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Signal-to-noise ratio estimation for M-QAM signals in $\eta - \mu$ and $\kappa - \mu$ fading channels

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Abstract

In this paper, signal-to-noise ratio (SNR) estimation is carried out by the method of moments (MOM) for fading channels modeled by probability distributions $\eta - \mu$ and $\kappa - \mu$, considering M-ary quadrature amplitude modulation (M-QAM) with constellation energy normalized to one. New expressions are presented for the SNR estimation and for the mean, variance, and normalized mean square error (NMSE) of the estimates, obtained by a statistical linearization argument. Additionally, it is shown how to obtain the SNR estimate for Nakagami-m channel from the estimation derived for the models $\eta - \mu$ and $\kappa - \mu$. The results obtained from the analytical expressions are corroborated by simulation results and show that the MOM is a suitable alternative for scenarios in which the mathematical tractability does not suggest the application of other estimation techniques.

Keywords: SNR estimation, Fading channels, $\eta - \mu$ fading, $\kappa - \mu$ fading, Method of moments

1 Introduction

Signal-to-noise ratio (SNR) estimation has been a recurrent research topic, due to the relevance of SNR for a variety of mobile communication systems. The a priori knowledge of the communication channel conditions is an important issue as long as those systems become more complex and widely required. In [1] for instance, the a priori knowledge of the channel, by means of the SNR, is proposed for evaluating the effective transmission rate (throughput) in a communication system with adaptive modulation and coding, while in [2] its use is considered in adaptive transmission systems.

In [3], the knowledge of the SNR is necessary for assessing the time-varying channel condition of an adaptive system with frequency hopping, and in [4] it is necessary for planning relay communication systems. In [5], SNR is a useful parameter in the scenario of turbo decoding systems, while in [6] it is useful in the context of low-density parity check (LDPC) codes.

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Usually, the choice of the estimation technique depends on the complexity of the mathematical model of the signal in the receiver [4, 7]. Depending on that complexity, one can use a method belonging to the class of estimators that use a training sequence (that is, a data-aided method (DA)) [8, 9], or a method belonging to the class of estimators that do not have a priori knowledge of the transmitted sequence of symbols (that is, a non-data-aided method (NDA)) [10].

In [11], for example, a new SNR estimator is derived, DA and NDA, for a slow time-varying channel with impulse response characterized by a polynomial function of order L_c and phase-shift keying (PSK) signals. In the NDA scenario, the expectation maximization (EM) algorithm is proposed for the calculation of the estimates and improvement of performance is observed in relation to the estimator with channel considered constant throughout the time of observation, at the cost of a moderate increase in computational complexity. The difference of that approach in relation to the method of moments (MOM) NDA technique considered in our article is that the MOM is not efficient in reaching the Cramér-Rao bound.

In [12], the least-squares (LS) technique is proposed for estimation of SNR in a received signal model composed

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by M-QAM symbols ($M = 16$) multiplied by samples of the impulse response of a time-varying channel added to samples of white Gaussian noise. The impulse response of the channel is approximated by a polynomial as a function of time with constant coefficients calculated by means of the LS technique, with the aid of a pilot sequence sent periodically and an array of antennas of N_a elements. The performance of the technique is evaluated by means of the normalized root mean square error (NRMSE), without comparison with the Cramér-Rao bound, and requires a much smaller number of samples than the required by the MOM without training sequence, in order to reach NRMSE values below 0.1 for an SNR above 10 dB.

In [13], one considers the maximum likelihood estimation (MLE) of the SNR in the demodulator of an orthogonal and non-coherent frequency-shift keying (FSK) signal, in which the transmitted signal is affected by Rayleigh fading and that this fading is constant in a block of k symbols, while in [14], the authors consider a binary frequency-shift keying (BFSK) affected by constant Rayleigh fading in a block of symbols whose BFSK carrier has frequency deviation. The MLE with and without pilot sequences are presented for the SNR, assisted by estimators by the MOM for the frequency deviation. Although the MLE is optimum in the sense of the minimum variance criterion, it depends on the expression of the probability density function (PDF) of the samples of the observed signal, and in the case of generalized fading models, such as those considered in this work, their PDFs make the PDFs of the received signal complicated for treatment by MLE.

In [15], the authors consider a model of received signal for a signal with frequency modulated differential chaos shift keying (FMDCSK) transmitted by a channel with multipath propagation and Rayleigh fading. The authors analyze the SNR estimators and their performance considering that the channel coefficients that characterize the fading are constant in a sequence of K symbols of the FMDCSK signal. The estimators are calculated for the data-aided (DA), non-data-aided (NDA), and joint DA-NDA cases, presenting good results in relation to the proximity of the Cramér-Rao Lower Bound (CRLB) for values of SNR above 20 dB.

In [16], an estimation method of SNR is presented for linearly modulated signals captured by an array of antennas in an environment with complex white additive Gaussian noise with spatial uncorrelation between the elements. In this NDA and single input multiple output (SIMO) estimator, based on the MOM, the performance is assessed in terms of the Normalized Mean Square Error (NMSE) for QAM signals and improves with the increase of the number of elements. The estimation of SNR in SIMO systems is also addressed in [17], in which the authors consider signal samples captured by an array of antennas in a channel model of constant gain by the time of observation of a sequence of symbols. A MLE based on the I/Q components of the received signal is evaluated, for the cases DA and NDA, that reach the CRLB for a wide range of SNR. In [18], the problem of estimation of the SNR is extended to a multiple input multiple output (MIMO) system in a channel model with block flat fading, in which the channel gain matrix is considered constant by a block of N symbols. From this consideration and reduction of the channel model to a Gaussian channel to each block of N symbols, the authors then present ML estimators for the DA and NDA cases, as well as the CRLB.

In [19], the main contribution of the authors is the analysis of the CRLB of SNR estimates of signals with minimum shift keying (MSK) modulation and QAM with turbo encoding. Even in a constant gain channel model over a K symbols window, the task of calculating the CRLB for the M-QAM symbols is the most laborious of the article and is solved from the analysis of the structure of Gray's mapping. The authors then show that the proposed DA ML estimator has a lower CRLB than the NDA CRLB case. The CRLB is also evaluated in [20] for a channel model of constant gain over the whole set of symbols observed under QAM modulation, for the cases DA and NDA. In the channel models of the aforementioned references, the channel gain is considered constant by a sequence of symbol intervals, which contributes to the mathematical treatment of estimation by maximizing the likelihood function. In our proposed model, in which fading can vary at each time interval in which a sample is obtained at the output of the receiver's matched filter, the PDF of the received signal modulus makes the likelihood function calculation more difficult.

In [21], the proposed solution consists of using the goodness-of-fit test of Kolmogorov-Smirnov (K-S) for the evaluation of the distance between empirical cumulative distribution function (CDF) generated from observed samples of the received signal and the theoretical curves of CDFs generated and stored in a file for different configurations of channel parameters. The proposed estimator was presented for a Gaussian channel, of constant gain by a sequence of symbol intervals, and was evaluated by means of the normalized root mean square error (NRMSE). For fading channels, such as the models considered in this manuscript, the method would depend on the calculation of the CDF of the envelope of the signal received, which would be a challenging task.

It is worth mentioning that in the paper by Bellili et al. [22], the authors do not make any other consideration about the channel coefficients than they are deterministic and unknown. In our article, the only consideration is that the fading is constant during the interval of the impulse response of the matched filter in the receiver. The main merit for our work is that the random behavior of the channel is taken into account by the fading probability distributions and the estimators obtained have simple expressions, encompass the estimators for other fading models, and are not aided by data sequence. In the flat fading channel model considered in [22], the NDA estimator needs the EM algorithm that converges for an optimum solution with a reduced number of iterations at cost of a greater computational complexity.

In propagation environments in which the fading is characterized by probability distributions such as $\eta-\mu$ and $\kappa-\mu$ [23], the SNR estimation is difficult by using estimators such as MLE, because the likelihood function for the problem becomes complicated. In these cases, the MOM is a good alternative.

Despite raising the mathematical complexity of the SNR estimation problem, general probability distributions, such as the $\eta-\mu$ and $\kappa-\mu$ distributions, model a wider variety of fading signals, such as signals received on Nakagami-m, Hoyt (Nakagami-q), Rice (Nakagami-n), unilateral Gauss, and Rayleigh channels. According to [23], the $\kappa-\mu$ distribution is better suited for line-of-sight applications and the $\eta-\mu$ distribution is more appropriate for non-line-of-sight applications. The SNR estimators obtained for the signal received under these two fading models encompass all the estimators obtained for the other fading models mentioned. The estimation of SNR by the MOM has been the considered technique, as shown, for example, in references [24, 25] and more recently in reference [26] in which the transmitted signal is modeled by a complex Gaussian random variable with zero mean by component. In [27], the MOM is proposed in a process of joint estimation of both the K parameter of the Rice fading distribution and SNR in a SIMO communication system.

In the present paper, the method of moments for SNR estimation is applied to channel models in which the fading is characterized by $\eta-\mu$ and $\kappa-\mu$ distributions and modulation scheme M-QAM [28] is considered.

1.1 Main contributions

The main contributions of this paper are as follows:

1. New expressions for the SNR estimates by NDA MOM for a received signal model for M-QAM signal under $\eta-\mu$ and $\kappa-\mu$ fading.
2. New expressions for the evaluation of the mean, variance, and NMSE of the estimates, obtained from a statistical linearization procedure.
3. Comparative analysis between the estimates obtained for the models $\eta-\mu$, $\kappa-\mu$, and Nakagami-m.
4. Exact expressions for the moments of order k of the envelope of the observed signal.

The remaining of the paper is organized as follows. In Section 2, methods used in the work and the problem definition are presented. In Section 3, the derivation of the kth moment of the absolute value of the samples of the observed signal is presented. In Section 4, the derivation of the SNR estimates is presented. In Section 5, it is shown how to obtain the SNR estimates for the signal under Nakagami fading from the estimates for the models $\eta-\mu$ and $\kappa-\mu$. In Section 6, the derivation of the mean, variance, and NMSE of the estimates is presented. In Section 7, the moments of order 2, 4, 6, and 8 of M-QAM are presented. In Section 8, a proposal is presented for evaluating the CRLB. In Section 9, theoretical curves corroborated by curves obtained by simulations are presented, and in Section 10, the conclusions of the work are provided.

2 Methods

The aim of this paper is to apply the MOM in the estimation of the SNR considering communication channels subject to $\eta-\mu$ and $\kappa-\mu$ generalized fading and M-QAM modulation schemes. Firstly, the channel model is described and the moments of the received signal are determined. Then, the estimation of the SNR is obtained from the ratio of the square of the second moment and the fourth moment considering the generalized fading models. The performance of the method is assessed by means of the normalized mean square error and the variance of the estimate.

2.1 Problem definition

The mathematical model for a signal sample $r[n]$ observed at the output of the detector with a matched filter, in a discrete time n, from a transmission through a fading channel, modeled by a multiplicative gain $g[n]$ and by additive noise $w[n]$, can be written as

$$r[n]=g[n]\,s[n]+w[n]\,. \tag{1}$$

In this model of received signal, $s[n]$ represents the equiprobable symbols of a M-QAM constellation, normalized such that the average energy per symbol be unitary, and $w[n]$ is a Gaussian complex random variable of zero mean and variance $2\sigma_w^2$. In this paper, the modulus of the random variable $g[n]$ is characterized by the probability distributions $\eta-\mu$ and $\kappa-\mu$ [23], which have been used to characterize the envelope of the fading in mobile communication channels since both of them can represent a wide variety of fading models. The average power of the envelope of $g[n]$ is given by $\sigma_g^2=E\left[|g[n]|^2\right]$, in which $E[\cdot]$ is the expected value operator.

The $\eta-\mu$ model is used to characterize small scale intensity variations in the faded signal in non-line-of-sight links and has two types, format I and format II. In format I, the parameter η varies in the interval $[1,\infty)$ and represents the ratio of the powers of the in-phase and quadrature components of the scattered electromagnetic wave, and in format II, the parameter η varies in the interval $(-1, 1)$ and represents the correlation coefficient between the in-phase and quadrature components of the electromagnetic wave in each cluster of multipath. The parameter μ has the same physical meaning in both formats and represents an extension of the number of clusters of multipath to the real numbers [23]. The PDF of $|g[n]|$ can be written as

$$f_{|g[n]|}(|g[n]|) = \frac{4\sqrt{\pi}\mu^{\mu+\frac{1}{2}}h^{\mu}}{\Gamma(\mu)H^{\mu-\frac{1}{2}}}\frac{1}{\sqrt{\sigma_g^2}}\left(\frac{|g[n]|^2}{\sigma_g^2}\right)^{\mu}\times \exp\left(-2\mu h\left(\frac{|g[n]|^2}{\sigma_g^2}\right)\right)\mathrm{I}_{\mu-\frac{1}{2}}\left(2\mu H\frac{|g[n]|^2}{\sigma_g^2}\right), \tag{2}$$

in which $h = \left(2+\eta^{-1}+\eta\right)/4$ and $H = \left(\eta^{-1}-\eta\right)/4$ for format I, while $h = \left(1-\eta^2\right)^{-1}$ and $H = \eta/\left(1-\eta^2\right)$ for format II. The term $\mathrm{I}_{\nu}(x)$ represents the modified Bessel function of first kind and order ν and $\Gamma(\mu)$ represents the Gamma function.

The probability distribution $\kappa-\mu$, on the other hand, can be applied in the characterization of small scale fading (variations in the signal intensity in short time intervals from propagation by multiple paths) and with line-of-sight. The parameter $\mu \geq 0$ represents the number of clusters of multipath in the environment. The parameter κ is defined as the ratio of the total power of the dominant components to the total power of the scattered waves. For an envelope $|g[n]|$ of root mean square (rms) value $\hat{r} = \sqrt{E\left[|g[n]|^2\right]} = \sqrt{\sigma_g^2}$, the PDF of the envelope $\kappa-\mu$ can be written as

$$f_{|g[n]|}\left(|g[n]|\right) = \frac{2\mu\left(1+\kappa\right)^{\frac{\mu+1}{2}}}{\kappa^{\frac{\mu-1}{2}}\exp^{(\kappa\mu)}\sqrt{\sigma_g^2}}\left(\frac{|g[n]|}{\sigma_g^2}\right)^{\mu} \times \exp\left(-\mu\left(1+\kappa\right)\left(\frac{|g[n]|^2}{\sigma_g^2}\right)\right) \times \mathrm{I}_{\mu-1}\left(2\mu\sqrt{\kappa\left(1+\kappa\right)}\left(\frac{|g[n]|}{\sqrt{\sigma_g^2}}\right)\right). \tag{3}$$

Considering that the variables $g[n]$, $s[n]$, and $w[n]$ are independent, it follows that a sequence of samples of $r[n]$ can be seen as a sequence of the faded signal $g[n]s[n]$ affected by a white Gaussian noise sequence $w[n]$. In this model of $r[n]$, the average power can be written as

$$P_r = E[r[n]r^*[n]] = E\left[|g[n]|^2\right]E\left[|s[n]|^2\right]+E\left[|w[n]|^2\right] \tag{4}$$

and the SNR can be written as

$$\mathrm{SNR} = \frac{E\left[|g[n]|^2\right]E\left[|s[n]|^2\right]}{E\left[|w[n]|^2\right]} = \frac{\sigma_g^2}{2\sigma_w^2}\sum_{i=1}^{M}p_i|A_i|^2, \tag{5}$$

in which $\sigma_g^2 = E\left[|g[n]|^2\right]$ represents the average power of the fading, M represents the order of the QAM constellation, and $|A_i|$ and p_i represent the amplitude and probability, respectively, of the occurrence of the ith symbol.

Since the QAM constellation symbols considered in the study are equiprobable, it follows that $\sum_{i=1}^{M}p_i|A_i|^2 = 1$. Thus, if one assumes $\gamma = \sigma_g^2/\sigma_w^2$, it follows that the estimate of the SNR γ can be calculated from a function $\mathrm{f}(\gamma)$ defined as the ratio between the square of the second moment M_2 and the fourth moment M_4 of $|r[n]|$. If the rational function $\mathrm{f}(\gamma)$ is algebraically inverted in terms of the ratio between the square of the second sample moment and the fourth sample moment of $|r[n]|$, then this inverse obtained is the estimate of γ and will be denoted by $\hat{\gamma}$. The problem is then to find the expressions of the moments of the variable $|r[n]|$.

3 Derivation of the *k*th moment of $|r[n]|$

For calculating the SNR estimate, $\hat{\gamma}$, one needs the moments of order 2 and 4 of $|r[n]|$. This is because the moment of order 2 of $|r[n]|$ is directly proportional to σ_w^2, while the moment of order 4 is directly proportional to the square of σ_w^2. Hence, the ratio of the square of the second moment and the fourth moment of $|r[n]|$ eliminates the dependence on the average power σ_w^2 and remains as a function of the SNR γ, which is the variable to be estimated in this study. In order to calculate the variance of the estimate $\hat{\gamma}$, the moments of order 2, 4, 6, and 8 of $|r[n]|$ are required. In this section, an exact expression is presented for the moment of order k, and then, the expressions of the individual moments are obtained in their simplified forms.

The kth moment of $|r[n]|$ can be calculated from the model $r[n]$ presented in (1) from the PDF of $|r[n]|$ conditioned on $|g[n]|$ and $|s[n]|$, written as [29]

$$f_{|r[n]|}(|r[n]|\,||g[n]|,|s[n]|) = \frac{|r[n]|}{\sigma_w^2} \times \exp\left(-\left(\frac{|r[n]|^2+|g[n]|^2|s[n]|^2}{2\sigma_w^2}\right)\right) \times \mathrm{I}_0\left(|r[n]|\frac{|g[n]||s[n]|}{\sigma_w^2}\right), \tag{6}$$

in which $I_0(x)$ represents the zero order modified Bessel function. For the sake of simplicity, it is appropriate to simplify the notation and represent the variables $|r[n]|$ and $|g[n]|$, respectively, by r and g.

Taking the mean of $f_r(r|g, |s[n]|)$ with respect to probability distribution of the modulus of the QAM constellation symbols considered and substituting the resulting expression in the integral that defines the conditional expectation $E\left[r^k|g\right]$, it follows, after some simplifications, that

$$E\left[r^k|g\right] = \frac{1}{\sigma_w^2}\sum_{i=1}^{M} p_i \exp\left(-\frac{g^2|A_i|^2}{2\sigma_w^2}\right) \times \int_0^\infty r^{k+1}\exp\left(-\frac{r^2}{2\sigma_w^2}\right)\mathrm{I}_0\left(2\frac{g|A_i|}{2\sigma_w^2}r\right)dr. \tag{7}$$

The integral in (7) can be solved by Expression 6.643 of the table of integrals ([30] pp. 709). Representing the Whittaker's function resulting from the solution of this integral by the Kummer's confluent hypergeometric function ${}_1\mathrm{F}_1(a;b;z)$ [30] and making the necessary algebraic simplifications, one can write the expression of kth moment of $|r[n]|$ conditioned on $|g[n]|$ as

$$E\left[r^k|g\right] = 2^{\frac{k}{2}}\left(\sigma_w^2\right)^{\frac{k}{2}+1}\Gamma\left(\frac{k}{2}+1\right) \times \sum_{i=1}^{M} p_i \exp\left(-\frac{g^2|A_i|^2}{2\sigma_w^2}\right){}_1\mathrm{F}_1\left(\frac{k}{2}+1;1;\frac{g^2|A_i|^2}{2\sigma_w^2}\right). \tag{8}$$

The kth moment of $|r[n]|$ can then be calculated taking the average of $E\left[r^k|g\right]$ with respect to PDF of the fading envelope, by the calculus of the integral

$$E\left[r^k\right] = \int_0^\infty E\left[r^k|g\right] f_g(g)dg. \tag{9}$$

3.1 Moments of $|r[n]|$ for the distribution $\eta - \mu$

The kth moment of $|r[n]|$ for $\eta-\mu$ fading can be obtained by substituting the expressions (2) and (8) in (9), so that one can write

$$E\left[|r[n]|^k\right] = \frac{\sqrt{2\pi}\,2^{\frac{k}{2}}\Gamma\left(\frac{k}{2}+1\right)}{\Gamma(\mu)(2H)^{\mu}}\left(\frac{H}{h}\right)^{\frac{1}{2}}\left(\sigma_w^2\right)^{\frac{k}{2}} \times \sum_{i=1}^{M}\frac{p_i}{(1+\gamma_i)^{\mu+\frac{1}{2}}}\int_0^\infty v^{\mu-\frac{1}{2}}\exp^{(-v)}\mathrm{I}_{\mu-\frac{1}{2}}\left[\frac{H}{h}\frac{v}{1+\gamma_i}\right] \times {}_1F_1\left(\frac{k}{2}+1;1;\frac{v}{1+\frac{1}{\gamma_i}}\right)dv, \tag{10}$$

in which

$$\gamma_i = \frac{|A_i|^2}{4h\mu}\frac{\sigma_g^2}{\sigma_w^2}. \tag{11}$$

The expressions of the moments of order 2, 4, 6, and 8, denoted by M_2, M_4, M_6 and M_8, respectively, can then be obtained from (10) by writing the Kummer's confluent hypergeometric function in terms of Laguerre polynomials [31],

$${}_1F_1(a;1;z) = e^z L_{a-1}(-z), \tag{12}$$

in which the polynomials $\mathrm{L}_n(x)$ can be generated from the Olinde Rodrigues formula [31], given by

$$L_n(x) = \frac{e^x}{n!}\frac{d^n}{dx^n}\left(e^{-x}x^n\right). \tag{13}$$

Using the representation (12) and simplifying the resulting integrals, one obtains, after the procedure of simplification, that the moments M_2, M_4, M_6, and M_8 can be written as polynomials as a function of the SNR γ as

$$\begin{aligned}
M_2 &= \left(\sigma_w^2\right)^1\left(\alpha_{21}\gamma^1+2\right),\\
M_4 &= \left(\sigma_w^2\right)^2\left(\alpha_{42}\gamma^2+\alpha_{41}\gamma+8\right),\\
M_6 &= \left(\sigma_w^2\right)^3\left(\alpha_{63}\gamma^3+\alpha_{62}\gamma^2+\alpha_{61}\gamma+48\right),\\
M_8 &= \left(\sigma_w^2\right)^4\left(\alpha_{84}\gamma^4+\alpha_{83}\gamma^3+\alpha_{82}\gamma^2+\alpha_{81}\gamma+384\right),
\end{aligned} \tag{14}$$

in which the parameters α_{ij} can be described by the group of expressions (15) and by the relations (16),

$$\begin{cases}
\alpha_{21} = C_2,\\
\alpha_{42} = \delta\left[2\mu+\xi^2+1\right]C_4,\\
\alpha_{63} = \delta^2\left[2\mu^2+3\left(\xi^2+1\right)\mu+3\xi^2+1\right]2C_6,\\
\alpha_{84} = \delta^3\left[4\mu^3+12(\xi^2+1)\mu^2+\left(3\xi^4+30\xi^2+11\right)\mu\right.\\
\qquad\left.+\,3\xi^4+18\xi^2+3\right]2C_8
\end{cases} \tag{15}$$

and

$$\begin{cases}
\alpha_{41} = 8\alpha_{21},\\
\alpha_{61} = 72\alpha_{21}, \quad \alpha_{62} = 18\alpha_{42},\\
\alpha_{81} = 768\alpha_{21}, \ \alpha_{82} = 288\alpha_{42}, \ \alpha_{83} = 32\alpha_{63},
\end{cases} \tag{16}$$

in which $\delta = 1/(2\mu)$, $\xi = H/h$, and C_k represent the kth moment of the constellation M-QAM.

3.2 Moments of $|r[n]|$ for the distribution $\kappa - \mu$

Similarly, substituting expressions (3) and (8) in (9), from the kth conditioned moment, one can write the kth moment of $|r[n]|$ under $\kappa - \mu$ fading as

$$E\left[|r[n]|^k\right] = \frac{2^{\frac{k}{2}}\left(\sigma_w^2\right)^{\frac{k}{2}}\Gamma\left(\frac{k}{2}+1\right)}{\kappa\mu^{\frac{\mu-1}{2}}\exp(\kappa\mu)}\sum_{i=1}^{M}p_i\left(\frac{1}{1+\gamma_i}\right)^{\frac{\mu}{2}+\frac{1}{2}} \times \int_0^\infty w^{\frac{\mu}{2}-\frac{1}{2}}\exp(-w){}_1F_1\left(\frac{k}{2}+1,1;\frac{w}{1+\frac{1}{\gamma_i}}\right) \times \mathrm{I}_{\mu-1}\left(2\sqrt{\frac{\kappa\mu w}{1+\gamma_i}}\right)dw, \tag{17}$$

in which

$$\gamma_i = \frac{\sigma_g^2}{\sigma_w^2}\frac{|A_i|^2}{2\mu(1+\kappa)}. \tag{18}$$

Expressions of the moments of order 2, 4, 6, and 8, denoted by M_2, M_4, M_6 and M_8, respectively, can be obtained from (17) writing the Kummer confluent hypergeometric function in terms of polynomials of Laguerre [31], so that the moments can be written as a function of the SNR γ,

$$\begin{aligned}
M_2 &= \left(\sigma_w^2\right)^1 \left(\beta_{21}\gamma^1 + 2\right),\\
M_4 &= \left(\sigma_w^2\right)^2 \left(\beta_{42}\gamma^2 + \beta_{41}\gamma + 8\right),\\
M_6 &= \left(\sigma_w^2\right)^3 \left(\beta_{63}\gamma^3 + \beta_{62}\gamma^2 + \beta_{61}\gamma + 48\right),\\
M_8 &= \left(\sigma_w^2\right)^4 \left(\beta_{84}\gamma^4 + \beta_{83}\gamma^3 + \beta_{82}\gamma^2 + \beta_{81}\gamma + 384\right),
\end{aligned} \tag{19}$$

in which the relations between the coefficients β_{ij} can be written as in the groups of expressions (20) and (21).

$$\begin{cases}
\beta_{21} = C_2,\\
\beta_{42} = \zeta^2\left[(\kappa+1)^2\mu^2 + (2\kappa+1)\mu\right]C_4,\\
\beta_{63} = \zeta^3\left[(\kappa+1)^3\mu^3 + 3(2\kappa^2+3\kappa+1)\mu^2\right.\\
\qquad \left.+2(3\kappa+1)\mu\right]C_6,\\
\beta_{84} = \zeta^4\left[\left(\kappa^4+64\kappa^2+4\kappa+1\right)\mu^4 + 6\left(2\kappa^3+5\kappa^2\right.\right.\\
\qquad \left.+4\kappa+1\right)\mu^3\\
\qquad \left.+(36\kappa^2+44\kappa+11)\mu^2 + 6(4\kappa+1)\mu\right]C_8,
\end{cases} \tag{20}$$

and

$$\begin{cases}
\beta_{41} = 8\beta_{21},\\
\beta_{61} = 72\beta_{21}, \quad \beta_{62} = 18\beta_{42},\\
\beta_{81} = 768\beta_{21},\ \beta_{82} = 288\beta_{42},\ \beta_{83} = 32\beta_{63},
\end{cases} \tag{21}$$

in which $\zeta = 1/(\mu(1+\kappa))$.

4 Derivation of SNR estimates

In Section 3, exact expressions were obtained for the moments of order 2, 4, 6, and 8 of the received signal envelope in discrete time, $|r[n]|$, for the fading models $\eta-\mu$ and $\kappa-\mu$. The estimation of the SNR can then be obtained from the ratio between the square of the second moment and the fourth moment for these models. This ratio can be expressed in terms of the SNR $\gamma = \sigma_g^2/\sigma_w^2$ and is denoted $\mathrm{f}(\gamma)$ for formats I and II of fading $\eta-\mu$ and for $\kappa-\mu$. Thus, if the ratio

$$\mathrm{f}(\gamma) = \frac{E\left[|r[n]|^2\right]^2}{E\left[|r[n]|^4\right]} = \frac{\mathrm{p}(\gamma)}{\mathrm{q}(\gamma)} \tag{22}$$

can be written as a function of γ and this function can be inverted in terms of the ratio of the square of the second sample moment and the fourth sample moment of $|r[n]|$, denoted by sample ratio and represented by $\hat{\mathrm{f}}$, then the inverse obtained is an estimate $\hat{\gamma}$ of the SNR γ.

In the first case, in which the fading is characterized by format I of the distribution $\eta-\mu$, for which $h = (2+\eta^{-1}+\eta)/4$ and $H = (\eta^{-1}-\eta)/4$, the function $\mathrm{f}(\gamma)$ can be written as the ratio of two polynomials of order 2 in γ,

$$\mathrm{f}(\gamma) = \frac{4 + 4C_2\gamma + C_2^2\gamma^2}{8 + 8C_2\gamma + 4\varrho C_4\gamma^2}, \tag{23}$$

in which ϱ is given in (24).

When inverted in terms of the sample ratio $\hat{\mathrm{f}}$, the estimate $\hat{\gamma}$ under the format I of fading $\eta-\mu$ may be written as

$$\text{Format I } \eta-\mu \begin{cases}
\hat{\gamma} = \frac{2(1-2\hat{\mathrm{f}})+2\sqrt{2\hat{\mathrm{f}}(1-2\hat{\mathrm{f}})(\epsilon-1)}}{C_2(2\epsilon\hat{\mathrm{f}}-1)}\\
\epsilon = 2\varrho\frac{C_4}{C_2^2}\\
\varrho = \frac{\mu(1+\eta)^2+(1+\eta^2)}{4\mu(1+\eta)^2}.
\end{cases} \tag{24}$$

In the second case, in which the fading is characterized by format II of distribution $\eta-\mu$, for which $h = 1/(1-\eta^2)$ and $H = \eta/(1-\eta^2)$, the parameter ϱ of the function $\mathrm{f}(\gamma)$ presented in (23) is given in (25). When inverted in terms of the sample ratio $\hat{\mathrm{f}}$, the estimate $\hat{\gamma}$ under format II of fading $\eta-\mu$ can be written as

$$\text{Format II } \eta-\mu \begin{cases}
\hat{\gamma} = \frac{2(1-2\hat{\mathrm{f}})+2\sqrt{2\hat{\mathrm{f}}(1-2\hat{\mathrm{f}})(\epsilon-1)}}{C_2(2\epsilon\hat{\mathrm{f}}-1)}\\
\epsilon = 2\varrho\frac{C_4}{C_2^2}\\
\varrho = \frac{1+2\mu+\eta^2}{8\mu}
\end{cases} \tag{25}$$

In the third case, in which the fading is characterized by the distribution $\kappa-\mu$, the parameter ϱ of the function $\mathrm{f}(\gamma)$ is given in (26). When inverted in terms of the sample ratio $\hat{\mathrm{f}}$, the estimate $\hat{\gamma}$ under $\kappa-\mu$ fading can be written as

$$\text{Model } \kappa-\mu \begin{cases}
\hat{\gamma} = \frac{2(1-2\hat{\mathrm{f}})+2\sqrt{2\hat{\mathrm{f}}(1-2\hat{\mathrm{f}})(\epsilon-1)}}{C_2(2\epsilon\hat{\mathrm{f}}-1)}\\
\epsilon = 2\varrho\frac{C_4}{C_2^2}\\
\varrho = \frac{(\kappa\mu(\kappa\mu+2\mu+2)+\mu^2+\mu)}{4\mu^2(1+\kappa)^2}.
\end{cases} \tag{26}$$

As the generalized distributions encompass the distribution of Nakagami-m as a particular case and this distribution is one of the most used to characterize the fading in mobile communication channels, the following section shows the relationship between the estimators obtained in this article and the SNR estimator for the received signal model under Nakagami-m fading.

5 Relation with the estimator for Nakagami fading channel

According to [23], the Nakagami-m distribution can be obtained from the distribution $\eta-\mu$ making $\mu = m$ and $\eta \to 0$ or $\eta \to \infty$ in format I or $\mu = m$ and $\eta \to \pm 1$ in format II. It can still be obtained making $\mu = \frac{m}{2}$ and $\eta \to 1$ in format I or $\mu = \frac{m}{2}$ and $\eta \to 0$ in format II. The Nakagami-m distribution can also be obtained from the $\kappa-\mu$ distribution making $\kappa = 0$ and $\mu = m$.

Using the estimates summarized in Table 1, one can obtain the parameters

$$\varrho = \frac{(m+1)}{4m}$$

and

$$\epsilon = \frac{(m+1)}{2m}\frac{C_4}{C_2^2}$$

for the Nakagami fading from the following possible substitutions

1. For $\eta-\mu$ fading format I: making $\mu = m$ and $\eta \to 0$ in the expression of ϱ.
2. For $\eta-\mu$ fading format I: making $\mu = \frac{m}{2}$ and $\eta = 1$ in the expression of ϱ.
3. For $\eta-\mu$ fading format II: making $\mu = m$ and $\eta \to \pm 1$ in the expression of ϱ.
4. For $\eta-\mu$ fading format II: making $\mu = \frac{m}{2}$ and $\eta \to 0$ in the expression of ϱ.
5. For $\kappa-\mu$: making $\kappa = 0$ and $\mu = m$ in the expression of ϱ.

6 Evaluation of the estimate by means of the NMSE

The normalized mean square error (NMSE) of a parameter θ, considering an estimate $\hat{\theta}$, is defined as [26]

$$\text{NMSE} = \frac{E\left[(\hat{\theta}-\theta)^2\right]}{\theta^2}, \tag{27}$$

and the variance of the estimate $\hat{\theta}$ is given by

$$\text{Var}\left[\hat{\theta}\right] = E\left[\hat{\theta}^2\right] - E^2\left[\hat{\theta}\right]. \tag{28}$$

Table 1 SNR estimates under generalized fading

fading	Estimate of SNR γ
Nakagami-*m*	$\hat{\gamma} = \frac{2(1-2\hat{f})+2\sqrt{2\hat{f}(1-2\hat{f})(\epsilon-1)}}{C_2(2\epsilon\hat{f}-1)}$ $\epsilon = 2\varrho\frac{C_4}{C_2^2}$ $\varrho = \frac{(m+1)}{4m}$
Format I $\eta-\mu$	$\hat{\gamma} = \frac{2(1-2\hat{f})+2\sqrt{2\hat{f}(1-2\hat{f})(\epsilon-1)}}{C_2(2\epsilon\hat{f}-1)}$ $\epsilon = 2\varrho\frac{C_4}{C_2^2}$ $\varrho = \frac{\mu(1+\eta)^2+(1+\eta^2)}{4\mu(1+\eta)^2}$
Format II $\eta-\mu$	$\hat{\gamma} = \frac{2(1-2\hat{f})+2\sqrt{2\hat{f}(1-2\hat{f})(\epsilon-1)}}{C_2(2\epsilon\hat{f}-1)}$ $\epsilon = 2\varrho\frac{C_4}{C_2^2}$ $\varrho = \frac{1+2\mu+\eta^2}{8\mu}$
$\kappa-\mu$	$\hat{\gamma} = \frac{2(1-2\hat{f})+2\sqrt{2\hat{f}(1-2\hat{f})(\epsilon-1)}}{C_2(2\epsilon\hat{f}-1)}$ $\epsilon = 2\varrho\frac{C_4}{C_2^2}$ $\varrho = \frac{(\kappa\mu(\kappa\mu+2\mu+2)+\mu^2+\mu)}{4\mu^2(1+\kappa)^2}$

Using $E\left[\hat{\theta}^2\right]$ from (28) and applying in (27), one can write

$$E\left[(\hat{\theta}-\theta)^2\right] = \text{Var}\left[\hat{\theta}\right] + \left(E\left[\hat{\theta}\right]-\theta\right)^2. \tag{29}$$

Hence, the NMSE of the estimate $\hat{\gamma}$ can be written as

$$\text{NMSE} = \frac{\text{Var}[\,\hat{\gamma}\,] + (E[\,\hat{\gamma}\,]-\gamma)^2}{\gamma^2}, \tag{30}$$

in which $\text{Var}[\,\hat{\gamma}\,]$ and $E[\,\hat{\gamma}\,]$ as a function of γ can be computed by a linearization process, as presented in [32]. According to that method, if an estimate $\hat{\theta}$ can be written as a function of the observed data vector $\mathbf{x}$,

$$\hat{\theta} = h^{-1}(\hat{\boldsymbol{\mu}}) = g(\mathbf{x}), \tag{31}$$

in which $\hat{\boldsymbol{\mu}}$ is the vector of sample moments of $\mathbf{x}$, it is possible to obtain approximate expressions for the mean and variance of the estimate to evaluate the performance of the estimator by means of a Taylor's expansion of $g(\mathbf{x})$ about a vector of statistics $\boldsymbol{\mu} = E[\,\boldsymbol{T}\,]$ [32]. The mean and variance values given by these expressions approximate the exact values as the number of samples observed in the vector $\mathbf{x}$ increases. The exact mean and variance, however, can only be determined by means of computational simulation.

In the context of the study presented in this paper, the estimated parameter is scalar, $\hat{\theta} = \hat{\gamma}$, and can be written in terms of the relation (31). Calculating approximate expressions for the mean and the variance of $\hat{\theta}$ can then be performed assuming that this estimate depends on the vector $\boldsymbol{T}$ of two statistics of the observed samples, $\boldsymbol{T} = [\,T_1(\mathbf{x})\;\, T_2(\mathbf{x})\,]$, respectively, the second sample moment and fourth sample moment of $|r[\,n]\,|$.

The Taylor's expansion of an estimate $\hat{\theta}$, about a vector $\boldsymbol{\mu}$ of two statistics, can be approximated by

$$\hat{\theta} \approx g(\boldsymbol{\mu}) + \left[\left.\frac{\partial g}{\partial \boldsymbol{T}}\right|_{\boldsymbol{T}=\boldsymbol{\mu}}\right]^T (\boldsymbol{T}-\boldsymbol{\mu}), \tag{32}$$

and

$$\hat{\theta}^2 \approx g^2(\boldsymbol{\mu}) + 2g(\boldsymbol{\mu})\left[\left.\frac{\partial g}{\partial \boldsymbol{T}}\right|_{\boldsymbol{T}=\boldsymbol{\mu}}\right]^T (\boldsymbol{T}-\boldsymbol{\mu}) + \left[\left.\frac{\partial g}{\partial \boldsymbol{T}}\right|_{\boldsymbol{T}=\boldsymbol{\mu}}\right]^T (\boldsymbol{T}-\boldsymbol{\mu})(\boldsymbol{T}-\boldsymbol{\mu})^T \left[\left.\frac{\partial g}{\partial \boldsymbol{T}}\right|_{\boldsymbol{T}=\boldsymbol{\mu}}\right]. \tag{33}$$

It follows that $\text{Var}(\hat{\theta})$ can be written as

$$\text{Var}(\hat{\theta}) = \left.\frac{\partial g}{\partial \boldsymbol{T}}\right|_{\boldsymbol{T}=\boldsymbol{\mu}}^T C_{\mathrm{T}} \left.\frac{\partial g}{\partial \boldsymbol{T}}\right|_{\boldsymbol{T}=\boldsymbol{\mu}}, \tag{34}$$

in which $C_{\mathrm{T}} = E\left[(\boldsymbol{T}-\boldsymbol{\mu})(\boldsymbol{T}-\boldsymbol{\mu})^T\right]$ represents the covariance matrix of the statistics vector $\boldsymbol{T}$.

In the context of SNR estimation, it is observed that all estimates $\hat{\gamma}$ summarized in Table 1 can be written from a single function, in terms of the parameters that characterize the type of fading and sample moments of $|r[n]|$, so that

$$\hat{\gamma} = g(\mathbf{T}) = 2(1-2q(T_1,T_2))(2\epsilon q(T_1,T_2)-1)^{-1} + 2\sqrt{2(\epsilon-1)q(T_1,T_2)(1-2q(T_1,T_2))}(2\epsilon q(T_1,T_2)-1)^{-1}, \tag{35}$$

in which $q(T_1,T_2) = \hat{\mathrm{f}} = \frac{T_1^2}{T_2}$,

$$\begin{aligned} T_1 &= \frac{1}{N}\sum_{n=1}^{N} |r[n]|^2 \\ T_2 &= \frac{1}{N}\sum_{n=1}^{N} |r[n]|^4. \end{aligned} \tag{36}$$

and

$$\begin{aligned} E[T_1] &= \frac{1}{N}\sum_{n=1}^{N} E\left[|r[n]|^2\right] = M_2 \\ E[T_2] &= \frac{1}{N}\sum_{n=1}^{N} E\left[|r[n]|^4\right] = M_4. \end{aligned} \tag{37}$$

If the constellation of the modulation scheme used has equiprobable symbols and with energy per symbol such that the second moment of the entire constellation is unitary, then $C_2 = 1$. Hence, the mean value of the estimate, $E[\hat{\gamma}] = g(\boldsymbol{\mu})$, can be written as

$$\begin{aligned} E[\hat{\gamma}] = {} & 2\left(1-2\frac{M_2^2}{M_4}\right)\left(2\epsilon\frac{M_2^2}{M_4}-1\right)^{-1} \\ & + 2\left(2(\epsilon-1)\frac{M_2^2}{M_4}\left(1-2\frac{M_2^2}{M_4}\right)\right)^{1/2} \\ & \times \left(2\epsilon\frac{M_2^2}{M_4}-1\right)^{-1}. \end{aligned} \tag{38}$$

For the evaluation of the variance of the estimates, it is necessary, according to the expression (34), the calculation of the partial derivatives of $\hat{\gamma}$ in relation to the statistics T_1 and T_2. These derivatives can be written as

$$\begin{aligned} \frac{\partial g}{\partial T_1} &= \frac{\partial g}{\partial q}\frac{\partial q}{\partial T_1} \\ \frac{\partial g}{\partial T_2} &= \frac{\partial g}{\partial q}\frac{\partial q}{\partial T_2}, \end{aligned} \tag{39}$$

and the partial derivative $\frac{\partial g}{\partial q}$ can be written as

$$\frac{\partial g}{\partial q} = h(q(T_1,T_2)), \tag{40}$$

in which the function $h(x)$ is given by

$$\begin{aligned} h(x) = {} & -4\epsilon(1-2x)(2\epsilon x-1)^{-2} - 4(2\epsilon x-1)^{-1} \\ & - 4\epsilon(2(\epsilon-1))^{1/2}(2\epsilon x-1)^{-2}x^{1/2}(1-2x)^{1/2} \\ & + 2(\epsilon-1)(2(\epsilon-1))^{-1/2}(1-4x)x^{-1/2}(1-2x)^{-1/2} \\ & \times (2\epsilon x-1)^{-1}. \end{aligned} \tag{41}$$

Using the fact that

$$\begin{aligned} \frac{\partial}{\partial T_1}q(T_1,T_2) &= 2\frac{T_1}{T_2} \\ \frac{\partial}{\partial T_2}q(T_1,T_2) &= -\frac{T_1^2}{T_2^2}, \end{aligned} \tag{42}$$

one can write the vector $\left.\frac{\partial g}{\partial T}\right|_{T=M}$ as

$$\left.\frac{\partial g}{\partial T}\right|_{T=M} = \begin{bmatrix} 2h(g(M_2,M_4))\frac{M_2}{M_4} \\ -h(g(M_2,M_4))\frac{M_2^2}{M_4^2} \end{bmatrix}. \tag{43}$$

The elements of the covariance matrix C_{T}, presented in (34), can be obtained from the expressions of the statistics T_1 and T_2, given in (36), and written as

$$\begin{aligned} \mathrm{Cov}(T_1,T_1) &= \frac{1}{N}\left(M_4 - M_2^2\right) \\ \mathrm{Cov}(T_1,T_2) &= \mathrm{Cov}(T_2,T_1) = \frac{1}{N}(M_6 - M_2M_4) \\ \mathrm{Cov}(T_2,T_2) &= \frac{1}{N}\left(M_8 - M_4^2\right). \end{aligned} \tag{44}$$

Substituting (43) and (44) in the expression of the variance of $\hat{\gamma}$, one can write $\mathrm{Var}(\hat{\gamma})$ as

$$\begin{aligned} \mathrm{Var}[\hat{\gamma}] = {} & \frac{h^2(g(M_2,M_4))}{N}\left(\frac{M_2}{M_4}\right)^2 \begin{bmatrix} 2 \\ -\frac{M_2}{M_4} \end{bmatrix}^T \\ & \times \begin{bmatrix} M_4 - M_2^2 & M_6 - M_2M_4 \\ M_6 - M_2M_4 & M_4 - M_8 - M_4^2 \end{bmatrix} \begin{bmatrix} 2 \\ -\frac{M_2}{M_4} \end{bmatrix}, \end{aligned} \tag{45}$$

which can be written as

$$\mathrm{Var}[\hat{\gamma}] = \frac{1}{N}h^2(f_M)\left[4f_M - f_M^2 - \frac{4M_6}{M_2M_4}f_M^2 + \frac{M_8}{M_4^2}f_M^2\right], \tag{46}$$

in which

$$f_M = \frac{M_2^2}{M_4}.$$

7 Moments of the *M*-QAM constellations

In this section, the moments of order 2, 4, 6, and 8 are determined for the M-QAM constellation. The moments of order 2 and 4 are necessary to calculate the SNR estimate, while the higher order moments are necessary to evaluate the variance of the estimate.

In this calculation, it is assumed that the symbols are equiprobable and normalized from the total energy of the constellation, obtained from the sum of the square modulus of M symbols. Writing these symbols in terms of the corresponding coordinates in the plane $\mathbb{R}^2$, $(2i-1-\sqrt{M}, 2j-1-\sqrt{M})$ for $i, j = 0, 1, \cdots, \sqrt{M}-1$, one has that the total energy is given by $E_{\text{t}} = \sum_{n=0}^{M-1} |s[n]|^2 = 2M(M-1)/3$, while the average energy per symbol, obtained by dividing the total energy by the number of symbols, can be written as $E_{\text{av}} = 2(M-1)/3$. The constellation can then be normalized by the constant $C = 1/\sqrt{E_{\text{av}}}$.

The moment of order $2k$ of the normalized constellation can then be obtained by multiplying the symbols by the constant of normalization C and calculating the sum

$$C_{2k} = \sum_{n=0}^{M-1} p_n (|s[n]|)^{2k}. \tag{47}$$

Writing $s[n]$ in terms of its coordinates in the plane $\mathbb{R}^2$ and developing the sum of (47) for k equal to 1, one has that the moment of order 2, C_2, can be written as

$$C_2 = \sum_{n=0}^{M-1} p_n |s[n]|^2 = \frac{1}{M} \sum_{n=0}^{M-1} |s[n]|^2 = \frac{E_{\text{t}}}{M} = 1. \tag{48}$$

Making k equal to 2, 3, and 4, one can obtain, respectively, the moments of order 4, 6, and 8, denoted by C_4, C_6, and C_8. Developing the summation (47) and making the necessary simplifications, one has

$$\begin{aligned} C_4 &= \frac{(7M^2 - 20M + 13)}{5(M-1)^2}, \\ C_6 &= \frac{9(9M^2 - 40M + 51)}{35(M-1)^2}, \\ C_8 &= \frac{9(83M^3 - 637M^2 + 1897M - 2183)}{175(M-1)^3}. \end{aligned} \tag{49}$$

8 Discussion on the Cramér-Rao bound

In this section, a discussion is presented about the calculation of the Cramér-Rao bound for the $\eta - \mu$ fading model. This proposal can be extended to $\kappa - \mu$ fading following the same procedure. The PDF of the samples modulus of the received signal $|r[n]|$ under $\eta - \mu$ fading can be obtained by taking the expected value of the conditioned PDF $f_{|r[n]|}(|r[n]|\,||g[n]|, |s[n]|)$ by the distributions of $|g[n]|$ and $|s[n]|$. Thus, the PDF can be written as

$$\begin{aligned} f_{|r[n]|}(x) = {} & \frac{\beta_{\eta-\mu}}{M} \frac{x}{\sigma_w^2} \exp\left(-\frac{x^2}{2\sigma_w^2}\right) \sum_{i=1}^{M} \int_0^\infty \\ & \nu^{\mu-\frac{1}{2}} \exp\left(-\left(2\mu h + \frac{|A_i|^2}{2} \frac{\sigma_g^2}{\sigma_w^2}\right)\nu\right) \\ & \times \mathrm{I}_{\mu-\frac{1}{2}}(2\mu\nu H) \mathrm{I}_0\left(x|A_i| \frac{\sigma_g}{\sigma_w^2}\sqrt{\nu}\right) d\nu, \end{aligned} \tag{50}$$

in terms of the average powers of the noise, σ_w^2, and fading, σ_g^2, for the normalized M-QAM constellation of equiprobable symbols. After searching in classical Laplace integrals and transform tables involving Bessel functions, it was not possible to find an exact solution to the integral in (50) because one of the Bessel functions has as argument a square root of the integration variable and this became the main constraint to calculating the Cramér-Rao bound.

A possible solution to this problem is to consider the average power, σ_g^2, of the fading of unit value, so that the SNR is $\gamma = 1/\sigma_w^2$. This consideration is appropriate from the point of view of computational simulation because it makes the generation of variables $\eta - \mu$ faster by the method of acceptance-rejection employed in this study, since the variation of σ_g^2 influences the PDF format of the $\eta - \mu$ distribution. In addition, this consideration allows to calculate only the Fisher information, instead of the Fisher information matrix, which is four times more laborious. Thus, by considering $\sigma_g^2 = 1$, $f_{|r[n]|}(x)$ can be written as

$$f_{|r[n]|}(x; \gamma) = \frac{\beta_{\eta-\mu}}{M} x\gamma \exp\left(-\frac{\gamma}{2}x^2\right) Q_{\eta-\mu}(x, \gamma), \tag{51}$$

in which

$$\begin{aligned} Q_{\eta-\mu}(x, \gamma) = {} & \sum_{i=1}^{M} \int_0^\infty \nu^{\mu-\frac{1}{2}} \exp\left(-\left(2\mu h + \frac{|A_i|^2}{2}\gamma\right)\nu\right) \\ & \times \mathrm{I}_{\mu-\frac{1}{2}}(2\mu\nu H) \mathrm{I}_0\left(x|A_i|\gamma\sqrt{\nu}\right) d\nu \end{aligned} \tag{52}$$

and

$$\beta_{\eta-\mu} = \frac{2\sqrt{\pi}\mu^{\mu+\frac{1}{2}} h^\mu}{\Gamma(\mu) H^{\mu-\frac{1}{2}}}. \tag{53}$$

Considering the N observed samples of $|r[n]|$ independent and identically distributed with PDF $f_{|r[n]|}(|r[n]|; \gamma)$, then the Fisher information $I(\gamma)$ can be written as

$$\begin{aligned} I(\gamma) &= -NE\left[\frac{\partial^2}{\partial\gamma^2} \ln f_{|r[n]|}(|r[n]|; \gamma)\right] \\ &= N \int_0^\infty \frac{\partial}{\partial\gamma} f_{|r[n]|}(x; \gamma) \frac{\partial}{\partial\gamma} \ln f_{|r[n]|}(x; \gamma) dx. \end{aligned} \tag{54}$$

in which the expected value was taken by the PDF $f_{|r[n]|}(|r[n]|; \gamma)$.

The integral in (54) can be written as

$$\begin{aligned}\int_0^\infty \frac{\partial}{\partial\gamma} f_{|r[n]|}(x,\gamma)\frac{\partial}{\partial\gamma}\ln f_{|r[n]|}(x,\gamma)dx \\ = \frac{1}{\gamma}\frac{\beta_{\eta-\mu}}{M}\underbrace{\int_0^\infty xQ_{\eta-\mu}(x,\gamma)\exp\left(-\frac{\gamma}{2}x^2\right)dx}_{I_1} \\ -\frac{\beta_{\eta-\mu}}{M}\underbrace{\int_0^\infty x^3Q_{\eta-\mu}(x,\gamma)\exp\left(-\frac{\gamma}{2}x^2\right)dx}_{I_2} \\ +\frac{\gamma}{4}\frac{\beta_{\eta-\mu}}{M}\underbrace{\int_0^\infty x^5Q_{\eta-\mu}(x,\gamma)\exp\left(-\frac{\gamma}{2}x^2\right)dx}_{I_3} \\ +2\frac{\beta_{\eta-\mu}}{M}\underbrace{\int_0^\infty xQ'_{\eta-\mu}(x,\gamma)\exp\left(-\frac{\gamma}{2}x^2\right)dx}_{I_4} \\ -\gamma\frac{\beta_{\eta-\mu}}{M}\underbrace{\int_0^\infty x^3Q'_{\eta-\mu}(x,\gamma)\exp\left(-\frac{\gamma}{2}x^2\right)dx}_{I_5} \\ +\underbrace{\gamma\frac{\beta_{\eta-\mu}}{M}\int_0^\infty x\frac{(Q'_{\eta-\mu}(x,\gamma))^2}{Q_{\eta-\mu}(x,\gamma)}\exp\left(-\frac{\gamma}{2}x^2\right)dx,}_{I_6}\end{aligned} \tag{55}$$

in which

$$\begin{aligned}\frac{\partial}{\partial\gamma}Q_{\eta-\mu}(x,\gamma) = \sum_{i=1}^{M}\int_0^\infty v^{\mu-\frac{1}{2}}\exp(-2\mu h v)\mathrm{I}_{\mu-\frac{1}{2}}(2\mu H v)\times \\ \frac{\partial}{\partial\gamma}\left[\exp\left(-\frac{|A_i|^2}{2}v\gamma\right)\mathrm{I}_0(x|A_i|\gamma\sqrt{v})\right]dv.\end{aligned} \tag{56}$$

Calculation of the expressions I_1, I_2, I_3, I_4, and I_5 could be performed and simplified without great difficulties because when $Q_{\eta-\mu}(x,\gamma)$ was considered, the change of order of integration in the resulting double integrals led to integrals that could be calculated as Laplace transforms involving the modified Bessel functions. Therefore, one has the following solutions.

$$I_1 = \frac{M}{\gamma}\frac{\Gamma(\mu)H^{\mu-\frac{1}{2}}}{2\sqrt{\pi}\mu^{\mu+\frac{1}{2}}}\frac{1}{\left[h^2-H^2\right]^{\mu}}. \tag{57}$$

$$\begin{aligned}I_2 = \frac{1}{\gamma}\sum_{i=1}^{M}|A_i|^2\left(\frac{\Gamma(\mu)H^{\mu-\frac{1}{2}}}{2\sqrt{\pi}\mu^{\mu+\frac{1}{2}}}\frac{h}{\left[h^2-H^2\right]^{\mu+1}}\right) \\ +\frac{2}{\gamma^2}\sum_{i=1}^{M}\left(\frac{\Gamma(\mu)H^{\mu-\frac{1}{2}}}{2\sqrt{\pi}\mu^{\mu+\frac{1}{2}}}\frac{1}{\left[h^2-H^2\right]^{\mu}}\right).\end{aligned} \tag{58}$$

$$\begin{aligned}I_3 = \frac{1}{2\gamma}\sum_{i=1}^{M}|A_i|^4\left(\frac{\Gamma(\mu)H^{\mu-\frac{1}{2}}}{2\sqrt{\pi}\mu^{\mu+\frac{1}{2}}}\frac{(H^2+(2\mu+1)h^2)}{\mu\left[h^2-H^2\right]^{\mu+2}}\right) \\ +\frac{8}{\gamma^2}\sum_{i=1}^{M}|A_i|^2\left(\frac{\Gamma(\mu)H^{\mu-\frac{1}{2}}}{2\sqrt{\pi}\mu^{\mu+\frac{1}{2}}}\frac{h}{\left[h^2-H^2\right]^{\mu+1}}\right) \\ +\frac{8}{\gamma^3}\sum_{i=1}^{M}\left(\frac{\Gamma(\mu)H^{\mu-\frac{1}{2}}}{2\sqrt{\pi}\mu^{\mu+\frac{1}{2}}}\frac{1}{\left[h^2-H^2\right]^{\mu}}\right).\end{aligned} \tag{59}$$

$$I_4 = \frac{1}{2\gamma}\sum_{i=1}^{M}|A_i|^2\left(\frac{\Gamma(\mu)H^{\mu-\frac{1}{2}}}{2\sqrt{\pi}\mu^{\mu+\frac{1}{2}}}\frac{h}{\left[h^2-H^2\right]^{\mu+1}}\right). \tag{60}$$

$$\begin{aligned}I_5 = \frac{1}{4\gamma}\sum_{i=1}^{M}|A_i|^4\left(\frac{\Gamma(\mu)H^{\mu-\frac{1}{2}}}{2\sqrt{\pi}\mu^{\mu+\frac{1}{2}}}\frac{(H^2+(2\mu+1)h^2)}{\mu\left[h^2-H^2\right]^{\mu+2}}\right) \\ +\frac{3}{\gamma^2}\sum_{i=1}^{M}|A_i|^2\left(\frac{\Gamma(\mu)H^{\mu-\frac{1}{2}}}{2\sqrt{\pi}\mu^{\mu+\frac{1}{2}}}\frac{h}{\left[h^2-H^2\right]^{\mu+1}}\right).\end{aligned} \tag{61}$$

The integral I_6 cannot be written in closed-form. It is rewritten as

$$I_6(\gamma) = \gamma\frac{\beta_{\eta-\mu}}{M}\int_0^\infty x\frac{(Q'_{\eta-\mu}(x,\gamma))^2}{Q_{\eta-\mu}(x,\gamma)}\exp\left(-\frac{\gamma}{2}x^2\right)dx. \tag{62}$$

In both Format I and Format II of the model $\eta-\mu$, one has $h/\left(h^2-H^2\right)=1$. Thus, after substituting the results I_k, $1\le k\le 5$ and $I_6(\gamma)$ in (55), Fisher information $I(\gamma)$ can be written, from (54), as

$$I(\gamma) = N\left[I_6(\gamma)+\frac{1}{\gamma^2}-\frac{C_2}{\gamma}-C_4\lambda\right]. \tag{63}$$

Therefore, one has

$$\mathrm{CRLB}[\hat{\gamma}] = \frac{1}{N\left[I_6(\gamma)+\frac{1}{\gamma^2}-\frac{C_2}{\gamma}-C_4\lambda\right]}, \tag{64}$$

in which

$$\lambda = \frac{1}{8\lambda}\left[\left(\frac{H}{h}\right)^2+(2\mu+1)\right], \tag{65}$$

$C_2=1$ for the normalized M-QAM constellation and C_4 is given by (47).

9 Results and discussion

In this section, results are presented for numerical evaluation of the mathematical expressions obtained for the estimates of the SNR and its NMSE, corroborated by simulations performed by the Monte Carlo method. For each

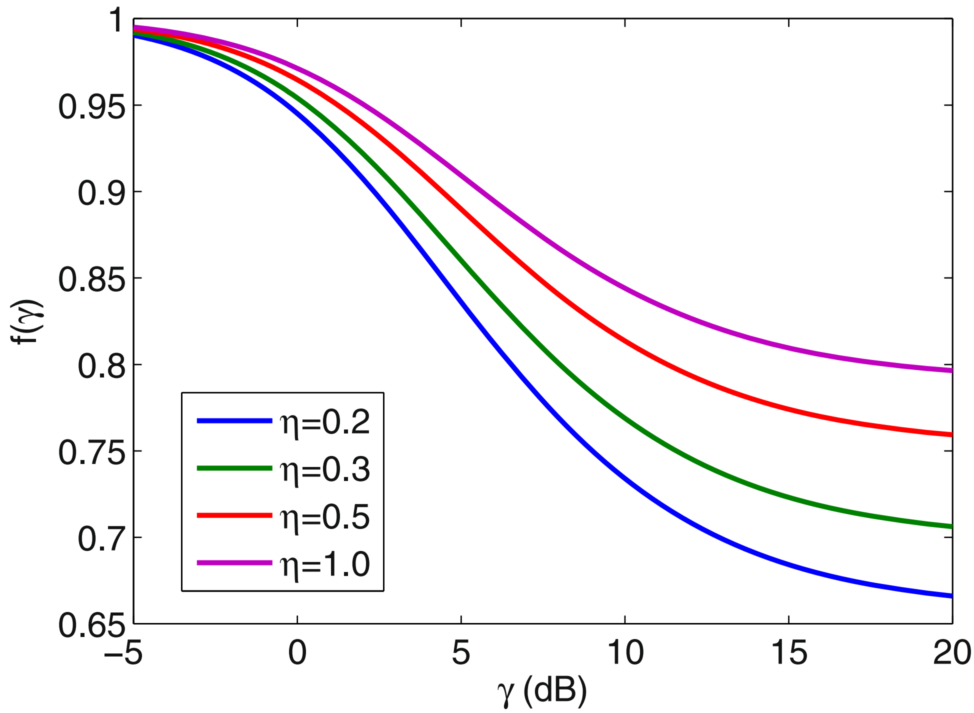

Fig. 1 Curves of the rational function f(γ), as a function of SNR γ (dB), for fading model $\eta - \mu$ with $\mu = 0.6$ for 64-QAM and different values of η

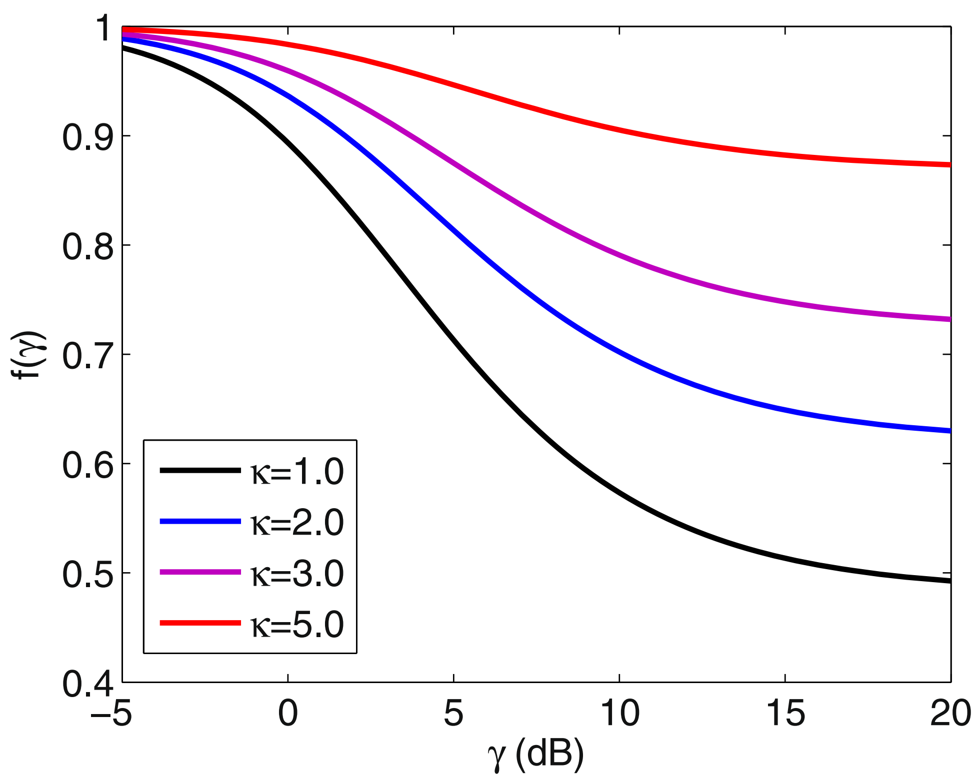

Fig. 2 Curves of the rational function f(γ), as a function of SNR γ (dB), for fading model $\kappa - \mu$ with $\mu = 0.5$ with 64-QAM modulation and different values of κ

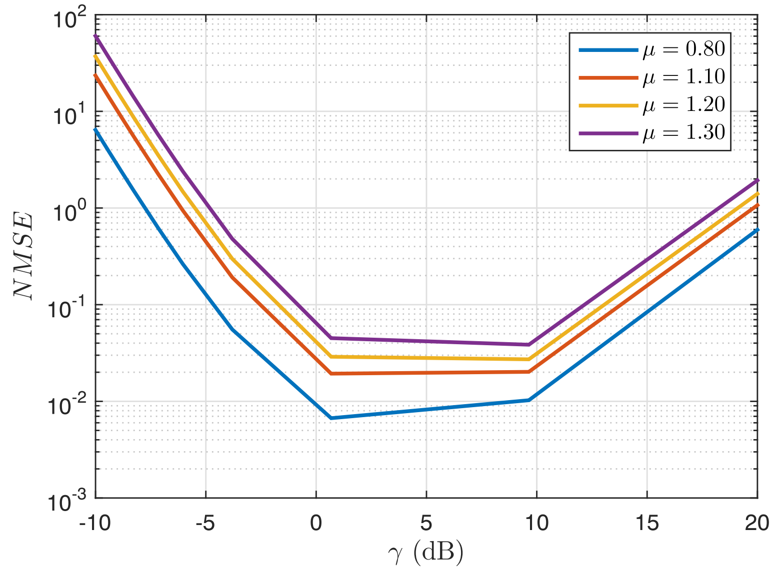

Fig. 3 NMSE of the estimate $\hat{\gamma}$ as a function of SNR in dB for fading $\eta - \mu$ with $\eta = 0.1$, different values of μ, $N = 5 \times 10^4$ and 64-QAM modulation

SNR value, the variance and the mean of each estimate were obtained from a mean of 10 realizations of N samples of $r[n]$. In Figs. 1 and 2, curves are presented for the rational function $f(\gamma)$ obtained from the ratio between the square of the second moment and the fourth moment of the magnitude of the received signal $|r[n]|$, for the fading models $\eta - \mu$ and $\kappa - \mu$, considering the 64-QAM modulation scheme.

The importance of these curves is that they allow to determine the interval within which the ratio between the square of the second sample moment and the fourth sample moment of $|r[n]|$ must lie within so that $f(\gamma)$ can be inverted and provide the estimate $\hat{\gamma}$ of the SNR. In both figures, one observes that the length of this interval increases as κ and η decrease. When these parameters decrease, for μ fixed, the probability of the normalized envelope of the fading take values close to zero, for example, is larger, and this behavior can be confirmed by means of the CDF of the distributions $\eta - \mu$ and $\kappa - \mu$ presented in [23].

The next group of curves, in Figs. 3 and 4, show the behavior of the NMSE and the mean of the estimate $\hat{\gamma}$

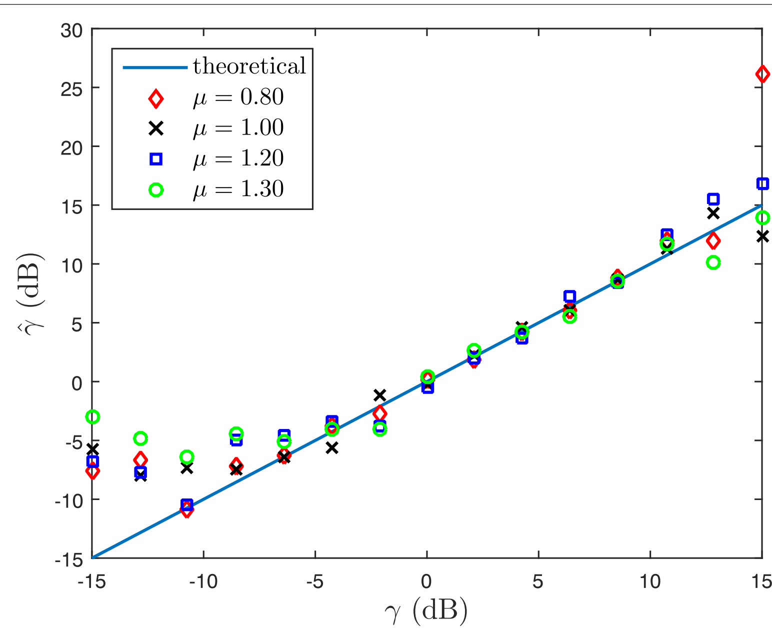

Fig. 4 Mean of the estimate $\hat{\gamma}$ as a function of SNR in dB for fading $\eta - \mu$ with $\eta = 0.1$, different values of μ, $N = 5 \times 10^4$ and 64-QAM modulation

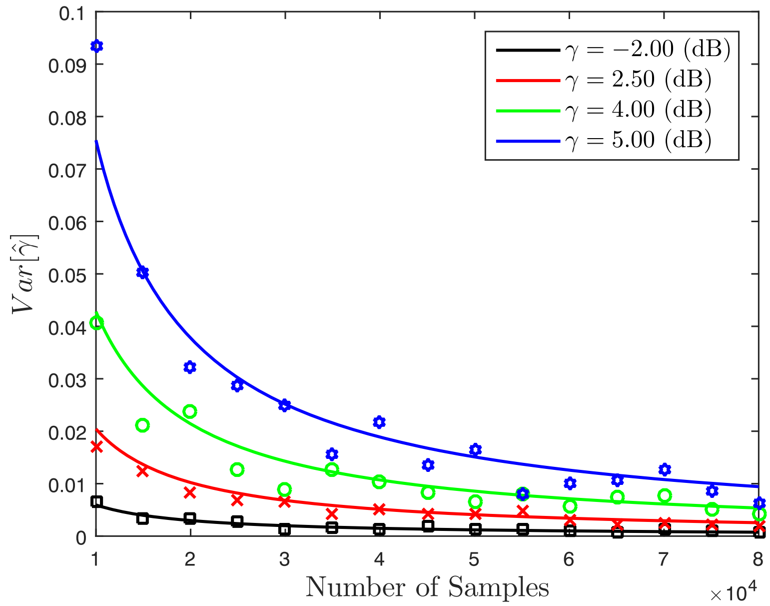

Fig. 5 Variance of the estimate $\hat{\gamma}$ as a function of the number of samples N for different values of SNR γ and 64-QAM modulation considering $\kappa - \mu$ fading with $\kappa = 1.1$ and $\mu = 0.3$

for the received signal model $r[n]$ under fading $\eta - \mu$ as a function of the SNR γ in dB. The theoretical curves of the NMSE were plotted for different values of η from the expressions (30), (38), (46), and the moments obtained for $|r[n]|$ under fading $\eta - \mu$. The modulation used was 64-QAM, and the number of samples N considered was 5×10^4, for the plot of the theoretical curves of the NMSE and for the the mean of the estimate obtained by simulation.

Figures 3 and 4, together, show the interval of SNR values for which the estimator presents its better performance. While the mean of the estimates approximates the desired values in the interval from 0 to 10 dB for all values of the parameter μ considered, the NMSE remains bellow 10^{-1} from -5 to 15 dB only for $\mu = 0.8$. It can be seen from Fig. 4 that the estimates adhere well to the expected average behavior in the range of SNR values

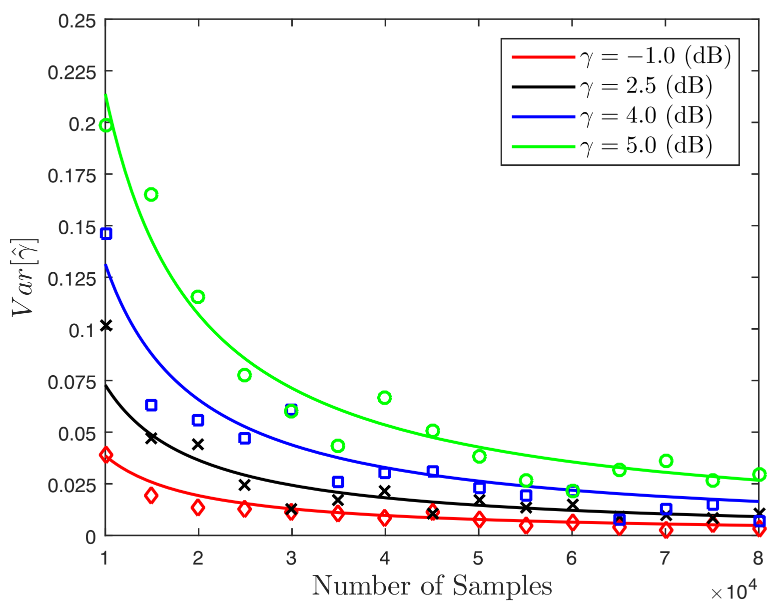

Fig. 6 Variance of the estimate $\hat{\gamma}$ as a function of the number of samples N for different values of SNR γ and 64-QAM modulation considering $\eta - \mu$ fading with $\eta = 0.1$ and $\mu = 0.8$

between 0 dB and approximately 15 dB for all values of the parameter μ, while the NMSE of the estimate is more sensitive to the variation of intensity of fading characterized by the different values μ. For η fixed, the probability of the intensity of the fading envelope decreases below a fixed value decreases as μ increases. This behavior of the fading envelope $\eta - \mu$ can be verified by means of its CDF, presented in [23], and means that the increase of parameter μ models fading situations of lesser intensity.

Figures 5 and 6 present curves of the variance of the estimate $\hat{\gamma}$ as a function of the number of samples N of the signal envelope $|r[n]|$ on a channel under fadings $\kappa - \mu$ and $\eta - \mu$. It was considered, both in the simulation and in the plot of theoretical curves, fading $\kappa - \mu$ with $\kappa = 1.1$ and $\mu = 0.3$ and fading $\eta - \mu$ with $\eta = 0.1$ and $\mu = 0.8$ for different values of γ (dB) and 64-QAM modulation scheme.

It is observed from Fig. 5 that the variance of $\hat{\gamma}$ is below 0.015 for $N = 8 \times 10^4$, for all values of SNR considered. The theoretical curves were all obtained from the expression (46) and corroborated with the points of the simulations carried out. By the curves, it is observed that the estimator performs better, in terms of the variance, for smaller values of SNR. For $\gamma = -2$ dB, the variance remains below 0.01 for the entire range of N values considered, indicating that with 10^4 samples, it is possible to get the estimate with a variance of less than 0.01 around the average.

From Fig. 6, it can be seen that for the SNR of –1 dB the variance of $\hat{\gamma}$ takes maximum value below 0.05 for a number of samples N of $|r[n]|$ equals 10^4, whereas for a SNR of 5 dB, the variance is only below 0.05 for $N = 4 \times 10^4$. It is also worth noting that the setting of parameters $\eta = 0.1$ and $\mu = 0.8$ models a case of more intense fading than configurations with values greater than μ. Therefore, the γ estimator of the $\eta - \mu$ model, presented in the second and third lines of Table 1, presents better performance for this more severe channel condition than the case where the SNR is larger and the fading is less intense. Although these MOM NDA estimators require much more samples of the received signal, compared to MLE estimators, it is still appropriate, due to its simplicity, to be implemented in hardware systems in which the processor capacity is reduced.

The group of curves of Figs. 7 and 8 shows the behavior of the NMSE and the mean of the estimate $\hat{\gamma}$ for different configurations of parameters of the $\kappa - \mu$ fading model. Each simulated value of the SNR estimate was obtained from $N = 5 \times 10^4$ samples of the envelope of the received signal $|r[n]|$, and the theoretical curves were obtained from the expressions (30), (38), and (46) and from the expressions of moments of $|r[n]|$ for the $\kappa - \mu$ model.

Regarding Fig. 7, it is observed that for $\mu = 0.8$ and fixed SNR γ, the NMSE of the estimate $\hat{\gamma}$ decreases with the decreasing of κ parameter. It can be observed, by means of the CDF of the envelope of $\kappa - \mu$ fading presented in [23], that with smaller values of κ, the distribution $\kappa - \mu$ models cases of greater fading depth. This conclusion can be obtained by fixing a value of envelope of the $\kappa - \mu$ fading and verifying the probability of the envelope taking values below that fixed value as κ decreases. With respect to Fig. 8, it can be observed that the mean of the estimate $\hat{\gamma}$ is close to the target value in the range of –5 to 15 dB, for the different parameter configurations used in simulations, while the interval of SNR values for

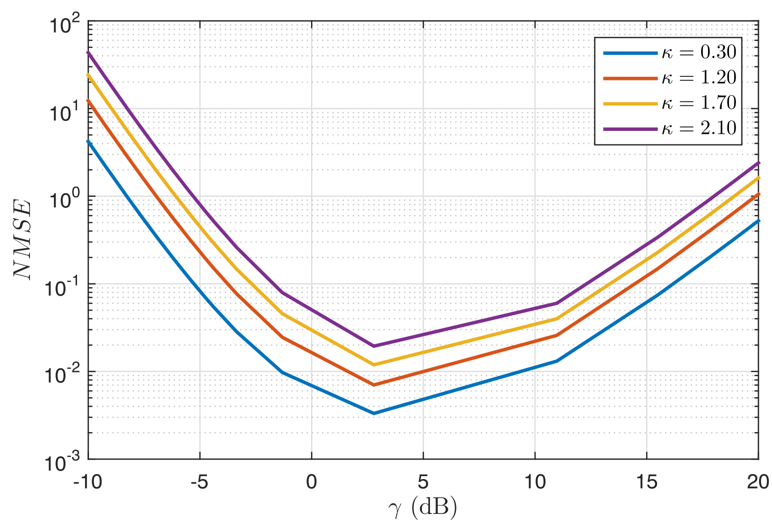

Fig. 7 NMSE of the estimate $\hat{\gamma}$ as a function of SNR in dB for fading $\kappa - \mu$ for $N = 5 \times 10^4$ and 64-QAM modulation considering different values of κ and $\mu = 0.8$

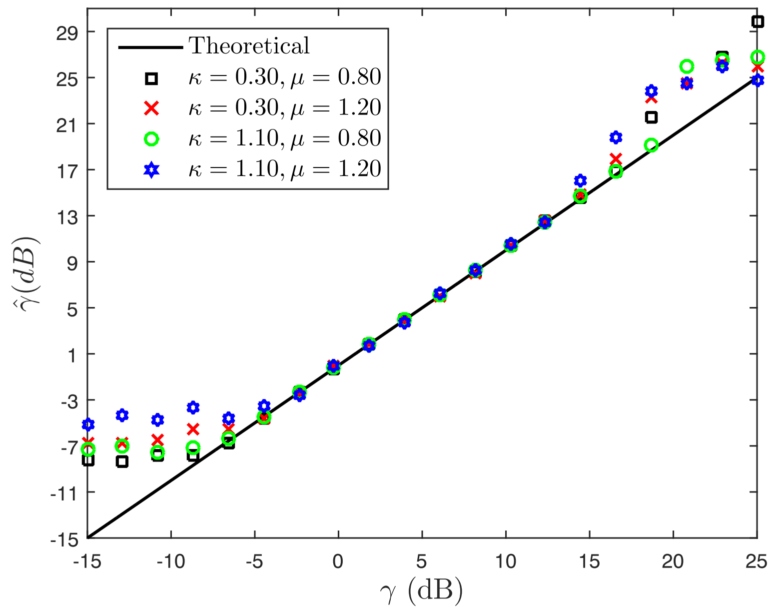

Fig. 8 Mean of the estimate $\hat{\gamma}$ as a function of SNR in dB for fading $\kappa - \mu$ for $N = 5 \times 10^4$ and 64-QAM modulation considering different values of κ and μ

which the NMSE remains below 10^{-1} becomes shorter as κ increases.

Either for the $\eta - \mu$ model or the $\kappa - \mu$ model, the SNR range for the best performance of the estimators is from −2 to 12 dB. Above 12 dB, the average values of the estimates begin to deviate from the exact value. Nevertheless, the simplicity of the expressions of the estimators justifies the number of samples of the observed signal greater than the necessary for the optimal MLE estimator. As can be seen in Section 8, the greater complexity of the fading model leads to a greater complexity of the likelihood function and the difficulty for obtaining both a MLE estimator and an exact expression for the CRLB.

Figures 9 and 10 present the curves of the NMSE for different values of the constellation order, as a function of the SNR γ, respectively, on channels under $\eta - \mu$ fading with $\eta = 0.1$ and $\mu = 0.6$ and $\kappa - \mu$ fading with $\kappa = 3$ and $\mu = 0.5$. In both channel models, the estimator performance is not affected as the constellation order increases. This happens for the M-QAM constellation because the value of its fourth moment changes a little as M increases, as can be seen from (49).

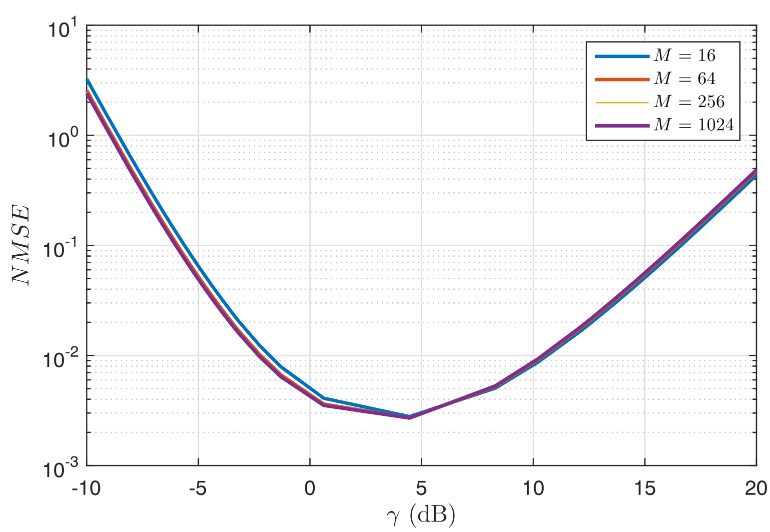

Fig. 9 NMSE of the estimate $\hat{\gamma}$ as a function of the SNR in dB, $N = 5 \times 10^4$, $\eta - \mu$ fading for $\eta = 0.1$ and $\mu = 0.6$, and different orders of the M-QAM constellation

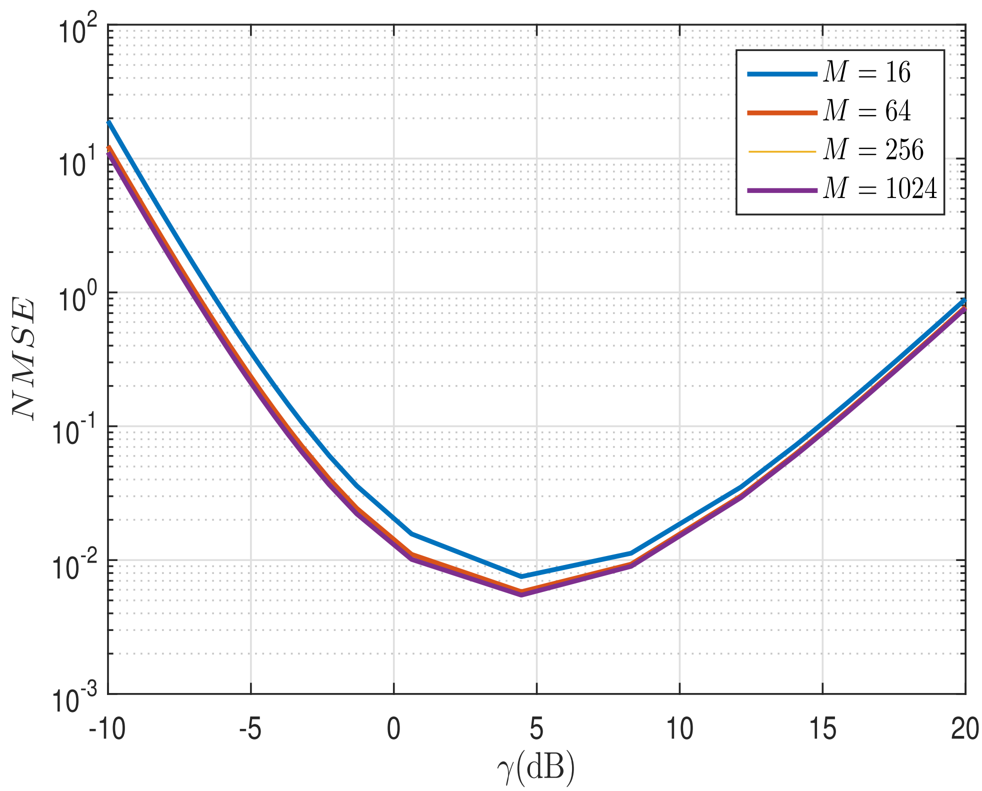

Fig. 10 NMSE of the estimate $\hat{\gamma}$ as a function of the SNR in dB, $N = 5 \times 10^4$, $\kappa - \mu$ fading for $\kappa = 3$ and $\mu = 0.5$, and different orders of the *M*-QAM constellation

10 Conclusion

In this article, new and exact expressions are presented for the estimation of SNR in generalized fading channels characterized by the probability distributions $\eta - \mu$ and $\kappa - \mu$, by using the method of moments. In the model of received signal considered, the modulation scheme M-QAM was used. The main advantage of using generalized models, such as $\eta - \mu$ and $\kappa - \mu$, is that their distributions encompass several other distributions commonly used to model fading. As an example, one of the scenarios considered in this article was that of the Nakagami fading, for which the estimate of SNR was obtained from expressions presented in this work. Regarding the performance of the estimators, the mean and the NMSE of the estimates were used. It is observed from the theoretical curves obtained, corroborated by the simulations, that the performance of the estimators is satisfactory in the range of SNR values in which the mean of the estimate adheres to the target SNR value. It is also noticed, from the curves presented, that the performance of the estimators is better for the parameter settings that characterize more intense fading and low SNR. As future works on SNR estimation, the authors will investigate the use of goodness-of-fit tests and kernel density estimation.

Abbreviations

BFSK: Binary frequency-shift keying; CDF: Cumulative distribution function; CRLB: Cramér-Rao lower bound; DA: Data-aided; EM: Expectation maximization; FMDCSK: Frequency modulated differential chaos shift keying; FSK: Frequency shift keying; LDPC: Low density parity check codes; LS: Least squares; MIMO: Multiple input multiple output; MLE: Maximum likelihood estimation; MOM: Method of moments; M-QAM: *M*-ary quadrature amplitude modulation; MSK: Minimum shift keying; NDA: Non-data-aided; NMSE: Normalized mean square error; NRMSE: Normalized root mean square error; PDF: Probability density function; PSK: Phase shift keying; SIMO: Single input multiple output; SNR: Signal-to-noise ratio

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Availability of data and materials

Not applicable.

Authors' contributions

WQ carried out the mathematical development of the paper, and simulations were done by DA. FM and WA performed the technical review and analysis of the results. WQ, FM, and WA wrote the paper. All authors read and approved the final manuscript.

Competing interests
The authors declare that they have no competing interests.

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