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An adaptive combination constrained proportionate normalized maximum correntropy criterion algorithm for sparse channel estimations

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Abstract

An adaptive combination constrained proportionate normalized maximum correntropy criterion (ACC-PNMCC) algorithm is proposed for sparse multi-path channel estimation under mixed Gaussian noise environment. The developed ACC-PNMCC algorithm is implemented by incorporating an adaptive combination function into the cost function of the proportionate normalized maximum correntropy criterion (PNMCC) algorithm to create a new penalty on the filter coefficients according to the devised threshold, which is based on the proportionate-type adaptive filter techniques and compressive sensing (CS) concept. The derivation of the proposed ACC-PNMCC algorithm is mathematically presented, and various simulation experiments have been carried out to investigate the performance of the proposed ACC-PNMCC algorithm. The experimental results show that our ACC-PNMCC algorithm outperforms the PNMCC and sparse PNMCC algorithms for sparse multi-path channel estimation applications.

Keywords: Sparse PNMCC algorithm, Mixed Gaussian noise environment, Zero-attracting technique, Adaptive combination constraint

1 Introduction

In recent years, wireless communication is the fastest growing and most widely used technology in the field of information communication. As for wireless communication channels, radio wave propagation through the channels will produce a direct wave and a ground reflection wave, which are the most common waves in the propagation process. In addition, there will also be scattered waves caused by various obstacles in the propagation paths [1, 2]. These phenomena eventually result in the multi-path effect [2–5], which often occurs and is often very serious. Such effect may lead to signal delays and to reduce communication quality. Furthermore, measurements show that the broadband channels are described as sparse multi-path channels, which are very common

in the field of wireless communication [6, 7]. The general characteristic of these sparse multi-path channels is that most of the channel impulse response (CIR) coefficients are equal to zero. The very few nonzero coefficients are termed active coefficients [2, 7, 8]. Adaptive filters (AF) have been widely applied to the estimation of sparse channels to improve the quality of wireless communications [9–12]. The most traditional AF algorithm is the least mean square (LMS), which has been derived to minimize the mean square estimation error [9, 13]. It is widely employed in real-world applications due to its simplicity. However, LMS performance degrades in low signal-to-noise (SNR) situations and in sparse channel estimation. Motivated by compressive sensing principles [14], a series of sparse LMS algorithms have been recently proposed by incorporating l_0 -norm, l_1 -norm, or l_p -norm penalties into the LMS cost function [14–21]. These algorithms have been shown to lead to superior performances than LMS both in steady-state error (SSE) and in convergence speed for sparse multi-path channel estimation. Nevertheless, the performances of such algorithms tend to degrade

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in non-Gaussian white noise environments (NGWNE), a frequently encountered situation in the real wireless communication systems [22–24].

Some recent efforts have also been directed to improve the performance of adaptive filters in the presence of non-Gaussian and impulsive disturbances. One of such efforts was to replace the MSE with information theoretic, entropy-related, cost functions [25–28]. At present, most entropy-based AF algorithms have been derived using the maximum correntropy criterion (MCC) and the minimum error entropy (MEE) criteria [25–27, 29–31]. Though MEE-based algorithms may lead to good estimation performance, they require a prohibitive computational effort for real-time application. MCC-based algorithms are less complex and have been successfully applied to channel estimation [26]. The MCC algorithm employs the correntropy as a local similarity measure, which tends to increase the robustness of the algorithm to outliers. It has been applied to channel estimation under NGWNE [26, 32, 33]. More recently, a generalized correntropy measure has been proposed which replaces the Gaussian kernel of the MCC with a generalized Gaussian kernel. The generalized MCC (GMCC) algorithm has been shown to outperform MCC in certain situations at the cost of an increase in computational complexity [34]. A normalized version of the MCC algorithm (NMCC) has also been proposed [35, 36] to make the convergence control less dependent on the input power.

To address the issue of sparse channel estimation in non-Gaussian noise, sparse MCC algorithms have been proposed by incorporating different norm penalties into the MCC cost function. Similar to the zero attracting LMS (ZA-LMS) algorithm [16], in [33], an l_1 -norm penalty was introduced into the MCC cost function as a zero attractor. The resulting algorithm was called zero-attracting MCC (ZA-MCC) algorithm. Then, a reweighting controlling factor ξ was incorporated into the ZA-MCC algorithm, yielding the reweighting ZA-MCC (RZA-MCC) algorithm [33]. The ZA-MCC and RZA-MCC were shown to improve the steady-state and convergence performances of MCC in estimating multi-path channels. Different zero attractors were added to MCC in [33, 37, 38] using an l_0 -norm and a correntropy-induced metric (CIM).

One drawback of the ZA-MCC algorithms is that it uniformly attracts all the channel coefficients to zero, which may result in a rather large estimation error for less sparse channels. Also, the selection of a good learning rate is not simple for these algorithms. Zero-attracting NMCC (ZA-NMCC) and reweighting ZA-MCC (RZA-NMCC) algorithms have also been developed in [39]. Moreover, a soft parameter function has also been introduced into the NMCC algorithm to attenuate the uniform attraction of coefficients to zero. The resulting algorithm is called

SPF-NMCC [40]. The SPF-NMCC algorithm improved the steady-state performance of NMCC, but tends to converge slowly due to the large discrepancy between coefficient values in sparse channels. To remedy this drawback, proportionate-type [41, 42] NMCC algorithms were proposed. The proportionate NMCC (PNMCC) algorithm was developed in [43, 44] and led to faster convergence speeds than the MCC and NMCC algorithms since the filter coefficients are proportionately updated according to the magnitude of the estimates of the channel coefficients. Unfortunately, however, the convergence of the PNMCC algorithm tends to slow down an initial convergence period [43, 44], a known effect of the proportionate type of adaptation.

In this paper, we propose a new adaptive algorithm by effectively combining some of the aforementioned strategies to derive a method that combines the properties of the l_0 -norm, the l_1 -norm, and the proportionate adaptation. This is done by integrating an adaptive combination function (ACF) to the PNMCC cost function to create a new adaptive zero attractor. The resulting algorithm is called Adaptive Combination Constrained Proportionate Normalized Maximum Correntropy Criterion (ACC-PNMCC) algorithm. The performance of the proposed ACC-PNMCC algorithm is investigated through several different simulation experiments, and it is compared to the MCC, NMCC, PNMCC, and sparse PNMCC algorithms in terms of the SSE and convergence speed. The simulation results illustrate the potential of the ACC-PNMCC algorithm to provide a superior estimation performance for sparse multi-path channel estimation under non-Gaussian and impulsive noise environments.

The paper is organized as follows. The NMCC and PNMCC algorithms are briefly reviewed in Section 2. In Section 2, we derive the proposed ACC-PNMCC algorithm in the context of sparse multi-path channel estimation. Simulation experiments are presented in Section 3 to illustrate the algorithm performance, and we finally summarize our work in Section 4.

2 Channel estimation algorithms

2.1 The NMCC and PNMCC algorithms

Consider the estimation of an unknown channel with impulse response given by $\mathbf{g}_0 = [g_1, g_2, g_3, \dots, g_M]^T$. The M -dimensional input vector is $\mathbf{x}(n) = [x(n), x(n-1), x(n-2), \dots, x(n-M+1)]^T$, and the observed channel output is $d(n) = \mathbf{x}^T(n) \mathbf{g}_0 + r(n)$, where $r(n)$ is the additive noise. Denoting $\hat{\mathbf{g}}(n)$ the channel response estimate, the estimation error is given by $e(n) = d(n) - \mathbf{x}^T(n) \hat{\mathbf{g}}(n)$.

It has been shown in [36] that the NMCC weight update equation is the solution of the following problem:

$$\begin{aligned} \min \frac{1}{2} \|\hat{\mathbf{g}}(n+1) - \hat{\mathbf{g}}(n)\|^2 \\ \text{subject to } \hat{e}(n) = \left[1 - \alpha_{\text{NMCC}} \exp\left(-\frac{e^2(n)}{2\sigma^2}\right) \right] e(n) \end{aligned} \quad (1)$$

where $\|\cdot\|^2$ represents the Euclidean norm of a vector, $\hat{e}(n) = d(n) - \mathbf{x}^T(n) \hat{\mathbf{g}}(n+1)$, the parameter $\sigma > 0$ is the bandwidth of the Gaussian kernel used to evaluate the instantaneous correntropy between the observed signal and the estimator output [26], and $\alpha_{\text{NMCC}} > 0$ is a design parameter that affects the quality of estimation.

Solving (1) using the Lagrange multiplier technique [36] leads to the NMCC weight update equation

$$\hat{\mathbf{g}}(n+1) = \hat{\mathbf{g}}(n) + \alpha_{\text{NMCC}} \frac{\exp\left(-\frac{e^2(n)}{2\sigma^2}\right)}{\|\mathbf{x}(n)\|^2} e(n) \mathbf{x}(n), \quad (2)$$

where α_{NMCC} is the NMCC step size. If $\alpha_{\text{NMCC}} = \beta \|\mathbf{x}(n)\|^2$, (1) becomes the MCC algorithm update [26].

One way to modify the NMCC algorithm to improve adaptation performance for sparse channels is to adjust each adaptive weight using a different step size, what can be done through a diagonal step size matrix

$$\mathbf{G}(n) = \text{diag}(m_1(n), m_2(n), m_3(n), \dots, m_M(n)). \quad (3)$$

The proportionate NMCC (PNMCC) algorithm assigns the step sizes m_i such that

$$m_i(n) = \frac{\gamma_i(n)}{\sum_{i=1}^M \gamma_i(n)}, \quad 1 \leq i \leq M \quad (4)$$

where $\gamma_i(n)$, $i = 1, \dots, M$ are given by

$$\gamma_i(n) = \max[\kappa \max[\eta, |\hat{g}_1(n)|, \dots, |\hat{g}_M(n)|], |\hat{g}_i(n)|] \quad (5)$$

with κ and η positive constants with typical values of $\kappa = 5/M$ and $\eta = 0.01$ [42–45]. Parameter κ is used to prevent the coefficients from stalling when they are much smaller than the largest one, while η avoids the stalling of all coefficients when $\hat{\mathbf{g}}(n) = \mathbf{0}_{M \times 1}$ at initialization. The weight update equation for the PNMCC algorithm is

$$\hat{\mathbf{g}}(n+1) = \hat{\mathbf{g}}(n) + \alpha_{\text{PNMCC}} \frac{\mathbf{G}(n) \exp\left(-\frac{e^2(n)}{2\sigma^2}\right)}{\mathbf{x}^T(n) \mathbf{G}(n) \mathbf{x}(n) + \theta} e(n) \mathbf{x}(n), \quad (6)$$

where θ is a regularization parameter, and α_{PNMCC} is the PNMCC step size. Although the PNMCC algorithm improves the performance of the NMCC, its convergence rate tends to slow down after an initial period of fast convergence, a well known issue affecting the conventional (MSE based) PNLMS algorithm [21]. This basically happens due to the dominant effect of the inactive taps, whose step sizes tend to be excessively reduced as their weight

estimates get closer to zero, reducing the convergence speed.

Among the several different strategies proposed to increase the PNLMS convergence rate after the initial convergence period, the l_1 norm penalty recently proposed in [21] has led to very interesting results. In [21], the l_1 norm of the weight vector pre-multiplied by the inverse of the step size matrix has been used. This makes the l_1 penalty to be governed primarily by the inactive taps. The constrained optimization problem solved in [21] to yield the ZA-PNLMS algorithm is

$$\begin{aligned} \hat{\mathbf{g}}(n+1) = \min_{\hat{\mathbf{g}}(n+1)} \|\hat{\mathbf{g}}(n+1) - \hat{\mathbf{g}}(n)\|_{\mathbf{G}^{-1}}^2 + \gamma \|\mathbf{G}^{-1} \hat{\mathbf{g}}(n+1)\|_1 \\ \text{subject to } d(n) - \mathbf{x}^T(n) \hat{\mathbf{g}}(n+1) = 0. \end{aligned}$$

One problem of enforcing sparsity through an l_1 norm constraint is that the resulting algorithm may lose performance when identifying responses with different levels of sparsity. One approach recently proposed in [46] to mitigate this problem for the LMS algorithm is to employ a so-called non-uniform norm constraint, inspired in the p -norm frequently used in compressive sensing. The sparsity of the weight vector is enforced through an adaptive combination function (ACF) that is a mixture of l_0 -norm and l_1 -norm and whose general form is

$$\|\hat{\mathbf{g}}(n)\|_q^q = \sum_{i=1}^M |\hat{g}_i(n)|^q \quad (7)$$

where $0 \leq q \leq 1$. We can see that the definition of the ACF is different from the classic Euclidean norm [47]. When $q \rightarrow 0$, it becomes to be

$$\lim_{q \rightarrow 0} \|\hat{\mathbf{g}}(n)\|_q^q = \|\hat{\mathbf{g}}(n)\|_0 \quad (8)$$

which approximates to be l_0 -norm that can count the number of non-zero coefficients in the sparse multipath channels. Besides, the ACF is the l_1 -norm, which is described as

$$\lim_{q \rightarrow 1} \|\hat{\mathbf{g}}(n)\|_q^q = \|\hat{\mathbf{g}}(n)\|_1 = \sum_{i=1}^M |\hat{g}_i(n)|. \quad (9)$$

2.2 The proposed ACC-PNMCC algorithm

To address the problems of estimating a sparse response with possibly different levels of sparsity, obtaining a good convergence rate from initialization to steady-state, and providing robustness to impulsive noise, we propose to integrate an adaptive combination function (ACF) into the PNMCC's cost function. This approach leads to the design of a new correntropy-based zero attractor algorithm that combines the techniques of [21, 46–50]. The proposed ACC-PNMCC algorithm is derived in the following.

Integrating the ACF into the cost function of the PNMCC algorithm with the weight scaling as in [21], we propose the following optimization problem:

$$\min_{\hat{\mathbf{g}}(n+1)} \frac{1}{2} \|\hat{\mathbf{g}}(n+1) - \hat{\mathbf{g}}(n)\|_{\mathbf{G}^{-1}(n)}^2 + \gamma_{\text{ACC}} \|\mathbf{G}^{-1}(n) \hat{\mathbf{g}}(n+1)\|_q^q$$

subject to $\hat{e}(n) = \left[1 - \alpha \exp\left(-\frac{e^2(n)}{2\sigma^2}\right)\right] e(n)$

(10)

where γ_{ACC} is a trade-off parameter between the steady-state performance and the ACF penalty. Using Lagrange multipliers, the cost function associated with this optimization problem is

$$J_{\text{ACC}}(n+1) = \frac{1}{2} (\hat{\mathbf{g}}(n+1) - \hat{\mathbf{g}}(n))^T \mathbf{G}^{-1}(n) (\hat{\mathbf{g}}(n+1) - \hat{\mathbf{g}}(n))$$

$$+ \gamma_{\text{ACC}} \|\mathbf{G}^{-1}(n) \hat{\mathbf{g}}(n+1)\|_q^q$$

$$+ \lambda_{\text{ACC}} \left[\hat{e}(n) - \left[1 - \alpha \exp\left(-\frac{e^2(n)}{2\sigma^2}\right)\right] e(n) \right]$$
(11)

where λ_{ACC} is the Lagrange multiplier.

As a first step to minimize (11) with respect to $\hat{\mathbf{g}}(n+1)$, we impose

$$\frac{\partial J_{\text{ACC}}(n+1)}{\partial \hat{\mathbf{g}}(n+1)} = \mathbf{0}$$
(12)

which yields the recursion

$$\hat{\mathbf{g}}(n+1) = \hat{\mathbf{g}}(n) + \lambda_{\text{ACC}} \mathbf{G}(n) \mathbf{x}(n)$$

$$- \gamma_{\text{ACC}} \mathbf{G}(n) \frac{\partial \|\mathbf{G}^{-1}(n) \hat{\mathbf{g}}(n+1)\|_q^q}{\partial \hat{\mathbf{g}}(n+1)}$$
(13)

Now, imposing the equality constraint

$$\hat{e}(n) = \left[1 - \alpha \exp\left(-\frac{e^2(n)}{2\sigma^2}\right)\right] e(n).$$
(14)

and solving for λ_{ACC} yields

$$\lambda_{\text{ACC}} = \frac{\alpha \exp\left(-\frac{e^2(n)}{2\sigma^2}\right) e(n) + \gamma_{\text{ACC}} \mathbf{x}^T(n) \mathbf{G}(n) \frac{\partial \|\mathbf{G}^{-1}(n) \hat{\mathbf{g}}(n+1)\|_q^q}{\partial \hat{\mathbf{g}}(n+1)}}{\mathbf{x}^T(n) \mathbf{G}(n) \mathbf{x}(n)}.$$
(15)

Substituting (15) in (13) and rearranging the terms yields

$$\hat{\mathbf{g}}(n+1) = \hat{\mathbf{g}}(n) + \alpha_{\text{ACC}} \frac{\exp\left(-\frac{e^2(n)}{2\sigma^2}\right) e(n) \mathbf{G}(n) \mathbf{x}(n)}{\mathbf{x}^T(n) \mathbf{G}(n) \mathbf{x}(n)}$$

$$- \gamma_{\text{ACC}} \left\{ \mathbf{I} - \frac{\mathbf{x}(n) \mathbf{x}^T(n) \mathbf{G}(n)}{\mathbf{x}^T(n) \mathbf{G}(n) \mathbf{x}(n)} \right\} \mathbf{G}(n) \frac{\partial \|\mathbf{G}^{-1}(n) \hat{\mathbf{g}}(n+1)\|_q^q}{\partial \hat{\mathbf{g}}(n+1)}.$$
(16)

In (16), we observe that the elements of the matrix $-\frac{\mathbf{x}(n) \mathbf{x}^T(n) \mathbf{G}(n)}{\mathbf{x}^T(n) \mathbf{G}(n) \mathbf{x}(n)}$ tend to be very small for reasonably large values of M . Hence, we simplify the algorithm recursion using the approximation $\mathbf{I} - \frac{\mathbf{x}(n) \mathbf{x}^T(n) \mathbf{G}(n)}{\mathbf{x}^T(n) \mathbf{G}(n) \mathbf{x}(n)} \approx \mathbf{I}$. The same

approximation has been successfully used in [21]. Then, (16) becomes

$$\hat{\mathbf{g}}(n+1) = \hat{\mathbf{g}}(n) + \alpha_{\text{ACC}} \frac{\exp\left(-\frac{e^2(n)}{2\sigma^2}\right) e(n) \mathbf{G}(n) \mathbf{x}(n)}{\mathbf{x}^T(n) \mathbf{G}(n) \mathbf{x}(n)}$$

$$- \gamma_{\text{ACC}} \mathbf{G}(n) \frac{\partial \|\mathbf{G}^{-1}(n) \hat{\mathbf{g}}(n+1)\|_q^q}{\partial \hat{\mathbf{g}}(n+1)}$$
(17)

Evaluation of the gradient vector in (17) yields

$$\frac{\partial \|\mathbf{G}^{-1}(n) \hat{\mathbf{g}}(n+1)\|_q^q}{\partial \hat{\mathbf{g}}(n+1)} = q \mathbf{G}^{-q}(n) \boldsymbol{\psi}(n+1) \odot \text{sgn}(\hat{\mathbf{g}}(n+1))$$
(18)

where $\odot$ denotes the Hadamard product, and

$$\boldsymbol{\psi}(n+1) = [|\hat{g}_1(n+1)|^{q-1}, \dots, |\hat{g}_M(n+1)|^{q-1}]^T.$$

As (18) is a function of $\hat{\mathbf{g}}(n+1)$, its substitution in (17) would not yield a recursive update for $\hat{\mathbf{g}}(n)$. To obtain an implementable recursive update, we replace $\hat{\mathbf{g}}(n+1)$ in (18) with $\hat{\mathbf{g}}(n)$. This approximation tends to be reasonable for practical step sizes. The resulting weight update equation is then

$$\hat{\mathbf{g}}(n+1) = \hat{\mathbf{g}}(n) + \alpha_{\text{ACC}} \frac{\exp\left(-\frac{e^2(n)}{2\sigma^2}\right) e(n)}{\mathbf{x}^T(n) \mathbf{G}(n) \mathbf{x}(n)} \mathbf{G}(n) \mathbf{x}(n)$$

$$- \gamma_{\text{ACC}} q (\mathbf{G}^{-1}(n))^{q-1} \boldsymbol{\psi}(n) \odot \text{sgn}(\hat{\mathbf{g}}(n)).$$
(19)

Though recursion (19) should fulfill the objective of minimizing (11) at convergence, it is still too complex for real-time implementations due to the last term on the r.h.s., which is responsible for the zero-attracting property of the algorithm. Since we propose to use either an l_0 -norm or an l_1 -norm, q must assume a value equal to zero or one, respectively. For $q = 0$, the last term in (19) is equal to zero. For $q = 1$, it is equal to $-\gamma_{\text{ACC}} \boldsymbol{\psi}(n) \odot \text{sgn}(\hat{\mathbf{g}}(n))$, which is a vector whose i th element is given by $-\gamma_{\text{ACC}} \text{sgn}(\hat{g}_i(n))$.

A final decision to be made is how to choose the value of q . According to the definitions (8) and (9), it should affect the norm of the whole vector $\hat{\mathbf{g}}(n)$. However, there is no clear measure to set the value of q to obtain good performance in practical applications. We propose instead to determine a different value of q for each element of $\hat{\mathbf{g}}(n)$ and based on a threshold that depends on the recent behavior of each weight estimate:

$$h_i(n) = E[|\hat{g}_i(n)|], \forall 1 \leq i \leq M.$$
(20)

Similar solutions have been employed in [47, 49, 50] to regularize different model misfit cost functions.

Given (20), and denoting becomes q_i the value of q applied to $q_i(n)$, we set $q_i = 0$ if $\hat{g}_i(n) > h_i(n)$, and $q_i = 1$ if $\hat{g}_i(n) < h_i(n)$. Hence, those coefficients for which

Fig. 1 The MSD performance of the proposed ACC-PNMCC algorithm with different γ_{ACC}

$\hat{g}_i(n) > h_i(n)$ will contribute to this term as if the norm l_0 had been used. Conversely, those coefficients for which $\hat{g}_i(n) < h_i(n)$ will contribute to this term as if the norm l_1 had been used.

We implement this strategy by defining a diagonal matrix

$$\mathbf{F} = \text{diag}(f_1, f_2, f_3, \dots, f_M), \quad (21)$$

with elements

$$f_i = \frac{\text{sgn}[h_i(n) - |\hat{g}_i(n)|] + 1}{2}, \quad \forall 1 \leq i \leq M \quad (22)$$

Fig. 2 The effects of α_{ACC} on the MSD performance of the proposed ACC-PNMCC algorithm at SNR = 30 dB and $\gamma_{ACC} = 5 \times 10^{-4}$

Fig. 3 The effects of ξ on the zero-attracting ability of the proposed ACC-PNMCC algorithm

so that the last r.h.s. term of (19) becomes $-\gamma_{\text{ACC}} \mathbf{F} \text{sgn}(\hat{\mathbf{g}}(n))$.

Finally, it has been verified [50] that the zero-attracting ability of $-\gamma_{\text{ACC}} \mathbf{F} \text{sgn}(\hat{\mathbf{g}}(n))$ can be improved by a reweighting factor used in the RZA-MCC algorithm. By doing that, the updating

equation of the proposed ACC-PNMCC algorithm becomes

$$\hat{\mathbf{g}}(n+1) = \hat{\mathbf{g}}(n) + \alpha_{\text{ACC}} \frac{\exp\left(-\frac{e^2(n)}{2\sigma^2}\right) e(n) \mathbf{G}(n) \mathbf{x}(n)}{\mathbf{x}^T(n) \mathbf{G}(n) \mathbf{x}(n) + \theta} - \gamma_{\text{ACC}} \mathbf{F} \frac{\text{sgn}(\hat{\mathbf{g}}(n))}{1 + \xi |\hat{\mathbf{g}}(n)|} \quad (23)$$

Fig. 4 The MSD performance of the proposed ACC-PNMCC algorithm with $K = 1$

Fig. 5 The MSD performance of the proposed ACC-PNMCC algorithm with $K = 2$

where $\theta > 0$ is a very small positive constant, ξ is the reweighting factor, and α_{ACC} denotes the step size.

The last r.h.s. term in (23) is the newly designed zero attractor. Each coefficient is treated as if a different norm penalty had been applied to the cost function. This is

done by comparing each channel coefficient estimate to the designed threshold. In the following, we will show that the proposed zero attractor speeds-up the convergence of the small coefficients after the initial convergence so that the proposed ACC-PNMCC algorithm outperforms

Fig. 6 The MSD performance of the proposed ACC-PNMCC algorithm with $K = 4$

the PNMCC algorithm. In addition, a better sparse multipath channel estimation performance can be obtained by properly selecting the value of the reweighting factor ξ .

3 Results and discussions

In this section, several different simulation experiments are carried out to investigate the performance of the proposed ACC-PNMCC algorithm. The input signal $\mathbf{x}(n)$ is a random signal, while the noise signal $r(n)$ is a mixed Gaussian signal with distribution [33, 37, 40]

$$(1 - \chi)N(\mu_1, v_1^2) + \chi N(\mu_2, v_2^2), \quad (24)$$

where $N(\mu_i, v_i^2)$ ($i = 1, 2$) denotes a Gaussian distribution with mean μ_i and variance v_i^2 . The mixing parameter χ is used to balance the two Gaussian noises. In our simulation experiments, these parameters are set to be $(\mu_1, \mu_2, v_1^2, v_2^2, \chi) = (0, 0, 0.05, 20, 0.05)$.

The performance of the proposed ACC-PNMCC algorithm is evaluated through the steady-state mean square deviation (MSD), which is defined as

$$\text{MSD}(\hat{\mathbf{g}}(n)) = E[\|\mathbf{g} - \hat{\mathbf{g}}(n)\|^2]. \quad (25)$$

From the updating Eq. (23), we notice that there are several key parameters which may affect the ACC-PNMCC performance and thus must be properly selected. To better select these parameters, we experimentally analyzed their effect on the MSD performance of the ACC-PNMCC algorithm. Herein, the regularization parameters include

α_{ACC} , γ_{ACC} , and ξ . In this paper, each parameter is optimized to obtain small MSD. At each optimization, only one parameter is changed while other parameters are set to the optimal values. In the experiments designed for this study, the number of nonzero coefficients is 1 and the total length of the unknown channel is set to be $M=16$. Firstly, γ_{ACC} is studied under different SNRs [51]. In this simulation, we have set $\alpha_{\text{ACC}} = 0.27$ and $\xi=5$. The simulation results are shown in Fig. 1. It is observed that the MSD with different values of γ_{ACC} decreases as the SNR increases from 1 to 40 dB. The SNR increases with different slopes for different values of γ_{ACC} . This means that the effect of γ_{ACC} is dependent on the SNR. As can be verified from Fig. 1, $\gamma_{\text{ACC}}=5 \times 10^{-4}$ yields the smallest MSD for a SNR of 30 dB. Hence, we study the effect of the step size value α_{ACC} on the MSD performance for SNR = 30 dB and $\gamma_{\text{ACC}}=5 \times 10^{-4}$. The corresponding MSD is shown in Fig. 2. We can verify that the MSD gradually decreases as the step size increases from 1×10^{-3} to 5×10^{-2} , while the MSD increases for step sizes greater than 5×10^{-2} . This indicates the importance of a proper choice of step size value. The effect of reweighting factor ξ is studied for a simple case on the zero attractor term. The results are shown in Fig. 3. From Fig. 3, we note that the reweighting factor mainly attracts to zero the coefficients that are smaller than the defined threshold, while the zero attractor term becomes zero for the coefficients with values greater than the threshold. In addition, the shrinkage ability of the reweighting factor decreases with

Fig. 7 The MSD performance of the proposed ACC-PNMCC algorithm with $K = 8$

ξ ranging from 0 to 25. Thus, the reweighting factor in the proposed ACC-PNMCC algorithm exerts strong zero attraction on the relatively small coefficients to improve their convergence after the initial transient.

Based on the above parameter selections, the MSD performance of the proposed ACC-PNMCC algorithm was verified for different sparsity levels at SNR = 30 dB. For comparison purposes, the conventional MCC, NMCC, PNMCC, and sparse PNMCC algorithms were also considered in the simulations. The sparse PNMCC (ZA-PNMCC and RZA-PNMCC) algorithms were obtained by using the l_1 -norm and are briefly described in the Appendix. In this experiment, the sparsity level of sparse multi-path channel was set as K . In other words, K represents the number of nonzero coefficients in the sparse multi-path channel. Firstly, there is only one nonzero channel coefficient ($K = 1$), which is randomly distributed within the unknown sparse multi-path channel. To obtain the same initial convergence speed for all the compared algorithms, the related parameters for the MCC, NMCC, PNMCC, ZA-PNMCC, and RZA-PNMCC algorithms were set as follows: $\alpha_{\text{MCC}}=0.03$, $\alpha_{\text{NMCC}}=0.4$, $\alpha_{\text{PNMCC}} = \alpha_{\text{ZA}} = \alpha_{\text{RZA}} = 0.3$, $\gamma_{\text{ZA}} = 5 \times 10^{-5}$, $\gamma_{\text{RZA}} = 1 \times 10^{-4}$, $\alpha_{\text{ACC}} = 0.27$, $\gamma_{\text{ACC}} = 5 \times 10^{-4}$, and $\xi=5$. α_{MCC} , α_{ZA} , and α_{RZA} are the step sizes of the MCC, ZA-PNMCC, and RZA-PNMCC algorithms, respectively. Moreover, γ_{ZA} and γ_{RZA} are the trade-off parameters of the ZA-PNMCC and RZA-PNMCC algorithms, respectively. The MSD performance of the

proposed ACC-PNMCC algorithm for $K = 1$ is given in Fig. 4. It is observed that ACC-PNMCC achieves the lowest steady-state MSD when all the compared algorithms have the same initial convergence speed. Then, the corresponding steady-state MSDs for $K = 2, 4$, and 8 are shown in Figs. 5, 6, and 7. We note that the steady-state MSDs of the ZA-PNMCC, RZA-PNMCC and ACC-PNMCC algorithms increased with the sparsity level increasing from $K = 1$ to $K = 8$. However, it is worth noting that the MSD of the ACC-PNMCC algorithm is still lower than those of the other algorithms. In addition, the effects on the ACC-PNMCC algorithm with a sparsity level K/M are shown in Fig. 8 to more intuitively understand the effect of sparsity level on the MSDs of compared algorithms. It is found that the MSDs of ZA-PNMCC, RZA-PNMCC, and ACC-PNMCC gradually increase as the sparsity level K/M increases from 0.0625 to 0.5, which is similar to the results in Figs. 4, 5, 6, and 7. The MSDs of the MCC, NMCC, and PNMCC algorithms remain almost invariant, as they do not have any zero attractor term that is sensitive to sparsity K/M . However, one should note that the MSDs of ZA-PNMCC and ACC-PNMCC are almost equal to 1×10^{-4} when $K/M = 0$. This is because the ACC-PNMCC algorithm uses l_1 -norm when $K/M = 0$, like the ZA-PNMCC algorithm. When K/M is greater than 0.0625, the ACC-PNMCC algorithm is always better than the other algorithms. Thus, we can say that the ACC-PNMCC algorithm is suitable for sparse multi-path channel estimation in practical applications.

Fig. 8 The MSD performance of the proposed ACC-PNMCC algorithm with different coefficient sparsities

The third experiment considered the application of ACC-PNMCC to echo cancellation. The typical echo path of a 256-tap channel with 16 non-zero coefficients is shown in Fig. 9, which is also known as block-sparse path [52]. However, the data reusing, sign, and memory schemes have been considered in [52], which has high complexity. Additionally, the sparsity is not given by a formula. Herein, the sparsity characteristic of the echo channel is measured by $\vartheta_{12} = \frac{M}{M-\sqrt{M}} (1 - \|\mathbf{g}\|_1 / \sqrt{M} \|\mathbf{g}\|_2)$ [40, 45, 49]. In this experiment, we use $\vartheta_{12} = 0.8222$ for the first 8000 iterations, while $\vartheta_{12} = 0.7362$ are set for the after 8000 iterations. Other simulation parameters are $\alpha_{\text{MCC}}=0.0055$, $\alpha_{\text{NMCC}}=1.3$, $\alpha_{\text{PNMCC}} = \alpha_{\text{ZA}} = \alpha_{\text{RZA}} = 0.9$, $\gamma_{\text{ZA}} = 1 \times 10^{-6}$, $\gamma_{\text{RZA}} = 1.5 \times 10^{-6}$, $\alpha_{\text{ACC}} = 0.8$, $\gamma_{\text{ACC}} = 5 \times 10^{-6}$, and $\xi=5$. The corresponding steady-state MSDs are shown in Fig. 9. It can be seen that ACC-PNMCC outperforms the other algorithms in terms of both steady-state MSD and convergence speed. Although the sparsity is changed from 0.8222 to 0.7362, the performance of the proposed ACC-PNMCC algorithm is still superior to that of the other algorithms, indicating the effectiveness of the ACC-PNMCC algorithm for echo cancellation.

At last, the computational complexity of the proposed ACC-PNMCC algorithm is presented in Table 1 in comparison with the corresponding algorithms. Herein, the numerical complexities of the algorithms include multiplications, additions, exponentiations, divisions, and

comparisons. From Table 1, it can be seen that the computational complexity of the developed ACC-PNMCC algorithm is slightly higher than that of the ZA-PNMCC and RZA-PNMCC algorithms, which is owing to the calculation of the ACF. However, the proposed ACC-PNMCC algorithm obviously increases the convergence speed and reduces the MSD.

Based on the above experiment analysis, we infer that the proposed ACC-PNMCC algorithm has a superior steady-state MSD performance and convergence speed for sparse multi-path channel estimation applications. This is because ACC-PNMCC utilizes the l_0 -norm penalty for the channel coefficients which are larger than our designed threshold, while it exerts the l_1 -norm penalty on the channel coefficients that are smaller than the designed threshold to attract relatively small coefficients to zero to improve the convergence. Thus, the proposed ACC-PNMCC algorithm performed better than the other algorithms for sparse channel estimation.

4 Conclusions

In this paper, an ACC-PNMCC algorithm has been proposed for sparse multi-path channel estimation. The proposed ACC-PNMCC algorithm exploits inherent sparsity features of sparse multi-path channels by utilizing the designed zero attractor. The performance of the algorithm was investigated and compared with the performances of the MCC, NMCC, PNMCC, and sparse

Fig. 9 The MSD performance of the proposed ACC-PNMCC algorithm in echo cancellation application

Table 1 Numerical complexities of the algorithms

Algorithms	Multiplications	Additions	Exponentiations	Divisions	Comparisons
MCC	$2M + 4$	$2M$	1	-	-
NMCC	$3M + 4$	$3M$	1	1	-
PNMCC	$4M + 7$	$6M + 2$	1	$M + 2$	-
ZA-PNMCC	$4M + 7$	$7M + 2$	1	$M + 2$	-
RZA-PNMCC	$4M + 7$	$8M + 2$	1	$M + 2$	-
ACC-PNMCC	$5M + 7$	$9M - 1$	1	$M + 1$	M

PNMCC algorithms for sparse multi-path channel estimation. Experimental results illustrated that the proposed ACC-PNMCC algorithm is superior to the competing algorithms in terms of both the steady-state MSD and convergence speed.

Appendix

Similar to the ZA-MCC and ZA-LMS algorithms [16, 33, 37, 38], the l_1 -norm is introduced into the cost function of the PNMCC algorithm to develop the zero-attracting PNMCC (ZA-PNMCC) algorithm. The cost function of the ZA-PNMCC algorithm is

$$J_{ZA}(n+1) = (\hat{\mathbf{g}}(n+1) - \hat{\mathbf{g}}(n))^T \mathbf{G}^{-1}(n) (\hat{\mathbf{g}}(n+1) - \hat{\mathbf{g}}(n)) + \gamma_{ZA} \|\mathbf{G}^{-1}(n) \hat{\mathbf{g}}(n+1)\|_1 + \lambda_{ZA} \left[\hat{e}(n) - \left[1 - \alpha \exp\left(-\frac{e^2(n)}{2\sigma^2}\right) \right] e(n) \right], \quad (26)$$

where λ_{ZA} is Lagrange multiplier. By using LMM, the updating equation of the ZA-PNMCC can be written as

$$\hat{\mathbf{g}}(n+1) = \hat{\mathbf{g}}(n) + \alpha_{ZA} \frac{\exp\left(-\frac{e^2(n)}{2\sigma^2}\right) e(n) \mathbf{G}(n) \mathbf{x}(n)}{\mathbf{x}^T(n) \mathbf{G}(n) \mathbf{x}(n) + \theta} - \gamma_{ZA} \text{sgn}(\hat{\mathbf{g}}(n)), \quad (27)$$

where α_{ZA} and γ_{ZA} are the step size and trade-off parameter of the ZA-PNMCC algorithm, respectively. Similarly to the RZA-MCC and RZA-LMS algorithms [16, 33, 37, 38], a reweighting factor is introduced into the ZA-PNMCC algorithm to develop the reweighting ZA-PNMCC (RZA-PNMCC) algorithm, the corresponding updating equation is

$$\hat{\mathbf{g}}(n+1) = \hat{\mathbf{g}}(n) + \alpha_{RZA} \frac{\exp\left(-\frac{e^2(n)}{2\sigma^2}\right) e(n) \mathbf{G}(n) \mathbf{x}(n)}{\mathbf{x}^T(n) \mathbf{G}(n) \mathbf{x}(n) + \theta} - \gamma_{RZA} \frac{\text{sgn}(\hat{\mathbf{g}}(n))}{1 + \xi_1 |\hat{\mathbf{g}}(n)|}, \quad (28)$$

where ξ_1 is reweighting factor, α_{RZA} denotes step size and γ_{RZA} represents trade-off parameter of the RZA-PNMCC algorithm.

Abbreviations

ACC-PNMCC: An adaptive combination constrained proportionate normalized maximum correntropy criterion; ACF: Adaptive combination function; AF: Adaptive filter; CIM: Correntropy induced metric; CIR: Channel impulse response; CS: Compressive sensing; GMCC: Generalized maximum correntropy criterion; LMS: Least mean square; MCC: Maximum correntropy criterion; MEE: Minimum error entropy; MSD: Mean square deviation MSE: Mean square error; NGWNE: Non-Gaussian white noise environments; PNMCC: Proportionate normalized maximum correntropy criterion; RZA-MCC: Reweighting zero attracting MCC; SNR: Signal-to-noise; SSE: Steady-state error; ZA-LMS: Zero-attracting LMS; ZA-MCC: Zero-attracting MCC; ZA-NMCC: Zero-attracting NMCC

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Availability of data and materials

Please contact authors for data requests.

Authors' contributions

YW devised the code and did the simulation experiments, then she wrote the draft of this paper. YL helped to check the codes and simulations, and he also put forward to the idea of the ACC-PNMCC algorithm. JCM and XH helped to check the derivation of the formulas and improved the English of this paper. All the authors wrote this paper together, and they have read and approved the final manuscript.

Ethics approval and consent to participate

This study does not involve human participants, human data, or human tissue.

Consent for publication

In the manuscript, there is no any individual person's data.

Competing interests

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