mainlobe and sidelobe, which have the same Doppler frequency or the same angle with the target. Hence, the conclusion derived before can be verified.

6 Conclusions

Reduced-dimension STAP is frequently employed in airborne radar systems to detect moving targets in the presence of fixed clutter, which is widely used in practical engineering application. In this paper, we proposed a novel RD STAP methodology. A parameter named ADC^2 is formulated to characterize the non-centrality parameter of AMF detector which has a direct impact on detection performance. Through estimating the Fourier basis offline, we select the auxiliary channels in the field of angular-Doppler corresponding to the largest value of ADC^2, so that the upper bound of performance will be achieved when the DOF is fixed. The performance of the proposed strategy was validated by simulations, which exhibited its advantage on target detection performance compared with some other RD and RR approaches. Furthermore, the location of the best auxiliary channels in the transform domain is predicted and verified by the simulation results.

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Competing interests
The authors declare that they have no competing interests.

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