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A novel non-negative matrix factorization technique for decomposition of Chinese characters with application to secret sharing

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Abstract

The decomposition of Chinese characters is difficult and has been rarely investigated in the literature. In this paper, we propose a novel non-negative matrix factorization (NMF) technique to decompose a Chinese character into several graphical components without considering the strokes of the character or any semantic or phonetic properties of the components. Chinese characters can usually be represented as binary images. However, traditional NMF is only suitable for representing general gray-level or color images. To decompose a binary image using NMF, we force all of the elements of the two matrices (obtained by factorizing the binary image/matrix to be decomposed) as close to 0 or 1 as possible. As a result, a Chinese character can be efficiently decomposed into several components, where each component is semantically unreadable. Moreover, our NMF-based Chinese character decomposition method is suitable for applications in visual secret sharing by distributing the shares (different character components) among multiple parties, so that only when the parties are taken together with their respective shares can the secret (the original Chinese character(s)) be reconstructed. Experimental results have verified the decomposition performance and the feasibility of the proposed method.

Keywords: Chinese characters, Matrix decomposition, Non-negative matrix factorization (NMF), Secret sharing

1 Introduction

A Chinese character is usually composed of several graphical components, including a radical and several other different parts. Radicals are the graphical components used to index Chinese characters in a dictionary. Characters with the same component may share similar semantic or phonetic properties [1–3]. The analysis and processing of Chinese characters have enjoyed a long history [4], and recently have garnered much attention with the development of artificial intelligence techniques, such as machine learning [5] and deep learning [6] as well as several related applications (e.g., recognition of Chinese characters [7] and natural language understanding of Chinese documents [8]).

1.1 Chinese character decomposition

To online process Chinese documents stored as binary images, it would be helpful if the Chinese characters could automatically be efficiently decomposed in advance. However, automatically decomposing a Chinese character (represented as a binary image) into different components is difficult and has rarely been investigated in the literature.

Based on our explorations of works relating to Chinese characters, most prior research has aimed at extracting Chinese character strokes via automatic character decomposition [9–13]. Strokes are the fundamental building blocks of Chinese characters and are needed to write Chinese characters in regular script [14]. To decompose a Chinese character into a set of strokes, a mathematical morphology-based method was proposed

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in [11]. A model of stroke extraction for Chinese characters was also proposed in [9] to extract primary strokes. In [13], a Chinese character is decomposed into isolated stroke structures based on the connections. Then, shape contexts of stroke structures are extracted, and the matched counterparts in a standard database are found through shape matching. Moreover [10], was proposed building a font skeleton manifold so that the most similar character could be always found as a template by traversing the locations in the manifold learned for the application of stroke extraction for Chinese characters. Meanwhile [12], was proposed starting with automatically decomposing a Chinese character image into strokes, and then separately resizing those strokes under the guidance of structure information to achieve structure-aware image resizing for Chinese characters.

These works on decomposing Chinese characters into strokes were mainly designed for applications in character recognition, writing style analysis, and new font synthesis [15]. Such methods usually rely on a standard database consisting of standard Chinese character templates or standard strokes, used for shape/template matching.

On the other hand, to better capture semantics of natural language words, low-dimensional distributed word representations, also known as word embeddings, were introduced recently [16, 17]. For example, a method, called cw2vec, for learning Chinese word embeddings with stroke n-gram information was proposed in [16]. More specifically, a minimalist approach was designed to exploit the stroke n-grams, which can capture semantic and morphological level information of Chinese words. Moreover, to improve word embeddings, a hybrid learning method, integrating compositional, and predictive models for word embeddings was presented in [17]. In general, word embedding techniques have been shown to be applicable in the tasks of word similarity, word analogy, text classification, and named entity recognition. However, the main goal of word embeddings is usually to better capture semantics of natural language words, which is essentially different from that of the proposed method described in the next subsection.

1.2 Main objective of this paper

In significant contrast to the goals and approaches of the above-mentioned works, the goal of this paper is to develop an automatic Chinese character decomposition framework to decompose a Chinese character into several graphical components without considering the strokes of the character or any semantic or phonetic properties of the components, as in the examples illustrated in Fig. 1. For example, the Chinese character '好' will be automatically decomposed into the components '女' and '子' without considering the semantic or phonetic properties of either of the two components. That is, we intend to automatically achieve graphical decomposition of Chinese characters represented as binary images without the need for any prior knowledge of semantic and phonetic properties of Chinese character components or character strokes.

To achieve our goal, we propose applying non-negative matrix factorization (NMF) technique to decompose a Chinese character into different components. In general, NMF (or non-negative matrix approximation) [18–21] intends to factorize a matrix into two matrices, with all three matrices having no negative elements. The property of non-negativity makes the resulting matrices easier to inspect; therefore, NMF has been successfully applied to several source decomposition applications in digital audio signals [22–24], digital visual signals [25–29], and document clustering [30].

However, from the viewpoint of image signal decomposition using NMF [15, 26–29], the standard NMF technique [18, 19] is only suitable for processing general

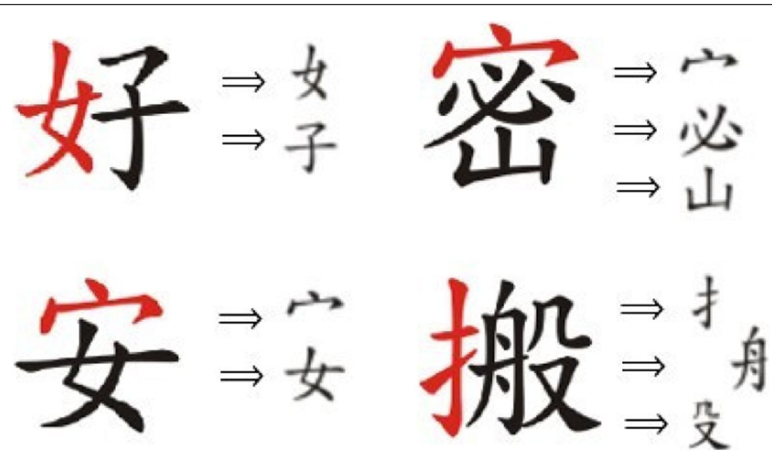

Fig. 1 Examples of Chinese character decomposition considered in this paper

digital grayscale or color images, not for binary images. In general, Chinese characters are usually presented in binary tones, black and white. Therefore, this paper presents a novel NMF framework to factorize a Chinese character represented as a binary image (or matrix). Furthermore, we evaluate the performance of the proposed NMF technique by applying it to the application of visual secret sharing (a visual cryptographic technique [31–35] in information security [36–38]) that securely shares secret messages encoded as binary images of partial Chinese characters.

1.3 Main contributions of this paper

The major innovations and contributions of this paper are three-fold: (a) to the best of our knowledge, we are among the first to propose an automatic Chinese character decomposition framework to break a character into graphical components without a need for prior knowledge of semantic or phonetic properties of Chinese character components or character strokes; (b) to achieve this goal, we propose a novel NMF framework to factorize a binary image into two matrices while forcing all of the elements of the two matrices as close to 0 or 1 as possible; and (c) we successfully apply our NMF-based Chinese character decomposition technique to visual secret sharing for the secure transmission of secret messages encoded by binary images of Chinese characters.

The rest of this paper is organized as follows. In Section 2, the standard NMF technique [18, 19] is briefly reviewed, followed by a presentation of the proposed novel NMF framework in Sections 3 and 4, we present the evaluation results by applying the proposed NMF-based Chinese character decomposition method to the application of visual secret sharing. Finally, Section 7 concludes this paper.

2 Standard non-negative matrix factorization algorithm

Non-negative matrix factorization (NMF) technique has been shown to be useful for the decomposition of multivariate data, where the ‘non-negativity’ is a useful constraint for matrix factorization being able to learn a part-based representation of the data. The learned non-negative basis vectors are used in distributed or sparse combinations to generate expressiveness in the data reconstructions [18, 19]. The basic NMF problem can be formally described as follows. Given a non-negative matrix V of size $n \times m$, the goal is to find two non-negative matrix factors, W of size $n \times r$, and H of size $r \times m$ such that

$$V \approx WH, \tag{1}$$

where V can be viewed as a matrix consisting of m data vectors (of dimension n for each) from a dataset. The term r is usually selected to be smaller than n or m, so that the two matrices, W and H, are smaller than the original matrix V. Based on Eq. 1, each data vector (column) v of V is approximated by a linear combination of the columns of W, weighted by the components of the corresponding column h of H. That is, W can be viewed as containing a basis that is optimized for the linear approximation of the data in V.

To solve Eq. 1, the standard NMF problem can be formulated as follows:

$$\min_{W\in\mathbf{R}^{n\times r},H\in\mathbf{R}^{r\times m}} \|V-WH\|_F^2, \text{subject to } W, H \geq 0, \tag{2}$$

The function defined Eq. 2 is convex in W only or H only, and therefore, it is not convex in both variables together. However, it has been found that the “multiplicative update rules” expressed below achieve a good compromise between convergence speed and ease of implementation for solving the problem [19].

For k = 1, 2, ...

$$H_{bj}^{k+1} = H_{bj}^{k} \times \frac{\left(\left(W^k\right)^{\mathbf{T}} V\right)_{bj}}{\left(\left(W^k\right)^{\mathbf{T}} W^k H^k\right)_{bj}}, \forall b, j, \tag{3}$$

$$W_{ia}^{k+1} = W_{ia}^{k} \times \frac{\left(V\left(H^{k+1}\right)^{\mathbf{T}}\right)_{ia}}{\left(W^k H^{k+1}\left(H^{k+1}\right)^{\mathbf{T}}\right)_{ia}}, \forall i, a, \tag{4}$$

where k denotes the iteration number, and i, a, b, j denote the matrix indices. The multiplicative update rules were also verified in [20]. The constraints of non-negativity allow for interpretation of the basis elements in image forms.

3 Proposed novel non-negative matrix factorization algorithm

To extend standard NMF to process a binary image (stored as a binary matrix), we propose forcing all of the elements of the two factorized matrices (e.g., W and H, defined equation 1) as close to 0 or 1 as possible. Similar to the basic NMF problem defined in Eq. 1, given a non-negative matrix V of size $n \times m$, our goal is to find two non-negative matrix factors, W of size $n \times r$, and H of size $r \times m$, such that $V \approx WH$, and all of the elements in W and H are approximately 0 or 1. We first define our objective function as

$$\min_{W\in\mathrm{R}^{n\times r},H\in\mathrm{R}^{r\times m}} \mathcal{L}(W,H) = \frac{1}{2}\sum\nolimits_{i=1}^{n}\sum\nolimits_{j=1}^{m}\left[V_{ij}-(WH)_{ij}\right]^2,$$

$$s.t. \;\; W_{ia}^2 - W_{ia} \approx 0 \text{ and } H_{bj}^2 - H_{bj} \approx 0, \forall i, a, b, j, \tag{5}$$

To realize the proposed idea, we introduce two penalty terms (with two parameters, λ_1 and λ_2) to increase the closeness between each W_{ia} or H_{bj} to 0 or 1, as

$$\min_{W\in\mathbf{R}^{n\times r},H\in\mathbf{R}^{r\times m}} \mathcal{L}(W,H) = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{m}\left[V_{ij}-(WH)_{ij}\right]^2 +\lambda_1\left(H_{bj}^2-H_{bj}\right)^2+\lambda_2\left(W_{ia}^2-W_{ia}\right)^2, \tag{6}$$

It should be noted that the main novelty of the proposed NMF algorithm is to enforce all of the elements in the matrices, W and H, to be approximately 0 or 1, for achieving the decomposition of the binary matrix. The idea is realized in our method by formulating it as Eq. 6. To solve the problem defined in Eq. 6, our framework performs the steps outlined below.

4 Proposed NMF algorithm for binary image factorization

1. Randomly initialize all of the elements in W and H to 0 or 1, and initialize a predefined number K of iterations for the algorithm to be performed.
2. Update W and H as defined in Eq. 6 based on the steepest descent method [39], described later.
3. If $(H_{bj}^2-H_{bj})^2+(W_{ia}^2-W_{ia})^2<\varepsilon$, or the number ($K$) of target iterations is reached, stop the algorithm, where ε is a predefined threshold. Otherwise, go to step 2.

In step 2 of our NMF algorithm, we apply the steepest descent method to iteratively update W and H, derived as follows. We first take the derivative of $\mathcal{L}(W,H)$, defined in Eq. 6, with respect to H, and obtain

$$\frac{\partial}{\partial H_{bj}}\mathcal{L}(W,H) = -\sum_{i=1}^{n}\left[V_{ij}-(WH)_{ij}\right]W_{ib} + 2\lambda_1\left(H_{bj}^2-H_{bj}\right)(2H_{bj}-1), \tag{7}$$

By letting the step size (denoted by α_{bj}) to be

$$\alpha_{bj} = \frac{H_{bj}}{\left(W^{\mathrm{T}}(WH)\right)_{bj}+4\lambda_1H_{bj}^3+2\lambda_1H_{bj}}, \tag{8}$$

the update rule of H can be defined as

$$H_{bj}^{k+1} = H_{bj}^k-\alpha_{bj}^k\times\frac{\partial}{\partial H_{bj}^k}\mathcal{L}(W,H)^k = H_{bj}^k \times\left[\frac{\left(W^{\mathrm{T}}V\right)_{bj}^k+6\lambda_1\left(H_{bj}^k\right)^2}{\left[W^{\mathrm{T}}(WH)\right]_{bj}^k+4\lambda_1\left(H_{bj}^k\right)^3+2\lambda_1H_{bj}^k}\right], \tag{9}$$

where the superscript k denotes the number of iterations (similar to the term k in Eqs. 3 and 4). To simplify the notations, from Eq. 5 to 8, the superscript k is ignored.

In the same way, W can be updated by

$$W_{ib}^{k+1} = W_{ib}^k-\alpha_{ib}^k\times\frac{\partial}{\partial W_{ib}^k}\mathcal{L}(W,H)^k = W_{ib}^k \times\left[\frac{\left(VH^{\mathrm{T}}\right)_{ib}^k+6\lambda_2\left(W_{bj}^k\right)^2}{\left[(WH)H^{\mathrm{T}}\right]_{ib}^k+4\lambda_2\left(W_{ib}^k\right)^3+2\lambda_2W_{ib}^k}\right], \tag{10}$$

The two update rules defined in Eqs. 9 10, respectively, will be used in step 2 of the proposed NMF algorithm for each iteration.

5 Proposed visual secret sharing method for Chinese characters via non-negative matrix factorization with experimental results

In this section, we apply the proposed NMF algorithm for binary image decomposition (described in Section 3 to visual secret sharing of Chinese characters, and present the experimental results to demonstrate the performance of our novel NMF algorithm.

5.1 Visual secret sharing

Visual cryptography is a kind of cryptographic technique allowing visual information (e.g., images, texts) to be encrypted in such a way that it becomes someone's job to decrypt the ciphertext via sight reading without a need for complex decryptographic computation. Visual secret sharing is a visual cryptographic technique, originally proposed in [34], where an image that is to be visually encrypted can be decomposed into n shares, so that only someone with all n shares can decrypt the image, while anybody with n − 1 shares cannot decrypt the original image. Based on this idea, several generalizations of the basic framework including t-out-of-n (or (t, n)-threshold) visual cryptography [31] were also developed. That is, any t shares can decrypt the secret, but no group of t − 1 or fewer can do so, where $t \le n$.

5.2 Visual secret sharing for Chinese characters via proposed NMF algorithm

To the best of our knowledge, research about visual secret sharing for Chinese characters is not commonly found in the literature. In this paper, we investigate the applicability of the proposed NMF algorithm to visual secret sharing for Chinese characters. It should be noted that the issues of (t, n)-threshold, as well as the security and robustness of secret sharing, are beyond the scope of this paper. Different from the general visual secret sharing scenario relying on the (t, n)-threshold principle, described in Section 5.2, the requirements of the target application considered in this paper are described as follows. First, the secret message to be shared in this paper should be a binary image containing a set of Chinese characters. Second, to reconstruct the secret message, all of the shares are required.

In the training stage of our method, it is necessary for our NMF model to learn a basis (or dictionary) for Chinese character decomposition. Therefore, we first randomly selected 30 Chinese characters represented by their corresponding 30 binary images, as shown in Fig. 2. Each binary image of a Chinese character is represented by a 100 × 100 matrix. To establish the matrix (V) to be factorized, each 100 × 100 matrix of a character is transformed to a column of size 10,000. As a result, the matrix (V) consists of 30 columns of size 10,000 each, where each column represents a training sample of a Chinese character, denoted by $V_{n\times m}$, n = 10,000, and m = 30. By applying the proposed NMF algorithm to factorize $V_{n\times m}$ into two matrices, W of size $n \times r$, and H of size $r \times m$, and setting r= 16, we can obtain the two matrices, where $W_{n\times r}$ represents the basis (or dictionary), and $H_{r\times m}$ contains the corresponding weights (coefficients) for constituting the Chinese characters. Figure 3 illustrates the r columns (basis elements or dictionary atoms) of W. It can be observed from Fig. 3 that the Chinese characters can be actually decomposed into several parts, which is consistent with the concept of recognition for human brains [40].

To achieve visual secret sharing of Chinese characters, we consider the problem that the secret message to be shared is represented by the Chinese characters shown in Fig. 2, and a set of shares can be obtained by the proposed NMF algorithm, shown in Fig. 3. Then, the shares can be distributed among a group of participants (or parties). That is, a set of Chinese characters (to be securely shared) forms the matrix V. By applying our NMF algorithm to factorize V, the two matrices, W (representing the basis) and H (including the corresponding coefficients), can be obtained. Finally, each column (basis element) of W can be viewed as a share to be distributed.

To decrypt the secret message to be shared, shares from different subsets of the set of shares printed on transparent slides only needs to be stacked upon each other, and then Chinese characters in the secret message can be revealed. For example, by stacking the shares (i.e., slides) of ‘口’ and ‘支’ the character ‘吱’ can be revealed, while stacking the shares ‘口’ and ‘巴’, the character ‘吧’ can be revealed.

To evaluate our method’s performance on computational reconstruction, we multiply the two matrices, W and H, to reconstruct the original message (V). The results are shown in Fig. 4. It is evident in Fig. 4 that good visual quality for the reconstruction results can be obtained. It should be noted that for the reconstruction of the secret message in visual secret sharing applications, the message can be visually decrypted by stacking the proper shares without any computation.

In the case of real applications for visual secret sharing, once the basis (or dictionary, say, the matrix “W”) is learned offline, it will be directly used to factorize the message formed by the Chinese character(s) to be shared to get the matrix “H”. Inspecting the coefficients of H can then determine which shares should be distributed. For each column, (a basis element) of W, when its corresponding coefficient in H is close to 1, it will be considered a share of the secret message for distribution. This means that the basis element has a significant contribution to the original message to be shared. Otherwise, that basis element will not be selected as a share.

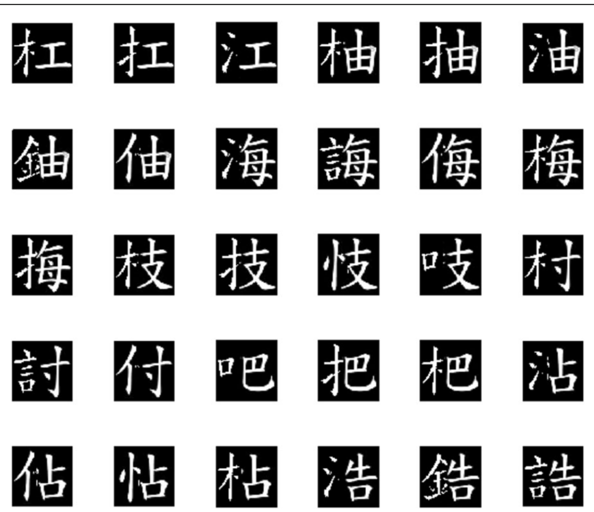

Fig. 2 Examples of training samples (binary images of Chinese characters) used in our experiments.

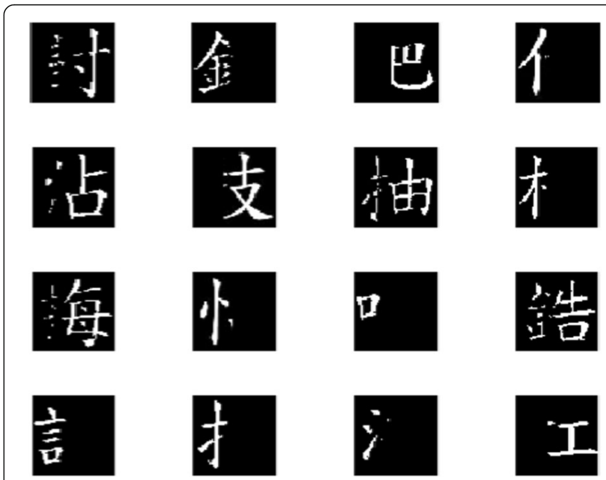

Fig. 3 Visualized basis elements (or dictionary atoms) in the matrix W, obtained by factorizing the matrix V, formed by the Chinese characters displayed in Fig. 2

Fig. 4 Reconstruction results based on the two matrices, *W* and *H*, for the matrix *V*, formed by the Chinese characters displayed in Fig. 2

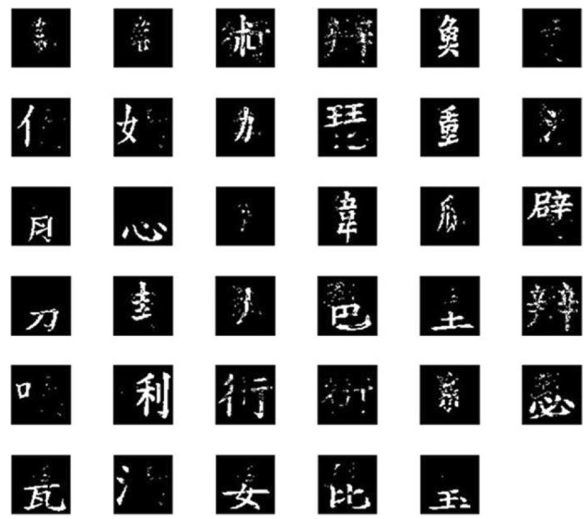

Fig. 6 Visualized basis elements in the matrix *W*, obtained by factorizing the matrix *V*, formed by the Chinese characters displayed in Fig. 5

5.3 More experimental results

This subsection presents more experimental results on the decomposition of Chinese characters when applied to visual secret sharing based on our method. Figure 5 depicts a set of Chinese characters forming the matrix *V*. By applying the proposed NMF algorithm, the basis matrix (*W*) can be obtained to become the shares of the message, as shown in Figure 6. It can be observed from Figs. 5 and 6 that the proposed NMF algorithm is able to properly factorize a binary matrix formed by a set of Chinese characters to find the basis matrix for determining a set of shares to be distributed for secret sharing applications.

Fig. 5 A set of Chinese characters represented as binary images

6 Discussion

Most prior research has aimed at extracting Chinese character strokes and these Chinese characters are needed to write in regular script. In contrast, the proposed method is to develop an automatic Chinese character decomposition framework to decompose a Chinese character into several graphical components without the need for any prior knowledge of semantic and phonetic properties of Chinese character components or character strokes. On the other hand, to compare the proposed method with traditional visual scrambling approaches (e.g. [41, 42],), the proposed method aims at decomposing a Chinese character into several graphical components, while the traditional visual scrambling approaches are designed to scramble image pixels to achieve image encryption. Furthermore, the proposed method is suitable to visual secret sharing applications for Chinese character messages, while the general image scrambling approaches (for encryption) cannot be directly used for visual secret sharing.

In this paper, we propose applying NMF technique to decompose a Chinese character in a binary image format into different components. However, conventional NMF technique is only suitable for processing general digital grayscale or color images, not for binary images, so the NMF framework should be totally redesigned. The above experimental results show the feasibility of the proposed method. We do not compare with other methods in this section because to the best of our knowledge, we are the first to propose an automatic Chinese character decomposition framework to break a character into graphical components without considering character strokes.

7 Conclusions and future work

In this paper, we have proposed a novel NMF algorithm for the decomposition of Chinese characters into different components. Relying on our key idea to force all of the elements from the two matrices (obtained by factorizing the image/matrix to be decomposed) as close to 0 or 1 as possible, our NMF algorithm can properly factorize a binary image, and furthermore, can be successfully applied to visual secret sharing of Chinese characters represented as binary images. Secret messages can be visually decrypted without performing computations, based on the collected distributed shares formed by the basis elements obtained by our NMF method. Moreover, the performance on computational reconstruction for the original message has been verified via matrix multiplication. Finally, as deep learning for NMF has recently been preliminarily investigated (e.g. [43],), deep NMF models for binary image representation and decomposition would be worth investigating in future work.

Abbreviations

NMF: Non-negative matrix factorization

Authors' contributions

In this paper, the idea and primary algorithm were proposed by CYL, LWK, and MKC. TYH conducted the simulation results. All authors read and approved the final manuscript.

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Availability of data and materials

Please contact authors for data requests.

Ethics approval and consent to participate

This study does not involve human participants, human data, or human tissue.

Consent for publication

In the manuscript, there is no any individual person's data.

Competing interests

The authors declare that they have no competing interests.

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