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Optimization of a MIMO amplify-and-forward relay system with channel state information estimation error and feedback delay

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Abstract
This paper addresses the robust design of a multiple-input multiple-output amplify-and-forward relay system against channel state information (CSI) mismatch due to estimation error and feedback delay. The estimation error and feedback delay are expressed using appropriate models, from which we derive the conditional mean square error (MSE) between the desired and the received signals upon the estimated CSI. The conditional MSE is then minimized to optimize the relay beamforming matrix with relay transmission power constraint. It is shown that the proposed optimization problem reduces to the conventional minimum MSE problem when CSI mismatch vanishes. By analyzing the structure of the optimal beamforming matrix, the optimization problem is simplified so that it can be directly solved using the genetic algorithm (GA). To further reduce the computational load, we develop a relaxed version of the optimization problem. It is found that the relaxation enables us to efficiently solve the problem using water filling strategy. Computer simulations show that both GA and water filling solutions are superior to conventional solutions without CSI mismatch consideration, while the water filling is 1000 times faster than the GA.

Keywords: Channel state information mismatch, Conditional expectation, Multiple-input multiple-output, Minimum mean square error, Relay network

1 Introduction

Relaying technique is capable of extending communication range and coverage by providing link to shadowed users via relay nodes. From the perspective of signal processing, the cooperative relays can be viewed as a virtual antenna array which provides spatial diversity to combat frequency/time fading of channels. Attracted by its obvious merits, relaying technique has received extensive study in recent years [1–3].

The advantages of multiple-input multiple-output (MIMO) systems can be exploited in relay communications by accommodating multiple antennas at the nodes [4]. Recently, many works concentrate on designing non-regenerative MIMO relay systems. With perfect channel state information (CSI) assumption of all hops, optimal designs of MIMO amplify-and-forward (AF) relay networks were proposed [5–9]. To take CSI mismatch into account, robust design of a three-node MIMO relay system against CSI mismatch for linear non-regenerative MIMO relays was established in [10], where quality of service was attained by minimizing the averaged transmission power subject to mean square error (MSE) constraints at each data stream. Assuming the CSI uncertainty lies in a norm bounded region, two performance metrics, mutual information and MSE, were adopted to design the MIMO AF relay precoders in [11]. With the goal to minimize the MSE between the transmitted and the estimated symbol, Zhang proposed a robust precoder for MIMO AF relay systems against channel estimation error [12]. In [13], MSE minimization criterion was proposed to deal with CSI mismatch, and a closed-form solution was derived. In [14], two schemes aiming to maximize the signal-to-interference noise ratio were proposed to deal with CSI mismatch. In [15], the channel quantization

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error was considered and robust precoding schemes were proposed based on zero forcing and minimum MSE criterion. Also, a novel precoding scheme for MIMO relay systems in the presence of imperfect CSI was introduced in [16], where a base station precoding matrix and a relay station precoding matrix were created.

All the aforementioned algorithms focus on enhancing robustness against CSI estimation error. Besides CSI estimation error, other factors can also cause mismatch. In [17], quatized CSI feedback was considered and a strategy of scaling quantization quality of both two-hop links was proposed. Besides, channel feedback delay also has a significant influence on the performance of an AF relay system [18]. Therefore, it is highly desirable to consider both CSI estimation error and feedback delay when designing the MIMO AF relay system.

This paper presents a robust design of a MIMO AF relay system against CSI feedback delay and estimation error. The conditional MSE between the desired and received signals upon the estimated CSI is derived based on models of CSI feedback delay and estimation error. The conditional MSE is minimized subject to relay power constraint to optimize the beamforming matrix. Two solutions, one global and one relaxed, are proposed. Computer simulations show that both solutions outperform the MMSE strategy in terms of bit error rate (BER). It is also shown that ignoring the feedback delay results in higher BER. Although the relaxed solution is not as good as the global solution, it requires 1000 times less CPU time than the global solution. Hence, the relaxed solution provides a fast option for practical use.

2 Relay system mode

Figure 1 depicts a relay network consisting of a source node, a relay node, and a destination node, where the source and destination nodes are equipped with M antennas, and the relay node is equipped with L antennas. It is assumed that $L \geq M$. Direct link between the source node and the destination node is supposed to be absent. Frequency flat fading channels are considered. The source to relay and relay to destination channel matrices are represented by $\mathbf{H}_1$ and $\mathbf{H}_2$, respectively. It is assumed that all elements of the two matrices are independent and identically distributed complex Gaussian variables with zero mean and variances $\sigma_{h_1}^2$ and $\sigma_{h_2}^2$.

The output of the system, denoted by $\widehat{\mathbf{s}}$, is given by

$$\widehat{\mathbf{s}} = \mathbf{W}\mathbf{H}_2\mathbf{Q}\mathbf{H}_1\mathbf{s} + \mathbf{W}\mathbf{n}, \tag{1}$$

where $\mathbf{s}$ is the transmitted signal, $\mathbf{W}$ denotes the equalization matrix at the destination node, $\mathbf{Q}$ denotes the relay beamforming matrix and $\mathbf{n} = \mathbf{H}_2\mathbf{Q}\mathbf{n}_1 + \mathbf{n}_2$ is the additive noise at the destination node. $\mathbf{n}_1$ and $\mathbf{n}_2$ are the additive complex Gaussian noises with zero mean and covariance $\sigma_1^2\mathbf{I}_{N_r}$ and $\sigma_2^2\mathbf{I}_{N_s}$, respectively.

In the conventional MMSE strategy, $\mathbf{Q}$ and $\mathbf{W}$ are optimized by minimizing $E\left(\| \mathbf{s} - \widehat{\mathbf{s}}\|_2^2\right)_{\mathbf{s},\mathbf{n}}$ subject to relay transmission power constraint [19]. This criterion is optimal if the estimated channel matrices are the same as the real ones. However, due to estimation error and feedback delay, there is usually certain mismatch between the estimated and the real channel matrices in practice. Therefore, a more realistic algorithm should take the CSI mismatch into account when designing $\mathbf{Q}$ and $\mathbf{W}$.

3 Models of CSI feedback delay and estimation error

3.1 Source to relay CSI

Denote the $(i,j)^{\text{th}}$ element of $\mathbf{H}_1$ as $h_1^{(i,j)}$. The maximum likelihood (ML) estimate of $h_1^{(i,j)}$ is represented by $\widehat{h}_1^{i,j}$. According to the stochastic error model [20], $h_1^{(i,j)}$ and $\widehat{h}_1^{i,j}$ are related by

$$\widehat{h}_1^{i,j} = h_1^{(i,j)} + \epsilon_1^{(i,j)}, \tag{2}$$

where $\epsilon_1^{(i,j)}$ denotes the estimation error which is modeled by a complex Gaussian variable with zero mean and variance $\sigma_{\epsilon_1}^2 = \sigma_1^2/(N_1 - 1)$. N_1 is the number of training samples to obtain $\widehat{h}_1^{(i,j)}$, and σ_1^2 is the noise level at relay node.

Based on the Bayesian theory, the conditional expectation of $h_1^{(i,j)}$ upon $\widehat{h}_1^{i,j}$ is given by

$$E\left[h_1^{(i,j)}|\widehat{h}_1^{(i,j)}\right] = \sigma_{h_1}^2\widehat{h}_1^{i,j}/\left(\sigma_{h_1}^2 + \sigma_{\epsilon_1}^2\right). \tag{3}$$

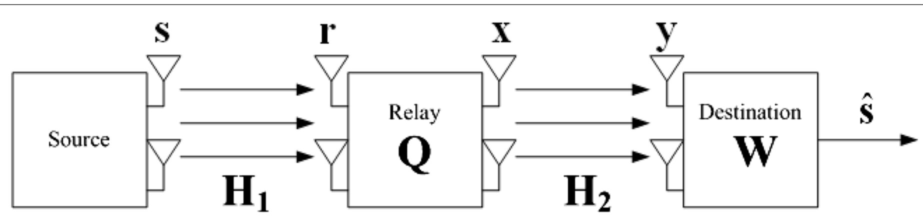

Fig. 1 A relay network with multiple antennas

Conditional correlation of $h_1^{(i,j)}$ and $h_1^{(k,l)}$ upon $\widehat{h}_1^{(i,j)}$ and $\widehat{h}_1^{(k,l)}$ is given by

$$E\left[h_1^{(i,j)}h_1^{*(k,l)}|\widehat{h}_1^{(i,j)},\widehat{h}_1^{(k,l)}\right] = \begin{cases} |E\left[h_1^{(i,j)}|\widehat{h}_1^{(i,j)}\right]|^2+\sigma^2_{h_1|\widehat{h}_1}, & i=k,j=l \\ E\left[h_1^{(i,j)}|\widehat{h}_1^{(i,j)}\right]E^*\left[h_1^{(k,l)}|\widehat{h}_1^{(k,l)}\right], & \text{otherwise,} \end{cases} \tag{4}$$

where $\sigma^2_{h_1|\widehat{h}_1}=\sigma^2_{h_1}-\sigma^4_{h_1}/\left(\sigma^2_{h_1}+\sigma^2_{\epsilon_1}\right)$.

3.2 Relay to destination CSI

Since the estimated CSI needs to be fed back from the destination to relay, due to channel temporal variation, the estimated CSI is actually an early version of the real CSI. Denote the $(i,j)^{\text{th}}$ element of $\mathbf{H}_2$ as $h_2^{(i,j)}$. According to the channel temporal variation model, $h_2^{\tau(i,j)}$ which represents an early version of $h_2^{(i,j)}$ is related to $h_2^{(i,j)}$ via [21]

$$h_2^{(i,j)}=\rho h_2^{\tau(i,j)}+\zeta^{(i,j)}, \tag{5}$$

where ρ is the normalized correlation coefficient calculated as $\rho = J_0(2\pi f_d\tau)$. J_0 denotes the first kind Bessel function of order zero, and f_d is the maximum Doppler frequency. $\zeta^{(i,j)}$ in (5) is a complex Gaussian variable with zero mean and variance $\left(1-\rho^2\right)\sigma^2_{h_2}$.

Denote the estimate of $h_2^{\tau(i,j)}$ by $\widehat{h}_2^{\tau(i,j)}$. Based on the stochastic error model, the relationship between $\widehat{h}_2^{\tau(i,j)}$ and $h_2^{\tau(i,j)}$ is given by

$$\widehat{h}_2^{\tau(i,j)}=h_2^{\tau(i,j)}+\epsilon_2^{(i,j)}, \tag{6}$$

where $\epsilon_2^{(i,j)}$ is a complex Gaussian variable with zero mean and variance $\sigma^2_{\epsilon_2}=\sigma^2_2/(N_2-1)$. N_2 and and σ^2_2 are the number of training samples and the noise level in estimating $\widehat{h}_2^{\tau(i,j)}$, respectively.

With (5) and (6), using the Bayesian theory, the conditional expectation of $h_2^{(i,j)}$ upon $\widehat{h}_2^{\tau(i,j)}$ is given by [21]

$$E\left[h_2^{(i,j)}|\widehat{h}_2^{\tau(i,j)}\right]=\rho\sigma^2_{h_2}\widehat{h}_2^{\tau(i,j)}/\left(\sigma^2_{h_2}+\sigma^2_{\epsilon_2}\right). \tag{7}$$

The conditional correlation of $h_2^{(i,j)}$ and $h_2^{(k,l)}$ upon $\widehat{h}_2^{(i,j)}$ and $\widehat{h}_2^{(k,l)}$ is given by

$$E\left[h_2^{(i,j)}h_2^{*(k,l)}|\widehat{h}_2^{(i,j)},\widehat{h}_2^{(k,l)}\right] = \begin{cases} |E\left[h_2^{(i,j)}|\widehat{h}_2^{\tau(i,j)}\right]|^2+\sigma^2_{h_2|\widehat{h}_2}, & i=k,j=l, \\ E\left[h_2^{(i,j)}|\widehat{h}_2^{\tau(i,j)}\right]E^*\left[h_2^{(k,l)}|\widehat{h}_2^{\tau(k,l)}\right], & \text{otherwise,} \end{cases} \tag{8}$$

where $\sigma^2_{h_2|\widehat{h}_2}=\sigma^2_{h_2}-\rho^2\sigma^4_{h_2}/\left(\sigma^2_{h_2}+\sigma^2_{\epsilon_2}\right)$.

4 Robust design of a MIMO AF relay system

4.1 Minimum conditional MSE criterion

Taking CSI mismatch of $\mathbf{H}_1$ and $\mathbf{H}_2$ into account, the proposed objective function is to minimize the conditional MSE between the transmitted and received signals subject to relay power constraint, which is given as follows:

$$\min_{\mathbf{Q},\mathbf{W}} E\left[\|\widehat{\mathbf{s}}-\mathbf{s}\|_2^2|\widehat{\mathbf{H}_1},\widehat{\mathbf{H}_2}\right], \tag{9a}$$

$$\text{subject to } tr\left(\mathbf{Q}\left(\sigma_s^2E\left[\mathbf{H}_1\mathbf{H}_1^H|\widehat{\mathbf{H}}_1\right]+\sigma_1^2\mathbf{I}_L\right)\mathbf{Q}^H\right)\le P_r, \tag{9b}$$

where P_r is the upper bound of relay transmission power.

Assuming that $\mathbf{s}$ and $\mathbf{n}$ are independent, based on (1), setting the gradient of (9a) with respect to $\mathbf{W}^H$ to zero yields the optimal $\mathbf{W}$

$$\mathbf{W}_{\text{opt}}=\sigma_s^2E^H\left[\mathbf{H}|\widehat{\mathbf{H}}_1,\widehat{\mathbf{H}}_2\right](\mathbf{R}_n)^{-1}, \tag{10}$$

where $\mathbf{W}_{\text{opt}}$ denotes the optimal solution of $\mathbf{W}$, and $\mathbf{R}_n = \sigma_s^2E\left[\mathbf{H}\mathbf{H}^H|\widehat{\mathbf{H}}_1,\widehat{\mathbf{H}}_2\right]+\sigma_1^2E\left[\mathbf{H}_2\mathbf{Q}\mathbf{Q}^H\mathbf{H}_2^H|\widehat{\mathbf{H}}_2\right]+\sigma_2^2\mathbf{I}_{N_s}$.

Define $\overline{\mathbf{H}}=E\left[\mathbf{H}|\widehat{\mathbf{H}}_1,\widehat{\mathbf{H}}_2\right]$, $\overline{\mathbf{H}}_1=E\left[\mathbf{H}_1|\widehat{\mathbf{H}}_1\right]$ and $\overline{\mathbf{H}}_2=E\left[\mathbf{H}_2|\widehat{\mathbf{H}}_2\right]$. Substituting (1) and (10) into (9a) yields (11) (shown on the top of next page).

$$\max_{\mathbf{Q}} tr\left(\overline{\mathbf{H}}^H\left(\sigma_s^2E\left[\mathbf{H}\mathbf{H}^H|\widehat{\mathbf{H}}_1,\widehat{\mathbf{H}}_2\right]+\mathbf{R}_n\right)^{-1}\overline{\mathbf{H}}\right), \tag{11a}$$

$$\text{subject to } tr\left(\mathbf{Q}\left(\sigma_s^2E\left[\mathbf{H}_1\mathbf{H}_1^H|\widehat{\mathbf{H}}_1\right]+\sigma_1^2\mathbf{I}_L\right)\mathbf{Q}^H\right)\le P_r. \tag{11b}$$

It is observed from (11a) that $\mathbf{Q}$ is contained in the inversion manipulation; therefore, direct optimization of (11) is difficult. To facilitate the solution of (11), the structure of optimal $\mathbf{Q}$ is analyzed and it is found that optimal $\mathbf{Q}$ has the form of

$$\mathbf{Q}_{\text{opt}}=\mathbf{V}_2\mathbf{\Phi}_1\mathbf{U}_1^H, \tag{12}$$

where $\mathbf{Q}_{\text{opt}}$ is the optimal $\mathbf{Q}$, $\mathbf{\Phi}_1$ is an $M\times M$ diagonal matrix, and $\mathbf{V}_2^H$ and $\mathbf{U}_1$ are unitary matrices constituted by right- and left-singular vectors of $\overline{\mathbf{H}}_2$ and $\overline{\mathbf{H}}_1$, respectively. The proof of (12) is provided in Appendix A. Using (12), (11) is equivalently expressed as

$$\max_{\mathbf{\Phi}_1} J\left(\mathbf{\Phi}_1\right), \tag{13a}$$

$$\text{subject to } \sum_{i=1}^{M}\gamma_i|\phi_i|^2\le P_r, \tag{13b}$$

where

$$J\left(\mathbf{\Phi}_1\right)=\sum_{i=1}^{M}\frac{\alpha_i|\phi_i|^2}{\beta_i|\phi_i|^2+\sigma_s^2b+c+\sigma_2^2}, \tag{14}$$

ϕ_i is the i^{th} diagonal element of $\mathbf{\Phi}_1$, $\alpha_i=\lambda_{1,i}^2\lambda_{2,i}^2$, $\beta_i = \sigma_s^2\left(\lambda_{1,i}^2+\sigma^2_{h_1|\widehat{h}_1}\right)\lambda_{2,i}^2+\sigma_1^2\lambda_{2,i}^2$, $\gamma_i = \sigma_s^2 \mid \lambda_{1,i} \mid^2 + \sigma_s^2\sigma^2_{h_1|\widehat{h}_1}+\sigma_1^2$,

$$b = \sigma^2_{h_2|\widehat{h}_2} \sum_{i=1}^{N_s} \left(\lambda^2_{1,i} + \sigma^2_{h_1|\widehat{h}_1}\right) \mid \phi_i \mid^2, \tag{15}$$

$$c = \sigma^2_{h_2|\widehat{h}_2} \sum_{i=1}^{N_s} \mid \phi_i \mid^2, \tag{16}$$

and $\lambda_{1,i}$ and $\lambda_{2,i}$ denote the i^{th} singular values of $\overline{\mathbf{H}}_1$ and $\overline{\mathbf{H}}_2$, respectively. Details of transformation from (11) to (13) are presented in Appendix B. The optimization problem (13) is easier to solve than (11). Once the optimal $\mathbf{\Phi}_1$ is derived, $\mathbf{Q}_{\text{opt}}$ and $\mathbf{W}_{\text{opt}}$ are computed using (12) and (10), respectively.

When $\sigma^2_{h_1|\widehat{h}_1} = 0$ and $\sigma^2_{h_2|\widehat{h}_2} = 0$, $b = 0$, $c = 0$, $\beta_i = \sigma_s^2\lambda^2_{1,i}\lambda^2_{2,i} + \sigma_1^2\lambda^2_{2,i}$, and $\gamma_i = \sigma_s^2 \mid \lambda_{1,i} \mid^2 + \sigma_1^2$,. (13) becomes

$$\max_{\phi_i} \sum_{i=1}^{N_s} \frac{\lambda^2_{1,i}\lambda^2_{2,i} \mid \phi_i \mid^2}{\left(\sigma_s^2\lambda^2_{1,i}\lambda^2_{2,i} + \sigma_1^2\lambda^2_{2,i}\right) \mid \phi_i \mid^2 + \sigma_2^2}, \tag{17a}$$

$$\text{subject to} \sum_{i=1}^{M} \left(\sigma_s^2 \mid \lambda_{1,i} \mid^2 + \sigma_1^2\right) \mid \phi_i \mid^2 \leq P_r, \tag{17b}$$

which are equivalent to (24) and (25) of [19]. Therefore, the minimum conditional MSE criterion reduces to the MSE criterion when the CSI mismatch vanishes.

4.2 Global solution by genetic algorithm

It is observed from (13) that multiplying ϕ_i by a constant maximizes the relay power and the value of the objective function (13a). Therefore, the optimal ϕ_i can be obtained while (13b) achieves equality. Based on this observation, the values of chromosomes in genetic algorithm (GA) is optimized within 0 and 1, then multiplied with a constant α which is obtained when (13b) achieves equality, i.e.,

$$\alpha = \frac{P_r}{\sum_{i=1}^{M} \sigma_s^2 \mid \phi_i \mid^2 \mid \lambda_{1,i} \mid^2 + \sigma_s^2\sigma^2_{h_1|\widehat{h}_1} \mid \phi_i \mid^2 + \sigma_1^2 \mid \phi_i \mid^2}. \tag{18}$$

Substituting $\alpha\phi_i$ into (13a) yields the value of the fitness function of GA.

4.3 Relaxed solution by water filling strategy

It is observed from (13a) that the terms of b and c contain $\phi_i, i = 1, \ldots, M$, which prevent deriving an analytical solution to (13). To avoid high computational loads of using the global searching algorithm, a relaxed version of (13) is proposed here.

From (13a), it is noted that increasing the values of b and c reduces the value of $J(\mathbf{\Phi}_1)$, which means

$$J(\mathbf{\Phi}_1) \geq J(\mathbf{\Phi}_1)_{\max}, \tag{19}$$

where $J(\mathbf{\Phi}_1)_{\max}$ is computed from (13a) using $b_{\max}$ and $c_{\max}$. Here, $b_{\max}$ and $c_{\max}$ denote the maximum values of b and c, respectively. From the relay power constraint (13b), the possible values of $b_{\max}$ and $c_{\max}$ are straightforward to derive and are given below:

$$b_{\max} = \sum_{i=1}^{M} \frac{P_r\sigma^2_{h_2|\widehat{h}_2}\left(\lambda^2_{1,i} + \sigma^2_{h_1|\widehat{h}_1}\right)}{\sigma_s^2 \mid \lambda_{1,i} \mid^2 + \sigma_s^2\sigma^2_{h_1|\widehat{h}_1} + \sigma_1^2}, \tag{20}$$

$$c_{\max} = \sum_{i=1}^{M} \frac{P_r\sigma^2_{h_2|\widehat{h}_2}}{\sigma_s^2 \mid \lambda_{1,i} \mid^2 + \sigma_s^2\sigma^2_{h_1|\widehat{h}_1} + \sigma_1^2}. \tag{21}$$

Substituting (20) and (21) into (13), and using the Lagrange multiplier technique, the solution of the relaxed version of (13) is given by

$$\mid \phi_i \mid^2 = \frac{1}{\sigma_s^2 \mid \lambda_{2,i} \mid^2 \left(\mid \lambda_{1,i} \mid^2 + \sigma^2_{h_1|\widehat{h}_1}\right) + \sigma_1^2 \mid \lambda_{2,i} \mid^2} \cdot \left(\sqrt{\frac{\sigma_s^2 \mid \lambda_{1,i} \mid^2 \mid \lambda_{2,i} \mid^2 \sigma^2_{2,\max}}{\mu\left(\sigma_s^2 \mid \lambda_{1,i} \mid^2 + \sigma_s^2\sigma^2_{h_1|\widehat{h}_1} + \sigma_1^2\right)}} - \sigma^2_{2,\max}\right)^+, \forall i, \tag{22}$$

where

$$\sigma^2_{2,\max} = \sigma_s^2 b_{\max} + \sigma_1^2 c_{\max} + \sigma_2^2, \tag{23}$$

and $(x)^+ = \max(x, 0)$, μ is the Lagrange constant which should be chosen such that (13b) is satisfied.

5 Computer simulations

To demonstrate validity and advantages of the proposed strategy, the following simulation scenarios are devised. The channel coefficients are assumed to be complex Gaussian variables with zero mean and variance $\sigma^2_{h_1}$ and $\sigma^2_{h_2}$. The signal-to-noise ratio (SNR) of the backward and forward channels are defined as $\text{SNR}_1 = \sigma_s^2\sigma^2_{h_1}/\sigma_1^2$ and $\text{SNR}_2 = P_r\sigma^2_{h_2}/\sigma_2^2 L$, respectively. The source is generated from a QPSK constellation. It is assumed that the number of data samples is 120, containing $N_{tr} = 20$ training samples. The number of M is 4. The relay transmission power P_r is 0 dB. The BER is derived from 100 independent trials for all the plots. The proposed algorithm is compared with the conventional MMSE strategy and the robust method which only considers CSI estimation error.

Figure 2 plots the BER versus SNR_1 with $L = 4$, $\text{SNR}_2 = 10$ dB, and $\rho = 0.90$. Because CSI estimation error exists, it is observed that the proposed methods outperform the conventional water filling method which assumes accurate estimation. Also, it is found that both of the proposed methods perform better than the robust method against estimation error, since the channel correlation coefficient is 0.90 in this situation, which has significant influence on the accuracy of CSI. Furthermore, because GA is global searching, the proposed algorithm

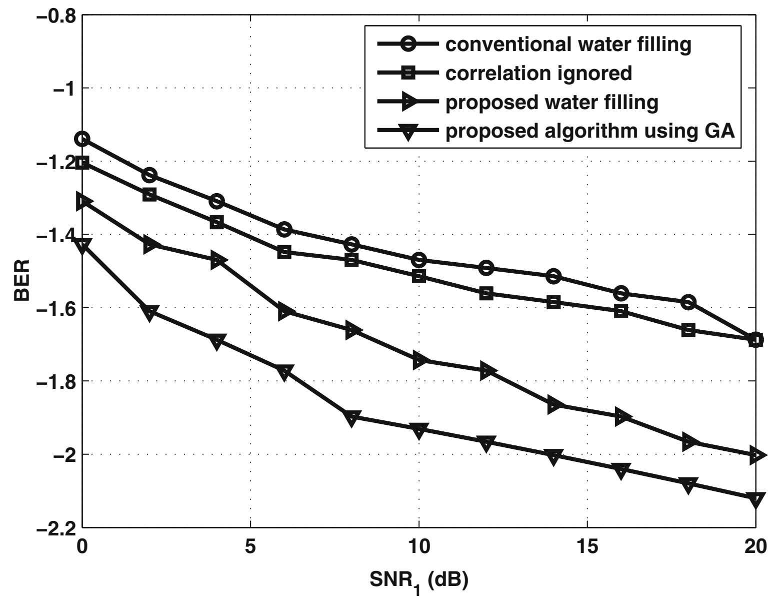

Fig. 2 BER versus SNR_2 with $L = 4$, $\mathrm{SNR}_2 = 10$ dB, and $\rho = 0.90$

using GA yields smaller BER than that of the proposed water filling method.

In Fig. 3, we assume a better communication environment. The SNR_1 is fixed at 20 dB, and the value of ρ is assumed as 0.95. Figure 3 shows the BER versus the value of SNR_2. It can be seen that the performance of all the methods becomes better as the value of SNR_2 increases. Compared with Fig. 2, all the methods give smaller BER when $\mathrm{SNR}_1 = 20$ dB and $\mathrm{SNR}_2 = 10$ dB, since the value of ρ increases. As SNR_2 increases beyond 10 dB, the performance of the conventional water filling technique and the robust method against CSI estimation error do not have significant improvement, while BER of the proposed methods continue to decrease.

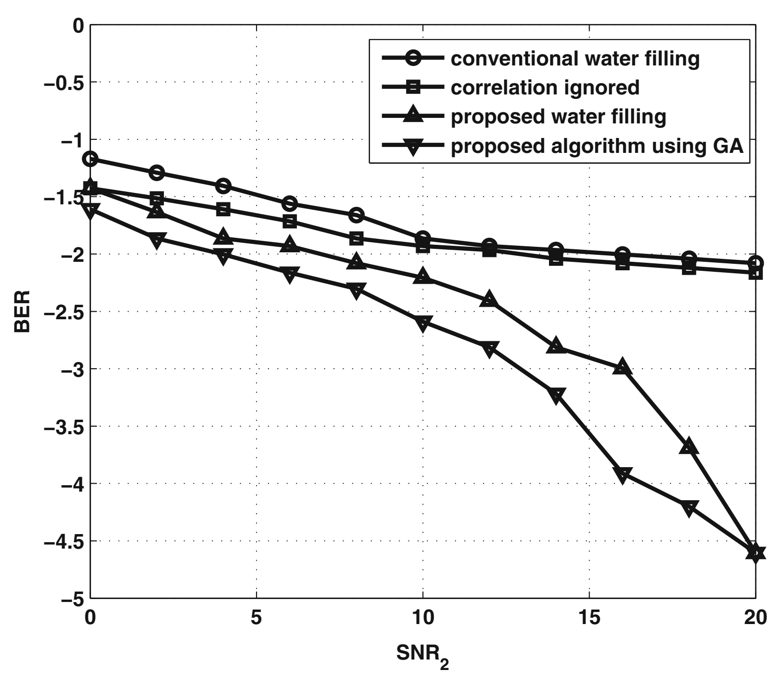

Fig. 3 BER versus SNR_1 with $L = 4$, $\mathrm{SNR}_1 = 20$ dB, and $\rho = 0.95$

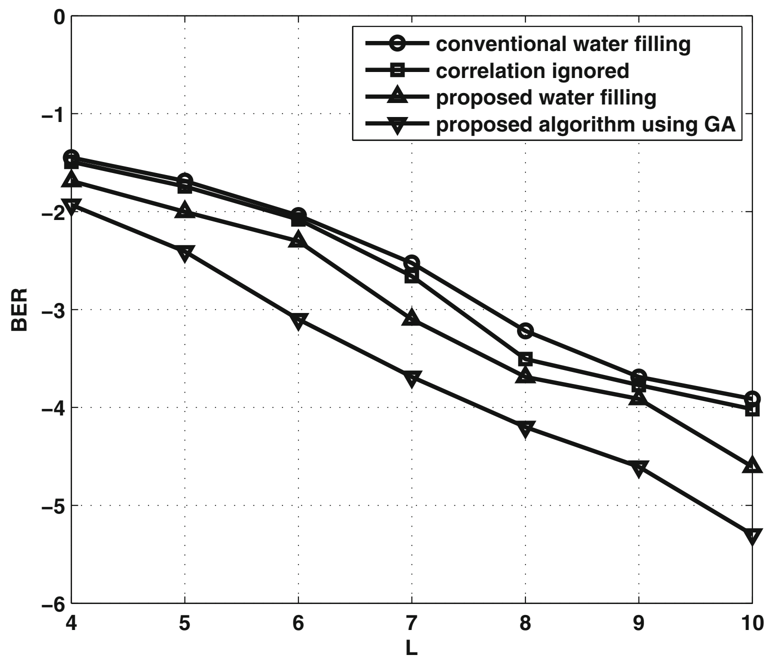

Fig. 4 BER versus L with $\mathrm{SNR}_1 = \mathrm{SNR}_2 = 10$ dB

Figure 4 plots BER versus the value of L with SNR_1 and SNR_2 equal to 10 dB. It is natural to see that the BER of all the methods decrease as the number of antennas equipped on relay increases. Furthermore, it is observed that the increase of L favors the improvement of performance of the conventional water filling technique and the robust algorithm against CSI estimation error. The gap between these two methods and the proposed methods becomes smaller.

In Fig. 5, plots of BER versus the value of ρ are given. BERs of all the methods increase as the value of ρ decreases. It is verified that channel correlation has

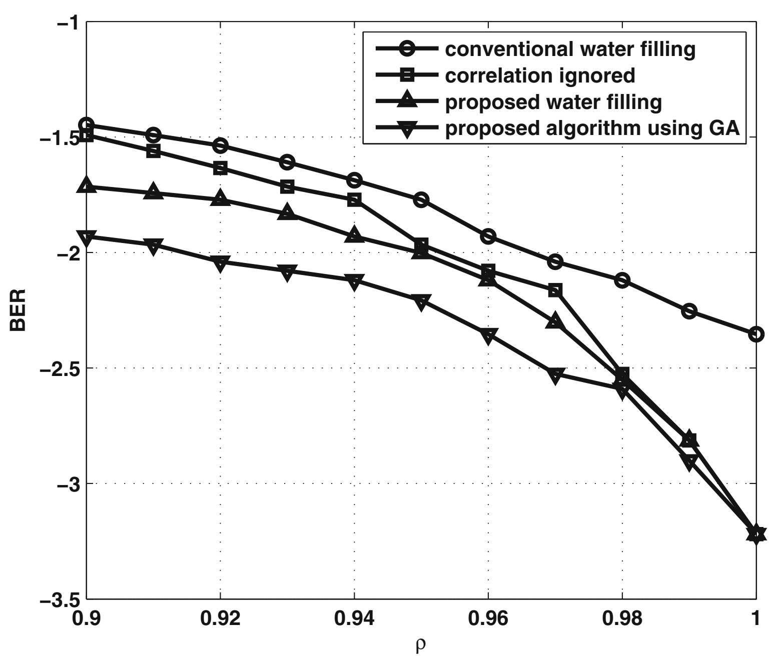

Fig. 5 BER versus ρ with $\mathrm{SNR}_1 = \mathrm{SNR}_2 = 10$ dB and $L = 4$

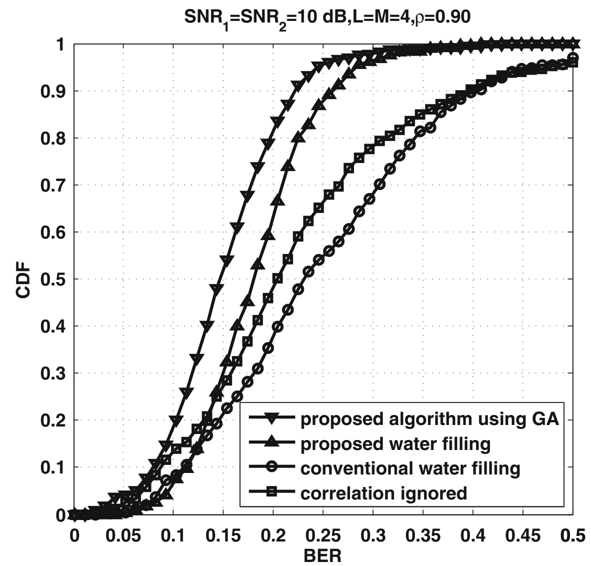

Fig. 6 CDF of BER with $\text{SNR}_1 = \text{SNR}_2 = 10$ dB, $\rho = 0.90$, and $L = 4$

significant compact on the performance of all the methods. The proposed methods outperform the conventional MMSE method for all values of ρ. When $\rho = 1$, performance of the proposed methods and the robust algorithm against CSI estimation error become same.

Figure 6 shows the cumulative distribution function (CDF) of BER with $\text{SNR}_1 = \text{SNR}_2 = 10$ dB, $\rho = 0.90$, and $L = 4$. It is observed that the proposed methods yield smaller BER at a high probability than that of the other methods.

To compare computational time of the GA and the relaxed solution, 100 independent trials are conducted. The number of antennas at all the nodes are assumed to be the same. From Table 1, it is observed that the relaxed solution cost far less computational time than that of the GA.

6 Conclusions

In this paper, we consider the robust design of an AF relay system against CSI mismatch. A relay system equipped with multiple antennas is considered. The conditional expectation of the mean square error is minimized with respect to precoding matrix and equalization coefficient. Computer simulations show that when the estimated CSI is different from the real CSI, the proposed strategy outperforms the MMSE strategies with perfect CSI assumption. The advantages include lower BER with different values of ρ, SNR_1, SNR_2, and L. Also, the proposed strategy is more likely to give small BER compared to the conventional MMSE strategy. Therefore, the proposed strategy is more reliable to be used in real applications.

Table 1 Computational time (second) comparison of 100 independent trials with $M = L$

Methods	$L = 4$	$L = 8$	$L = 16$	$L = 32$
GA	140.78	176.75	396.56	2739.67
Relaxed solution	0.103	0.164	0.214	0.360

Appendix A

Proof of the Theorem Assume $\mathbf{Q} = \mathbf{V}_2\mathbf{\Phi}\mathbf{U}_1^H$, where $\mathbf{\Phi} = \begin{pmatrix} \mathbf{\Phi}_1 & \mathbf{\Phi}_2 \\ \mathbf{\Phi}_3 & \mathbf{\Phi}_4 \end{pmatrix}$ and $\mathbf{\Phi}_1 \in \mathcal{C}^{N_s \times N_s}$.

From (4), we have

$$E\left[\mathbf{H}_1\mathbf{H}_1^H\right] = \overline{\mathbf{H}}_1\overline{\mathbf{H}}_1^H + \begin{pmatrix} \sigma^2_{h_1|\widehat{h}_1}\mathbf{I}_{N_s} & \mathbf{O} \\ \mathbf{O} & \mathbf{O} \end{pmatrix} \tag{24}$$

Substituting $\mathbf{\Phi}$ and (24) into $E\left[\mathbf{H}\mathbf{H}^H|\widehat{\mathbf{H}}_1, \widehat{\mathbf{H}}_2\right]$ yields (25) shown as follows,

$$\begin{aligned} E\left[\mathbf{H}\mathbf{H}^H|\widehat{\mathbf{H}}_1, \widehat{\mathbf{H}}_2\right] = {} & \overline{\mathbf{H}}_2\mathbf{V}_2\mathbf{\Phi}\mathbf{\Lambda}_1\mathbf{\Lambda}_1^H\mathbf{\Phi}^H\mathbf{V}_2^H\overline{\mathbf{H}}_2^H + \\ & \sigma^2_{h_2|\widehat{h}_2}\sum_{i=1}^{N_s} \mid \lambda_{1,i} \mid^2 \|\mathbf{\Phi}_i^T\|_2^2 \begin{pmatrix} \mathbf{I}_{N_s} \\ \mathbf{O} \end{pmatrix} + \\ & \overline{\mathbf{H}}_2\mathbf{V}_2\mathbf{\Phi}\mathbf{U}_1^H \begin{pmatrix} \sigma^2_{h_1|\widehat{h}_1}\mathbf{I}_{N_s} & \mathbf{O} \\ \mathbf{O} & \mathbf{O} \end{pmatrix} \mathbf{U}_1\mathbf{\Phi}^H\mathbf{V}_2^H\overline{\mathbf{H}}_2^H + \\ & \sigma^2_{h_1|\widehat{h}_1}\sigma^2_{h_2|\widehat{h}_2}\sum_{i=1}^{N_s} \mid \lambda_{1,i} \mid^2 \|\mathbf{\Phi}_i^T\|_2^2 \begin{pmatrix} \mathbf{I}_{N_s} \\ \mathbf{O} \end{pmatrix} \end{aligned} \tag{25}$$

where $\mathbf{\Phi}_i^T$ denotes the i^{th} row of $\mathbf{\Phi}$.

Similarly, we have

$$\mathbf{R}_n = \sigma_1^2\overline{\mathbf{H}}_2\mathbf{V}_2\mathbf{\Phi}\mathbf{\Phi}^H\mathbf{V}_2^H\overline{\mathbf{H}}_2^H + \sigma_2^2\sigma^2_{h_2|\widehat{h}_2}\sum_{i=1}^{N_r} \|\mathbf{\Phi}_i\|^2 \begin{pmatrix} \mathbf{I}_{N_s} \\ \mathbf{O} \end{pmatrix}. \tag{26}$$

Define

$$f(\mathbf{z} \mid \mathbf{\Phi}) = \mathbf{z}^H\left(\sigma_s^2E\left[\mathbf{H}\mathbf{H}^H|\widehat{\mathbf{H}}_1, \widehat{\mathbf{H}}_2\right] + \mathbf{R}_n\right)\mathbf{z}, \tag{27}$$

where $\mathbf{z} \in \mathcal{C}^{N_s \times 1}$.

From (26) and (27), it is straightforward to show that

$$\forall \mathbf{z}, f(\mathbf{z} \mid \mathbf{\Phi}) \geq f(\mathbf{z} \mid \mathbf{\Phi} = \mathbf{\Phi}_0). \tag{28}$$

Suppose $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{\Delta}$ are non-negative matrices, then

$$tr\left((\mathbf{A} + \mathbf{\Delta})^{-1}\mathbf{B}\right) \leq tr\left(\mathbf{A}^{-1}\mathbf{B}\right). \tag{29}$$

Therefore, (28) and (29) gives the result that

$$tr\left(\overline{\mathbf{H}}^H\left(\sigma_s^2E\left[\mathbf{H}\mathbf{H}^H \mid \widehat{\mathbf{H}}_1, \widehat{\mathbf{H}}_2\right] + \mathbf{R}_n\right)^{-1}\overline{\mathbf{H}}\right)$$

reaches its maximum when $\mathbf{\Phi} = \mathbf{\Phi}_0$.

Also, it is easy to show that $\mathbf{\Phi} = \mathbf{\Phi}_0$ helps to save relay transmission power. Therefore, (11) has a solution as $\mathbf{Q} = \mathbf{V}_2\mathbf{\Phi}_0\mathbf{U}_1^H$. □

Appendix B

Proof of Lemma 1 From the theorem, we have

$$E\left[\mathbf{H}\mathbf{H}^H \mid \widehat{\mathbf{H}}_1, \widehat{\mathbf{H}}_2\right] = \sum_{i=1}^{N_s}\left(\left(\lambda_{1,i}^2+\sigma^2_{h_1|\widehat{h}_1}\right)\lambda_{2,i}^2 \mid \phi_i \mid^2 + b\right)\mathbf{u}_{2i}\mathbf{u}_{2i}^H, \tag{30}$$

where $b = \sigma^2_{h_2|\widehat{h}_2}\sum_{i=1}^{N_s}\left(\lambda_{1,i}^2+\sigma^2_{h_1|\widehat{h}_1}\right) \mid \phi_i \mid^2$, and $\mathbf{u}_{2,i}$ denotes the i^{th} column of $\mathbf{U}_2$.

Similarly, it can be derived that

$$\mathbf{R}_n = \sum_{i=1}^{N_s}\left(\sigma_1^2\lambda_{2,i}^2 \mid \phi_i \mid^2 + c + \sigma_2^2\right)\mathbf{u}_{2i}\mathbf{u}_{2i}^H, \tag{31}$$

where $c = \sigma^2_{h_2|\widehat{h}_2}\sum_{i=1}^{N_s} \mid \phi_i \mid^2$.

Furthermore, we have

$$\overline{\mathbf{H}\mathbf{H}}^H = \sum_{i=1}^{N_s}\lambda_{1,i}^2\lambda_{2,i}^2 \mid \phi_i \mid^2 \mathbf{u}_{2,i}\mathbf{u}_{2,i}^H. \tag{32}$$

Using (30)–(32), we have (33)

$$tr\left(\overline{\mathbf{H}}^H\left(\sigma_s^2E\left[\mathbf{H}\mathbf{H}^H \mid \widehat{\mathbf{H}}_1, \widehat{\mathbf{H}}_2\right]+\mathbf{R}_n\right)^{-1}\overline{\mathbf{H}}\right) = \sum_{i=1}^{N_s}\frac{\lambda_{1,i}^2\lambda_{2,i}^2 \mid \phi_i \mid^2}{\left(\sigma_s^2\left(\lambda_{1,i}^2+\sigma^2_{h_1|\widehat{h}_1}\right)\lambda_{2,i}^2 \mid \phi_i \mid^2 + \sigma_s^2 b + \sigma_1^2\lambda_{2,i}^2 \mid \phi_i \mid^2 + c + \sigma_2^2\right)}. \tag{33}$$

In a similar way, (11b) can be calculated as (13b). □

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Competing interests
The authors declare that they have no competing interests.

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