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Consensus-based distributed adaptive target tracking in camera networks using Integrated Probabilistic Data Association

Khaled Obaid Al Ali[1,2], Nemanja Ilić[3,2*], Miloš S. Stanković[4,2] and Srdjan S. Stanković[5,2]

Abstract

In this paper, a novel consensus-based adaptive algorithm for distributed target tracking in large scale camera networks is presented, aimed at situations characterized by limited sensing range, high-level clutter, and possibly occulted targets. The concept of Integrated Probabilistic Data Association (IPDA) is introduced in the distributed adaptive tracker design so that the proposed algorithm, named IPDA Adaptive Consensus Filter (IPDA-ACF), incorporates probabilities of acquiring target-originated measurements, conditioned on either target perceivability or target existence. A distributed adaptation scheme represents the core element of the algorithm, allowing fast convergence under a large variety of operating conditions, emphasizing the influence of the nodes with the highest probability of obtaining target-originated measurements. A theoretical analysis of stability and reduction of noise influence allows getting an insight into the relationship between the local trackers and the global consensus scheme. A comparison with analogous existing methods done by extensive simulations shows that the proposed method achieves the best performance, in spite of lower communication and computation requirements.

Keywords: Camera networks, Distributed target tracking, Integrated Probabilistic Data Association, Consensus, Decentralized adaptation

1 Introduction

Recent rapid improvement in quality and resolution of imaging sensors and availability of low-cost smart cameras, together with the development of *sensor network technology*, have paved the way for creation of *large scale camera networks*. Examples of very successful applications are more and more numerous, especially in the fields of wide-area surveillance, disaster response, and environmental monitoring. *Detection* and *tracking* of objects of interest is one of the fundamental functions of camera networks, see, e.g., [1] and the references therein.

In this context, *distributed estimation schemes* are becoming increasingly popular, due to their high precision and scalability to large number of sensors, with no need for centralized actions, e.g., [2–4]. In spite of the fact that the theoretical fundamentals of target tracking are basically the same for different types of sensors (radars, sonars, cameras), camera networks impose some unique and interesting challenges, arising from the fact that (1) most of the cameras are *directional* and have *limited sensing range (LSR) or field-of-view (FoV)* so that at each time instant there might be a significant share of sensors that do not observe the target (see Fig. 1 as an illustration) and (2) environment is highly cluttered and targets can be temporarily occulted. Majority of the existing distributed estimation algorithms of general type based on *consensus* cannot be directly applied in this context. The Kalman-consensus filter (KCF) [5] has been applied to target tracking problems by sensor networks with LSR in [6]. The algorithm assumes internode exchange of local target state estimates, together with the so-called information vectors and matrices, which depend on local measurements and their covariances. The information-weighted consensus filter (ICF) has been found to outperform KCF under limited local observability scenarios [4]. It uses the fact that the local information contained in the nodes becomes correlated as

*Correspondence: nemili@etf.rs
[2]Vlatacom Institute, Bulevar Milutina Milankovica 5, 11000 Belgrade, Serbia
[3]College of Technics and Technology, 37000 Kruševac, Serbia
Full list of author information is available at the end of the article

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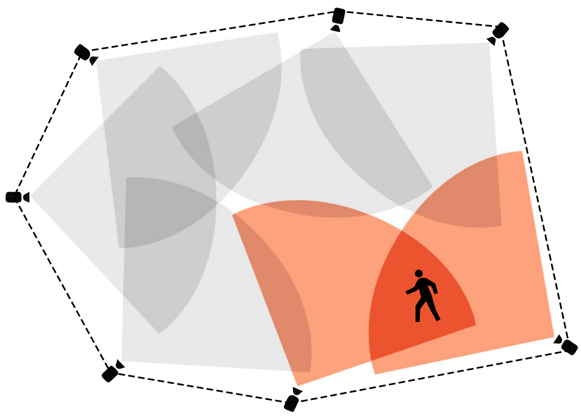

Fig. 1 Camera network with seven nodes with limited sensing and communication ranges. The dashed lines represent the communication channels between different nodes. The target is observable by two cameras at the given time instant

the state estimates converge. The communication requirements incorporate, besides those of KCF, the local error covariance matrices. Both KCF and ICF have been applied to distributed tracking problems in cluttered environment [7, 8], incorporating elements of the Probability Data Association (PDA) methodology [9–11]. In [12], an algorithm for decentralized state estimation has been proposed, based on a combination of local estimators of Luenberger type and a flexible dynamic consensus strategy. It assumes exchange of only state estimates between neighboring nodes. Its adaptive modification has been proposed in [2] in the context of target tracking problems with sensors having limited sensing range, but without assuming presence of clutter. We shall refer to this algorithm as Adaptive Consensus Filter (ACF). It has been demonstrated that ACF largely outperforms KCF and slightly outperforms ICF, preserving, at the same time, significantly lower communication bandwidth requirements [3].

In this paper, we assume cluttered environment and possibly temporarily occulted targets and propose a new distributed adaptive tracking algorithm for camera networks, representing an extension of ACF from [2] based on Integrated Probabilistic Data Association (IPDA); this algorithm will be denoted further as IPDA-ACF. The concept of IPDA has been introduced by Musicki, Evans, and Stankovic in [13] and further extended in different directions, see, e.g., [9–11, 14, 15]. The notion of *target perceivability* [14, 15] has been utilized to construct two distinct consensus-based tracking algorithms, having the form of the basic Probabilistic Data Association (PDA) recursion [9–11]. The algorithms deal with two types of the so-called "β"-parameters, representing probabilities of getting a target-originated measurement conditioned either on target *perceivability* or on target *existence*; in this sense, we have the algorithms denoted as IPDA-ACF[1] and IPDA-ACF[2], respectively. The core element of the proposed algorithm, making it competitive and even superior to the algorithms with higher communication requirements proposed in [7, 8], is the *distributed adaptation scheme*, developed as a substantial generalization of the adaptation strategy originally described in [2, 3] (see also [16]). The adaptation scheme, aiming at giving emphasis to the nodes with the highest probability of obtaining target-originated measurements, is based on the locally obtained "β"-parameters as indicators of target observability in cluttered environment; these parameters are used to improve both tracking accuracy and convergence rate of achieving consensus among the nodes. Stability and noise immunity of the whole distributed tracking method are also theoretically analyzed in detail, by applying the methodology introduced in [2, 12], as well as the recent results concerning properties of the modified deterministic discrete Riccati Eq. [12, 17, 18]. The paper also introduces appropriate modifications of the original versions of KCF and ICF incorporating IPDA methodology [7, 8], necessary for a fair comparison with the proposed algorithm (IPDA-KCF and IPDA-ICF). Tracking quality of the proposed algorithm has been analyzed in detail by simulation. The presented results give an insight into the main properties of the algorithm and show that it *outperforms both alternative algorithms*, in spite of lower communication and computation requirements.

The outline of the paper is as follows. Section 2 is devoted to the problem definition and the basic notions related to the IPDA methodology. Section 3 contains a description of the new distributed tracking algorithm (IPDA-ACF), including the algorithm definition and presentation of the adaptation strategy. A theoretical analysis of stability of the proposed adaptive tracking algorithm, as well as of the reduction of noise influence is given in Section 4. Section 5 is devoted to the formulation of IPDA-KCF and IPDA-ICF algorithms. Section 6 contains the results of simulation analysis, and Section 7 concluding remarks.

Notation. In the paper, we shall use t for discrete time and subscript i to indicate ith node (sensor) in a network; more specifically, we shall use the following notations:

x	Target state vector
F	State transition matrix
e	Process noise vector
G	Process noise covariance matrix
$\mathcal{G}$	Directed graph reflecting the network communication topology
$\mathcal{N}_i$	In-neighborhood of ith node
$\mathcal{J}_i$	Closed in-neighborhood of ith node

$z_{i,j}$	j-th measurement
v_i	Measurement noise vector
H_i	Measurement noise covariance matrix
$Z_i(t)$	Set of all measurements at time t
Z_i^t	Set of all measurements up to time t
O_i^t	Event that the target is perceivable at time t
$\pi_i(t\|t-1)$	Predicted probability of target perceivability
$\pi_i(t\|t)$	Updated probability of target perceivability
P_i^D	Probability of detection
P_i^g	Probability of target-originated measurement falling inside the gate (FoV)
$\tilde{z}_{i,j}$	Residual of the jth measurement
S_i	Covariance of $\tilde{z}_{i,j}$
λ_i	Clutter spatial density
V_i	Gate volume
$\theta_{i,j}$	Event that the j-th measurement is target originated
$\theta_{i,0}$	Event that the target-originated measurement is not in the gate or nonexistent
$\beta_{i,j}^{[1]}$	Probability of $\theta_{i,j}, j = 0, 1, \ldots, m_i$, conditioned on O_i^t and on Z_i^t
$\beta_{i,j}^{[2]}$	Probability of $\theta_{i,j}, j = 0, 1, \ldots, m_i$, conditioned on the event that the target is present and on Z_i^t
$\xi_i(t\|t-1)$	Predicted state estimate
$\xi_i(t\|t)$	Updated state estimate
$P_i(t\|t-1)$	Predicted estimation error covariance matrix
$P_i(t\|t)$	Updated estimation error covariance matrix
L_i	Kalman gain matrix
$\mathcal{C}, \mathcal{A}$	Single step consensus operators
ω_i	Scalar defining node importance
u_i	Information vector
U_i	Information matrix
y_i	Sum of information vectors across neighbors
B_i	Sum of information matrices across neighbors
$\\|\cdot\\|$	Spectral norm

2 Problem definition; Integrated Probabilistic Data Association

Consider a target modeled as a dynamic stochastic linear time-invariant discrete-time system, given by

$$x(t+1) = Fx(t) + Ge(t), \tag{1}$$

where $t \in \mathcal{I}^+$, $\mathcal{I}^+$ is the set of nonnegative integers, $x \in \mathbb{R}^n$ is the target state vector, e zero-mean white Gaussian noise with covariance matrix Q, and F and G constant matrices with appropriate dimensions (see, e.g., [19]).

Assume that we have N *intelligent sensors* forming a *network*, formally represented by a *directed graph* $\mathcal{G} = (\mathcal{N}, \mathcal{E})$, where $\mathcal{N}$ is the set of nodes and $\mathcal{E}$ the set of arcs. Assume also that $\mathcal{N}_i$ is the in-neighborhood of the node i, containing the head nodes of the arcs entering the node i; define also $\mathcal{J}_i = \mathcal{N}_i \cup \{i\}$. Each *node* (camera) is supposed to have, in general, *limited sensing and communication ranges*, which determine at each t the set of nodes that can observe the target and the sets of neighboring nodes exchanging messages (these sets may contain, in general, low percentage of the total number of nodes). We assume that real measurements are *cluttered*, i.e., in addition to data originating from the target, a set of measurements corresponds to *no targets* [9–11, 19]. At time t, node i gets $m_i(t)$ measurements, denoted as $z_{i,j}(t)$, $j = 1, \ldots, m_i(t)$. Under the hypothesis that the measurement $z_{i,j}(t)$ originates from the target, the *sensing model* for the ith node is given by

$$z_{i,j}(t) = H_i x(t) + v_i(t), \tag{2}$$

where $z_{i,j}(t) \in \mathbb{R}^{p_i}$ is the measurement vector, H_i a constant output matrix, and v_i zero-mean white Gaussian measurement noise with covariance matrix R_i. Let $Z_i(t) = \{z_{i,1}(t), z_{i,2}(t), \ldots, z_{i,m_i(t)}(t)\}$ denotes the set of all $m_i(t) = m_i^T(t) + m_i^f(t)$ measurements obtained by node i at time t, where $m_i^T(t)$ is the number of target-originated measurements, and $m_i^f(t)$ the number of false clutter-originated measurements. Let $Z_i^t = \{Z_i(1), \ldots, Z_i(t)\}$.

The purpose of tracking is to estimate the state of a target in real time, based on a set of current measurements. For tracking in clutter, conventional state estimation techniques cannot be used because several measurements are available at every scan, and at most, one measurement can arise from the target. The elegant Probabilistic Data Association (PDA) methodology has been successfully applied in this case, including a variety of very successful applications to radars, sonars, and electro-optic systems, e.g., [9–11, 20–25]. This methodology is based on the implicit strong assumption that the target is always *perceivable*. The concept of *target perceivability* is a part of the general methodology of Integrated Probabilistic Data Association (IPDA) and represents a refinement of the concept of track existence (which does not address the possibility that a target cannot be detected) and of target observability (which presumes the presence of a target) [13–15]. According to [15], *a target is perceivable if it is present and can be detected by the sensors used.* If O_i^t and $\bar{O}_i^t$ denote the events that the target is perceivable and not perceivable, respectively, at time t by node i, we shall follow [13] and assume that the target perceivability can be modeled by a *first order Markov model*, with $p_i^{11} = P\left(O_i^t|O_i^{t-1}\right)$ and $p_i^{21} = P\left(O_i^t|\bar{O}_i^{t-1}\right)$, see [14, 15]. If $\pi_i(t|t-1)$ and $\pi_i(t|t)$ are the predicted and updated probabilities of target perceivability for a given track, respectively, the total probability theorem gives

$$\pi_i(t|t-1) = p_i^{11}\pi_i(t-1|t-1)+p_i^{21}[1-\pi_i(t-1|t-1)]\,. \tag{3}$$

According to the basic references related to the IPDA methodology [13–15], the updated and predicted *target perceivability probabilities* are connected by

$$\pi_i(t|t) = \frac{[1-\phi_i(t)]\,\pi_i(t|t-1)}{1-\phi_i(t)\pi_i(t|t-1)}, \tag{4}$$

where

$$\phi_i(t) = P_i^D P_i^g \left[1 - m^{-1}\sigma_i(t)V_i(t)\frac{\mu_i(t,m-1)}{\mu_i(t,m)}\right]$$

for $m_i(t) = m \neq 0$, and

$$\phi_i(t) = P_i^D P_i^g$$

for $m_i(t) = 0$, P_i^D is the detection probability (assuming that the target is perceivable), P_i^g is the probability that a target-originated measurement falls inside the gate (FoV) of the sensor i,

$$\sigma_i(t) = \left(P_i^g\right)^{-1} \Sigma_{j=1}^m \mathcal{N}(\tilde{z}_{i,j}(t);0,S_i(t))$$

for $m_i(t) \neq 0$, and

$$\sigma_i(t) = 0$$

for $m_i(t) = 0$, where $\tilde{z}_{i,j}(t) = z_{i,j}(t) - \hat{z}_{i,j}(t)$ is the residual of the jth measurement by node i ($\hat{z}_{i,j}(t)$ denotes the optimal estimate of $z_{i,j}(t)$), and $S_i(t)$ its covariance,

$$\mu_i(t,m) = \frac{(\lambda_i(t)V_i(t))^m}{m!}e^{-\lambda_i(t)V_i(t)},$$

$\mu_i(t,m) = P\left\{m_i^f(t) = m|Z_i^{t-1}\right\}$, $\lambda_i(t)$ is the clutter spatial density and $V_i(t)$ the gate volume (under the classical *Gaussian assumption* for the target and the *Poisson model* for clutter [14, 15]).

Let $\theta_{i,j}$ be the event that the jth measurement of the ith node is target originated, and $\theta_{i,0}$ that the target-originated measurement is not in the gate (FoV) or nonexistent. Define

$$\beta_{i,j}^{[1]}(t) = P\left\{\theta_{i,j}|O_i^t, Z_i^t\right\}, \quad j = 0,1,\ldots,m_i(t). \tag{5}$$

It is possible to show that the probabilities $\beta_{i,j}^{[1]}(t)$ are identical to the well-known weights ("β"-parameters) of the classical PDA filter [10, 11]:

$$\begin{aligned}\beta_{i,0}^{[1]}(t) &= \frac{\lambda_i(t)}{\lambda_i(t)+\sigma_i(t)q_i},\\ \beta_{i,j}^{[1]}(t) &= \frac{q_i\left(P_i^g\right)^{-1}\mathcal{N}(\tilde{z}_{i,j}(t);0,S_i(t))}{\lambda_i(t)+\sigma_i(t)q_i},\end{aligned} \tag{6}$$

for $j = 1,\ldots,m_i(t)$, where $q_i = P_i^D P_i^g / \left(1 - P_i^D P_i^g\right)$. Probabilities $\beta_{i,j}^{[1]}(t)$ are used within the formulation of the target perceivability-based PDAF in [15].

In general, it can be of interest to estimate the state of a target that is *present no matter whether it can be detected or not*. One may define the following probabilities [15]:

$$\beta_{i,j}^{[2]}(t) = P\left\{\theta_{i,j}|T, Z_i^t\right\}, \quad j = 0,1,\ldots,m_i(t), \tag{7}$$

where T stands for the event that a *target is present*. Following [15], one can show that

$$\begin{aligned}\beta_{i,0}^{[2]}(t) &= \frac{\lambda_i(t)}{\lambda_i(t)+\sigma_i(t)r_i(t)},\\ \beta_{i,j}^{[2]}(t) &= \frac{r_i(t)\left(P_i^g\right)^{-1}\mathcal{N}(\tilde{z}_{i,j}(t);0,S_i(t))}{\lambda_i(t)+\sigma_i(t)r_i(t)},\end{aligned} \tag{8}$$

for $j = 1,\ldots,m_i(t)$, where $r_i(t) = P_i^D P_i^g \pi_i(t|t-1)/\left[1 - P_i^D P_i^g \pi_i(t|t-1)\right]$. The probabilities (8) represent a part of the target existence-based PDAF formulated in [15] and depend on the probabilities (6) and the perceivability probabilities (4). Recently, the concept of target existence probability has been used in the context of multi-target tracking in multi-static passive radar systems [26]. Notice that $\beta_{i,j}^{[2]}(t) \leq \beta_{i,j}^{[1]}(t)$, $j = 1,\ldots,m_i(t)$ and that, consequently, $\beta_{i,0}^{[2]}(t) \geq \beta_{i,0}^{[1]}(t)$ (see [14, 15] for a more complete discussion).

In the context of distributed target tracking by sensor networks considered in this paper, all the above statements hold locally, for each sensor. It will be seen below that the choice between $\beta_{i,j}^{[1]}(t)$ and $\beta_{i,j}^{[2]}(t)$ offers an additional possibility for adaptation to the current situation concerning the target, the entire sensor network, and its environment.

3 Adaptive consensus filter: IPDA-ACF

3.1 Tracking algorithm

In this subsection, we shall present the main IPDA-ACF algorithm, with its two versions (IPDA-ACF^l, $l = 1,2$), depending on the choice of the "β"-parameters: for $l = 1$, one assumes target perceivability (IPDA-ACF^1) and for $l = 2$ target existence (IPDA-ACF^2). The algorithm has the following form:

$$\begin{aligned}\xi_i(t|t) &= \xi_i(t|t-1) + L_i(t)\tilde{z}_i(t),\\ \xi_i(t+1|t) &= F\mathcal{C}(\xi_i(t|t)),\end{aligned} \tag{9}$$

$i = 1,\ldots,N$, where ξ_i is an estimate of x generated by the ith node, $\tilde{z}_i(t) = \sum_{j=1}^{m_i(t)} \beta_{i,j}^{[l]}(t)\tilde{z}_{i,j}(t)$, $\tilde{z}_{i,j}(t) = z_{i,j}(t) - H_i\xi_i(t|t-1)$, $l = 1,2$, $\mathcal{C}(\cdot)$ is a single step consensus operator

$$\mathcal{C}(\xi_i(t|t)) = \sum_{j\in\mathcal{J}_i} c_{ij}(t)\xi_j(t|t), \tag{10}$$

where $c_{ij}(t)$, $i,j = 1,\ldots,N$, are (generally) time varying weights, such that $N \times N$ matrix $C(t) = [c_{ij}(t)]$ (*consensus matrix*) is *row-stochastic* for all t, and

$$L_i(t) = P_i(t|t-1)H_i^T S_i(t)^{-1} \tag{11}$$

is the local Kalman gain obtained from (1) and (2), using

$$\begin{aligned}
P_i(t|t) &= P_i(t|t-1) \\
&\quad + \left[\beta_{i,0}^{[l]}(t) - 1\right] L_i(t)S_i(t)L_i(t)^T + \tilde{P}_i(t,t), \\
\tilde{P}_i(t,t) &= L_i(t)\left[\sum_{j=1}^{m_i(t)} \beta_{i,j}^{[l]}(t)\tilde{z}_{i,j}(t)\tilde{z}_{i,j}(t)^T - \tilde{z}_i(t)\tilde{z}_i(t)^T\right] L_i(t)^T, \\
P_i(t+1|t) &= FP_i(t|t)F^T + GQG^T, \\
S_i(t) &= H_iP_i(t|t-1)H_i^T + R_i.
\end{aligned} \tag{12}$$

The above algorithm follows structurally the consensus-based overlapping estimation algorithm proposed in [12]. Instead of the local estimators of Luenberger type from [12], we have here trackers of PDA type [10]. The algorithm contains two main parts:

1) The filtering part, in which the local measurements are processed
2) The prediction part, in which the agreement between the nodes is enforced through a convex combination of the estimates communicated by the neighboring nodes $\mathcal{C}(\xi_i(t|t))$.

Obviously, the algorithm requires only the exchange of state estimates (size $n \times 1$). The choice of l in $\beta_{i,j}^{[l]}(t)$ results from a predefined setting. For $l = 1$, one obtains the estimation scheme resulting directly from the PDAF [10], while for $l = 2$, the algorithm follows the existence-based PDAF from [15] and becomes more capable of coping with high level of clutter and possible target occlusions (typical for camera networks - see, e.g., [27] for a general discussion).

The choice of the consensus matrix C(t) can be based, in general, on different principles, including the well-known standard Metropolis weights (see, e.g., [28] and the references therein) and optimization procedures providing the fastest convergence to consensus [29, 30]. However, in the above context (limited FoV, clutter, target occlusions), such approaches provide insufficient tracking accuracy of the proposed algorithm in real-time applications. One should bear in mind that we have, in the context of the proposed algorithm, the so-called "running consensus" [31, 32], consisting of an inseparable pair estimation algorithm - consensus algorithm, each possessing its own dynamics. It would be possible, in principle, to formulate the problem of defining the consensus matrix in a theoretically unified way, including both the estimation algorithm and the consensus scheme; however, such methodologies can hardly provide results applicable in real time. In order to provide a practically efficient solution, the next subsection contains a presentation of a distributed adaptation applicable in real time, aiming at providing adequate time-varying consensus weights in (10), using the locally available realizations of the β-parameters, in accordance with the basic principles of the IPDA methodology.

3.2 Distributed adaptation strategy

Distributed adaptation strategy represents the core element of the IPDA-ACF algorithm. In the context of the scenario considered in this paper, we follow the basic line of thought from [2, 3, 12] and construct a novel-improved fully distributed adaptation procedure generating $C(t)$ in (10), dynamically giving more importance to the nodes with *higher probability of receiving target-originated measurements*, enabling, at the same time, *fast information flow* through the network. In the sequel, we shall first give a short outline of the original adaptive scheme from [2], and then, we shall present the general adaptation strategy for the tracking algorithm presented above.

1) *Basic adaptation scheme: no clutter*

The distributed adaptive tracking algorithm ACF in [2] has been derived from [12]; it requires *additional inter-node communication* of $\beta_{i,0}^{[1]}(t) \in \{0,1\}$ that are available locally (indicators of measurement availability at time t). Using this information, each node computes $|\mathcal{J}_i|$ scalars $\chi_j^i(t)$, representing observation histories of the neighboring nodes. The nonnormalized consensus gains are given by $c_{ij}^{\chi}(t) = c_{ij}^{\chi}(0)k^{\chi_j^i(t)}$, for $\chi_j^i(t) \geq 0$, and $c_{ij}^{\chi}(t) = c_{ij}^{\chi}(\infty) - \left(c_{ij}^{\chi}(\infty) - c_{ij}^{\chi}(0)\right)k^{|\chi_j^i(t)|}$, for $\chi_j^i(t) \leq 0$, where $c_{ij}^{\chi}(0)$ are the initial values and $c_{ij}^{\chi}(\infty)$ the desired final values; parameter $k \in (0,1]$ determines the rate of change of $c_{ij}^{\chi}(t)$. The normalized consensus gains, ensuring row stochasticity, are obtained from $c_{ij}(t) = \frac{c_{ij}^{\chi}(t)}{\sum_{j\in\mathcal{J}_i} c_{ij}^{\chi}(t)}$, when $j \notin \mathcal{J}_i$, $c_{ij}(t) = 0$. It has been found that the resulting ACF algorithm from [2] outperforms KCF from [6].

2) *General adaptation strategy*

The adaptation procedure proposed in this paper essentially extends and modifies the basic adaptation scheme from [2] in two principal directions: (1) instead of binary indicators of target presence, we utilize real numbers $\beta_{i,0}^{[l]}(t)$ defined by (6) or (8) since they reflect the uncertainty of obtaining a target-originated measurement; (2) in order to enable fast diffusion of the target state estimates throughout the network, the adaptation scheme incorporates its own, specially designed dynamics.

Formally, we introduce the *node importance vector* $\omega(t) = (\omega_1(t),\ldots,\omega_N(t))^T$, generated by the following recursion:

$$\omega(t+1) = \alpha A\omega(t) + \gamma(t+1), \tag{13}$$

which starts from $\omega(0) = (\omega_1(0), \ldots, \omega_N(0))^T = (0, \ldots, 0)^T$, where $\gamma(t) = (\gamma_1(t), \ldots, \gamma_N(t))^T$, $\gamma_i(t) = 1 - \beta_{i,0}^{[l]}(t)$, $A = \left(I + A_{\text{adj}}\right)_{\text{rs}}$, where A_{adj} represents the *adjacency matrix* of the underlying network graph, and $Y = (X)_{\text{rs}}$ denotes, for $N \times N$ matrices X and Y, that Y is obtained from X in such a way that the elements of its rows are divided by the corresponding row-sums, while $\alpha > 0$ is a sufficiently small real number. Using the node importance vector, the elements of the consensus matrix $C(t)$ are given by

$$c_{ij}(t) = \frac{\omega_j(t)}{\sum_{j \in \mathcal{J}_i} \omega_j(t)} \tag{14}$$

for $j \in \mathcal{J}_i$; $c_{ij}(t) = 0$ otherwise.

The algorithm requires that each node receives at each t real numbers $\omega_j(t)$ from its neighbors. It is obvious that the recursion (13) is implementable in a *distributed way*, having in mind that locally

$$\omega_i(t+1) = \frac{\alpha}{1+\nu_i} \sum_{j \in \mathcal{J}_i} \omega_j(t) + \gamma_i(t+1), \tag{15}$$

where ν_i is the number of nonzero elements of ith row of A_{adj}.

The input vector $\gamma(t+1)$ in (13) reflects the *current network-wide perceivability*, i.e., its elements represent the local *probabilities of getting target-originated measurements* (in the case of no clutter, $\gamma_i(t)$ is a binary random variable, equal to one when the target is observed, and to zero otherwise [2]). It can be considered that $\gamma_i(t)$ provides a direct measure of "quality" of the current estimate of the ith node, obtained after the local filtering phase (prior to the application of the consensus scheme). Namely, when a target is successfully being tracked and ith node receives measurement from it, $\beta_{i,j}^{[l]}(t)$ connected to that measurement has high values (close to one), while all the other $\beta_{i,j}^{[l]}(t)$ are generally small, as well as $\beta_{i,0}^{[l]}(t)$ (close to zero). On the other hand, if the ith node does not receive measurement from the target, all $\beta_{i,j}^{[l]}(t)$ will be generally close to zero and $\beta_{i,0}^{[l]}(t)$ close to one. Obviously, the difference between the choices $l = 1$ and $l = 2$ lies in the fact that in the second case, the resulting tracking algorithm is less sensitive to the availability of measurements than in the case when $l = 1$; this choice is well adapted to the high level of clutter and to the targets that are temporarily not perceivable (as stated above). Another choice for $\gamma_i(t)$ could be $\gamma_i(t) = \max_{j=1,\ldots,m_i(t)} \beta_{i,j}^{[l]}(t)$, which yielded in our experiments the results comparable to those obtained by the above formulated algorithm.

The role of the first term at the right hand side of (13) containing αA is to enable fast *decentralized diffusion* of the local state estimates *throughout the whole network*. Under the standard assumption that the graph $\mathcal{G}$ has a center node (a node from which all the nodes are reachable), matrix A has one simple eigenvalue at one, while the remaining ones are inside the unit circle. In this case, we have that A^k converges, when $k \to \infty$, to a constant matrix with equal rows composed of positive elements. This means, having in mind (13), that all the nodes will have asymptotically positive values $\omega_i(\infty)$, $i = 1, \ldots, N$. The multiplying constant α defines the memory length of the algorithm; obviously, for $\alpha < 1$, the recursion (13) is asymptotically stable. The chosen value of α should be small, in order to be able to efficiently exploit current measurements, and large enough, in order to sufficiently smooth out stochastic variations in the random sequence $\{\gamma(t)\}$.

In order to better clarify the basic effects of choosing α and A, consider w.l.o.g. the situation in which $\gamma(t+1) = \gamma^{[1]}$, where $\gamma_1^{[1]} \approx 1$ and the remaining $\gamma_i^{[1]} \approx 0$, $i = 2, \ldots, N$. The asymptotic response of (13) to such an input signal is given by

$$\lim_{t \to \infty} \omega(t+1) = \lim_{T \to \infty} \sum_{t=0}^{T} (\alpha A)^t \gamma^{[1]}. \tag{16}$$

Having in mind (16) and (14), there are now two main possibilities: (1) the rows of $C(\infty)$ with $c_{i1} \neq 0$ contain, in fact, $c_{i1} \approx 1$, while the remaining elements are of the order of magnitude of α; (2) the rows of $C(\infty)$ with $c_{i1} = 0$ contain the nonzero elements having different order of magnitude, depending on graph topology, irrespective of α. In the first case, the convex combination from (9) will give the highest emphasis to the neighbors observing the target with high probability, while in the second, node priorities result from the given graph. In the first case, we have prompt reaction for small α, while in the second, diffusion rate of the state estimates is maximized. In our experiments, we adopted α to be around 0.05.

Asymptotic properties of the proposed adaptation scheme capture the perceivability history in a more efficient way than the original scheme from [2]. For example, in the cases when two nodes have the same γ_i, the first term at the right hand side of (15) gives more weight to the node that previously received information, coming more likely from the nodes that had observed the target (see the example below and the simulation results).

The proposed algorithm is summarized in Algorithm 1.

Example. In order to demonstrate clearly the positive effect of introducing specially designed dynamics into the original adaptive scheme from ACF, we shall consider the following simple illustrative example of a sensor network with the ring topology and $N = 6$ nodes, where each node is connected to 2 neighboring nodes and only the first node observes the target; there are no false measurements. The adaptation parameters have been chosen as $c_{ij}^{\chi}(\infty) = 1$, $c_{ij}^{\chi}(0) = 0.01$, $k = 0.2$ (original adaptive

Algorithm 1 IPDA-ACF^l for node i at time t

Input: $\xi_i(t|t-1)$, $P_i(t|t-1)$, $\tilde{\omega}_i(t-1)$
Get measurements: $z_{i,j}(t), j = 1, \ldots, m_i(t)$
Compute:
$\tilde{z}_{i,j}(t) = z_{i,j}(t) - H_i\xi_i(t|t-1)$, $S_i(t) = H_iP_i(t|t-1)H_i^T + R_i$
$\beta_{i,0}^{[l]}(t)$, $\beta_{i,j}^{[l]}(t)$ from (6) or (8) ($l = 1$ or $l = 2$, respectively)
$\tilde{z}_i(t) = \sum_{j=1}^{m_i(t)} \beta_{i,j}^{[l]}(t)\tilde{z}_{i,j}(t)$
$L_i(t) = P_i(t|t-1)H_i^T S_i(t)^{-1}$, $P_i(t|t)$ from (12)
$\xi_i(t|t) = \xi_i(t|t-1) + L_i(t)\tilde{z}_i(t)$
$\gamma_i(t) = 1 - \beta_{i,0}^{[l]}(t)$, $\omega_i(t) = \frac{\alpha}{1+\nu_i}\tilde{\omega}_i(t-1) + \gamma_i(t)$
Send data: $\xi_i(t|t)$, $\omega_i(t)$
Receive data: $\xi_j(t|t)$, $\omega_j(t)$, $j \in \mathcal{J}_i$
Compute:
$\tilde{\omega}_i(t) = \sum_{j\in\mathcal{J}_i} \omega_j(t)$
$c_{ij}(t) = \frac{\omega_j(t)}{\tilde{\omega}_i(t)}$ for $j \in \mathcal{J}_i$, $c_{ij}(t) = 0$ else
Perform consensus: $\mathcal{C}(\xi_i(t|t)) = \sum_{j\in\mathcal{J}_i} c_{ij}(t)\xi_j(t|t)$
Compute:
$\xi_i(t+1|t) = F\mathcal{C}(\xi_i(t|t))$
$P_i(t+1|t) = FP_i(t|t)F^T + GQG^T$
Output: $\xi_i(t+1|t)$, $P_i(t+1|t)$, $\tilde{\omega}_i(t)$

scheme from [2]), and $\alpha = 0.03$, $\beta_{i,0}^{[l]}(t) = 0$ if the ith node observes the target at time instant t and $\beta_{i,0}^{[l]}(t) = 1$ otherwise (the proposed general adaptation strategy). Figure 2 shows the time evolution of the resulting nonzero consensus weights connected to the original scheme (ACF_1) and the proposed algorithm (ACF_2). It can be seen that the main difference between the algorithms is in the consensus weights connected to the nodes that do not have measuring nodes in their neighborhoods. The original algorithm assigns equal consensus weights in the consensus matrix rows connected to these nodes (3rd, 4th, and 5th row in Fig. 2a), while the proposed general algorithm gives more weight to the nodes that are closer to the measuring node (3rd, 4th, and 5th row in Fig. 2b), enabling better information flow through the network which, in turn, allows faster convergence and better tracking.

Remark 1 *The exposed methodology for distributed adaptive generation of the consensus weights in (10) is mainly motivated by the idea to exploit the locally available "β"-parameters in order to assign higher importance to the nodes with higher probabilities of getting target-originated measurements; it has no other direct connection with the main state estimation algorithm based on the PDAF. Therefore, it is easy to conclude that the results of this section can be readily used in the context of any nonlinear estimation algorithm, provided an estimate of the probabilities contained in the "β"-parameters is available. In this sense, it is straightforward to construct adaptive algorithms based, e.g., on the extended Kalman filter (like in [8]), on the unscented Kalman filter, and particle filters. It is also possible to apply the proposed methodology within the framework of the ML-PDA method applicable in real time [20–25]. The adaptation strategy should again cope mainly with the limited FoVs (or sensing ranges).*

4 Stability and reduction of noise influence

4.1 Stability

Stability and noise reduction of the proposed distributed tracking algorithm will be analyzed starting from the following global network model:

$$X(t+1|t) = \Phi(t)X(t|t-1) + \Psi(t)Z(t), \tag{17}$$

where $X(t+1|t) = \left(\xi_1(t+1|t)^T, \ldots, \xi_N(t+1|t)^T\right)^T$, $\Phi(t) = [\,\Phi_{ij}(t)]$, $i,j = 1,\ldots,N$, in which $\Phi_{ij}(t) = c_{ij}(t)F\left[I - \left(1-\beta_{j,0}^{[l]}(t)\right)L_j(t)H_j\right]$ are $m \times m$ blocks, $\Psi(t) = [\,\Psi_{ij}(t)]$, in which $\Psi_{ij}(t) = c_{ij}(t)FL_j(t)$ are $m \times p$ blocks defined by (11), and $Z(t) = \left(\sum_{k=1}^{m_1(t)} \beta_{1,k}^{[l]}(t)\tilde{z}_{1,k}(t), \ldots, \sum_{k=1}^{m_n(t)} \beta_{N,k}^{[l]}(t)\tilde{z}_{N,k}(t)\right)^T$.

To the best of the authors knowledge, no general methodology is directly applicable to the stability analysis of the model (17), not to mention many open questions related to the local recursions of PDA type [17]. We shall here make an attempt to apply the methodology proposed in [2, 12] in relation with distributed estimation schemes with overlapping subsystems and to get an additional insight into the complex relationship between the local trackers and the adaptive consensus scheme. Using the basic results from [12, 33–35], we define the norm of the block-matrix $\Phi(t) = [\,\Phi_{ij}(t)]$ as

$$\|\Phi(t)\|_\tau^* = \|[\,\|\Phi_{ij}(t)\|_\tau]\,\|_\infty,$$

where $\|Y\|_\tau$ denotes the norm of a square matrix Y defined as $\|Y\|_\tau = \left\|D_\tau U^T Y U D_\tau^{-1}\right\|_\infty$, in which U is an orthogonal matrix in the representation $Y = U\Delta U^T$, where Δ is an upper triangular matrix containing the eigenvalues of Y at the diagonal (according to the Schur's theorem [35]) and $D_\tau = \text{diag}\{\tau, \tau^2, \tau^3, \ldots, \tau^n\}$. It is essential to notice that it is possible to find, for any given $\varepsilon > 0$, such a $\tau_\varepsilon > 0$ that, for all $\tau > \tau_\varepsilon$, $\rho(Y) \le \|Y\|_\tau \le \rho(Y) + \varepsilon$, where $\rho(Y)$ is the spectral radius of a square matrix Y [12, 35]. Without going into details related to specific properties of the adopted norm in the case of (17), notice that the main idea behind the adopted definition is to take $\|\Phi(t)\|_\tau^*$ as an estimate of $\rho(\Phi(t))$ based on the local algorithm properties expressed by $\|\Phi_{ij}(t)\|_\tau$.

Concentrating on $\Phi(t)$, we shall start from the following sufficient conditions for asymptotic stability of the linear time-varying system (17) [36, 37]:

1) $\Phi(t)$ is strictly Schur for all t
2) For some constant $\mu \in [0, 1)$

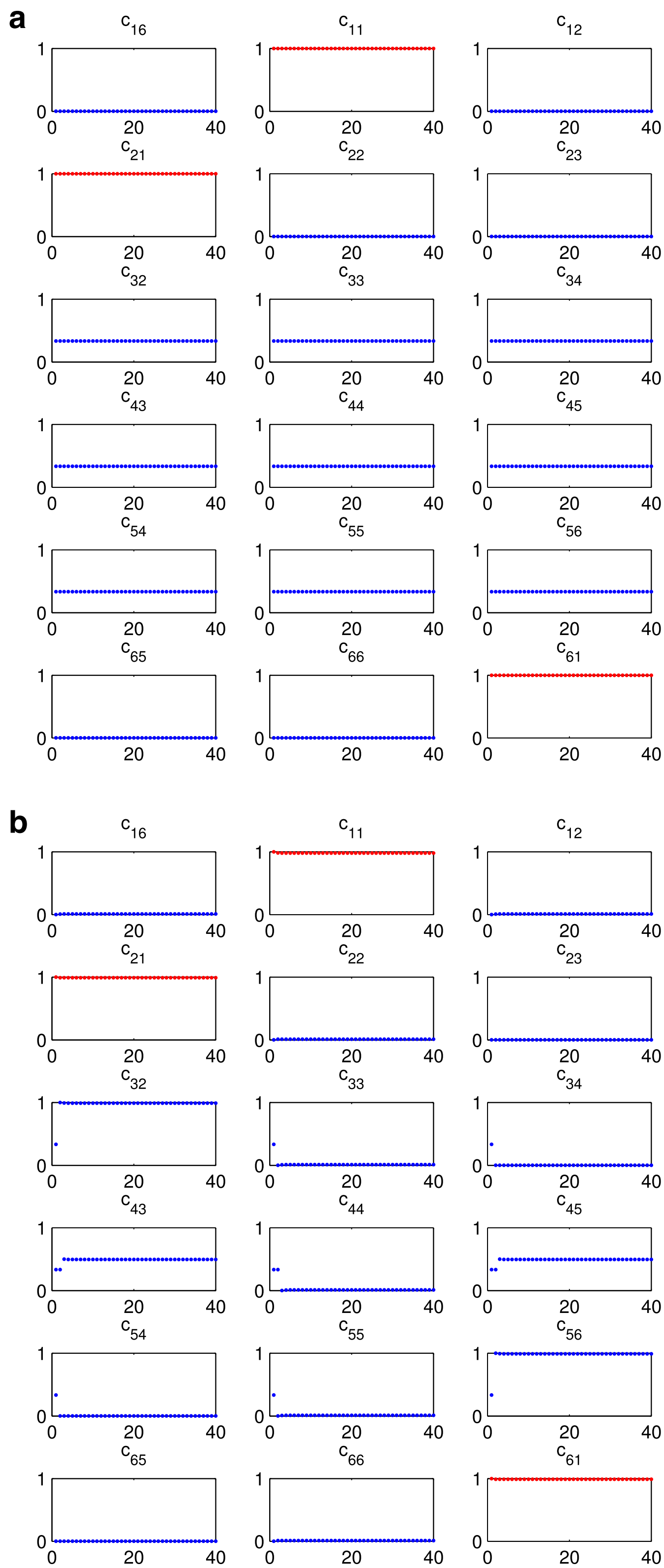

Fig. 2 Time evolution of the nonzero consensus weights. Weights corresponding to the measuring node are shown in red. **a** The original adaptation procedure (ACF_1). **b** The refined adaptation procedure (ACF_2)

$$\|\Delta(\Phi(t)\otimes\Phi(t))\| \leq \frac{\mu}{(nN)^{\frac{1}{2}}}\sigma_{\min}(\Phi(t)\otimes\Phi(t)-I) \cdot\sigma_{\min}(\Phi(t+1)\otimes\Phi(t+1)-I), \qquad (18)$$

where $\Delta(\Phi(t)\otimes\Phi(t))=\Phi(t+1)\otimes\Phi(t+1)-\Phi(t)\otimes\Phi(t)$, $\otimes$ denotes the Kronecker's product and $\sigma_{\min}(\cdot)$ the minimal singular value [37].

For τ large enough, $\left\|F\left[I-\left(1-\beta_{j,0}^{[l]}(t)\right)L_j(t)H_j\right]\right\|_\tau$ becomes a good estimate of $\rho\left(F\left[I-\left(1-\beta_{j,0}^{[l]}(t)\right)L_j(t)H_j\right]\right)$, i.e., there exists such a $\tau'>0$ that for all $\tau>\tau'$, we have $\left\|F\left[I-\left(1-\beta_{j,0}^{[l]}(t)\right)L_j(t)H_j\right]\right\|_\tau < \rho\left(F\left[I-\left(1-\beta_{j,0}^{[l]}(t)\right)L_j(t)H_j\right]\right)+\varepsilon$, for any $\varepsilon>0$. Coming back to the system (17), we derive that

$$\lim_{\tau\to\infty}\|\Phi(t)\|_\tau^* = \left\|\left[c_{ij}(t)\rho\left(F\left[I-\left(1-\beta_{j,0}^{[l]}(t)\right)L_j(t)H_j\right]\right)\right]\right\|_\infty. \qquad (19)$$

At this point, an insight into the local recursions (12) and (11) provides the following general conclusions:

1) When $P_i(t|t)$ diverges (see [17] for general observations concerning stability properties of PDAF), $\rho\left(F\left[I-\left(1-\beta_{j,0}^{[l]}(t)\right)L_j(t)H_j\right]\right)$ diverges and, consequently, $\|\Phi(t)\|_\tau^*$ defined by (19) diverges, as well;
2) When $P_i(t|t)$ remains bounded, and condition (18) holds for some $\mu\in[0,1)$, the corresponding $L_i(t)$ could be not stabilizing in the sense of ensuring the condition $\rho\left(F\left[I-\left(1-\beta_{j,0}^{[l]}(t)\right)L_j(t)H_j\right]\right)<1$ for all $i,j=1,\ldots,N$. However, in spite of this, the condition $\lim_{\tau\to\infty}\|\Phi(t)\|_\tau^*<1$ in (19) may still be achieved, provided the coefficients $c_{ij}(t)$ $\left(\sum_{j=1}^N c_{ij}(t)=1\right)$ are chosen appropriately. The overall stabilizing property results from the network; from this point of view, it is obvious that the adaptation procedure in the proposed algorithm should be such that the nodes with higher detection probability (lower $\beta_{i,0}^{[l]}(t)$) should be given higher priority (see the previous subsection for more details). This is exactly provided by the proposed adaptation scheme given in Section 3.2;
3) When $P_i(t|t)$ is stabilizing in the sense that $\rho\left(F\left[I-\left(1-\beta_{j,0}^{[l]}(t)\right)L_j(t)H_j\right]\right)<1$ for all $i,j=1,\ldots,N$ [38], $\lim_{\tau\to\infty}\|\Phi(t)\|_\tau^*<1$ for admissible choice of the consensus weights.

Having in mind the stochastic time-varying nature of the basic recursions (12), the above general conclusions require verification in all particular cases; one should also bear in mind that, to the authors best knowledge, there is no rigorous stability analysis of the PDAF in the general case which would give a possibility to estimate $\rho(\Phi_{ij}(t))$ [17, 39]. In order to provide additional clarifications to the above qualitative statements, we shall focus our attention on an approximation of (12), in which the random matrices $P_i(t|t)$ and the probabilities $\beta_{i,0}^{[l]}(t)$ are replaced by their expected values (see, e.g., [17, 39]). Without going here into details, we shall utilize the results from [17, 18] and obtain that $P_i(t|t)$ in (12) gives rise to the following modified deterministic discrete Riccati equation

$$\bar{P}_i(t|t)=\bar{P}_i(t|t-1)-q_2^{(i)}\bar{L}_i(t)\bar{S}_i(t)\bar{L}_i(t)^T, \qquad (20)$$

where $q_2^{(i)}$ is a constant $\left(q_2^{(i)}\in(0,1]\right)$, while $\bar{L}_i(t)$ and $\bar{S}_i(t)$ follow analogously from the formulae for $L_i(t)$ and $S_i(t)$ in (12), $i=1,\ldots,N$. Under the standard detectability condition of the pairs (F,H_i) and positive definiteness of the noise covariance matrices R_i, the important result from [18] leads to the conclusion that the recursion (20) converges (irrespective of the initial condition) to the unique positive definite solution of the following modified algebraic Riccati equation, resulting from (12) and (11):

$$\bar{P}_i=F\bar{P}_iF^T-q_2^{(i)}F\bar{P}_iH_i^T(H_i\bar{P}_iH_i^T+R_i)^{-1}H_i\bar{P}_iF^T+GQG^T \qquad (21)$$

for all values of $q_2^{(i)}\in\left(\tilde{q}_2^{(i)},1\right]$, where $1-\frac{1}{(\rho(F))^2}\leq\tilde{q}_2^{(i)}\leq 1-\frac{1}{(M(F))^2}$ is the critical value of the characteristic parameter $q_2^{(i)}$, $M(F)=\prod_{l=1}^n\max\{1,|\lambda_l(F)|\}$. Let $p_1^{(i)}=1-\left(1-q_2^{(i)}\right)^{\frac{1}{2}}$ and $p_2^{(i)}=1+\left(1-q_2^{(i)}\right)^{\frac{1}{2}}$; then, every positive definite solution of (21) renders the matrix

$$\bar{F}_i\left(p^{(i)}\right)=F-p^{(i)}F\bar{P}_iH_i^T\left(H_i\bar{P}_iH_i^T+R_i\right)^{-1}H_i^T \qquad (22)$$

to be Schur for all $p^{(i)}\in\left[p_1^{(i)},p_2^{(i)}\right]$ (the stabilizing solution of (21)) [18] .

Assume next that the described approximations are adopted instead the original local PDAFs and that the recursions (20) converge to the unique positive definite matrices $\bar{P}_i$ defined by (21). According to (9), asymptotic stability of the resulting approximation of the tracker model depends exclusively on the eigenvalues of $\bar{F}_i(p^{(i)})$, where $p^{(i)}=1-\bar{\beta}_i^{[l]}=P_D^{(i)}P_G^{(i)}$, $P_D^{(i)}$ representing the local detection probability and $P_G^{(i)}$ the local probability that the correct measurement (if the target is detected) lies within the predefined gate [17]. Accordingly, we immediately conclude that for $\bar{\beta}_i^{[l]}<\left(1-q_2^{(i)}\right)^{\frac{1}{2}}$, the adopted approximation of the tracker model is asymptotically stable; however, for $\bar{\beta}_i^{[l]}>\left(1-q_2^{(i)}\right)^{\frac{1}{2}}$, this model may be not asymptotically stable, in spite of the fact that (21) has a positive definite solution [18]. Using (19), we obtain

$$\lim_{\tau\to\infty} \|\Phi(t)\|_\tau^* = \left\| \left[c_{ij}(t) \lim_{\tau\to\infty} \left\| F \left[I - \left(1 - \bar{\beta}_{j,0}^{[l]}\right) \bar{L}_j H_j \right] \right\|_\tau \right] \right\|_\infty . \quad (23)$$

As above, the condition $\lim_{\tau\to\infty} \|\Phi(t)\|_\tau^* < 1$ may be achieved for (17) not only when $\bar{F}_i(1-\bar{\beta}_{i,0})$ is Schur for all i but also in the situations when for some nodes, matrix $\bar{F}_i(1-\bar{\beta}_{i,0})$ is not Schur. In this sense, the overall stability of the estimator depends directly on the choice of $c_{ij}(t)$: the better the coefficients $c_{ij}(t)$ follow the essentially time varying detection probabilities, expressed through the importance of the nodes, the closer the whole system is to stability. A more detailed analysis could be connected to specific target and observation models, taking into account the corresponding structure of the solution of (21) and its dependence on the underlying parameters (notice, for example, that $\rho(F) = 1$ for standard kinematic target models). Such an analysis would be beneficial for practice not only from the point of view of stability (which is expected to hold for a large variety of definitions of $c_{ij}(t)$) but also from the point of view of tracking accuracy.

4.2 Reduction of noise influence

In order to provide an insight into the influence of measurement noise to the overall estimation error, we can write simply $E(t+1|t) = X(t+1|t) - X(t+1)$, where $X(t) = \left(x(t)^T, \ldots, x(t)^T\right)^T$, and obtain from (17) that the stochastic driving term in the relation for $E(t+1|t)$, appearing as a consequence of the measurement noise, is given by $V_c(t) = \Psi(t)v(t)$, where $v(t) = \left(v_1(t)^T, \ldots, v_N(t)^T\right)^T$. The effect of this driving term to the overall estimation error covariance can be immediately seen from the expression for $P_c(t) = E\left\{V_c(t)V_c(t)^T\right\}$. Assuming the deterministic modified Riccati Eq. (20) and the state transition matrix (22), as well as uncorrelatedness of the local measurement noises, we obtain that the covariance of $V_c(t)$ at node i, conditioned by the consensus weights, is given by

$$P_c^{(i)}(t) = \sum_{j=1}^{N} c_{ij}(t)^2 \left(p^{(j)}\right)^2 F L_j R_j L_j^T F^T. \quad (24)$$

For equal noise covariances $R_j = R$, we obtain the following inequality $\sum_{j=1}^{N} c_{ij}(t)^2 \left(p^{(j)}\right)^2 FLRL^TF^T \leq FLRL^TF^T$, having in mind row stochasticity of $C(t)$. The last term represents the covariance of the driving term in the case when only one node has access to measurements ($p^{(i)} = 1$). Evidently, the applied consensus scheme decreases the noise influence by averaging over the set of measuring nodes.

5 Alternative IPDA-based distributed trackers

5.1 Kalman-consensus filter: IPDA-KCF

We shall define information vectors and information matrices as $u_i(t) = H_i^T R_i^{-1} \sum_{j=1}^{m_i(t)} \beta_{ij}^{[l]}(t) z_{ij}(t)$ and $U_i(t) = H_i^T R_i^{-1} H_i$, respectively, as well as their sums across neighbors: $y_i(t) = \sum_{j\in\mathcal{J}_i} u_j(t)$ and $B_i(t) = \sum_{j\in\mathcal{J}_i} U_j(t)$ [6, 7].

The so-called JPDA Kalman-consensus filter (JPDA-KCF) has been proposed in [7]. In the context of the present paper and the IPDA methodology, it is given in the following form:

$$\begin{aligned}
\xi_i(t|t) &= \xi_i(t|t-1) + \left[P_i(t|t-1)^{-1} + B_i(t)\right]^{-1} \\
&\quad \cdot \Big[y_i(t) - \left(1 - \beta_{i,0}^{[l]}\right) B_i(t)\xi_i(t|t-1) \\
&\quad + M_\varepsilon(t) \sum_{j\in\mathcal{N}_i} \left(\xi_j(t|t-1) - \xi_i(t|t-1)\right) \Big], \\
\xi_i(t+1|t) &= F\xi_i(t|t), \\
M_\varepsilon(t) &= \varepsilon \frac{P_i(t|t)}{1 + \|P_i(t|t)\|}, \\
P_i(t|t) &= \beta_{i,0}^{[l]}(t) P_i(t|t-1) \\
&\quad + \left[1 - \beta_{i,0}^{[l]}(t)\right]\left[P_i(t|t-1)^{-1} + B_i(t)\right]^{-1} + \tilde{P}_i(t), \\
P_i(t+1|t) &= FP_i(t|t)F^T + GQG^T,
\end{aligned} \quad (25)$$

where ε is a small positive scalar. The algorithm is derived by decomposing the global Kalman filter for the whole network and adding the consensus term at the filtering level [5]. Notice that the algorithm requires communication of information vectors (size $n \times 1$) and information matrices (size $n \times n$) between the neighbors, in addition to the exchange of state estimates (size $n \times 1$).

5.2 Information-weighted consensus filter: IPDA-ICF

The tracking algorithm which takes into account cross-covariances between the state estimates across different nodes is the so-called information-weighted consensus filter (ICF) [4]. A multi-target version of this algorithm based on the PDA methodology is described in [8]. In the context of the present paper and the IPDA methodology, the single consensus step version of this algorithm is represented by:

$$\begin{aligned}
\xi_i(t|t) &= \xi_i(t|t-1) + \left[\mathcal{A}\left(\frac{P_i(t|t-1)}{N}\right) + \mathcal{A}(U_i(t))\right]^{-1} \\
&\quad \cdot \Bigg[\mathcal{A}(u_i(t)) - \mathcal{A}(U_i(t))\xi_i(t|t-1) \\
&\quad + \mathcal{A}\left(\beta_{i,0}^{[l]}(t) U_i(t) \xi_i(t|t-1)\right) \\
&\quad + \varepsilon \sum_{j\in\mathcal{N}_i} \frac{P_j^{-1}(t|t-1)}{N} \left(\xi_j(t|t-1) - \xi_i(t|t-1)\right) \Bigg], \\
\xi_i(t+1|t) &= F\xi_i(t|t),
\end{aligned} \quad (26)$$

where

$$
\begin{aligned}
P_i(t|t)^{-1} &= \mathcal{A}(P_i^{-1}(t|t-1)) + N\mathcal{A}(G_i(t)),\\
G_i(t) &= P_i(t|t-1)^{-1}L_i(t)\\
&\quad \cdot \left[C_i(t)^{-1} - L_i(t)^T P_i(t|t-1)^{-1}L_i(t)\right]^{-1}\\
&\quad \cdot L_i(t)^T P_i(t|t-1)^{-1},\\
C_i(t) &= \left[1-\beta_{i,0}^{[l]}(t)\right] S_i(t) - \tilde{P}_i(t),\\
P_i(t+1|t) &= FP_i(t|t)F^T + GQG^T;
\end{aligned}
\tag{27}
$$

by $\mathcal{A}(x_i(t))$, where $x_i(t)$ is a local variable at the instant t, we denote the single step averaging consensus operation

$$\mathcal{A}(x_i(t)) = x_i(t) + \varepsilon \sum_{j\in\mathcal{N}_i}(x_j(t) - x_i(t)). \tag{28}$$

The algorithm requires inter-node communication of one $n \times 1$ vector and two $n \times n$ matrices.

Notice that ICF was originally designed to be applied with multiple consensus steps between two consecutive time instants. This requirement represents a significant communication burden for the whole tracking scheme; in this paper, our focus is on the single consensus step version, compatible with the trackers formulated above.

Remark 2 *To summarize, the communication requirements of the above considered algorithms are as follows (per node and per time step): IPDA-ACF: $O(n)$ (more exactly, $n+1$), IPDA-KCF: $O\left(n^2\right)$ (more exactly, n^2+2n), and IPDA-ICF: $O\left(n^2\right)$ (more exactly, $2n^2+n$). Regarding the computational requirements, complexity of all the methods is, in general, $O\left(n^3\right)$. However, the algorithms IPDA-KCF and IPDA-ICF require computation of matrix inversions (size $n\times n$, see* (25) *and* (26)*, respectively), which is not the case with the IPDA-ACF algorithm. Therefore, in addition to favorable communication requirements (which are of crucial importance for the efficiency of large sensor networks), the proposed algorithm exhibits also the lowest computational demands.*

6 Simulation results

We shall consider a network of $N = 15$ cameras (nodes) aimed at distributed tracking of a target moving within a 500×500 space [4, 8]. The cameras are randomly distributed in space with random orientations resulting in overlapping field-of-views (FoVs), represented by equilateral triangles with 300 units height. The communication range is set to 200 units. The dynamics of a target is modeled using the constant speed model. The process covariance Q is set to diag(10, 10, 1, 1) [4]. Target's initial position is randomly selected within the given square area, and its initial speed is set to 2 units per time step, with random direction. The measurement noise covariances are set to $100I_2$. The initial state estimates are randomly set around actual target's initial state, with covariance $10Q$. The initial error covariance matrices are set to diag(100, 100, 10, 10) for all the nodes.

First, we shall examine scenarios with no clutter. An experiment has been performed with 250 Monte Carlo runs, over which estimation errors (average distance per node between the target position estimates and the actual position) and disagreements between the nodes (variance of the estimates) have been computed. Each run uses different target trajectory and network topology, designed in the way described above. In the chosen simulation setting, at each time iteration, 2.3 nodes on average receive measurements from the target, and one node communicates with 5.3 nodes for each network topology. Parameter ε in ICF has been set to $1/\max_i(|\mathcal{N}_i|+1)$ at each run; parameter ε in KCF has been set to 0.07. ACF has been simulated in two described variants, with ACF_1 parameters chosen as in the example from Section 3.2, and with α set to 0.05 for ACF_2. Figure 3 shows the resulting estimation errors (top) and disagreements (bottom). It is obvious that, owing to the introduced refined adaptation procedure, ACF outperforms both KCF and ICF, in spite of much lower communication requirements.

In the situations with clutter, P_i^D has been calculated individually for each node at each time instant by integrating the probability density function of the predicted estimate over the triangular area visible to the camera; P_i^g has been set to 0.999 for all the nodes and $\lambda_i(t)V_i(t) = 1/32$ [8]. The target perceivability has been modeled using $p_i^{11} = 0.98$, $p_i^{21} = 0$, and $\pi_i(0|0) = 0.2$ for all the nodes. IPDA-ACF with the refined adaptation scheme has been simulated for two choices of l (IPDA-ACF^l, with $l = 1$ corresponding to the classical PDA filter, and $l = 2$ to the target-existence-based IPDA filter), together with the IPDA-ICF algorithm (both choices of l have produced very similar results so that only the case $l = 1$ has been considered). The IPDA-KCF algorithm does not give comparable performances. Figure 4 illustrates performance of IPDA-ACF^2, giving snapshots of the position estimates of all the nodes, together with the target trajectory for one simulation run. It shows that the proposed algorithm gives network-wide accurate target state estimates. The resulting average estimation errors (top) and disagreements (bottom) of different algorithms are shown in Fig. 5. The optimal centralized Kalman filter (CKF) has also been simulated. It can be seen that the proposed algorithm with the target existence-based IPDA filter again outperforms ICF. CKF obviously represents a reference scheme with the best tracking results; however, it assumes that all the measurements are available to all the nodes. It is far from being realistic to perform all the necessary communications before the next updating of the state estimates. However, we have also investigated achievable performance

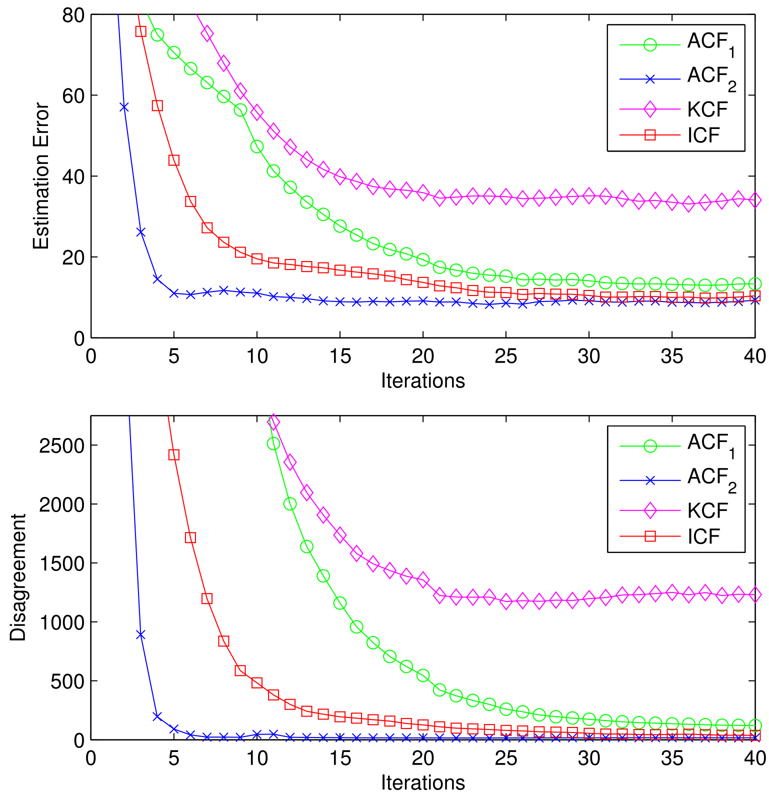

Fig. 3 Performance of the algorithms: no clutter

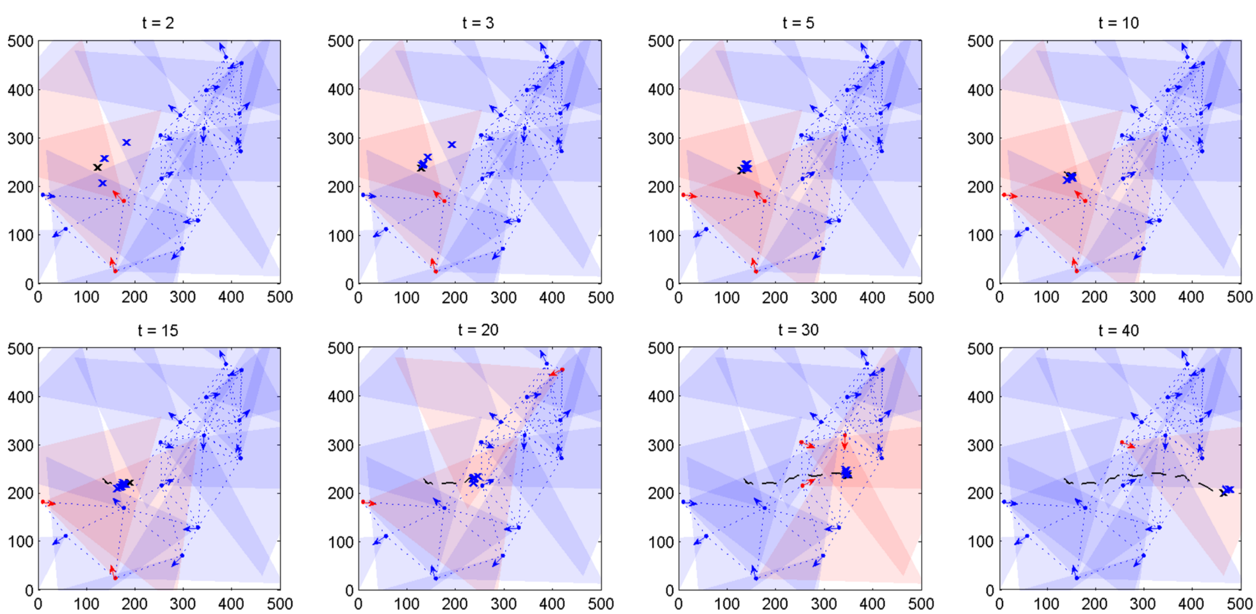

Fig. 4 Snapshots of the target position estimates of all the nodes (blue x), together with the target trajectory up to the given time instants (black dashed line) and the current target positions (black x). Cameras' positions, orientations, and FoVs are also shown (dots, arrows, and triangles, respectively)—in red for cameras that observe the target, in blue for the others. Communication topology is represented by dotted lines

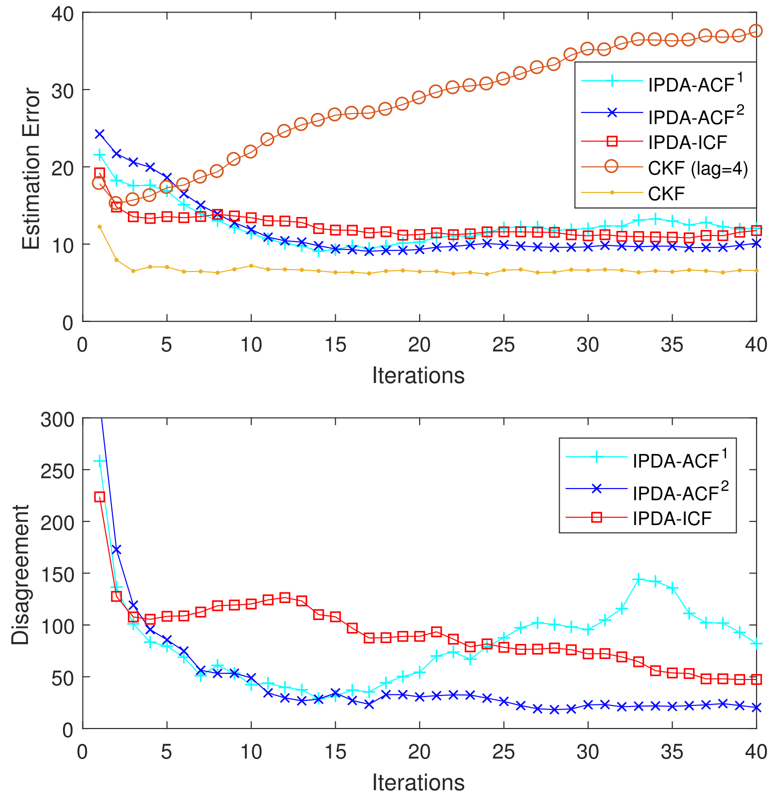

Fig. 5 Performance of the algorithms: clutter is present

in the case of the *flooding protocols* realizable by the simulated networks (see, e.g., [40]), regardless of the communication cost. Obviously, depending on the adopted network diameter, different results can be obtained, represented typically as delayed estimates generated by CKF. For the networks simulated in the way described above, the results are much inferior to those obtained by the proposed algorithm (having in mind the intrinsic need for tracking *moving* targets); the average diameter in our experiments has been found to be 4.4. The results presented in Fig. 5 give an obvious illustration. The proposed algorithm has been found to be superior even in the case of networks with the average diameter equal to one.

To assess the performance of the algorithms in case of different communication topologies, the original communication networks have been pruned to their spanning trees, and random edges progressively added, so that the resulting networks have the average node degree δ ranging from 2 to 14. Parameter ε in IPDA-ICF has been set to $1/(\delta+1)$. We can see in Fig. 6 that in terms of the tracking error, all the algorithms exhibit similar behavior, while IPDA-ACF^2 outperforms IPDA-ICF in terms of the disagreement between the nodes. IPDA-ICF slightly outperforms IPDA-ACF^2 in terms of the tracking error in the case of dense communication graphs.

In order to illustrate the idea underneath the consensus scheme design in the proposed adaptation algorithm, we have plotted in Fig. 7, for one simulation run, the actual target perceivabilities of the nodes (represented by binary signals equal to 1 when the target is visible and to 0 otherwise), together with the corresponding values of $\gamma_i(t)$ used in (15). It can be seen that, for each node, $\gamma_i(t)$ follows the binary perceivability signal, resulting in a consensus scheme which efficiently potentiates the nodes receiving measurements from the target.

As a part of the rationale behind the choice of $l = 2$ in the design of the proposed algorithm, we have simulated a situation when a target is temporarily occulted in the circle with 50 units radius (Fig. 8). One target trajectory crossing the circle has been fixed, and 250 experimental runs have included random network topologies. Figure 9 shows the resulting average estimation errors (top) and disagreements (bottom) for different algorithms. We can see that choosing $l = 2$ instead of $l = 1$ yields substantially lower disagreement between the local estimates when the target is occulted since in these situations, the introduced target perceivability probability is small, resulting in the state estimates relying on prediction, rather than on the measurements generated by clutter.

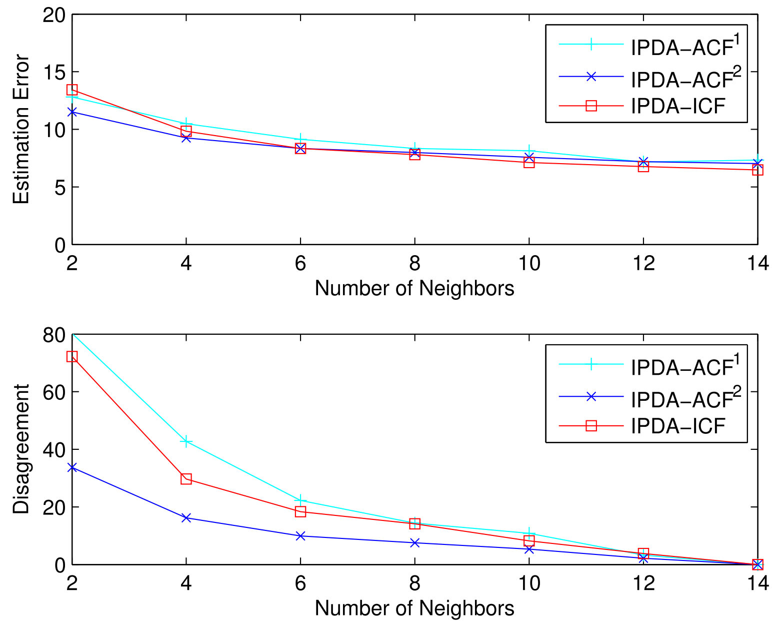

Fig. 6 Performance of the algorithms at $t = 40$ vs. network connectivity

7 Conclusion

In this paper, a new distributed adaptive consensus-based tracking algorithm has been proposed for camera networks in the case of limited FoVs, high-level clutter and occulted targets, using the methodology of (IPDA) [13]. The algorithm is defined either in the perceivability-based or the existence-based form, representing distributed consensus-based versions of the classical PDA tracker from [10, 11] and the existence-based tracker from [15], respectively. Special care is

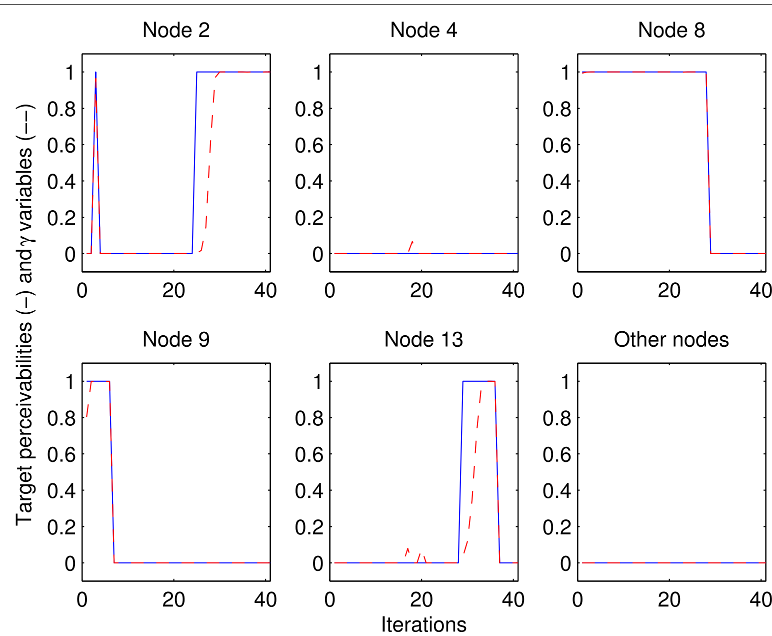

Fig. 7 Target perceivabilities and consensus scheme parameters corresponding to these perceivabilities

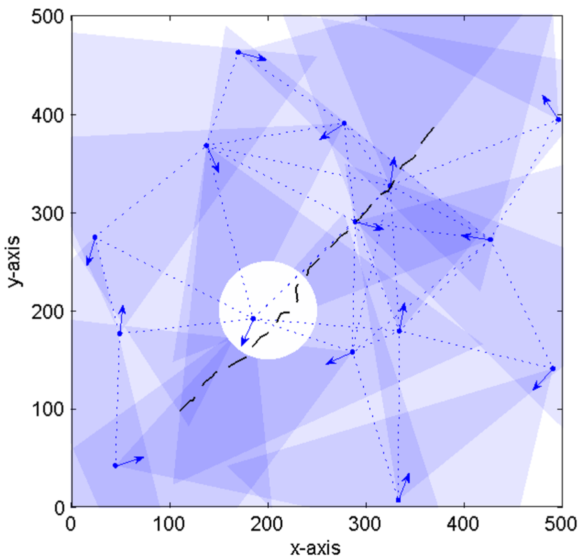

Fig. 8 One realization of a sensor network together with the target trajectory (black dashed line). The target is occulted when crossing the white circle

taken of the design of a distributed adaptation procedure, based on utilization of the "β''-parameters as a measure of the probabilities of observing the target by particular nodes. A new distributed adaptation algorithm, based on the generation of *node importance*, is proposed for defining appropriate weights in the consensus matrix, providing high tracking precision and fast convergence of the estimates over the network. The paper contains a theoretical analysis of stability and noise rejection capabilities of the proposed algorithm. The applied methodology of analysis is based on the idea from [12] and on recent results related to the properties of the modified Riccati equations resulting from the PDA recursions [18]. It has been shown that the algorithm provides convergence even in the case when some nodes may be unstable, owing to a proper choice of the consensus weights. The proposed algorithm has been verified by extensive simulations. It has been shown that it outperforms similar algorithms known from the literature—IPDA-KCF from [7] and IPDA-ICF from [8]—in spite of lower communication requirements; derivation of one-step consensus-based versions of these algorithms, consistent with the form of the proposed algorithm, is given in a separate section. The results related to occulted objects and high-level clutter are of special interest. The proposed algorithm can be readily applied to all distributed target tracking problems (radars, sonars) under analogous assumptions. We do hope that it can become a simple and efficient tool for engineering practice.

Further efforts could be oriented in the direction of the development of complementary distributed track initiation, confirmation, and termination algorithms using IPDA methodology [13, 15]. Generalization of the algorithm to the multi-target case in the sense of applying the basic methodology of JPDAF [10, 41] in the distributed multi-agent context represents a complex but straightforward task.

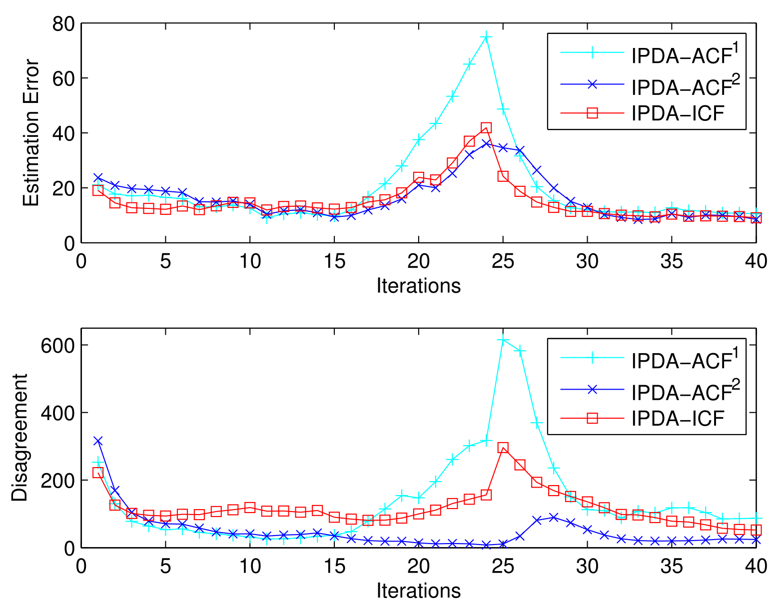

Fig. 9 Performance of the algorithms: occulted target

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Author details
[1]Etimad, Abu Dhabi, United Arab Emirates. [2]Vlatacom Institute, Bulevar Milutina Milankovica 5, 11000 Belgrade, Serbia. [3]College of Technics and Technology, 37000 Kruševac, Serbia. [4]Innovation Center, Faculty of Electrical Engineering, University of Belgrade, 11000 Belgrade, Serbia. [5]Faculty of Electrical Engineering, University of Belgrade, 11000 Belgrade, Serbia.

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References

1. DE Maggio, DA Cavallaro, *Video tracking: theory and practice*, 1st edn. (Wiley Publishing, Hoboken, 2011)
2. Ilić, MS Stanković, SS Stanković, Adaptive consensus-based distributed target tracking in sensor networks with limited sensing range. IEEE Trans. Control Syst. Technol. **22**(2), 778–785 (2014)
3. N Ilić, KO Al-Ali, M Stanković, S Stanković, in *Proc. 3rd IcETRAN Conf*. Adaptive consensus based distributed target tracking in camera networks (ETRAN Society, Zlatibor, 2016)
4. AT Kamal, JA Farrell, AK Roy-Chowdhury, Information weighted consensus filters and their application in distributed camera networks. IEEE Trans. Autom. Control. **58**(12), 3112–3125 (2013)
5. R Olfati-Saber, in *Proc. IEEE Conf. Decision and Control*. Distributed Kalman filtering for sensor networks (IEEE, New Orleans, 2007), pp. 5492–5498
6. R Olfati-Saber, NF Sandell, in *Proc. American Control Conference*. Distributed tracking in sensor networks with limited sensing range (IEEE, Seattle, 2008), pp. 3157–3162
7. NF Sandell, R Olfati-Saber, in *Proc. 47th IEEE Conf. Dec. Control*. Distributed data association for multi-target tracking in sensor networks (IEEE, Cancun, 2008), pp. 1085–1090
8. AT Kamal, JH Bappy, JA Farrell, AK Roy-Chowdhury, Distributed multi target tracking and data association in vision networks. IEEE Trans. Patt. Anal. Mach. Intel. **38**, 1397–1410 (2016)
9. Y Bar-Shalom, Tracking methods in a multitarget environment. IEEE Trans. Aut. Control. **23**, 618–626 (1978)
10. Y Bar-Shalom, F Daum, J Huang, The probabilistic data assocation filter. IEEE Contr. Syst. Magaz. **7**, 82–100 (2009)
11. TE Fortmann, Y Bar-Shalom, M Scheffe, Sonar tracking of multiple targets using joint probabilistic data association. IEEE J. of Oceaninc Eng. **OE-8**, 173–184 (1983)
12. SS Stanković, MS Stanković, DM Stipanović, Consensus based overlapping decentralized estimation with missing observations and communication faults. Automatica. **45**, 1397–1406 (2009)
13. D Musicki, R Evans, S Stankovic, Integrated Probabilistic Data Association. IEEE Trans. Automat. Contr. **39**, 1237–1241 (1994)
14. J Dezert, N Li, XR Li, in *Proc. Euro. Contr. Conf*. A new formulation of IPDAF for tracking in clutter (IEEE, Karlsruhe, 1999)
15. N Li, XR Li, Target perceivability and its applications. IEEE Trans. Sign. Proc. **11**, 2588–2604 (2001)
16. SS Stanković, N Ilić, KOA Ali, MS Stanković, in *2016 IEEE International Conference on Multisensor Fusion and Integration for Intelligent Systems (MFI)*. Distributed consensus based IPDAF for tracking in vision networks (IEEE, Baden-Baden, 2016), pp. 304–309
17. TE Fortmann, Y Bar-Shalom, M Scheffe, S Gelfand, Detection thresholds for tracking in clutter—a connection between estimation and signal processing. IEEE Trans. Aut. Control. **30**, 221–229 (1985)
18. MZQ Chen, L Zhang, H Su, G Chen, Stabilizing solution and parameter dependence of modified algebraic Riccati equations with application to discrete-time network synchronization. Trans. Aut. Control, IEEE. **61**, 228–233 (2016)
19. Y Bar-Shalom, XR Li, *Multitarget-multisensor tracking: principles and techniques, vol. December*. (YBS Publishing, Storrs, Connecticut, 1995)
20. C Jauffret, Y Bar-Shalom, Track formation with bearing and frequency measurements in clutter. IEEE Trans. Aerosp. Electro. Syst. **26**(6), 999–1010 (1990)
21. D Ciuonzo, PK Willett, Y Bar-Shalom, Tracking the tracker from its passive sonar ML-PDA estimates. IEEE Trans. Aerosp. Electr. Syst. **50**(1), 573–590 (2014)
22. MR Chummun, Y Bar-Shalom, T Kirubarajan, Adaptive early-detection ML-PDA estimator for LO targets with EO sensors. IEEE Trans. Aerosp. Electro. Syst. **38**(2), 694–707 (2002)
23. D Ciuonzo, S Horn, in *2012 IEEE Aerospace Conference*. A hash-tree based approach for a totally distributed track oriented multi hypothesis tracker (IEEE, Big Sky, 2012), pp. 1–9
24. WR Blanding, PK Willett, Y Bar-Shalom, RS Lynch, Directed subspace search ML-PDA with application to active sonar tracking. IEEE Trans. Aerosp. Electro. Syst. **44**(1), 201–216 (2008)
25. S Schoenecker, P Willett, Y Bar-Shalom, ML-PDA and ML-PMHT: Comparing multistatic sonar trackers for VLO targets using a new multitarget implementation. IEEE J. Oceanic Eng. **39**(2), 303–317 (2014)
26. YF Shi, SH Park, TL Song, Multitarget tracking in cluttered environment for a multistatic passive radar system under the DAB/DVB network. EURASIP J. Adv. Signal Process. **2017**(1), 11 (2017)
27. W Qu, D Schonfeld, M Mohamed, Distributed Bayesian multiple-target tracking in crowded environments using multiple collaborative cameras. EURASIP J. Adv. Signal Process. **2007**(1), 038373 (2006)
28. R Olfati-Saber, A Fax, R Murray, Consensus and cooperation in networked multi-agent systems. Proc. IEEE. **95**, 215–233 (2007)
29. L Xiao, S Boyd, Fast linear iterations for distributed averaging. Syst. Control Lett. **53**, 65–78 (2004)
30. A Olshevsky, JN Tsitsiklis, Convergence speed in distributed consensus and averaging. SIAM Review. **53**(4), 747–772 (2011)
31. P Braca, S Marano, V Matta, Enforcing consensus while monitoring the environment in wireless sensor networks. IEEE Trans. Signal Process. **56**, 3375–3380 (2008)
32. P Braca, S Marano, V Matta, P Willett, Asymptotic optimality of running consensus in testing binary hypotheses. IEEE Trans. Signal Process. **58**, 814–825 (2010)
33. IF Pierce, Matrices with dominating diagonal blocks. J Economic Theory. **9**, 159–170 (1974)
34. Y Ohta, D Siljak, Overlapping block diagonal dominance and existence of Lyapunov functions. J. Math. Anal. Appl. **112**, 396–410 (1985)
35. RA Horn, CA Johnson, *Matrix analysis*. (Cambridge Univ. Press, Cambridge, 1985)
36. WJ Rugh, *Linear system theory*. (Prentice Hall, Upper Saddle River, 1996)
37. F Amato, G Celentano, F Garofalo, New sufficient conditions for the stability of slowly varying linear systems. IEEE Trans. Autom. Control. **38**, 1409–1411 (1993)
38. RR Bitmead, MR Gevers, IR Petersen, Monotonicity and stabilizability properties of solutions of the Riccati difference equation: propositions, lemmas, theorems, fallacious conjectures and counterexamples. Syst. Control Lett. **5**, 309–315 (1985)
39. Y-S Kim, K-S Hong, in *Proc. 39the Conf. Decision and Control*. Stability of the modified probabilistic data association filter: Lyapunov function analysis (IEEE, Sydney, 2000), pp. 3896–3901
40. T Li, JM Corchado, J Prieto, Convergence of distributed flooding and its application for distributed Bayesian filtering. IEEE Trans. Signal Inf. Process. Over Netw. **3**(3), 580–591 (2017)
41. D Musicki, R Evans, Clutter map information for data association and track initialization. IEEE Trans. Aerosp. Electr. Syst. **40**, 387–398 (2004)